

# Probability Assignment 4

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**Question :** In an examination, 20 questions of true-false type are asked. Suppose a student tosses a fair coin to determine his answer to each question. If the coin falls heads, he answer true; if it falls tails, he answer false. Find the probability that he answers at least 12 questions correctly.

**Solution :** Let  $X$  denote the number of correct answers out of 20 questions. Clearly,  $X$  has the binomial distribution with  $n = 20$  and  $p$  being the probability of answering question correctly. Since, answering correctly depends on coin toss. Therefore,

$$p = \frac{1}{2} \quad (1)$$

$$q = 1 - p = \frac{1}{2} \quad (2)$$

Since  $X$  has binomial distribution,

$$\Rightarrow P_X(r) = {}^nC_r (p)^r (q)^{n-r} \quad (3)$$

$$\Rightarrow F_X(r) = \sum_{r=0}^n {}^nC_r (p)^r (q)^{n-r} \quad (4)$$

Therefore,

$$\Pr(X \geq 12) = 1 - F_X(11) \quad (5)$$

$$= 1 - \sum_{r=0}^{11} {}^{20}C_r \left(\frac{1}{2}\right)^r \left(\frac{1}{2}\right)^{20-r} \quad (6)$$

$$= 1 - \left(\frac{1}{2}\right)^{20} \sum_{r=0}^{11} {}^{20}C_r \quad (7)$$

$$= 0.2517 \quad (8)$$