# **Recommender Systems**

#### Disclaimer

### The slides mostly taken from:

 Jannach & Freidrich (2013), Tutorial Recommendation Systems, International Joint Conference on Artificial Intelligence.

## Other sources/references:

- Charu C. Aggarwal (2015), Data Mining The Textbook, Springer. Chapter 18.5
- (2019), Recommendation Systems Explained, Crossing Mind

## **Recommendation System - Example**

Facebook-"People You May Know"

**Netflix**-"Other Movies You May Enjoy"

**LinkedIn**-"Jobs You May Be Interested In"

**Amazon**-"Customer who bought this item also bought ..."

YouTube-"Recommended Videos"

Google-"Search results adjusted"

Pinterest-"Recommended Images"

[Source: Crossing Mind, 2019]

#### **Recommendation Problem**

#### User versus Items

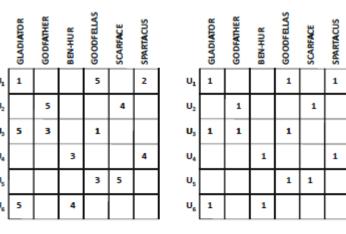
- Utility Matrix of n users and d items. Matrix D of utility values
- The Value of: Positive Preferences only vs Positive and Negative Preferences

### Recommendation System

Content-based Recommendation: users and items are ascociated with

feature-based descriptions

Collaborative Filtering



(a) Ratings-based utility

(b) Positive-preference utility

Figure 18.4: Examples of utility matrices.

## **Collaborative Filtering (CF)**

#### Collaborative Filtering (CF)

- Pure CF approaches
- User-based nearest-neighbor
- The Pearson Correlation similarity measure
- Memory-based and model-based approaches
- Item-based nearest-neighbor
- The cosine similarity measure
- Data sparsity problems
- Recent methods (SVD, Association Rule Mining, Slope One, RF-Rec, ...)
- The Google News personalization engine
- Discussion and summary
- Literature

## **Collaborative Filtering (CF)**

### The most prominent approach to generate recommendations

- used by large, commercial e-commerce sites
- well-understood, various algorithms and variations exist
- applicable in many domains (book, movies, DVDs, ..)

## Approach

use the "wisdom of the crowd" to recommend items



## Basic assumption and idea

- Users give ratings to catalog items (implicitly or explicitly)
- Customers who had similar tastes in the past, will have similar tastes in the future

## **Pure CF Approaches**

#### Input

Only a matrix of given user—item ratings

### Output types

- A (numerical) prediction indicating to what degree the current user will like or dislike a certain item
- A top-N list of recommended items

## User-based nearest-neighbor collaborative filtering (1)

## The basic technique

- Given an "active user" (Alice) and an item i not yet seen by Alice
  - find a set of users (peers/nearest neighbors) who liked the same items as Alice in the past and who have rated item i
  - use, e.g. the average of their ratings to predict, if Alice will like item i
  - do this for all items Alice has not seen and recommend the best-rated

## Basic assumption and idea

- If users had similar tastes in the past they will have similar tastes in the future
- User preferences remain stable and consistent over time

## User-based nearest-neighbor collaborative filtering (2)

## Example

A database of ratings of the current user, Alice, and some other users is given:

	ltem1	Item2	Item3	Item4	Item5
Alice	5	3	4	4	?
User1	3	1	2	3	3
User2	4	3	4	3	5
User3	3	3	1	5	4
User4	1	5	5	2	1

Determine whether Alice will like or dislike *Item5*, which Alice has not yet rated or seen

## User-based nearest-neighbor collaborative filtering (3)

### Some first questions

- How do we measure similarity?
- How many neighbors should we consider?
- How do we generate a prediction from the neighbors' ratings?



	ltem1	Item2	Item3	Item4	ltem5
Alice	5	3	4	4	?
User1	3	1	2	3	3
User2	4	3	4	3	5
User3	3	3	1	5	4
User4	1	5	5	2	1

## Measuring user similarity (1)

A popular similarity measure in user-based CF: Pearson correlation

a, b: users

 $r_{a,p}$ : rating of user a for item p

P: set of items, rated both by a and b

- Possible similarity values between -1 and 1

$$sim(a,b) = \frac{\sum_{p \in P} (r_{a,p} - \bar{r}_a)(r_{b,p} - \bar{r}_b)}{\sqrt{\sum_{p \in P} (r_{a,p} - \bar{r}_a)^2} \sqrt{\sum_{p \in P} (r_{b,p} - \bar{r}_b)^2}}$$

## Measuring user similarity (2)

## A popular similarity measure in user-based CF: Pearson correlation

a, b: users

 $r_{a,p}$ : rating of user a for item p

P: set of items, rated both by a and b

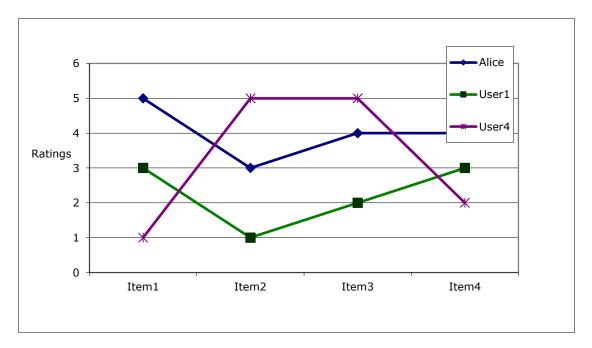
- Possible similarity values between -1 and 1

	Item1	Item2	Item3	Item4	Item5
Alice	5	3	4	4	?
User1	3	1	2	3	3
User2	4	3	4	3	5
User3	3	3	1	5	4
User4	1	5	5	2	1

sim = 0.85 sim = 0.00 sim = 0.70sim = -0.79

### **Pearson correlation**

Takes differences in rating behavior into account



- Works well in usual domains, compared with alternative measures
  - such as cosine similarity

## **Making predictions**

A common prediction function:

$$pred(a,p) = \overline{r_a} + \frac{\sum_{b \in N} sim(a,b) * (r_{b,p} - \overline{r_b})}{\sum_{b \in N} sim(a,b)}$$



- Calculate, whether the neighbors' ratings for the unseen item i are higher or lower than their average
- Combine the rating differences use the similarity with a as a weight
- Add/subtract the neighbors' bias from the active user's average and use this as a prediction

## Improving the metrics / prediction function

## Not all neighbor ratings might be equally "valuable"

- Agreement on commonly liked items is not so informative as agreement on controversial items
- Possible solution: Give more weight to items that have a higher variance

#### Value of number of co-rated items

 Use "significance weighting", by e.g., linearly reducing the weight when the number of co-rated items is low

### Case amplification

 Intuition: Give more weight to "very similar" neighbors, i.e., where the similarity value is close to 1.

### Neighborhood selection

Use similarity threshold or fixed number of neighbors

## Memory-based and model-based approaches

### User-based CF is said to be "memory-based"

- the rating matrix is directly used to find neighbors / make predictions
- does not scale for most real-world scenarios
- large e-commerce sites have tens of millions of customers and millions of items

### Model-based approaches

- based on an offline pre-processing or "model-learning" phase
- at run-time, only the learned model is used to make predictions
- models are updated / re-trained periodically
- large variety of techniques used
- model-building and updating can be computationally expensive
- item-based CF is an example for model-based approaches

## **Item-based collaborative filtering**

#### Basic idea:

Use the similarity between items (and not users) to make predictions

## Example:

- Look for items that are similar to Item5
- Take Alice's ratings for these items to predict the rating for Item5

	Item1 Item2 Item3		Item4	Item5	
Alice	5	3	4	4	<u>;</u>
User1	3	1	2	3	3
User2	4	3	4	3	5
User3	3	3	1	5	4
User4	1	5	5	2	1

## The cosine similarity measure

- Produces better results in item-to-item filtering
- Ratings are seen as vector in n-dimensional space
- Similarity is calculated based on the angle between the vectors

$$sim(\vec{a}, \vec{b}) = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| * |\vec{b}|}$$



- Adjusted cosine similarity
  - take average user ratings into account, transform the original ratings
  - U: set of users who have rated both items a and b

$$sim(\vec{a}, \vec{b}) = \frac{\sum_{u \in U} (r_{u,a} - \overline{r_u}) (r_{u,b} - \overline{r_u})}{\sqrt{\sum_{u \in U} (r_{u,a} - \overline{r_u})^2} \sqrt{\sum_{u \in U} (r_{u,b} - \overline{r_u})^2}}$$



## **Making predictions**

A common prediction function:

$$pred(u, p) = \frac{\sum_{i \in ratedItem(u)} sim(i, p) * r_{u, i}}{\sum_{i \in ratedItem(u)} sim(i, p)}$$



- Neighborhood size is typically also limited to a specific size
- Not all neighbors are taken into account for the prediction
- An analysis of the MovieLens dataset indicates that "in most real-world situations, a neighborhood of 20 to 50 neighbors seems reasonable" (Herlocker et al. 2002)

## **Pre-processing for item-based filtering**

- Item-based filtering does not solve the scalability problem itself
- Pre-processing approach by Amazon.com (in 2003)
  - Calculate all pair-wise item similarities in advance
  - The neighborhood to be used at run-time is typically rather small, because only items are taken into account which the user has rated
  - Item similarities are supposed to be more stable than user similarities

### Memory requirements

- Up to N<sup>2</sup> pair-wise similarities to be memorized (N = number of items) in theory
- In practice, this is significantly lower (items with no co-ratings)
- Further reductions possible
  - Minimum threshold for co-ratings
  - Limit the neighborhood size (might affect recommendation accuracy)

## **More on ratings – Explicit ratings**

- Probably the most precise ratings
- Most commonly used (1 to 5, 1 to 7 Likert response scales)
- Research topics
  - Optimal granularity of scale; indication that 10-point scale is better accepted in movie dom.
  - An even more fine-grained scale was chosen in the joke recommender discussed by Goldberg et al. (2001), where a continuous scale (from −10 to +10) and a graphical input bar were used
    - No precision loss from the discretization
    - User preferences can be captured at a finer granularity
    - Users actually "like" the graphical interaction method
  - Multidimensional ratings (multiple ratings per movie such as ratings for actors and sound)
- Main problems
  - Users not always willing to rate many items
    - number of available ratings could be too small → sparse rating matrices → poor recommendation quality
  - How to stimulate users to rate more items?

## More on ratings – Implicit ratings

- Typically collected by the web shop or application in which the recommender system is embedded
- When a customer buys an item, for instance, many recommender systems interpret this behavior as a positive rating
- Clicks, page views, time spent on some page, demo downloads ...
- Implicit ratings can be collected constantly and do not require additional efforts from the side of the user
- Main problem
  - One cannot be sure whether the user behavior is correctly interpreted
  - For example, a user might not like all the books he or she has bought; the user also might have bought a book for someone else
- Implicit ratings can be used in addition to explicit ones; question of correctness of interpretation

## **Data sparsity problems**

### Cold start problem

– How to recommend new items? What to recommend to new users?

### Straightforward approaches

- Ask/force users to rate a set of items
- Use another method (e.g., content-based, demographic or simply nonpersonalized) in the initial phase
- Default voting: assign default values to items that only one of the two users to be compared has rated (Breese et al. 1998)

## **Example algorithms for sparse datasets**

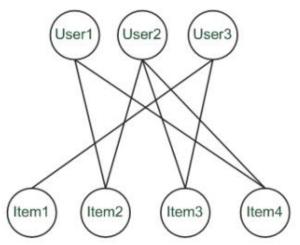
- Recursive CF (Zhang and Pu 2007)
  - Assume there is a very close neighbor n of u who however has not rated the target item i yet.
  - Idea:
    - Apply CF-method recursively and predict a rating for item i for the neighbor
    - Use this predicted rating instead of the rating of a more distant direct neighbor

	Item1	Item2	Item3	Item4	Item5	
Alice	5	3	4	4	? 🗖	-i 0.05
User1	3	1	2	3	?	sim = 0.85
User2	4	3	4	3	5	Predict
User3	3	3	1	5	4	rating for
User4	1	5	5	2	1	User1

## **Graph-based methods (1)**

### "Spreading activation" (Huang et al. 2004)

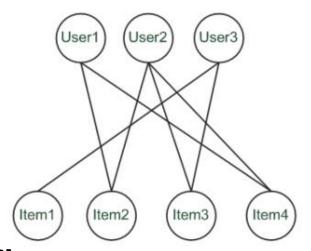
- Exploit the supposed "transitivity" of customer tastes and thereby augment the matrix with additional information
- Assume that we are looking for a recommendation for User1
- When using a standard CF approach, *User2* will be considered a peer for *User1* because they both bought *Item2* and *Item4*
- Thus Item3 will be recommended to User1 because the nearest neighbor, User2, also bought or liked it



## **Graph-based methods (2)**

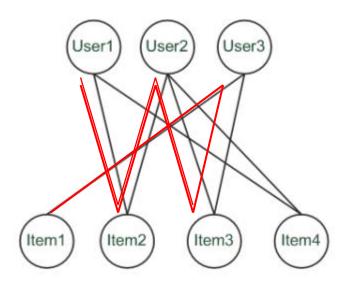
### "Spreading activation" (Huang et al. 2004)

- In a standard user-based or item-based CF approach, paths of length 3 will be considered that is, *Item3* is relevant for *User1* because there exists a three-step path (*User1–Item2–User2–Item3*) between them
- Because the number of such paths of length 3 is small in sparse rating databases, the idea is to also consider longer paths (indirect associations) to compute recommendations
- Using path length 5, for instance



## **Graph-based methods (3)**

- "Spreading activation" (Huang et al. 2004)
  - Idea: Use paths of lengths > 3
     to recommend items
  - Length 3: Recommend Item3 to User1
  - Length 5: Item1 also recommendable



## More model-based approaches

## Plethora of different techniques proposed in the last years, e.g.,

- Matrix factorization techniques, statistics
  - singular value decomposition, principal component analysis
- Association rule mining
  - compare: shopping basket analysis
- Probabilistic models
  - clustering models, Bayesian networks, probabilistic Latent Semantic Analysis
- Various other machine learning approaches

### Costs of pre-processing

- Usually not discussed
- Incremental updates possible?

### **Matrix factorization**

Informally, the SVD theorem (Golub and Kahan 1965) states that a given matrix M can be decomposed into a product of three matrices as follows

$$M = U \times \Sigma \times V^T$$

- where U and V are called *left* and *right singular vectors* and the values of the diagonal of  $\Sigma$  are called the *singular values*
- We can approximate the full matrix by observing only the most important features – those with the largest singular values
- In the example, we calculate U, V, and  $\Sigma$  (with the help of some linear algebra software) but retain only the two most important features by taking only the first two columns of U and  $V^T$

## **Example for SVD-based recommendation**

• SVD:  $\boldsymbol{M}_k = \boldsymbol{U}_k \times \boldsymbol{\Sigma}_k \times \boldsymbol{V}_k^T$ 

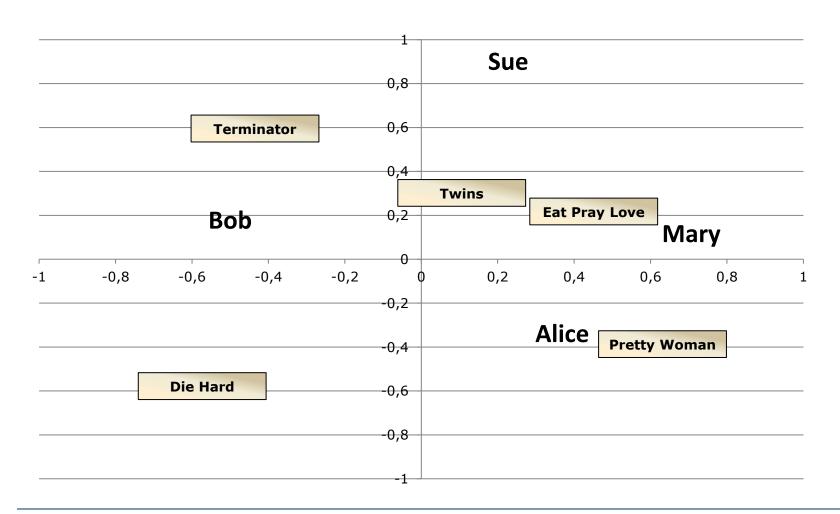
U <sub>k</sub>	Dim1	Dim2
Alice	0.47	-0.30
Bob	-0.44	0.23
Mary	0.70	-0.06
Sue	0.31	0.93

1	6	riminator	Die Hard	Twins	Pray	AKY Woman
	$V_k^{T}$				Ove	an
1	Dim1	-0.44	-0.57	0.06	0.38	0.57
1	Dim2	0.58	-0.66	0.26	0.18	-0.36

•	Prediction:	$\hat{r}_{ui} = \overline{r}_u + U_k(Alice) \times \Sigma_k \times V_k^T(EPL)$ = 3 + 0.84 = 3.84
		= 3 + 0.84 = <b>3.84</b>

$\sum_{k}$	Dim1	Dim2
Dim1	5.63	0
Dim2	0	3.23

## The projection of U and $V^T$ in the 2 dimensional space $(U_2, V_2^T)$



## Discussion about dimensionality reduction (Sarwar et al. 2000a)

#### Matrix factorization

- Generate low-rank approximation of matrix
- Detection of latent factors
- Projecting items and users in the same n-dimensional space

#### Prediction quality can decrease because...

the original ratings are not taken into account

#### Prediction quality can increase as a consequence of...

- filtering out some "noise" in the data and
- detecting nontrivial correlations in the data

#### Depends on the right choice of the amount of data reduction

- number of singular values in the SVD approach
- Parameters can be determined and fine-tuned only based on experiments in a certain domain
- Koren et al. 2009 talk about 20 to 100 factors that are derived from the rating patterns

## **Association rule mining**

## Commonly used for shopping behavior analysis

aims at detection of rules such as
 "If a customer purchases beer then he also buys diapers in 70% of the cases"

### Association rule mining algorithms

- can detect rules of the form X → Y (e.g., beer  $\rightarrow$  diapers) from a set of sales transactions D =  $\{t_1, t_2, ... t_n\}$
- measure of quality: support, confidence
  - used e.g. as a threshold to cut off unimportant rules

- let 
$$\sigma(X) = \frac{|\{x | x \subseteq ti, ti \in D\}|}{|D|}$$

- support = 
$$\frac{\sigma(X \cup Y)}{|D|}$$
, confidence =  $\frac{\sigma(X \cup Y)}{\sigma(X)}$ 

## **Recommendation based on Association Rule Mining**

## Simplest approach

transform 5-point ratings into binary ratings (1 = above user average)

#### Mine rules such as

Item1 → Item5

	Item1	Item2	Item3	Item4	Item5
Alice	1	0	0	0	?
User1	1	0	1	0	1
User2	1	0	1	0	1
User3	0	0	0	1	1
User4	0	1	1	0	0

support (2/4), confidence (2/2) (without Alice)

### Make recommendations for Alice (basic method)

- Determine "relevant" rules based on Alice's transactions (the above rule will be relevant as Alice bought Item1)
- Determine items not already bought by Alice
- Sort the items based on the rules' confidence values.

### Different variations possible

dislike statements, user associations ...

#### **Probabilistic methods**

## Basic idea (simplistic version for illustration):

- given the user/item rating matrix
- determine the probability that user Alice will like an item i
- base the recommendation on such these probabilities

### Calculation of rating probabilities based on Bayes Theorem

- How probable is rating value "1" for Item5 given Alice's previous ratings?
- Corresponds to conditional probability P(Item5=1 | X), where
  - X = Alice's previous ratings = (Item1 =1, Item2=3, Item3= ... )
- Can be estimated based on Bayes' Theorem

$$P(Y|X) = \frac{P(X|Y) \times P(Y)}{P(X)} \qquad P(Y|X) = \frac{\prod_{i=1}^{d} P(X_i|Y) \times P(Y)}{P(X)}$$



Assumption: Ratings are independent (?)

## Calculation of probabilities in simplistic approach

	ltem1	Item2	Item3	Item4	Item5
Alice	1	3	3	2	?
User1	2	4	2	2	4
User2	1	3	3	5	1
User3	4	5	2	3	3
User4	1	1	5	2	1

$$P(X|Item5 = 1)$$

$$= P(Item1 = 1|Item5 = 1) \times P(Item2 = 3|Item5 = 1)$$

$$\times \textit{P(Item3} = 3 | \textit{Item5} = 1) \times \textit{P(Item4} = 2 | \textit{Item5} = 1) = \frac{2}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$$

$$\approx 0.125$$

$$P(X|Item5 = 2)$$

$$= P(Item1 = 1|Item5 = 2) \times P(Item2 = 3|Item5 = 2)$$

$$= P(Item1 = 1 | Item5 = 2) \times P(Item2 = 3 | Item5 = 2)$$

$$\times P(Item3 = 3 | Item5 = 2) \times P(Item4 = 2 | Item5 = 2) = \frac{0}{0} \times \dots \times \dots \times \dots$$

$$= 0$$



#### More to consider

- Zeros (smoothing required)
- like/dislike simplification possible

## **Practical probabilistic approaches**

- Use a cluster-based approach (Breese et al. 1998)
  - assume users fall into a small number of subgroups (clusters)
  - Make predictions based on estimates
    - probability of Alice falling into cluster c
    - probability of Alice liking item i given a certain cluster and her previous ratings
    - $P(C = c, v_1, ..., v_n) = P(C = c) \prod_{i=1}^n P(v_i | C = c)$
  - Based on model-based clustering (mixture model)
    - Number of classes and model parameters have to be learned from data in advance (EM algorithm)

#### Others:

Bayesian Networks, Probabilistic Latent Semantic Analysis, ....

### Empirical analysis shows:

- Probabilistic methods lead to relatively good results (movie domain)
- No consistent winner; small memory-footprint of network model

## Slope One predictors (Lemire and Maclachlan 2005)

- Idea of Slope One predictors is simple and is based on a popularity differential between items for users
- Example:

	Item1		Item5		5	
Alice		2			?	
User1		1	)		2	<b></b>
				5		

- p(Alice, Item5) = 2 + (2 1) = 3
- Basic scheme: Take the average of these differences of the co-ratings to make the prediction
- In general: Find a function of the form f(x) = x + b
  - That is why the name is "Slope One"

## RF-Rec predictors (Gedikli et al. 2011)

- Idea: Take rating frequencies into account for computing a prediction
- Basic scheme:  $\hat{r}_{u,i} = \arg \max_{v \in R} f_{user}(u,v) * f_{item}(i,v)$ 
  - R: Set of all rating values, e.g.,  $R = \{1,2,3,4,5\}$  on a 5-point rating scale
  - $f_{user}(u, v)$  and  $f_{item}(i, v)$  basically describe how often a rating v was assigned by user u and to item i resp.

#### Example:

	ltem1	ltem2	Item3	Item4	ltem5
Alice	1	1	?	5	4
User1	2		5	5	5
User2			1	1	
User3		5	2		2
User4	3		1	1	
User5	1	2	2		4

p(Alice, Item3) = 1

## **2008:** Factorization meets the neighborhood: a multifaceted collaborative filtering model, Y. Koren, ACM SIGKDD

- Stimulated by work on Netflix competition
  - Prize of \$1,000,000 for accuracy improvement of 10% RMSE compared to own Cinematch system
  - Very large dataset (~100M ratings, ~480K users , ~18K movies)
  - Last ratings/user withheld (set K)



- Metrics measure error rate
  - Mean Absolute Error (MAE) computes the deviation between predicted ratings and actual ratings

$$MAE = \frac{1}{n} \sum_{i=1}^{n} |p_i - r_i|$$

Root Mean Square Error (RMSE) is similar to MAE, but places more emphasis on larger deviation

$$MAE = \frac{1}{n} \sum_{i=1}^{n} |p_i - r_i|$$
 $RMSE = \sqrt{\frac{1}{n} \sum_{i=1}^{n} (p_i - r_i)^2}$ 

Welcome!

## **Collaborative Filtering Issues**

#### Pros:



well-understood, works well in some domains, no knowledge engineering required

#### Cons:



 requires user community, sparsity problems, no integration of other knowledge sources, no explanation of results

#### What is the best CF method?

 In which situation and which domain? Inconsistent findings; always the same domains and data sets; differences between methods are often very small (1/100)

### How to evaluate the prediction quality?

- MAE / RMSE: What does an MAE of 0.7 actually mean?
- Serendipity (novelty and surprising effect of recommendations)
  - Not yet fully understood

### What about multi-dimensional ratings?

## The Google News personalization engine



Search News

Search the Web

Search and browse 4,500 news sources updated continuously.

News archive search | Advanced news search | Blog search

Auto-generated 13 minutes ago

>Top Stories Recommended U.S. Business Flections World Entertainment Sci/Tech Health Sports Most Popular

☑ News Alerts

Text Version

Standard Version

Image Version

RSS | Atom About Feeds Top Stories | Personalized News

Go

## Tibet's Communist Party Leader Denounces

MSN UK News

warned of a "life and death struggle" with the Dalai Lama, as China struggles to bring an end to several days of protests in the Himalayan region.

Dalai Lama threatens to resign Los Angeles Times

By VOA News The head of Tibet's Communist Party has

Comment by Jamie Metzl Executive Vice President, Asia Society BBC News - Forbes - Reuters - Washington Post

all 5,998 news articles »

Exiled Dalai Lama

Voice of America - 43 minutes ago

#### Forex - Dollar resumes weak trend on expectations Fed to cut rates ...

CNNMoney.com - 2 hours ago

HONG KONG, Mar. 19, 2008 (Thomson Financial delivered by Newstex) -- The dollar resumed its weak tone against other key currencies in afternoon Asian trade on Wednesday as investors bet the Federal Reserve will further cut interest rates to lift the ...

Commentary by John M. Berry Bloomberg

Stocks soar after Federal Reserve trims rate Houston Chronicle

Los Angeles Times - New York Times - Sacramento Bee - Financial Times

all 805 news articles »

#### Edit this personalized page

#### Fed cuts key interest rate

Los Angeles Times - all 510 news articles »

#### Obama on race

Los Angeles Times - all 200 news articles »

#### US, Russia Politely Dug In Over Missile Defense

Washington Post - all 1.096 news articles »

#### Sci-fi guru Sir Arthur C. Clarke dies

Vancouver Sun - all 976 news articles »

#### Facebook Beefs Up Privacy Options, Readies Online Chat

Washington Post - all 297 news articles »

#### Mills' Money Can't Buy Her Love

E! Online - all 3.490 news articles »

#### Boeing confident of winning back tanker deal

Reuters - all 200 news articles »

#### In The News

Dalai Lama Barack Obama

Windows Vista Halle Berry

## **Google News portal (1)**

- Aggregates news articles from several thousand sources
- Displays them to signed-in users in a personalized way
- Collaborative recommendation approach based on
  - the click history of the active user and
  - the history of the larger community
- Main challenges
  - Vast number of articles and users
  - Generate recommendation list in real time (at most one second)
  - Constant stream of new items
  - Immediately react to user interaction
- Significant efforts with respect to algorithms, engineering, and parallelization are required

## **Google News portal (2)**

- Pure memory-based approaches are not directly applicable and for model-based approaches, the problem of continuous model updates must be solved
- A combination of model- and memory-based techniques is used
- Model-based part: Two clustering techniques are used
  - Probabilistic Latent Semantic Indexing (PLSI) as proposed by (Hofmann 2004)
  - MinHash as a hashing method
- Memory-based part: Analyze story co-visits for dealing with new users
- Google's MapReduce technique is used for parallelization in order to make computation scalable

## Literature (1)

- [Adomavicius and Tuzhilin 2005] Toward the next generation of recommender systems: A survey of the state-of-the-art
  and possible extensions, IEEE Transactions on Knowledge and Data Engineering 17 (2005), no. 6, 734–749
- [Breese et al. 1998] Empirical analysis of predictive algorithms for collaborative filtering, Proceedings of the 14th Conference on Uncertainty in Artificial Intelligence (Madison, WI) (Gregory F. Cooper and Seraf'in Moral, eds.), Morgan Kaufmann, 1998, pp. 43–52
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- [Goldberg et al. 2001] Eigentaste: A constant time collaborative filtering algorithm, Information Retrieval 4 (2001), no. 2, 133–151
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