UNIVERSITY OF LORRAINE

DECENTRALISED SMART ENERGY SYSTEMS

ENERGY COVERSION PROCESSES:

HEAT AND FLUID FOR ENERGY

Thermodynamic Cycles

Topic C

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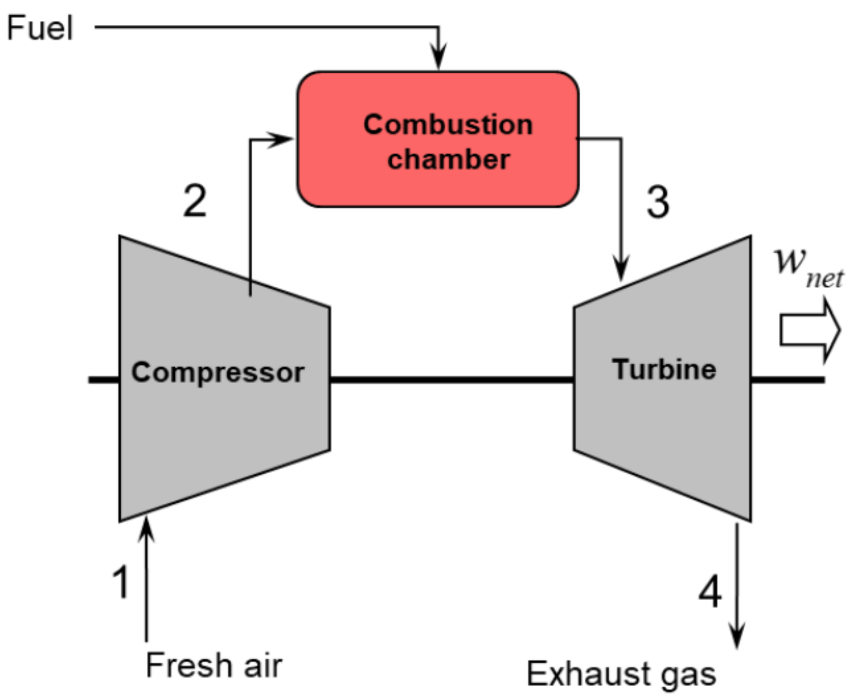
Study on the optimization of a gas turbine

Consider a gas turbine with an isentropic compression stage and a non-isentropic expansion stage. The synoptic diagram of the installation is given below:



All gases are assimilated to a perfect diatomic gas with one single constant specific heat capacity equal to that of air (γ = 1.4, R = 8.314 J.K-1 .mol-1 , r = R/M_{air}, M_{air} being the molar mass of air for a composition of 79% nitrogen and 21% oxygen). It is assumed that there is no change in the nature and mass of the gas during combustion.

All graphical representations have to be done numerically using Matlab, Python or an equivalent program. Only graphics in image format have to be included in the final report.

3. Study of a simple gas turbine

The entire gas turbine is assumed to operate according to an ideal Brayton power cycle. Irreversibilities during the expansion stage are taken into account through an isentropic efficiency η_{is} = 85 %. Pressure losses during heat exchange phases are neglected.

The gas enters the compressor at T_1 = 45°C and at atmospheric pressure p_{atm} = 1bar. The total compression ratio is denoted $\rho = p_2/p_1$. Moreover, stationary flow is assumed and the variations in kinetic and potential energy are negligible compared to Δh and $V_m$dp (V_m being the mass volume in m^3 /kg). The heat input is made by the combustion of natural gas for which we give the LHV = 42.3 MJ/kg. The air-fuel ratio is $[m_{air} + m_{fuel}]$ = [16 + 1].

3.1 Draw the cycle in a $T - S$ diagram and indicate the energies involved for each stage.

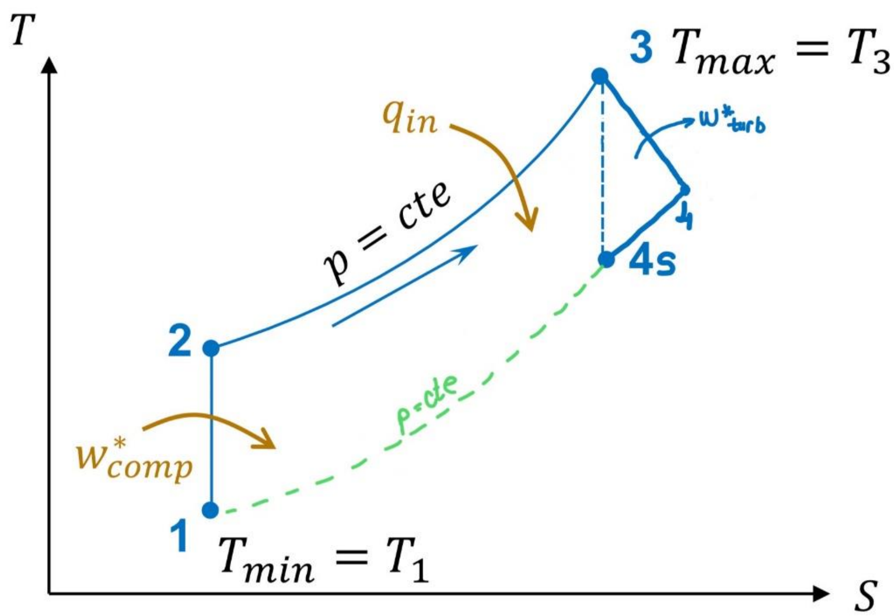


Figure 1. T-s diagram of the Simple Brayton Cycle

3.2 Express the temperature of the individual points of the power cycle as a function of T_1, ρ, LHV, η_{is}, γ, and r (not necessarily all at once). Intermediate results may be used to lighten writing. Comment the impact of the irreversibility in the expansion stage compared to the isentropic process.

Using the given data: $R = 8.314 \frac{J}{Kmol}, M_{air} = 0.02897 \frac{kg}{mol}$ and $\gamma = 1.4$ we calculate the following required values:

$$r = \frac{R}{M} = \frac{8.314}{0.02897} = 288 \frac{J}{kg.K} \quad \text{and} \quad c_p = \frac{\gamma \times r}{\gamma - 1} = 1009 \frac{J}{kg.K}$$

For Stage 1 to 2, since the compression is reversible (isentropic), we know that:

$$\frac{T_2}{T_1} = \rho^{\frac{\gamma-1}{\gamma}} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} \rightarrow T_2 = T_1 \times \rho^{\frac{\gamma-1}{\gamma}}$$

For Stage 2 to 3, isobaric heating happens, thus:

$$Q_{2-3} = m_T \times c_p \times (T_3 - T_2) = m_{fuel} \times LHV$$

**Here LHV is expressed in* $\left[\frac{J}{kg}\right]$.

$$T_3 = \frac{m_{fuel} \times \mathrm{LHV}}{\mathrm{m_T} \times c_p} + T_2$$

Using the already expressed T_2, we can express the temperature at Stage 3 as:

$$T_3 = \frac{m_{fuel}}{\mathrm{m_T}} \times LHV \times \frac{\gamma - 1}{\gamma \times r} + \ T_1 \times \rho^{\frac{\gamma-1}{\gamma}}$$

For Stage 3 to 4, non-isentropic expansion happens, therefore:

$$\begin{cases} \dfrac{T_{4s}}{T_3} = \rho^{\frac{1-\gamma}{\gamma}} \\ \eta_{is} = \dfrac{w^*_{turb}}{w^*_{turb,s}} = \dfrac{(h_4 - h_3\ )}{(h_{4s} - h_3)} = \dfrac{T_4 - T_3}{T_{4s} - T_3} \end{cases} \rightarrow T_4 = (T_{4s} - T_3)\eta_{is} + T_3 = T_3(\rho^{\frac{1-\gamma}{\gamma}} - 1)\eta_{is} + T_3$$

So,

$$T_4 = T_3\left(\left(\rho^{\frac{\gamma-1}{\gamma}} - 1\right)\eta_{is} + 1\right) = \left(\frac{m_{fuel}}{\mathrm{m_T}} \times LHV \times \frac{\gamma-1}{\gamma \times r} + \ T_1 \times \rho^{\frac{\gamma-1}{\gamma}}\right)\left(\left(\rho^{\frac{\gamma-1}{\gamma}} - 1\right)\eta_{is} + 1\right)$$

Comment: As seen in Figure 1., the turbine output reduces when the expansion is irreversible. Due to the irreversibility, entropy is generated which causes the increase in outlet temperature of the turbine, thus reducing the useful work extracted from the turbine. Hence, the Net Work obtained from the system is also reduced.

3.3 Express the specific work (in J/kg) to be given to the compressor $w^*_{comp}$and the one provided by the turbine w^*_{turb} as a function of T_1, T_3, ρ, γ, r and η_{is}.

For the compressor:

$$w^*_{comp} = c_p(T_2 - T_1) = \frac{\gamma \times r}{\gamma - 1} \times \left(T_1 \times \rho^{\frac{\gamma-1}{\gamma}} - T_1\right) = \frac{\gamma \times r}{\gamma - 1} T_1 \left(\rho^{\frac{\gamma-1}{\gamma}} - 1\right)$$

For the turbine, since the expansion is irreversible (non-isentropic), we use the following equation $\eta_{is} = \frac{w^*_{turb}}{w^*_{turb,s}} = \frac{T_4 - T_3}{T_{4s} - T_3}$, thus:

$$w^*_{turb} = \eta_{is}(T_{4s} - T_3) = \eta_{is} T_3 (\rho^{\frac{1-\gamma}{\gamma}} - 1)$$

3.3.1 **Comment the impact of the irreversibility in the expansion stage compared to the isentropic process.**

Comment: As seen in Figure 1., the turbine output reduces when the expansion is irreversible. Due to the irreversibility, entropy is generated which causes the increase in outlet temperature of the turbine, thus reducing the useful work extracted from the turbine. Hence, the Net Work obtained from the system is also reduced.

3.3.2 **Draw the evolution of the compressor and the turbine work as well as the ratio between of work of the compressor to the turbine work as a function of ρ for $1 \leq \rho \leq 20$. Comment the result.**

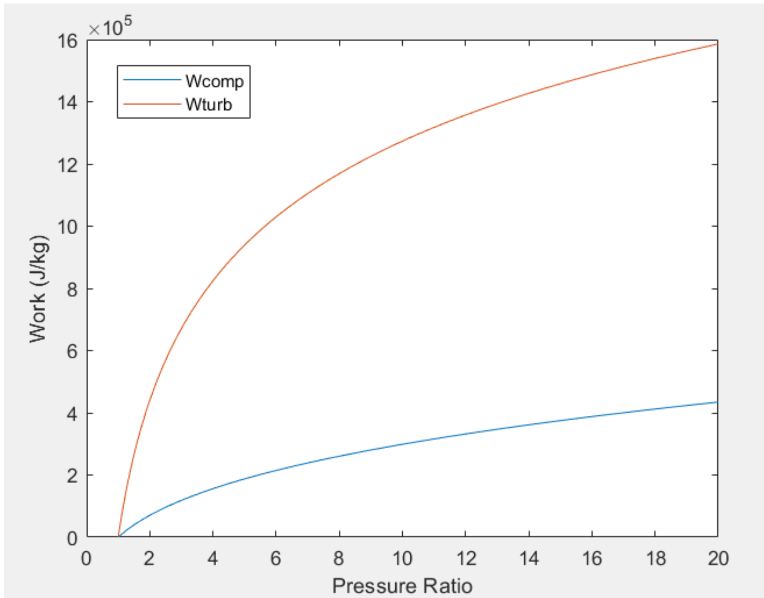


Figure 2. Evolution of Compressor and Turbine specific works against the Pressure Ratio

Comment: As shown in Figure 2., as the pressure ratio increases, both the compressor work provided, and the turbine output received by the system increases. Both compressor and turbine works are proportional to the pressure ratio. Furthermore, taking a physical approach it is also understandable that higher pressure ratio means more energy must be supplied to the system to achieve the pressure ratio. Consequently, more work is obtained from the turbine as the pressure ratio between its inlet and outlet increases.

We can express the work ratio between the compressor and turbine work as a function of compression ratio using the following formula:

$$Work\ ratio = \frac{w^*_{comp}}{-w^*_{turb}} = \frac{\frac{\gamma \times r}{\gamma - 1} T_1 \left(\rho^{\frac{\gamma-1}{\gamma}} - 1\right)}{\eta_{is} T_3 (\rho^{\frac{1-\gamma}{\gamma}} - 1)}$$

The following figure represents this formulation graphically:

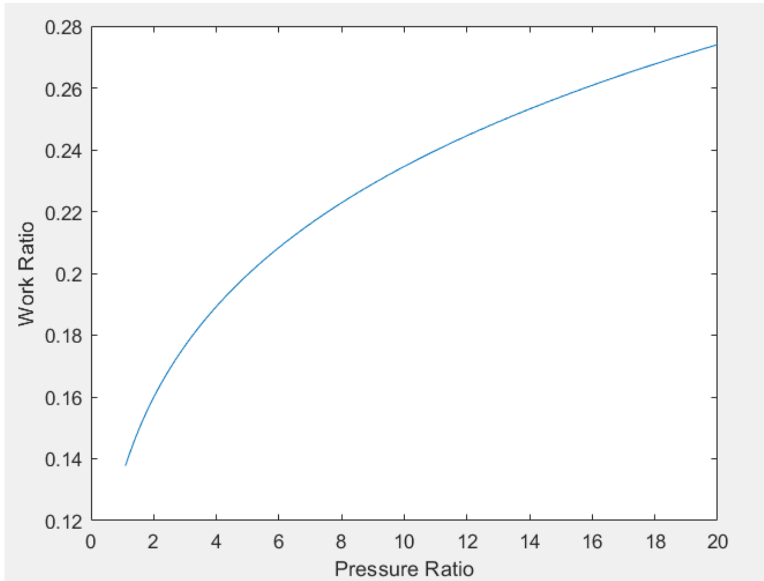


Figure 3. Evolution of Work Ratio against the Pressure Ratio

Comment: From the Figure 3. we can see that as the pressure ratio increases, the work ratio increases. As we have discussed earlier, both the compressor and turbine work increase with the pressure ratio which is mainly the reason why Work Ratio increases. However, the decrease in slope indicates that for the given change in pressure ratio, the change in turbine work is greater compared to that of compressor work.

3.4 Express the Net work output of the gas turbine w_{net}^*(in J/kg) and its thermal efficiency $\eta_{th,simple}$ as a function of the known characteristics. Draw their evolution as a function of ρ and comment the results.

$$w_{net}^* = w_{comp}^* + w_{turb}^* = \frac{\gamma \times r}{\gamma - 1} T_1 \left(\rho^{\frac{\gamma-1}{\gamma}} - 1\right) + \eta_{is} T_3 (\rho^{\frac{1-\gamma}{\gamma}} - 1)$$

$$\eta_{th,simple} = \frac{-w_{net}^*}{q_{in}} = \frac{-(w_{comp}^* + w_{turb}^*)}{c_p(T_3 - T_2)} = \frac{-(\frac{\gamma \times r}{\gamma - 1} T_1 \left(\rho^{\frac{\gamma-1}{\gamma}} - 1\right) + \eta_{is} T_3 \left(\rho^{\frac{1-\gamma}{\gamma}} - 1\right))}{\frac{\gamma \times r}{\gamma - 1} \times \frac{m_{fuel}}{m_T} \times LHV}$$

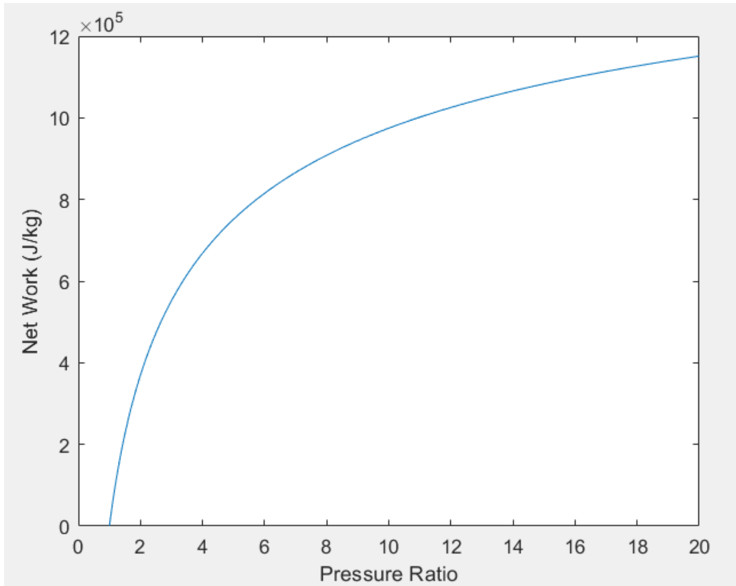


Figure 4. Evolution of Net specific Work Output against the Pressure Ratio

Comment: Figure 4. shows the evolution of Net specific Work Output against the Pressure Ratio. The Net specific Work Output of the system increases as the Pressure Ratio increases, and it is maximum at Pressure Ratio "20" for the given range of Pressure Ratio (1 to 20). The reasoning can be explained using the impact of Pressure Ratio on Compressor and Turbine work. As the Pressure Ratio increases, the change in Turbine Work is greater compared to change in Compressor Work. Therefore, we see an increase in Net specific work output. This reasoning can further be supported by decreasing slope of curve of Work Ratio in Figure 3. Additionally, from Figure 2., we can see that the difference between the Turbine work and the compressor work is increasing for the given interval. Thus, we can conclude that the Net Work increases for the given interval.

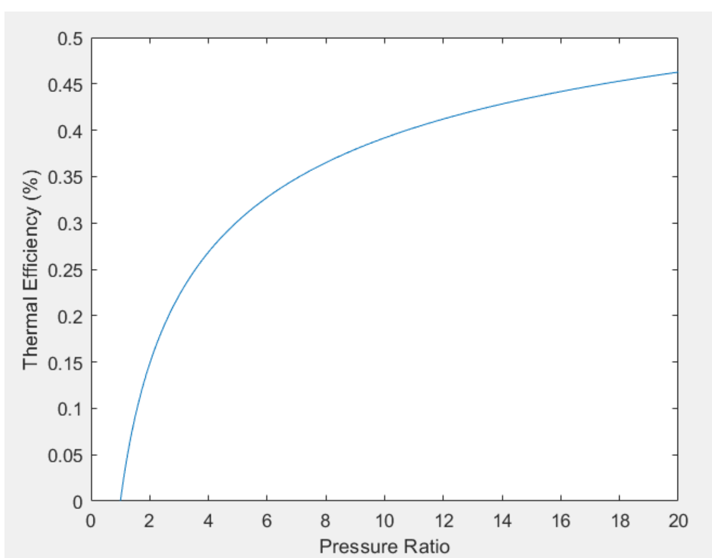


Figure 5. Evolution of Thermal Efficiency against the Pressure Ratio

Comment: From the figure 5., it can be observed that increasing the Pressure Ratio has positive impact on Thermal Efficiency of the system. The thermal efficiency is directly proportional to the net specific work output while it is inversely proportional to the heat provided to the system at the combustion chamber. We have already seen that there is positive impact of increasing the Pressure Ratio on the net specific output of the system. On the other hand, it causes the decrease in the heat provided to the system because when the Pressure Ratio increases, the outlet temperature of the compressor increases. Therefore, to achieve the same maximum temperature of the system at turbine inlet, less heat input is needed at the combustion chamber.

3.5 Determine the optimum compression ratio ρ_{opt} giving the maximum net Work output and make the digital application for w^*_{comp}, w^*_{turb}, w^*_{net} and $\eta_{th,\,simple}$ for ρ_{opt}.

For maximum power $\frac{\mathrm{dW_{net}}}{d\rho} = 0$

The methodology to find the optimum Pressure Ratio and required values in MATLAB is: the maximum Net Work was found using the function "Net_Work_opt = max(Net_Work')" and then coordinates of the maximum value were extracted using "[row, col] = find(Net_Work' == Net_Work_opt)" function. Later, those coordinates were used to retrieve the values of the required parameters. For instance, "rho_opt = rho_values(col,row)"

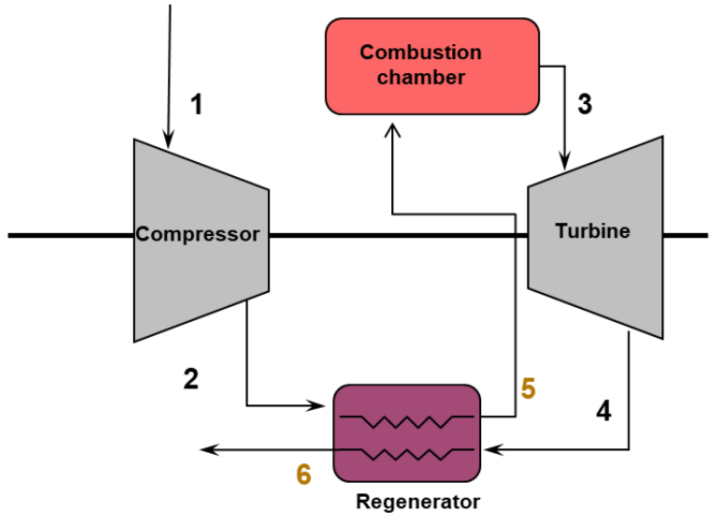
The values obtained from the code are obtained as.

Parameter	Value
ρ_{opt}	20
w^*_{net}	1155300 [J/kg]
w^*_{comp}	407180 [J/kg]
w^*_{turb}	1562500 [J/kg]
$\eta_{th,\,simple}$	46.43 %

4. Study of the optimization of the gas turbine

2.3. Heat regeneration

In the following, we consider the presence of a heat regenerator with an efficiency η_{reg}. All numerical values of the simple cycle are to be set to those of ρopt. The new synoptic diagram of the installation is given below:



4.1.1. **Draw the cycle in a T-S diagram and indicate the changes for the energies involved compared to the simple Brayton cycle.**

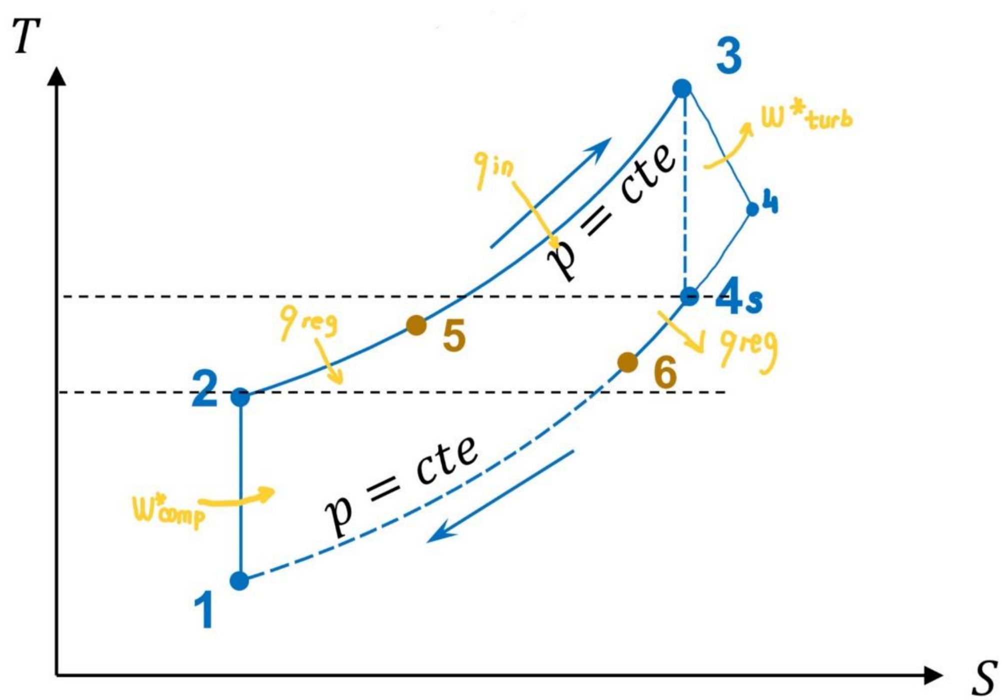


Figure 6. T-S diagram of Brayton Cycle with Regenerator

Comparison: Comparing the T-S diagram of a Simple Brayton Cycle and the Cycle with Heat Regenerator, we can see a few differences. The most important difference is the heat regenerated from the exhaust fumes, denoted as q_{reg} in the Figure 6. We can clearly see that the heat required to be provided to the system from the combustion in the process 5-3 is lower than in the Simple Brayton Cycle, as the portion of it is covered by the regenerated heat.

4.1.2. **Express the inlet temperature of the combustion chamber T_5 as a function of η_{reg} and the temperatures of the simple cycle. Draw the evolution of T_5 for $0 \leq \eta_{reg} \leq 1$. Comment your observations, including the higher and lower limits $\eta_{reg} = 0$ and $\eta_{reg} = 1$.**

We know that the efficiency of the regenerator can be written as: $\eta_{reg} = \frac{T_5 - T_2}{T_4 - T_2}$.

Therefore, the inlet temperature of combustion chamber can be given as:

$$T_5 = \eta_{reg}(T_4 - T_2) + T_2$$

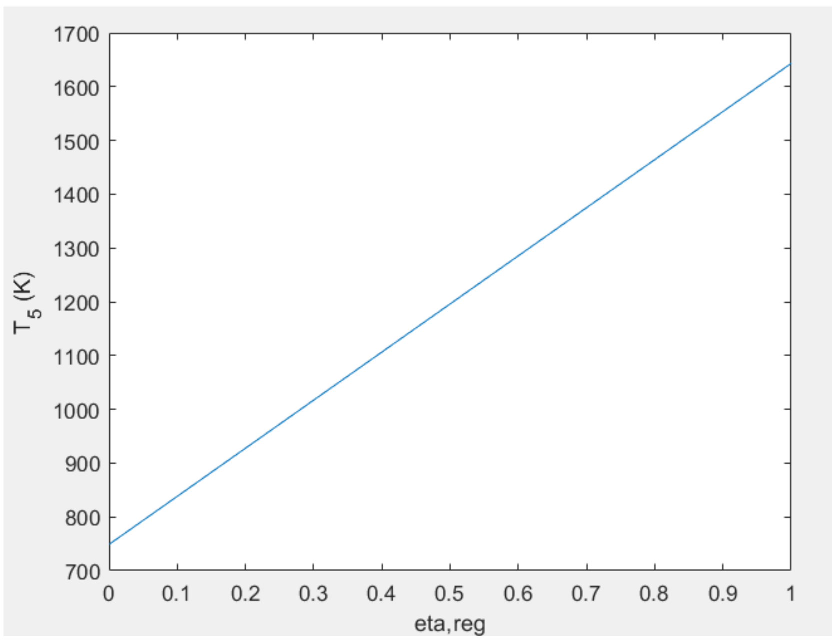


Figure 6. Evolution of T_5 against η_{reg}

Comment: From the derived mathematical equation, we can see that T_5 is linearly dependent on the regenerator efficiency. If the regenerator efficiency is zero, the temperature T_5 is going to be equal to T_2, and similarly, if the regenerator efficiency is equal to 1, the temperature T_5 is going to be equal to T_4. The graph substantiates our mathematical conclusions. Physically, the higher the regenerator efficiency, the more energy the exhaust gases can exchange with the air entering the combustion chamber. Therefore, we see T_5 approaching the temperature of exhaust gases as the efficiency of the regenerator is increasing.

4.1.3. **Express the new specific heat to be supplied $q_{in,reg}$ and the thermal efficiency $\eta_{th,reg}$ as a function of η_{reg}. Draw their evolution of $0 \leq \eta_{reg} \leq 1$.**

Before entering the combustion chamber, the air is pre-heated using the exhaust gases. The amount of heat that air receives can be obtained as:

$$q_{in,reg} = c_p(T_3 - T_5) = c_p(T_3 - \eta_{reg}(T_4 - T_2) - T_2)$$

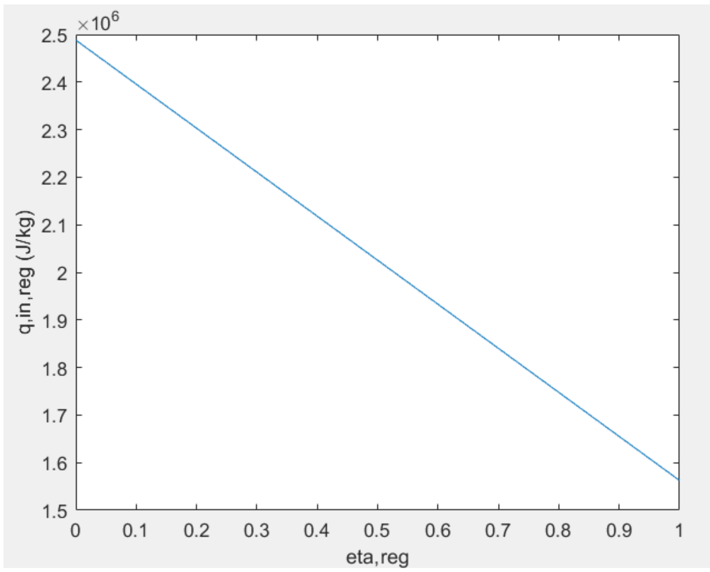


Figure 7. Evolution of $q_{in,reg}$ against η_{reg}

The thermal efficiency can be given as:

$$\eta_{th,reg} = \frac{-w_{net}^*}{q_{in,reg}} = \frac{-w_{net}^*}{c_p(T_3 - T_5)} = \frac{-(w_{comp}^* + w_{turb}^*)}{c_p(T_3 - \eta_{reg}(T_4 - T_2) - T_2)}$$

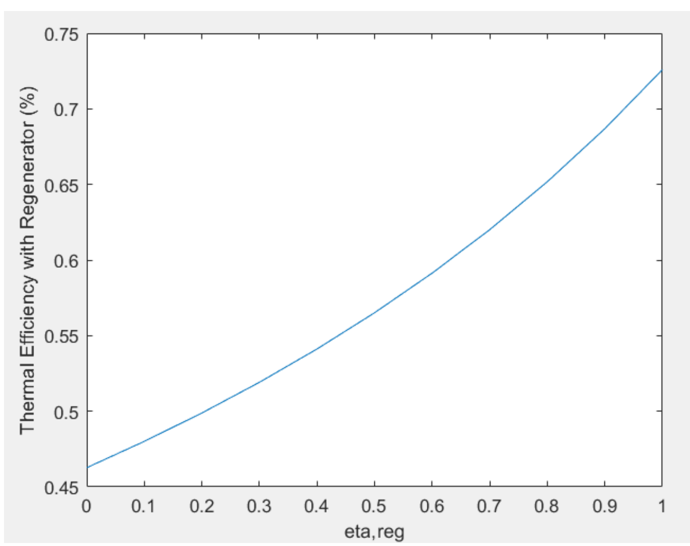


Figure 8. Evolution of $\eta_{th,reg}$ against η_{reg}

<u>*4.1.4.*</u> Make the digital application of these characteristics for η_{reg} = 0.7. Comment the results compared to the simple cycle.

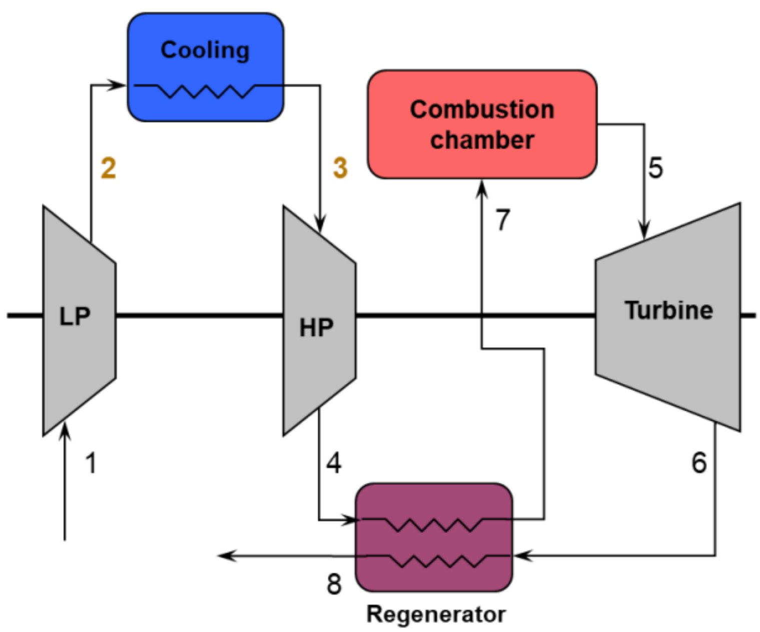
The MATLAB approach for calculation of these values is same as in the first part. Hence, the obtained values are as follows:

Parameter	Simple Cycle	Regeneration Cycle
ρ_{opt}	20	20
w^*_{net}	1155300 [J/kg]	1155300 [J/kg]
w^*_{comp}	407180 [J/kg]	407180 [J/kg]
w^*_{turb}	1562500 [J/kg]	1562500 [J/kg]
q_{in}	2488200 [J/kg]	1840200 [J/kg]
η_{th}	46.43 %	62.78%

Comparison: The most important thing that can be deduced from this comparison is that the thermal efficiency is higher for the system with regeneration because the heat required by the system at combustion chamber is less than that of the simple cycle because a significant portion of it is recovered using regenerator.

2.4. Staged compression

Here, we consider the presence of staged compression. After a first LP compression stage to $p_{int} = p_2 = p_3$ the gas is cooled down at constant pressure to its initial temperature. It is then compressed to is final pressure in a second HP compression stage where $p_4 = \rho_{opt} p_1$. The new synoptic diagram of the installation is given below:



2.4.1. Draw the cycle in a T-S diagram and indicate the changes for the energies involved compared to the Brayton cycle with heat regeneration only.

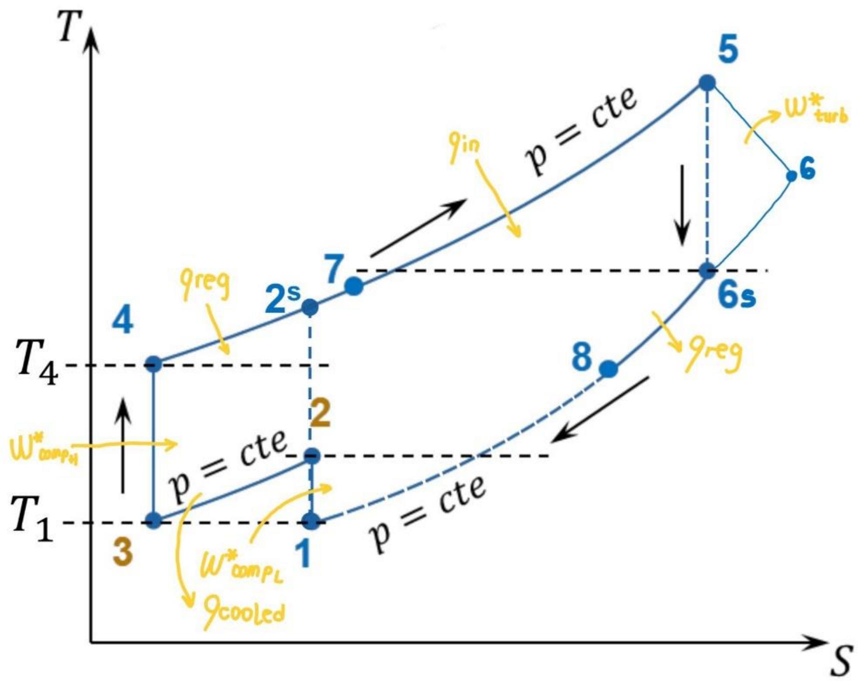


Figure 9. T-S diagram of Staged Compression

Comparison: Compared to the Brayton Cycle with Regeneration, the Staged Compression divides the total compressor work into two parts. Also, the heat is rejected from the system between two stages of compressor using a intercooler.

2.4.2. Express the specific total work (in J/kg) to be given to the compressor as a function of p_{int}, p_1, T_1, ρ_{opt} and γ. Draw its evolution as well as the ratio between of the compressor to the turbine work as a function of pint for $p_1 \leq p_{int} \leq p_4$. Comment the result.

$$T_2 = T_1\left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = T_1\left(\frac{p_{int}}{p_1}\right)^{\frac{\gamma-1}{\gamma}}$$

$$T_4 = T_3\left(\frac{\rho_{opt}\cdot p_1}{p_4}\right)^{\frac{\gamma-1}{\gamma}} = T_1\left(\frac{\rho_{opt}\cdot p_1}{p_{int}}\right)^{\frac{\gamma-1}{\gamma}} \ where\ p_4 = \rho_{opt}\cdot p_1$$

The total work given to the compressors can written as:

$$w^*{}_{comp,total} = c_p(T_2 - T_1) + c_p(T_4 - T_3) = \ c_p(T_4 + T_2 - 2\cdot T_1)$$
$$= \ c_p\left(T_1\left(\frac{\rho_{opt}\cdot p_1}{p_{int}}\right)^{\frac{\gamma-1}{\gamma}} + T_1\left(\frac{p_{int}}{p_1}\right)^{\frac{\gamma-1}{\gamma}} - 2\cdot T_1\right)$$

$$w^{*}{}_{comp,total} = c_p T_1 \left(\left(\frac{\rho_{opt} \cdot p_1}{p_{int}} \right)^{\frac{\gamma-1}{\gamma}} + \left(\frac{p_{int}}{p_1} \right)^{\frac{\gamma-1}{\gamma}} - 2 \right)$$

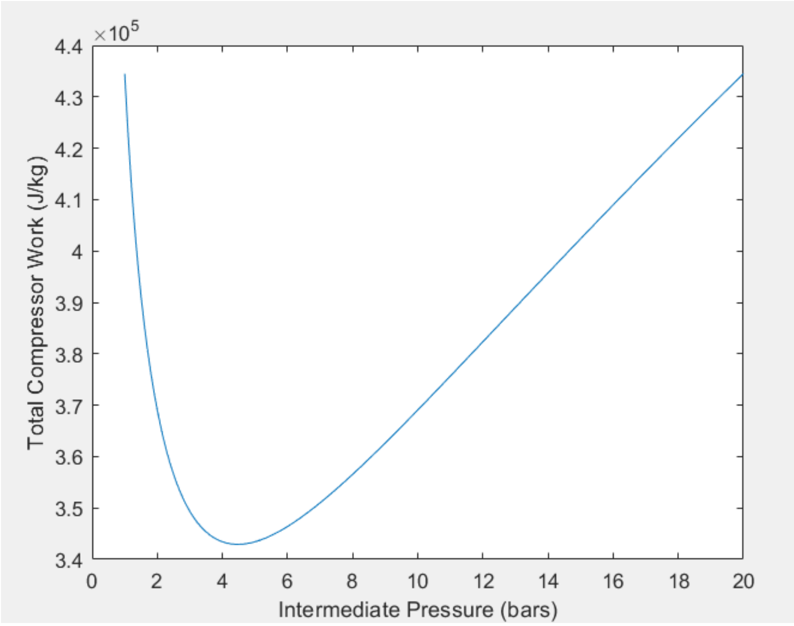


Figure 10. Specific total work as a function of intermediate pressure p_{int}

Comment: The Figure 10. shows the impact of intermediate pressure on the specific total work of the compressors. The total work input first decreases for the range of intermediate pressure (1 to 4.47 bars), it reaches its minimum value at 4.47 bars and then it starts increasing for the range of intermediate pressure (4.47 to 20 bars). Thus, we can conclude that the optimum intermediate pressure is 4.47 bars (less than halfway between 1 & 20 bars). The reason for this value to be 4.47 bars is because the compressor work depends on inlet temperature and the pressure ratio. Since the inlet temperature of both compressor is constant the only factor on which compressor depends is the pressure ratio. Therefore, we want to keep the pressure ratio of both compressor equal and only at this value the pressure ratio of both compressors becomes equal which is 4.47.

Since the Turbine work is equal to the optimum value which we obtained from the Simple Brayton Cycle, the Work Ratio can be given as:

$$Work\ ratio = \frac{w^{*}_{comp,total}}{-w^{*}_{turb\_opt}}$$

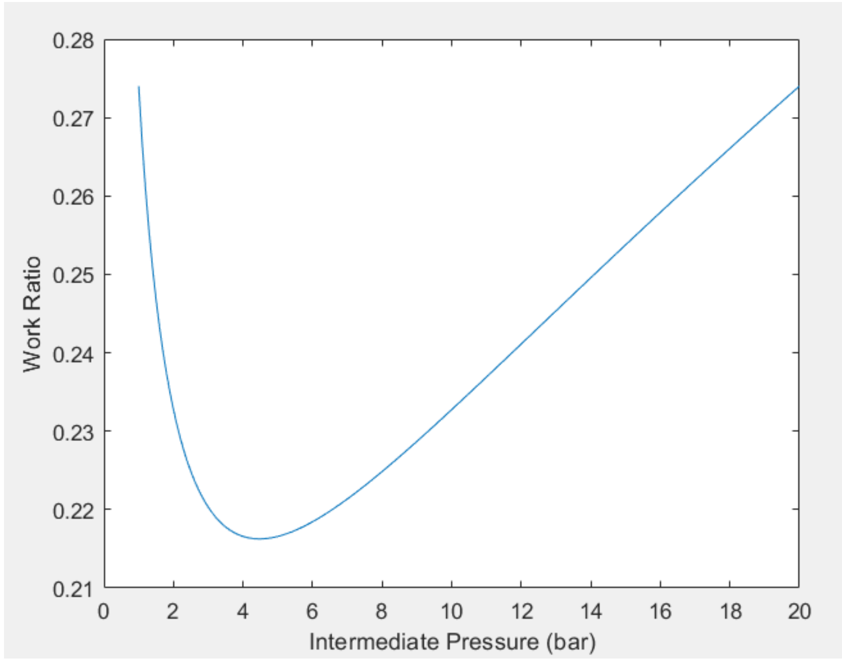


Figure 11. Work Ratio as a function of intermediate pressure $\boldsymbol{p_{int}}$

Comment: The Figure 11. shows the impact of intermediate pressure on Work Ratio. The trend seen in the figure is like the one we saw in Figure 10. And indeed, the reasoning is also the same. The optimum work of turbine is constant, so only factor on which our Work Raito depends is the total work input of the compressors. And we saw that the optimum intermediate pressure was 4.47 bars in case of Figure 10. Therefore, it is also the same in this case.

2.4.3. Determine the optimum intermediate pressure $\boldsymbol{p_{opt}}$ **allowing to minimize the compression work.**

- The optimum intermediate pressure p_{opt} can be calculated in two ways. First way is to use the formula.

$$p_{int} = \sqrt{p_1 \cdot p_4} = \sqrt{p_1 \cdot \rho_{opt} \cdot p_1} = p_1\sqrt{\rho_{opt}}$$

- From these calculations we obtain that $p_{opt} = 4.47\ bar.$

- The second way to calculate the optimum intermediate pressure is to find the minimum of the function

$$w^*{}_{comp} = f(p_{int})$$

- To find the minimum of the function, we need to find it's derivative and equalise it with zero:

$$\frac{dw^*{}_{comp}}{dp_{int}} = 0$$

and from this equation we can obtain p_{opt}. To get the optimum intermediate pressure in MATLAB, we use the MATLAB function "${w^*_{comp}}_{opt} = \min(w^*_{comp})$" and the result we obtained is again $p_{opt} = 4.47\ bar$. Figure 10. clearly strengthens our conclusion, as the minimum of the function is found at $p_{opt} = 4.47\ bar$.

2.4.4. Make the digital application for the work involved (compression, expansion, their ratio, net work output) and the thermal efficiency for p_{opt}. Comment the results in comparison to the cycle using only heat regeneration.

Parameter	Regeneration Cycle	Staged Compression along with Regeneration Cycle
ρ_{opt}	20	20
w^*_{net}	1155300 [J/kg]	1241100 [J/kg]
w^*_{comp}	407180 [J/kg]	321360 [J/kg]
w^*_{turb}	1562500 [J/kg]	1562500 [J/kg]
$Work\ Ratio$	0.2606	0.2057
q_{in}	1840200 [J/kg]	1962300[J/kg]
η_{th}	62.78%	63.25%

Comparison: Few observations can made from the above comparison: The Net Work of the system has increased due to reduced work done on the system by compressing the air in stages. As the result, positive impact is also seen on the Work Ratio (it has reduced). However, we observe that the heat provided to the system has increased at the combustion. The reason is when we compressed the air in stages, we also employed the intercooler to reduce the temperature at the inlet of second compressor. As the consequence, the temperature at outlet of the compressor also reduced, followed by reduced temperature at the inlet of combustion chamber after passing through regenerator. Therefore, more energy to achieve the same maximum temperature at the inlet of turbine. But overall, the thermal efficiency has increased. Thus, it can be concluded that the system with the Staged Compression has exhibited the improved performance compared to the one with only Regeneration.