Probability Assignment 2

EE22BTECH11210 - KUMAR ARYAN

Question : The number lock of a suitcase has 4 wheels, each labelled with ten digits i.e., from 0 to 9. The lock opens with a sequence of four digits with no repeats. What is the probability of a person getting the right sequence to open the suitcase ?
Solution : Define random variable X_i such that

$$X_i = \begin{cases} 1, & \text{correct number choosen in } i^{th} \text{ wheel} \\ 0, & \text{otherwise} \end{cases} \tag{1}$$

and since repetition is not allowed, sample space for every next wheel will reduce by 1 unit. Therefore,

$$P_{X_i}(1) = \frac{1}{11-i} \tag{2}$$

$$P_{X_i}(0) = 1 - \frac{1}{11-i} \tag{3}$$

$$= \frac{10-i}{11-i} \tag{4}$$

Therefore,

$$\implies P_{X_1}(n) = \begin{cases} \frac{1}{10}, & \text{n = 1} \\ \frac{9}{10}, & \text{n = 0} \end{cases} \tag{5}$$

$$\implies P_{X_2}(n) = \begin{cases} \frac{1}{9}, & \text{n = 1} \\ \frac{8}{9}, & \text{n = 0} \end{cases} \tag{6}$$

$$\implies P_{X_3}(n) = \begin{cases} \frac{1}{8}, & \text{n = 1} \\ \frac{7}{8}, & \text{n = 0} \end{cases} \tag{7}$$

$$\implies P_{X_4}(n) = \begin{cases} \frac{1}{7}, & \text{n = 1} \\ \frac{6}{7}, & \text{n = 0} \end{cases} \tag{8}$$

Let E be event of getting the right sequence. Since X_i are independent of each other, hence

$$\Pr(E) = \prod_{i=1}^{4} P_{X_i}(1) \tag{9}$$

$$= P_{X_1}(1) \times P_{X_2}(1) \times P_{X_3}(1) \times P_{X_4}(1)$$

$$= \frac{1}{10} \times \frac{1}{9} \times \frac{1}{8} \times \frac{1}{7}$$

$$= \frac{1}{5040}$$