Asynchronous Finite Difference Scheme for PDEs

MA14M004
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1D Advection Diffusion Equation

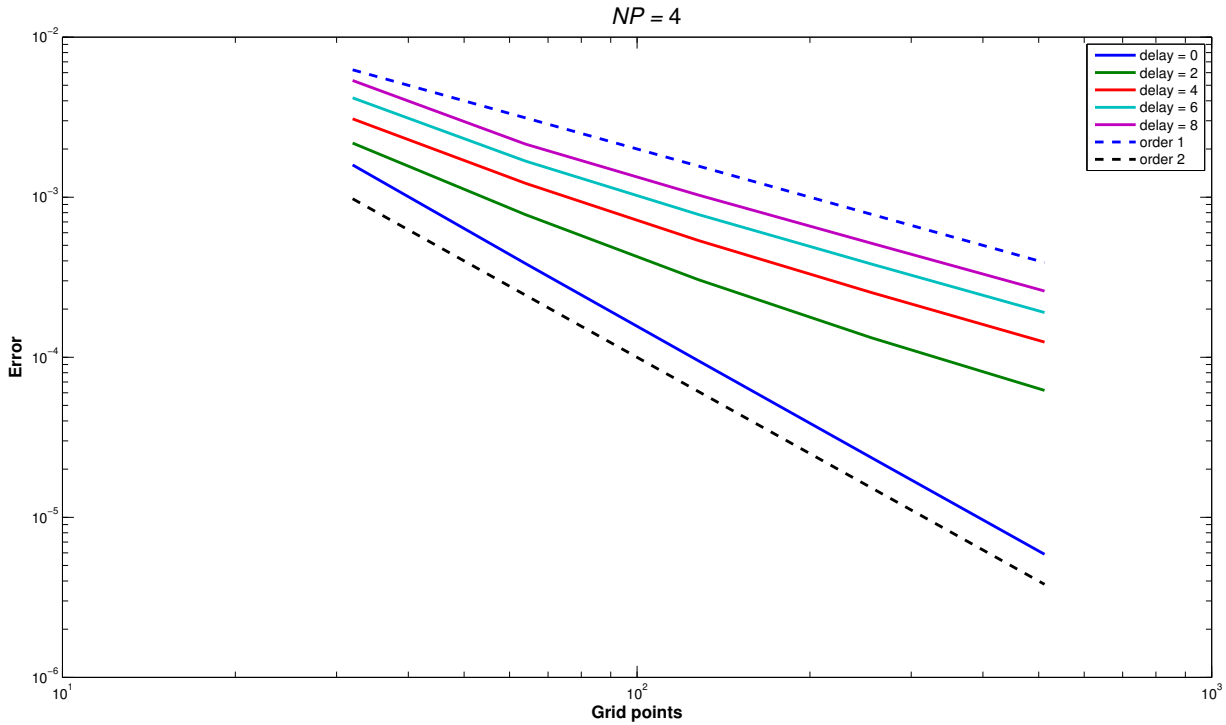


Figure : Result of asynchronous Scheme

Order Recovery

Scheme

Interior nodes:

$$\frac{1}{\triangle t}(u_i^{n+1} - u_i^n) + \frac{c}{2\triangle x}(u_{i+1}^n - u_{i-1}^n) = \frac{\alpha}{\triangle x^2}(u_{i+1}^n - 2u_i^n + u_{i-1}^n) \tag{1}$$

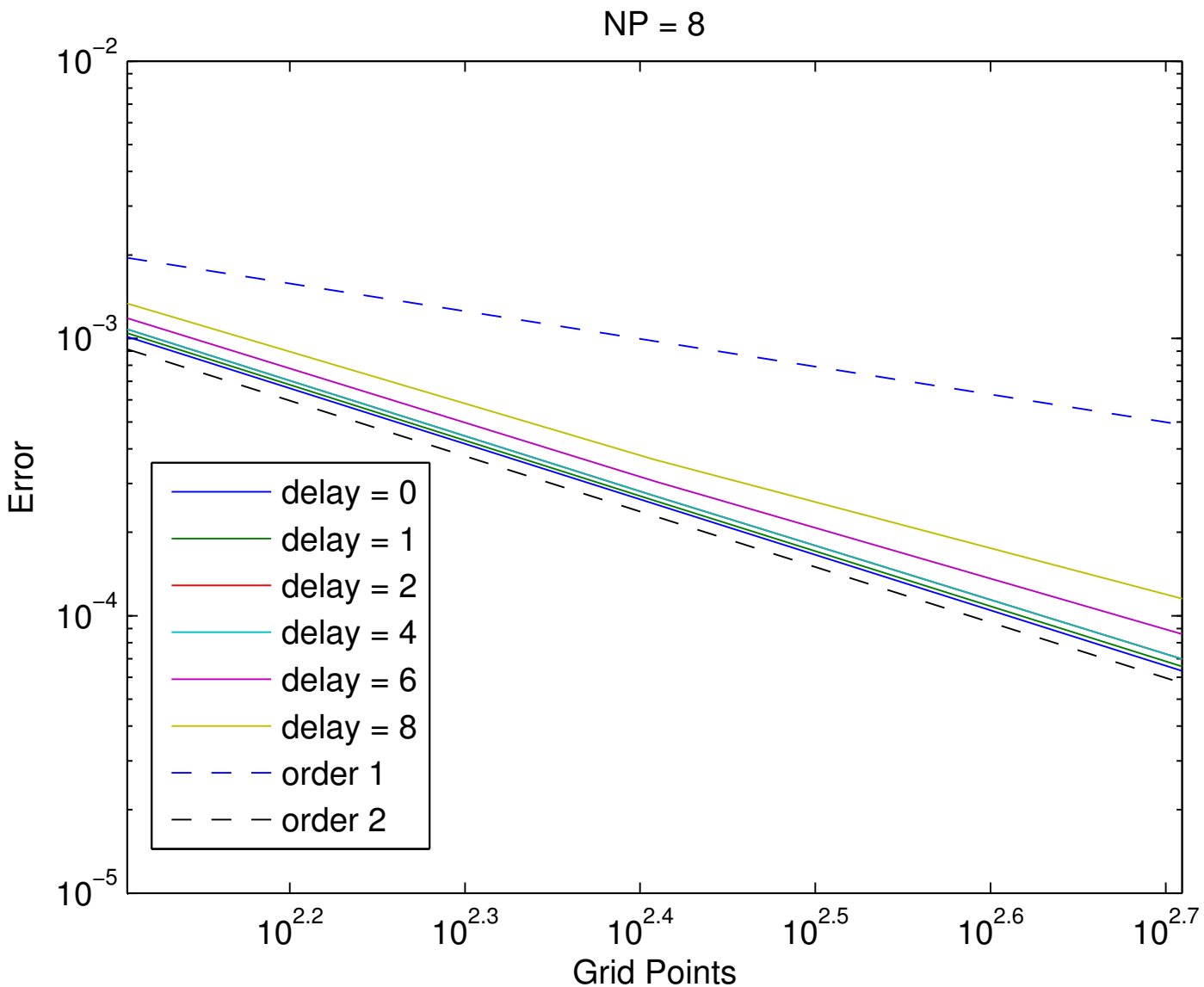
Left Boundary node:

$$\frac{1}{\triangle t}(u_i^{n+1} - u_i^n) + \frac{c}{2\triangle x}(u_{i+1}^{\tilde{n}} - u_{i-1}^n) = \frac{\alpha}{2\triangle x^2}(u_{i+2}^{\tilde{n}} - u_{i+1}^{\tilde{n}} + u_i^n - u_{i-1}^n) \tag{2}$$

Right Boundary node:

$$\frac{1}{\triangle t}(u_i^{n+1} - u_i^n) + \frac{c}{2\triangle x}(u_{i+1}^n - u_{i-1}^{\tilde{n}}) = \frac{\alpha}{2\triangle x^2}(u_{i+2}^n - u_{i+1}^n + u_i^n - u_{i-1}^{\tilde{n}}) \tag{3}$$

Result of Order Recovery Scheme



Order Recovery

Delay	Async	Order Recovery
0(sync)	-2.0195	-2.0018
1	-1.0764	-1.9958
2	-1.0371	-1.9764
4	-1.0117	-1.8925
6	-1.0033	-1.7687
8	-0.9995	-1.6586

- The order decreases to the first order as the delay increases, even with the order recovery scheme.
- The magnitude of the error in case of order recovery scheme is small as compared to the asynchronous scheme.

2D Diffusion Problem

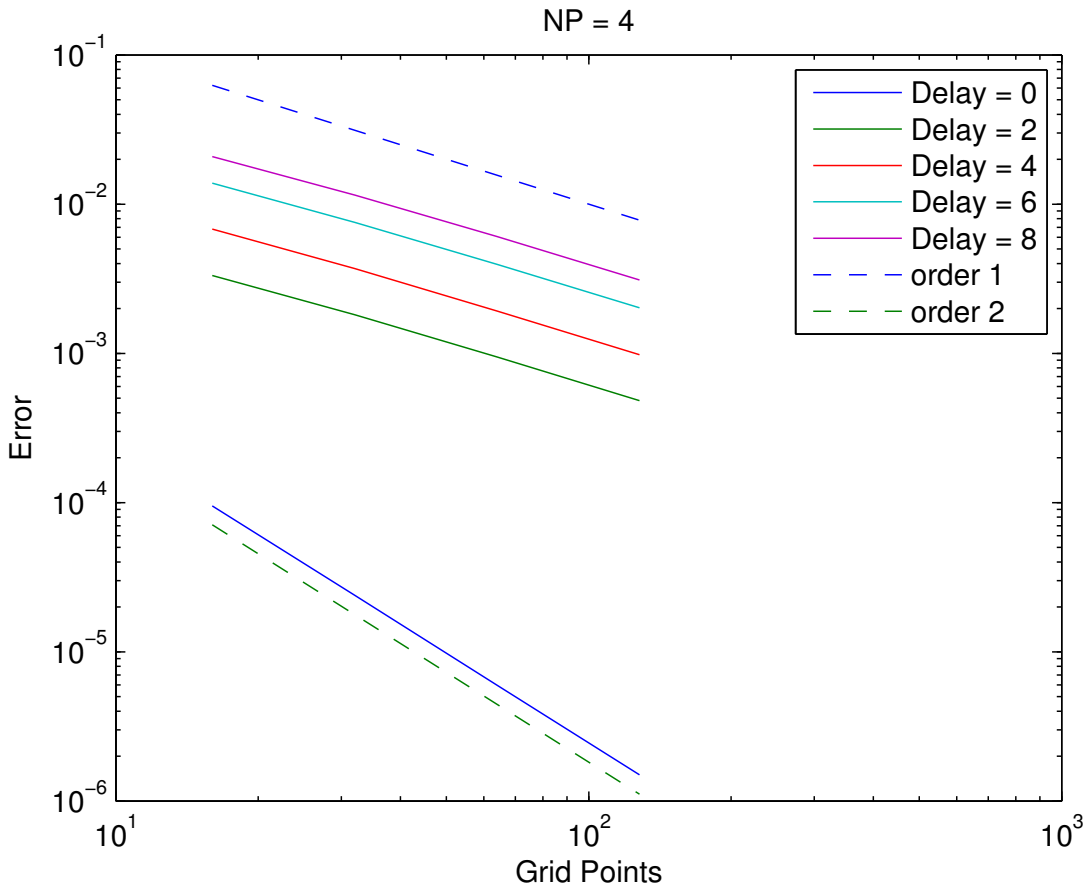
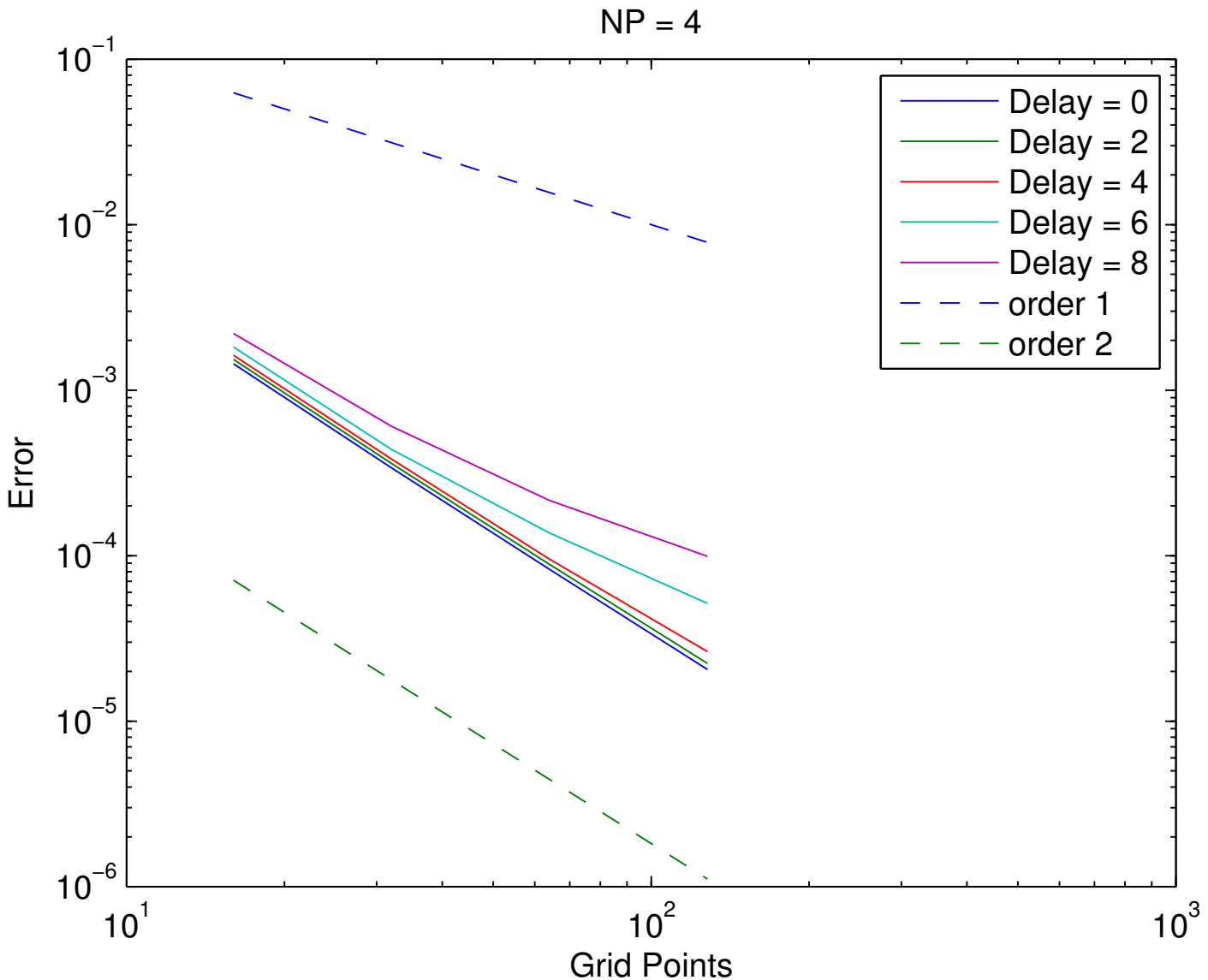


Figure : Result of Asynchronous Scheme

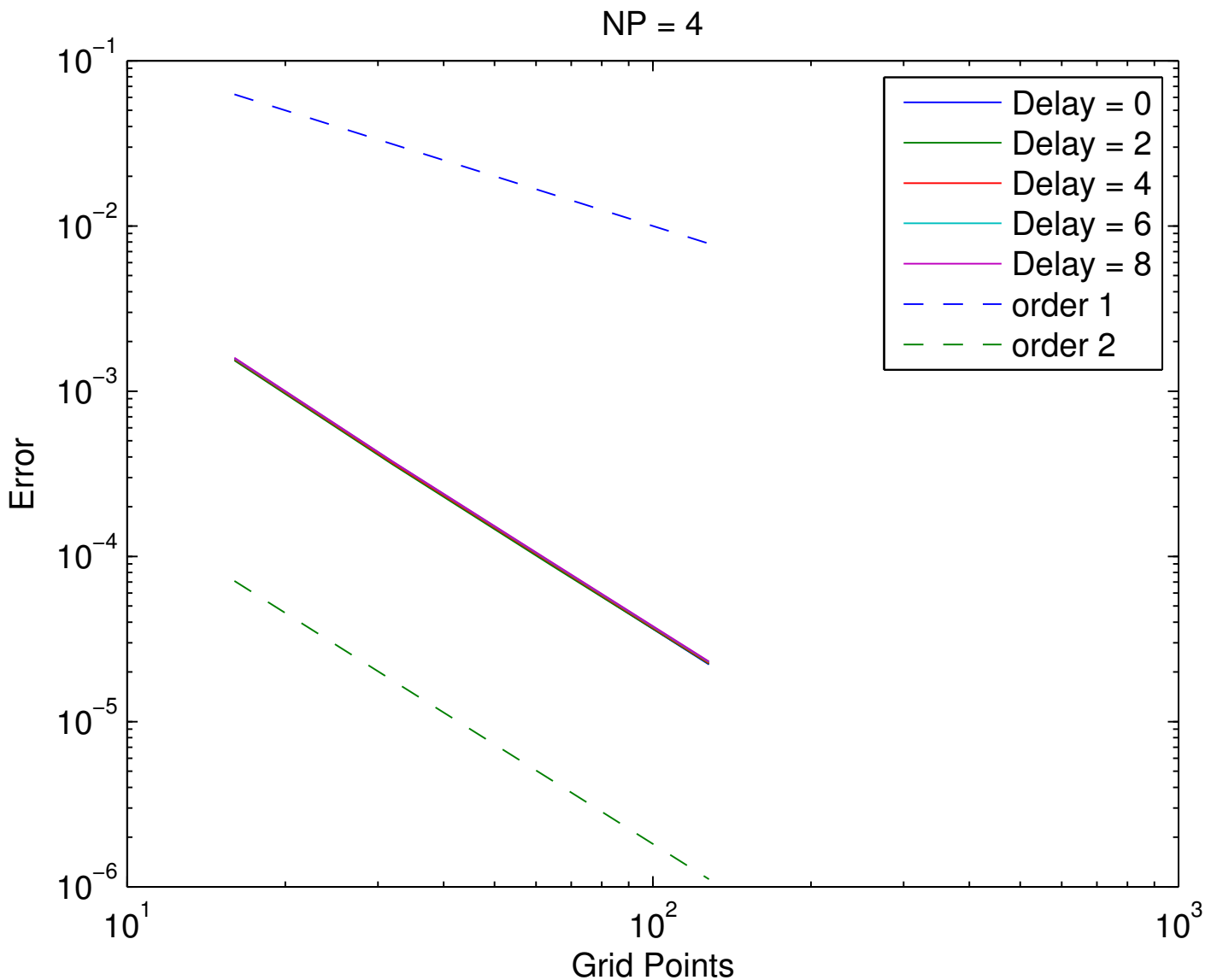
Result of Order Recovery Scheme



New Scheme

- Disretization in space according to the Order Recovery Scheme.
- Deciding the value of Δt according to value of Delay.
- Multiplying by a factor of $\frac{2}{Delay+1}$ to the previous value of Δt

Result of New Scheme



2D Diffusion Problem

Delay	Async	Order Recovery	New Scheme
0(sync)	-1.9959	-2.0172	-2.028
1	-0.9314	-1.9725	-2.0048
2	-0.9329	-1.7933	-2.0136
4	-0.9262	-1.3788	- 2.0124
6	-0.9174	-1.1966	-2.0118
8	-0.9061	-1.1038	-2.0115

- In practice, it is not feasible to know the Delay.
- Decreasing the value of Δt increases the load on computation.

Refrences

- Diego A. Donzis and Konduri Aditya. Asynchronous Finite Difference Scheme for Partial Difference Equations *Journal of Computational Physics*. 274(0):370-392,2014
- Thomas Camminady. CES Seminar Paper on Asynchronous Finite Difference Scheme for Partial Difference Equation. January 9,2015
- MPICH , `http://www.mpich.org/`, 4 12 2015.