Questions

Chapter 1

Solved questions

1 Analysis

1. What is the relationship between harmonic functions on $\mathbb{R}^2$ and holomorphic functions?

Chapter 2

Unsolved questions

1 Analysis

1. If f is a Lebesgue integrable function on $[a, b]$, for any ε there exists a continuous function g such that $\int_a^b |f - g|\, d\mu < \varepsilon$.

2. When is $\frac{d}{dx} \int_a^x f(t)\, dt = f(x)$? When are you allowed to differentiate under an integral sign?

3. Compare the bounds for harmonic functions with the bounds for holomorphic functions.

4. What definite integrals are amenable to contour integration?

5. If two holomorphic functions agree on a subset with a limit point, then they agree everywhere.

6. Find effective bounds for the Stone-Weierstrass Theorem. See here: `http://en.wikipedia.org/wiki/Bernstein_polynomial`.

7. For Fourier expansion: what happens if we choose a different orthonormal basis? Are we similarly doomed?

8. PDE: heat equation. Does it always approach steady-state; in what sense; how does energy decrease?

2 Number theory

1. Try to prove the following statement for as small a y as possible: $\pi(x+y) - \pi(x) \sim \frac{y}{\ln x}$.

2. Find the asymptotics of the numbers $n < x$ that can be expressed as the sum of 2 squares.

3 Geometry

1.

Chapter 3

Research questions

1 Geometry

1. Consider a ball rolling in a polygonal pool table. Suppose its trajectory is not periodic. Is it ergodic?

2. Ravi's problem.