CSE471 - Homework 3

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Exercise 1.1 (a)

(b)

(c) i. Shown from this truth table, there are 12 models, 12 out of the 16 worlds that are true.

A	B	C	D	$B \vee C$
0	0	0	0	0
0	0	0	1	0
0	0	1	0	1
0	0	1	1	1
0	1	0	0	1
0	1	0	1	1
0	1	1	0	1
0	1	1	1	1
1	0	0	0	0
1	0	0	1	0
1	0	1	0	1
1	0	1	1	1
1	1	0	0	1
1	1	0	1	1
1	1	1	0	1
1	1	1	1	1

ii. Shown from this truth table, there are 15 models, 15 out of

the 16 worlds that are true.

A	B	C	D	$\neg A \vee \neg B \vee \neg C \vee \neg D$
0	0	0	0	1
0	0	0	1	1
0	0	1	0	1
0	0	1	1	1
0	1	0	0	1
0	1	0	1	1
0	1	1	0	1
0	1	1	1	1
1	0	0	0	1
1	0	0	1	1
1	0	1	0	1
1	0	1	1	1
1	1	0	0	1
1	1	0	1	1
1	1	1	0	1
1	1	1	1	0

iii. Shown from this truth table, there are 0 models, 0 out of the 16 worlds that are true.

A	B	C	D	$(A \implies B) \wedge A \wedge \neg B \wedge C \wedge D$
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	0
0	1	0	1	0
0	1	1	0	0
0	1	1	1	0
1	0	0	0	0
1	0	0	1	0
1	0	1	0	0
1	0	1	1	0
1	1	0	0	0
1	1	0	1	0
1	1	1	0	0
1	1	1	1	0

Exercise 1.2 (a) This axiom entails if the left and right are both true, and in this one it is, so they entail.

p	q	r	s	$(p \to q) \wedge (r \to s)$	$(p \vee r) \to (q \vee s)$
0	0	0	0	1	1
0	0	0	1	1	1
0	0	1	0	0	0
0	0	1	1	1	1
0	1	0	0	1	1
0	1	0	1	1	1
0	1	1	0	0	1
0	1	1	1	1	1
1	0	0	0	0	0
1	0	0	1	0	1
1	0	1	0	0	0
1	0	1	1	0	1
1	1	0	0	1	1
1	1	0	1	1	1
1	1	1	0	0	1
1	1	1	1	1	1

(b) This axiom entails if the left and right are both true, and in this one it is, so they entail.

p	q	$(p \vee (q \to p)) \wedge q$	p
0	0	0	0
0	1	0	0
1	0	1	1
1	1	1	1

(c) This axiom entails if the left and right are both true, and in this

one it is, so they entail.

p	q	r	s	$p \to (q \vee r) \wedge (q \to s) \wedge (r \to s)$	$(p \to s)$
0	0	0	0	1	1
0	0	0	1	1	1
0	0	1	0	1	1
0	0	1	1	1	1
0	1	0	0	1	1
0	1	0	1	1	1
0	1	1	0	1	1
0	1	1	1	1	1
1	0	0	0	0	0
1	0	0	1	0	1
1	0	1	0	0	0
1	0	1	1	1	1
1	1	0	0	0	0
1	1	0	1	1	1
1	1	1	0	0	0
1	1	1	1	1	1

Exercise 1.3 (a)

Exercise 1.4 (a)

(b) i. $(W_{1,1}, \wedge\neg W_{1,2} \wedge \neg W_{1,3} \wedge \neg W_{1,4} \wedge \neg W_{2,1}, \wedge\neg W_{2,2} \wedge \neg W_{2,3} \wedge \neg W_{2,4} \wedge \neg W_{3,1}, \wedge\neg W_{3,2} \wedge \neg W_{3,3} \wedge \neg W_{3,4} \wedge \neg W_{4,1}, \wedge\neg W_{4,2} \wedge \neg W_{4,3} \wedge \neg W_{4,4}) \vee (\neg W_{1,1}, \wedge W_{1,2} \wedge \neg W_{1,3} \wedge \neg W_{1,4} \wedge \neg W_{2,1}, \wedge\neg W_{2,2} \wedge \neg W_{2,3} \wedge \neg W_{2,4} \wedge \neg W_{3,1}, \wedge\neg W_{3,2} \wedge \neg W_{3,3} \wedge \neg W_{3,4} \wedge \neg W_{4,1}, \wedge\neg W_{4,2} \wedge \neg W_{4,3} \wedge \neg W_{4,4}) \vee (\neg W_{1,1}, \wedge\neg W_{1,2} \wedge W_{1,3} \wedge \neg W_{1,4} \wedge \neg W_{2,1}, \wedge\neg W_{2,2} \wedge \neg W_{2,3} \wedge \neg W_{2,4} \wedge \neg W_{3,1}, \wedge\neg W_{3,2} \wedge \neg W_{3,3} \wedge \neg W_{3,4} \wedge \neg W_{4,1}, \wedge\neg W_{4,2} \wedge \neg W_{4,3} \wedge \neg W_{4,4}) \vee (\neg W_{1,1}, \wedge\neg W_{1,2} \wedge \neg W_{1,3} \wedge W_{1,4} \wedge \neg W_{2,1}, \wedge\neg W_{2,2} \wedge \neg W_{2,3} \wedge \neg W_{2,4} \wedge \neg W_{3,1}, \wedge\neg W_{3,2} \wedge \neg W_{3,3} \wedge \neg W_{3,4} \wedge \neg W_{4,1}, \wedge\neg W_{4,2} \wedge \neg W_{4,3} \wedge \neg W_{4,4}) \vee (\neg W_{1,1}, \wedge\neg W_{1,2} \wedge \neg W_{1,3} \wedge \neg W_{1,4} \wedge W_{2,1}, \wedge\neg W_{2,2} \wedge \neg W_{2,3} \wedge \neg W_{2,4} \wedge \neg W_{3,1}, \wedge\neg W_{3,2} \wedge \neg W_{3,3} \wedge \neg W_{3,4} \wedge \neg W_{4,1}, \wedge\neg W_{4,2} \wedge \neg W_{4,3} \wedge \neg W_{4,4}) \vee (\neg W_{1,1}, \wedge\neg W_{1,2} \wedge \neg W_{1,3} \wedge \neg W_{1,4} \wedge \neg W_{2,1}, \wedge W_{2,2} \wedge \neg W_{2,3} \wedge \neg W_{2,4} \wedge \neg W_{3,1}, \wedge\neg W_{3,2} \wedge \neg W_{3,3} \wedge \neg W_{3,4} \wedge \neg W_{4,1}, \wedge\neg W_{4,2} \wedge \neg W_{4,3} \wedge \neg W_{4,4}) \vee (\neg W_{1,1}, \wedge\neg W_{1,2} \wedge \neg W_{1,3} \wedge \neg W_{1,4} \wedge \neg W_{2,1}, \wedge\neg W_{2,2} \wedge W_{2,3} \wedge \neg W_{2,4} \wedge \neg W_{3,1}, \wedge\neg W_{3,2} \wedge \neg W_{3,3} \wedge \neg W_{3,4} \wedge \neg W_{4,1}, \wedge\neg W_{4,2} \wedge \neg W_{4,3} \wedge \neg W_{4,4}) \vee (\neg W_{1,1}, \wedge\neg W_{1,2} \wedge \neg W_{1,3} \wedge \neg W_{1,4} \wedge \neg W_{2,1}, \wedge\neg W_{2,2} \wedge$

$\neg W_{2,3} \wedge W_{2,4} \wedge \neg W_{3,1}, \wedge \neg W_{3,2} \wedge \neg W_{3,3} \wedge \neg W_{3,4} \wedge \neg W_{4,1}, \wedge \neg W_{4,2} \wedge$
$\neg W_{4,3} \wedge \neg W_{4,4}) \vee (\neg W_{1,1}, \wedge \neg W_{1,2} \wedge \neg W_{1,3} \wedge \neg W_{1,4} \wedge \neg W_{2,1}, \wedge \neg W_{2,2} \wedge$
$\neg W_{2,3} \wedge \neg W_{2,4} \wedge W_{3,1}, \wedge \neg W_{3,2} \wedge \neg W_{3,3} \wedge \neg W_{3,4} \wedge \neg W_{4,1}, \wedge \neg W_{4,2} \wedge$
$\neg W_{4,3} \wedge \neg W_{4,4}) \vee (\neg W_{1,1}, \wedge \neg W_{1,2} \wedge \neg W_{1,3} \wedge \neg W_{1,4} \wedge \neg W_{2,1}, \wedge \neg W_{2,2} \wedge$
$\neg W_{2,3} \wedge \neg W_{2,4} \wedge \neg W_{3,1}, \wedge W_{3,2} \wedge \neg W_{3,3} \wedge \neg W_{3,4} \wedge \neg W_{4,1}, \wedge \neg W_{4,2} \wedge$
$\neg W_{4,3} \wedge \neg W_{4,4}) \vee (\neg W_{1,1}, \wedge \neg W_{1,2} \wedge \neg W_{1,3} \wedge \neg W_{1,4} \wedge \neg W_{2,1}, \wedge \neg W_{2,2} \wedge$
$\neg W_{2,3} \wedge \neg W_{2,4} \wedge \neg W_{3,1}, \wedge \neg W_{3,2} \wedge W_{3,3} \wedge \neg W_{3,4} \wedge \neg W_{4,1}, \wedge \neg W_{4,2} \wedge$
$\neg W_{4,3} \wedge \neg W_{4,4}) \vee (\neg W_{1,1}, \wedge \neg W_{1,2} \wedge \neg W_{1,3} \wedge \neg W_{1,4} \wedge \neg W_{2,1}, \wedge \neg W_{2,2} \wedge$
$\neg W_{2,3} \wedge \neg W_{2,4} \wedge \neg W_{3,1}, \wedge \neg W_{3,2} \wedge \neg W_{3,3} \wedge W_{3,4} \wedge \neg W_{4,1}, \wedge \neg W_{4,2} \wedge$
$\neg W_{4,3} \wedge \neg W_{4,4}) \vee (\neg W_{1,1}, \wedge \neg W_{1,2} \wedge \neg W_{1,3} \wedge \neg W_{1,4} \wedge \neg W_{2,1}, \wedge \neg W_{2,2} \wedge$
$\neg W_{2,3} \wedge \neg W_{2,4} \wedge \neg W_{3,1}, \wedge \neg W_{3,2} \wedge \neg W_{3,3} \wedge \neg W_{3,4} \wedge W_{4,1}, \wedge \neg W_{4,2} \wedge$
$\neg W_{4,3} \wedge \neg W_{4,4}) \vee (\neg W_{1,1}, \wedge \neg W_{1,2} \wedge \neg W_{1,3} \wedge \neg W_{1,4} \wedge \neg W_{2,1}, \wedge \neg W_{2,2} \wedge$
$\neg W_{2,3} \wedge \neg W_{2,4} \wedge \neg W_{3,1}, \wedge \neg W_{3,2} \wedge \neg W_{3,3} \wedge \neg W_{3,4} \wedge \neg W_{4,1}, \wedge W_{4,2} \wedge$
$\neg W_{4,3} \wedge \neg W_{4,4}) \vee (\neg W_{1,1}, \wedge \neg W_{1,2} \wedge \neg W_{1,3} \wedge \neg W_{1,4} \wedge \neg W_{2,1}, \wedge \neg W_{2,2} \wedge$
$\neg W_{2,3} \wedge \neg W_{2,4} \wedge \neg W_{3,1}, \wedge \neg W_{3,2} \wedge \neg W_{3,3} \wedge \neg W_{3,4} \wedge \neg W_{4,1}, \wedge \neg W_{4,2} \wedge$
$W_{4,3} \wedge \neg W_{4,4}) \vee (\neg W_{1,1}, \wedge \neg W_{1,2} \wedge \neg W_{1,3} \wedge \neg W_{1,4} \wedge \neg W_{2,1}, \wedge \neg W_{2,2} \wedge$
$\neg W_{2,3} \wedge \neg W_{2,4} \wedge \neg W_{3,1}, \wedge \neg W_{3,2} \wedge \neg W_{3,3} \wedge \neg W_{3,4} \wedge \neg W_{4,1}, \wedge \neg W_{4,2} \wedge$
$\neg W_{4,3} \wedge W_{4,4})$

(c)

Exercise 1.5 (a)

(b)