CSE471 - Homework 6

Kumal Patel

April 6, 2021

1.1 (a) Draw the Bayesian network corresponding to this setup and define the necessary CPTs.

Coin	**P(Coin)**
a	20%
b	60%
c	80%

Table 1: Given Data with the probability of coming up heads

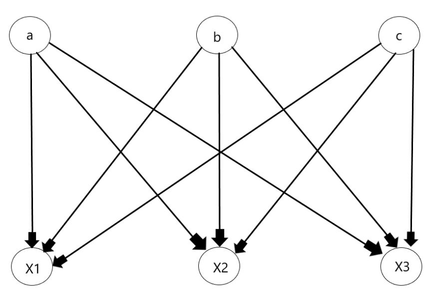

Figure 1: The outcomes X1, X2 and X3 have an equal probability of landing either heads or tails

Table 2: Bayesian network where a random variable C denoting which coin to draw has X_1, X_2, X_3 as child nodes.

Coin	**P(Coin)**
a	1/3
b	1/3
c	1/3

Coin	X_i	**P(Coin)**
a	heads	0.2
b	heads	0.6
c	heads	0.8

(b) Given Information: $P(H) = \frac{2}{3}$ and $P(T) = \frac{1}{3}$. The probability for the three types of coins can be calculated as $P(C|2\ heads, 1\ tails)$.

$$\begin{aligned}
P(C|2\ heads, 1\ tails) &= P(2\ heads, 1\ tails|C)P(C)/P(2\ heads, 1\ tails) \\
&\propto P(2\ heads, 1\ tails|C)P(C) \\
&\propto P(2\ heads, 1\ tails)P(C) \\
P(X_1 = tails|C = a)P(X_2 = heads|C = a)P(X_3 = heads|C = a) \\
&= 0.8 * 0.2 * 0.2 = 0.032 \\
P(2\ heads, 1\ tails|C = a) &= 3 * 0.032 = 0.096 \\
P(X_1 = tails|C = b)P(X_2 = heads|C = b)P(X_3 = heads|C = b) \\
&= 0.4 * 0.6 * 0.6 = 0.144 \\
P(2\ heads, 1\ tails|C = b) &= 3 * 0.144 = 0.432 \\
P(X_1 = tails|C = c)P(X_2 = heads|C = c)P(X_3 = heads|C = c) \\
&= 0.2 * 0.8 * 0.8 = 0.128 \\
P(2\ heads, 1\ tails|C = c) &= 3 * 0.128 = 0.384
\end{aligned}$$

So, since $P(2\ heads, 1\ tails|C = b)$ has the highest probability among the three coins. So we can conclude coin b is most likely to be drawn from the bag.

1.2 (a) B = Burglary E = Earthquake

$$P(B, E) = P(B|parents(B))P(E|parents(E)) = P(B)P(E)$$

Therefore, Burglary and Earthquake are independent.

A non-descendants shows that each variable is conditionally independent of each other. The Bayesian network Burglary is a non-descendant of Earthquake therefore, is independent.

(b)

$$P(B, E|A) = P(B|A)P(E|A)$$
$$P(A) = P(A|B, E)P(B)P(E) + P(A|B, \neg E)P(B)P(\neg E) + P(A|\neg B, E)P(\neg B)P(E)$$
$$+P(A|\neg B, \neg E)P(\neg B)P(\neg E)$$
$$= 0.95 * 0.001 * 0.002 + 0.94 * 0.001 * 0.998 + 0.29 * 0.999 * 0.002+$$
$$0.001 * 0.999 * 0.998$$
$$P(A) = 0.00251$$

$$P(B|A) = P(A|B)P(B)/P(A)$$
$$= P(A|B, E)P(B)P(E) + P(A|B, \neg E)P(B)P(\neg E)/P(A)$$
$$= 0.95 * 0.001 * 0.002 + 0.94 * 0.001 * 0.998/0.00251$$
$$P(B|A) = 0.37451$$

$$P(E|A) = P(A|E)/P(A)$$
$$= P(A|E, B)P(B) + P(A|E, \neg B)P(\neg B)/P(A)$$
$$= P(A|B, E)P(B)P(E) + P(A|\neg B, E)P(\neg B)P(E)/P(A)$$
$$= 0.95 * 0.001 * 0.002 + 0.29 * 0.999 * 0.002/0.00251$$
$$P(E|A) = 0.2316$$

$$P(B|A)P(E|A) = 0.37451 * 0.2316 = 0.0867$$
$$P(B, E|A) = P(A|B, E)P(B, E)/P(A)$$
$$= P(A|B, E)P(B)P(E)/P(A)$$
$$= 0.95 * 0.001 * 0.002/0.00251$$
$$P(B, E|A) = 0.000757$$

$$P(B,E|A) = P(B|A)P(E|A)$$
$$0.000757 \neq 0.0867$$

Therefore, if we observe Alarm = true, Burglary and Earthquake are not independent.

1.3 (a)

(b)

1.4 (a) Determine if the following are to be asserted by the network structure.

i.
$$P(B,I,M) = P(B)P(I)P(M)$$
Is not asserted because you would I, B, M would need to have independent paths with each other.

ii.
$$P(J|G) = P(J|G,I)$$
Is asserted

iii.
$$P(M|G,B,I) = P(M|G,B,I,J)$$
Is asserted

(b)

$$P(b,i,\neg m,g,\neg j) = P(b)P(\neg m)P(i|b,\neg m)P(g|b,i,\neg m)P(\neg j|g)$$
$$= 0.9 * 0.9 * 0.5 * 0.8 * 0.1 = 0.0324$$

(c)

$$P(j|b,\neg i,m) = P(g|b,\neg i,m)P(j|g) + P(\neg g|b,\neg i,m)P(j|\neg g)/$$
$$P(g|b,\neg i,m)P(j|g) + P(\neg g|b,\neg i,m)P(j|\neg g) + P(g|b,\neg i,m)$$
$$P(\neg j|g) + P(\neg g|b,\neg i,m)P(\neg j|\neg g)$$
$$P(j|b,\neg i,m) = 0.0 * 0.9 + 1 * 0.0/0.0 * 0.9 + 1 * 0.0 + 1 + 1 * 1 = 0$$

1.5 (a)

(b)

$$P(E_0) =< 0.7, 0.3 >$$
$$P(E_1) = \sum_{E_0} P(E_1|E_0)P(E_0)$$
$$= (< 0.8, 0.2 > 0.7+ < 0.3, 0.7 > 0.3)$$
$$=< 0.65, 0.35 >$$
$$P(E_1|e_1) = P(e_1|E_1)P(E_1)$$
$$=< 0.8 * 0.9, 0.3 * 0.7 >< 0.65, 0.35 >$$
$$=< 0.72, 0.21 >< 0.65, 0.35 >$$
$$=< 0.468, 0.0735 >$$
$$0.468 + 0.0735 = 0.5415$$
$$=< 0.468/0.5415, 0.0735/0.5415 >$$
$$=< 0.8643, 0.1357 >$$
$$P(E_2|e_1) = \sum_{E_1} P(E_2|E_1)P(E_1|e_1)$$
$$=< 0.7321, 0.2679 >$$
$$P(E_2|e_{1:2}) = P(e_2|E_2)P(E_2|e_1)$$
$$=< 0.5010, 0.4990 >$$
$$P(E_3|e_{1:2}) = \sum_{E_3} P(E_3|E_2)P(E_2|e_{1:2})$$
$$=< 0.5505, 0.4495 >$$
$$P(E_3|e_{1:3}) = P(e_3|E_3)P(E_3|e_{1:2})$$
$$=< 0.1045, 0.8955 >$$