Find the solution to ODE

$$f''' + ff'' + 1 - (f')^2 = 0$$

And the boundary conditions are

$$f_0 = f(0) = 0 = f_0' = f'(0)$$
$$f_n' = f'(10) = 1$$

To simplify our calculations we take $f' = F$

Now our equations become

$$f_0 = f(0) = 0 = F_0 = F(0)$$
$$F_n = F(10) = 1$$

And we solve for the equation with

$$X_i = \begin{bmatrix} f_i \\ F_i \end{bmatrix}$$

In [3]:

```
import matplotlib.pyplot as plt
import numpy as np
import pandas as pd
%matplotlib inline

plt.rcParams['figure.figsize'] = [10, 15]
```

In [4]:

```
def BlockTridiagonal(A, B, C, D):
    n = len(D)
    _B = np.zeros((n, 2, 2))
    _C = np.zeros(A.shape)
    _D = np.zeros((n,2,1))
    D_out = np.zeros(_D.shape)
    _C[0] = np.linalg.inv(B[0]).dot(C[0])
    _D[0] = np.linalg.inv(B[0]).dot(D[0])
    for i in range(1, n-1):
        _B[i] = B[i] - A[i-1].dot(_C[i-1])
        _C[i] = np.linalg.inv(_B[i]).dot(C[i])
        _D[i] = np.linalg.inv(_B[i]).dot(D[i] - A[i-1].dot(_D[i-1]))
    _D[n-1] = np.linalg.inv(B[n-1] - A[n-2].dot(_C[n-2])).dot(D[n-1] - A[n-2].dot(_I
    D_out[n-1] = np.copy(_D[n-1])
    for i in range(n-2, -1, -1):
        D_out[i] = _D[i] - _C[i].dot(D_out[i+1])

    return D_out
```

In [13]:

```
BVP(x0, xn, h, epsilon = 0.001):
n = int(np.ceil((xn - x0)/h)+1)
print(n)
A = np.zeros((n-2, 2, 2))
B = np.zeros((n-1, 2, 2))
C = np.zeros((n-2, 2, 2))
D = np.zeros((n-1, 2, 1))
f = np.zeros(n)
F = np.zeros(n)
F[n-1] = 1
f[n-1] = h/2
Solution = np.stack((f, F))
print("Solution Shape", Solution.shape)
count = 0
delta_X = np.ones(Solution.shape)
while(np.amax(np.absolute(delta_X))>epsilon):
    print("Iteration : ", count+1)
    B[0] = np.array([[1, -h/2],
                     [(Solution[1][2] - Solution[1][0])/(2*h), -2/h**2 - 2*Solution
    C[0] = np.array([[0, 0],
                     [0, 1/h**2 + Solution[0][1]/(2*h)]])
    A[n-3] = np.array([[-1, -h/2],
                       [0, 1/h**2 - Solution[0][-2]/2*h]])
    B[n-2] = np.array([[1, -h/2],
                       [(Solution[-1][-1] - Solution[1][-3])/2*h, -2/h**2 - 2*Solu
    D[0] = np.array([[-Solution[0][1] + Solution[0][0] + h*(Solution[1][1] + Solutio
                       [Solution[1][1]**2 - 1 - Solution[0][1]*(Solution[1][2] - S
    D[n-2] = np.array([[-Solution[0][-2] + Solution[0][-3] + h*(Solution[1][-2] + So
                       [Solution[1][-2]**2 - 1 - Solution[0][-2]*(Solution[1][-1]

    for i in range(1, n-2):
        A[i-1] = np.array([[-1, -h/2],
                           [0, 1/h**2 - Solution[0][i]/2*h]])
        B[i] = np.array([[1, -h/2],
                         [(Solution[1][i+2] - Solution[1][i])/2*h, -2/h**2 - 2*Solu
        C[i] = np.array([[0, 0],
                         [0, 1/h**2 + Solution[0][i+1]/2*h]])
        D[i] = np.array([[-Solution[0][i+1] + Solution[0][i] + h*(Solution[1][i+1] +
                         [Solution[1][i+1]**2 - 1 - Solution[0][i+1]*(Solution[1][i-
    print(delta_X.shape)
    delta_X = np.reshape(BlockTridiagonal(A, B, C, D), (n-1, 2)).T
    print(delt)
    b = np.array([[0],[0]])
    delta_X = np.concatenate([b, delta_X], axis=1)

    delta_X[1][-1] = 0
    delta_X[0][-1] = delta_X[0][-2] + h*(delta_X[1][-1] + delta_X[1][-2])/2
    Solution = Solution + delta_X
      print(Solution)
    count+= 1

print(Solution)
return Solution[0, :]
```

In [17]:

```
def func(x0, xn, h = 0.1):
    lst = np.arange(x0, xn, h)
    lst = np.append(lst, xn)
    return lst
x0 = 0
xn = 10
x = func(x0, xn, h = 0.4)
solution = BVP(x0, xn, h=0.4, epsilon=0.05)
```

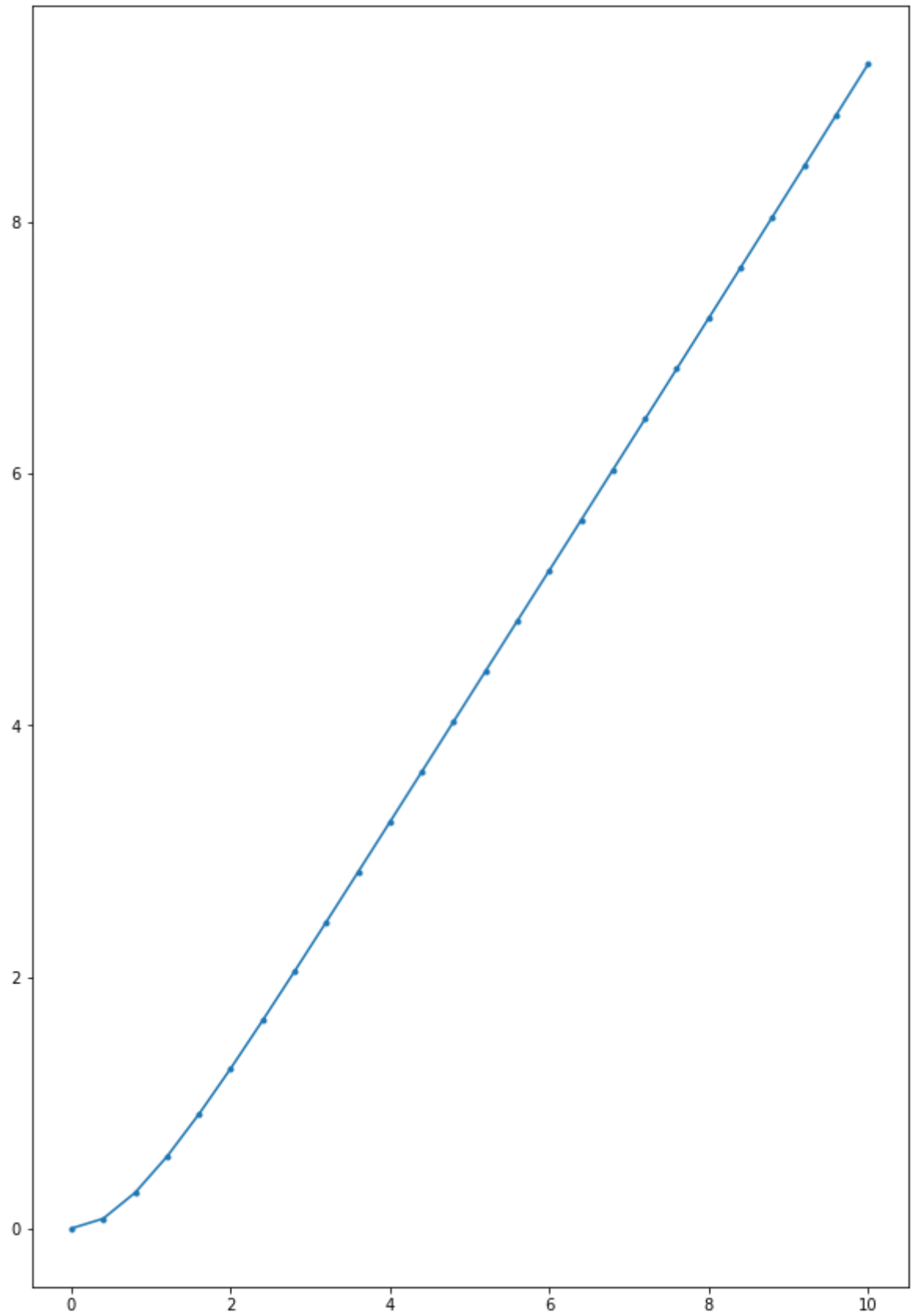
```
26
Solution Shape (2, 26)
Iteration :  1
(2, 26)
(2, 25)
Iteration :  2
(2, 26)
(2, 25)
Iteration :  3
(2, 26)
(2, 25)
Iteration :  4
(2, 26)
(2, 25)
Iteration :  5
(2, 26)
(2, 25)
Iteration :  6
(2, 26)
(2, 25)
Iteration :  7
(2, 26)
(2, 25)
Iteration :  8
(2, 26)
(2, 25)
[[0.         0.0778352  0.28386767 0.5710348  0.90707113 1.2715566
  1.65219039 2.04180576 2.4363298  2.83349484 3.23206186 3.63136698
  4.0310631  4.43097615 4.83102695 5.2311905  5.63147613 6.03191984
  6.43258376 6.83356083 7.23498349 7.63703658 8.03997465 8.44414467
  8.85001456 9.25344153]
 [0.         0.38917601 0.64098632 0.79484935 0.88533228 0.93709507
  0.96607386 0.98200303 0.99061717 0.99520803 0.99762706 0.99889849
  0.99958214 0.99998309 1.00027093 1.00054681 1.00088139 1.00133712
  1.00198248 1.00290286 1.00421047 1.00605494 1.00863542 1.01221466
  1.01713482 1.        ]]
```

In [18]:

```
plt.plot(x, solution, '.-')
```

Out[18]:

```
[<matplotlib.lines.Line2D at 0x11dcb79e8>]
```



In [ ]:

```
```