Sol 2 Let $f_i(x)$ $i = 1, 2, 3, 4$ be four bivariate gaussian distributions with mean $\mu_1, \mu_2, \mu_3, \mu_4$ and covariance matrix $\Sigma_1, \Sigma_2, \Sigma_3, \Sigma_4$

define

$$z_i = \begin{cases} i & \text{if } X_i \sim f_i \\ 0 & \text{otherwise} \end{cases}$$

$$P\{x_i < X_i < x_i + \Delta x_i, z_i | \theta\}$$

$$P\left[\{x_i < X_i < x_i + \Delta x_i | z_i, \theta\} P[z_i = z_i]\right]$$

$$= \begin{cases} p_1 f_1(x_i) & x_i \sim f_1 \\ p_2 f_2(x_i) & x_i \sim f_2 \\ p_3 f_3(x_i) & x_i \sim f_3 \\ p_4 f_4(x_i) & x_i \sim f_4 \end{cases}$$

$p_1 + p_2 + p_3 + p_4 = 1.$

$$L(\theta) = p_1 f_1(x_i) I(z_i = 1) + p_2 f_2(x_i) I(z_i = 2) + p_3 f_3(x_i) I(z_i = 3) + p_4 f_4(x_i) I(z_i = 4)$$

$$= (p_1 f_1(x_i))^{I(z_i=1)} [p_2 f_2(x_i)]^{I(z_i=2)} [p_3 f_3(x_i)]^{I(z_i=3)} \cdots [p_4 f_4(x_i)]^{I(z_i=4)}$$

Soll. multinom $(20, 0.3, 0.4, 0.1)$

Clearly $0.3+0.4+0.1 = 0.8 \neq 1$

so, we've another component P_4 s.t.

$$P_1+P_2+P_3+P_4 = 1.$$

$$\Rightarrow P_4 = 1-0.8 = 0.2$$