



# INDIAN INSTITUTE OF TECHNOLOGY, KHARAGPUR

Mid-Spring Semester 2016-17

Date of Examination : \_\_\_\_\_ Session (FN/AN) \_\_\_\_\_ Duration 2 hrs

Subject No. : Advanced Numerical Techniques Subject Name : MA 3011

Department/Center/School : MATHEMATICS

Special Instructions (if any) : Answer ALL the Questions

Marks are indicated within the parenthesis

1. Obtain second-order accurate forward difference and backward difference formula for a first-order derivative.

Use the above formulae to discretize the boundary conditions and obtain a second-order accurate solution for the following BVP when  $h=0.25$

$$\frac{d^2 y}{dx^2} - 2x \frac{dy}{dx} - 2y = -4x; \quad y(0) - y'(0) = 0 \text{ \& } 2y(1) - y'(1) = 1. \quad [4+6]$$

2. Describe an iterative method to solve the following nonlinear BVP by a second-order accurate finite difference technique

$$\frac{d^3 y}{dx^3} + 2y \frac{d^2 y}{dx^2} + 1 - \left( \frac{dy}{dx} \right)^2 = 0; \quad y(0) = y'(0) = 0 \text{ \& } y'(1) = 1.$$

Derive the ensuing block tri-diagonal system of equations at each iteration level when the step size is  $h=0.2$

Describe an algorithm to solve the block tri-diagonal system of equations.

Solutions are not required. [4+3+3]

- 3a. Describe the cubic spline interpolation technique by deriving all the governing equations.
- 3b. Use the cubic spline interpolation technique to solve the following boundary value problem with  $h=0.5$

$$\frac{d^2 y}{dx^2} + 2 \frac{dy}{dx} + y = 30x; \quad y(0) = 0 \text{ \& } y(1) = 0. \quad [5+5]$$

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