

Probability Assignment 2

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Question : The number lock of a suitcase has 4 wheels, each labelled with ten digits i.e., from 0 to 9. The lock opens with a sequence of four digits with no repeats. What is the probability of a person getting the right sequence to open the suitcase ?

Solution : Define random variable X_i such that

$$X_i = \begin{cases} 1, & \text{correct number choosen in } i^{th} \text{ wheel} \\ 0, & \text{otherwise} \end{cases} \quad (1)$$

and

$$P_{X_i}(1) = \frac{1}{11-i} \quad (2)$$

$$P_{X_i}(0) = 1 - \frac{1}{11-i} \quad (3)$$

$$= \frac{10-i}{11-i} \quad (4)$$

Therefore,

$$\Rightarrow P_{X_1}(n) = \begin{cases} \frac{1}{10}, & n = 1 \\ \frac{9}{10}, & n = 0 \end{cases} \quad (5)$$

$$\Rightarrow P_{X_2}(n) = \begin{cases} \frac{1}{9}, & n = 1 \\ \frac{8}{9}, & n = 0 \end{cases} \quad (6)$$

$$\Rightarrow P_{X_3}(n) = \begin{cases} \frac{1}{8}, & n = 1 \\ \frac{7}{8}, & n = 0 \end{cases} \quad (7)$$

$$\Rightarrow P_{X_4}(n) = \begin{cases} \frac{1}{7}, & n = 1 \\ \frac{6}{7}, & n = 0 \end{cases} \quad (8)$$

Let E be event of getting the right sequence. Since X_i are independent of each other, hence

$$\begin{aligned} \Pr(E) &= \prod_{i=1}^4 P_{X_i}(1) \\ &= P_{X_1}(1) \times P_{X_2}(1) \times P_{X_3}(1) \times P_{X_4}(1) \\ &= \frac{1}{10} \times \frac{1}{9} \times \frac{1}{8} \times \frac{1}{7} \\ &= \frac{1}{5040} \end{aligned} \quad (9)$$