Thomas Algorithm

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1 Advanced Numerical Techniques Assignment 1

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```
The differential equation to be solved is
(y'')(x^2) + xy' = 1
```

```
In [9]: import matplotlib.pyplot as plt
        import numpy as np
        %matplotlib inline
        plt.rcParams['figure.figsize'] = [10, 15]
```

```
Tridiagonal Matrix Algorithm (TDMA) is also known as Thomas Algorithm which is implemented in th
```

```
In [10]: def A(x):
             return (1./x)
         def B(x):
             return 0.
         def C(x):
             return(1./x**2)

         def TDMA(diag, sub, sup, d):
             """
             All the parameters are numpy arrays
             diag --> Diagonal entries of the tri-diagonal matrix
             sub --> Sub-Diagonal entries of the tri-diagonal matrix
             sup --> Super-Diagonal entries of the tri-diagonal matrix
             """
             n = len(diag)
             sup[0] = sup[0]/diag[0]
             d[0] = d[0]/diag[0]
             for i in range(1, n):
                 sup[i] = sup[i]/(diag[0] - sup[i - 1]*sub[i])
                 d[i] = (d[i] - d[i - 1]*sub[i])/(diag[0] - sup[i - 1]*sub[i])

             y = np.zeros(n)
```

```
    y[n - 1] =  d[n - 1]
    for i in range(n - 2, -1, -1):
        y[i] = d[i] - sup[i] * y[i + 1]
    return y
```

Finding the solution of Boundary Value problem

```
In [11]: def BVPsolution(y0, yn, x0, xn, h):
             n = int((xn -x0)/h) + 1
             diag = [1 for i in range(1, n)]
             sub = [1 for i in range(1, n)]
             sup = [1 for i in range(1, n)]
             d = [1 for i in range(1, n)]
             for i in range(1, n):
                 x = x0 + i*h
         #         print(sub, diag, sup)
                 sub[i-1] = (1.0 / (h ** 2))- (A(x) / (2.0 * h))
                 diag[i-1] = (-2.0 / (h ** 2)) + B(x)
                 sup[i-1] = (1.0 / (h ** 2)) + (A(x) / (2.0 * h))
                 if i == 1:
                     d[i-1] = C(x) - sub[i-1] * y0
                 elif i == n - 1 :
                     d[i-1] = C(x) - sup[i-1] * yn
                 else:
                     d[i-1] = C(x)
             y = TDMA(diag, sub, sup, d)
             np.insert(y, 1, y0)
         #     print("The SOLN: ", y)
             return y
```

```
In [15]: # Take Differential Equation as an inputs
         # Initializing boundary conditions y(1) = 0, y(1.4) = 0.0566
         x0 = 1
         xn = 1.4
         y0 = 0
         yn = 0.0566
         steps = [0.1, 0.05, 0.01]
         # n = int((xn - x0)/steps[1]) + 1
         # for i in range(1, n):
         #     print(x0 + i*steps[1])
         y_0 = np.insert(BVPsolution(y0, yn, x0, xn, steps[0]), 0, 0)
         y_1 = np.insert(BVPsolution(y0, yn, x0, xn, steps[1]), 0, 0)
         y_2 = np.insert(BVPsolution(y0, yn, x0, xn, steps[2]), 0, 0)
         print('Values of xi wrt step = 0.1')
         print(y_0)
         print('Values of xi wrt step = 0.05')
         print(y_1)
```

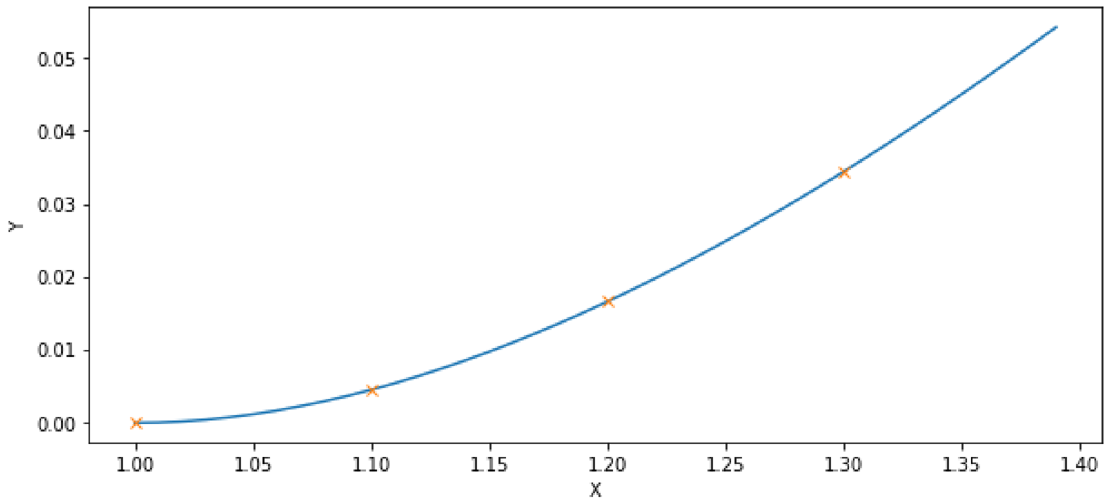
```
        print('Values of xi wrt step = 0.01')
        print(y_2)
```

```
Values of xi wrt step = 0.1
[0.         0.00457418 0.01665575 0.03443745]
Values of xi wrt step = 0.05
[0.         0.00119469 0.00454866 0.00977376 0.01662666 0.02490052
 0.03441855 0.04502879]
Values of xi wrt step = 0.01
[0.00000000e+00 4.93560595e-05 1.95772553e-04 4.36408608e-04
 7.68523908e-04 1.18947432e-03 1.69670776e-03 2.28776025e-03
 2.96025219e-03 3.71188485e-03 4.54043696e-03 5.44376152e-03
 6.41978281e-03 7.46649343e-03 8.58195161e-03 9.76427853e-03
 1.10116559e-02 1.23223235e-02 1.36945769e-02 1.51267656e-02
 1.66172905e-02 1.81646021e-02 1.97671990e-02 2.14236254e-02
 2.31324701e-02 2.48923644e-02 2.67019806e-02 2.85600307e-02
 3.04652649e-02 3.24164700e-02 3.44124685e-02 3.64521168e-02
 3.85343045e-02 4.06579532e-02 4.28220150e-02 4.50254718e-02
 4.72673341e-02 4.95466401e-02 5.18624551e-02 5.42138697e-02]
```

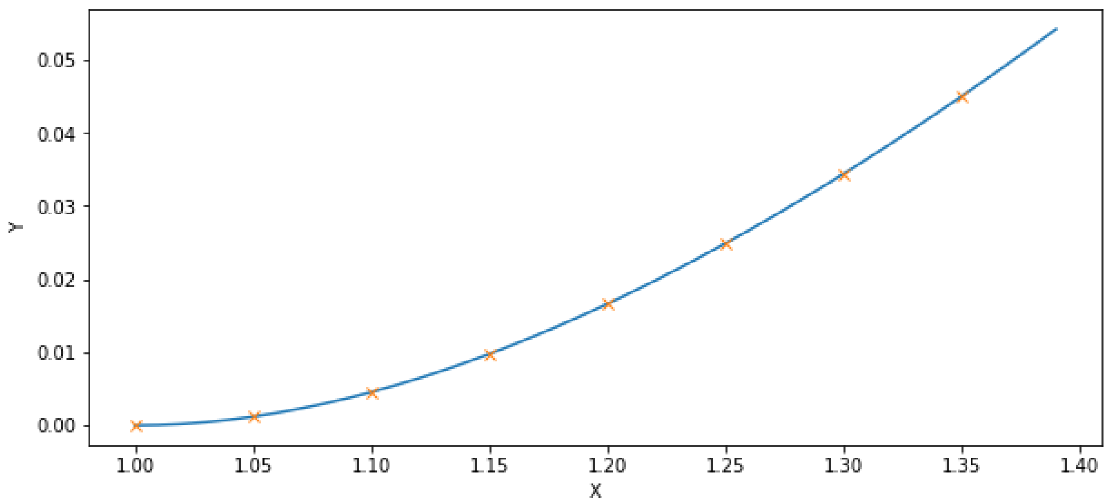
```
In [14]: def f(x0, xn, h = 0.1):
             return np.arange(x0, xn, h)

         def func(arr):
             return (np.power(np.log(arr), 2)/2)

         x_range0 = f(x0, xn, h = steps[0])
         x_range1 = f(x0, xn, h = steps[1])
         x_range2 = f(x0, xn, h = steps[2])

         y_range0 = func(x_range0)
         y_range1 = func(x_range1)
         y_range2 = func(x_range2)

         print("Plotting wrt h = 0.1, h = 0.05, h =0.01")
         # print((y_range0 - y_0)/y_range0)
         # print((y_range1 - y_1)/y_range1)
         # print((y_range2 - y_2)/y_range2)
         #Plotting step = 0.1

         plt.subplot(3, 1, 1)
         plt.xlabel('X')
         plt.ylabel('Y')
         plt.plot(x_range2, y_range2, '-', x_range0, y_0, 'x')
         #Plotting step = 0.05
         plt.subplot(3, 1, 2)
         plt.xlabel('X')
```

```
plt.ylabel('Y')
plt.plot(x_range2, y_range2, '-', x_range1, y_1, 'x')
#Plotting step = 0.001
plt.subplot(3, 1, 3)
plt.xlabel('X')
plt.ylabel('Y')
plt.plot(x_range2, y_range2, '-', x_range2, y_2, 'x')
plt.show()
```

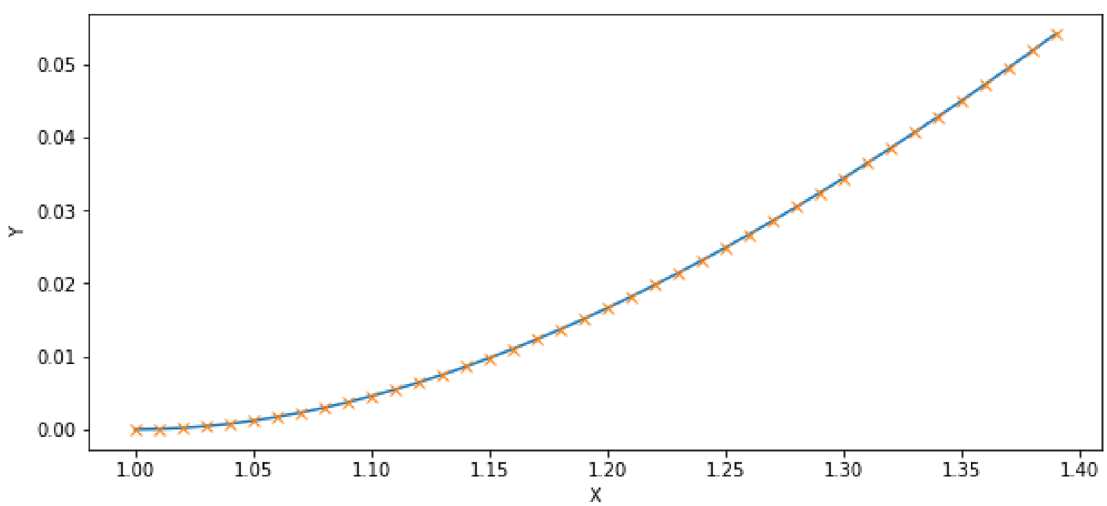
```
Plotting wrt h = 0.1, h = 0.05, h =0.01
```



0.05
0.04
0.03
0.02
0.01
0.00
Y
1.00
1.05
1.10
1.15
1.20
1.25
1.30
1.35
1.40
X



0.05
0.04
0.03
0.02
0.01
0.00
Y
1.00
1.05
1.10
1.15
1.20
1.25
1.30
1.35
1.40
X



0.05
0.04
0.03
0.02
0.01
0.00
Y
1.00
1.05
1.10
1.15
1.20
1.25
1.30
1.35
1.40
X