Stream Function -Vorticity Approach to the Channel Flow

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VORTICITY AND STREAM FUNCTION EQUATIONS

Incompressible Navier-Stokes Equation in 2-D

- X – momentum equation

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \upsilon\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)$$

- Y – momentum equation

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial y} + \upsilon\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right)$$

Non-dimensionalization

- X – momentum equation:

$$\tilde{u} = \frac{u}{U_0}, \tilde{v} = \frac{v}{U_0}, \tilde{x} = \frac{x}{H}, \tilde{y} = \frac{y}{H}$$

$$\tilde{t} = \frac{t}{H/U_0}, \tilde{p} = \frac{p}{\rho U_0^{\,2}}$$

$$\frac{U_0^{\,2}}{H}\left(\frac{\partial \tilde{u}}{\partial \tilde{t}} + \tilde{u}\frac{\partial \tilde{u}}{\partial \tilde{x}} + \tilde{v}\frac{\partial \tilde{v}}{\partial \tilde{y}}\right) = -\frac{U_0^{\,2}}{H}\frac{\partial \tilde{p}}{\partial \tilde{x}} + \frac{\upsilon U_0}{H^2}\left(\frac{\partial^2 \tilde{u}}{\partial \tilde{x}^2} + \frac{\partial^2 \tilde{v}}{\partial \tilde{y}^2}\right)$$

$$\frac{\partial \tilde{u}}{\partial \tilde{t}} + \tilde{u}\frac{\partial \tilde{u}}{\partial \tilde{x}} + \tilde{v}\frac{\partial \tilde{u}}{\partial \tilde{y}} = -\frac{\partial \tilde{p}}{\partial \tilde{x}} + \frac{1}{\mathrm{Re}}\left(\frac{\partial^2 \tilde{u}}{\partial \tilde{x}^2} + \frac{\partial^2 \tilde{u}}{\partial \tilde{y}^2}\right)$$

Non-dimensionalization

- Y – momentum equation:

$$\frac{\partial \tilde{v}}{\partial \tilde{t}} + \tilde{u}\frac{\partial \tilde{v}}{\partial \tilde{x}} + \tilde{v}\frac{\partial \tilde{v}}{\partial \tilde{y}} = -\frac{\partial \tilde{p}}{\partial \tilde{y}} + \frac{1}{\mathrm{Re}}\left(\frac{\partial^2 \tilde{v}}{\partial \tilde{x}^2} + \frac{\partial^2 \tilde{v}}{\partial \tilde{y}^2}\right)$$

The vorticity Equation

- Differentiate X – momentum Equation with respect to y:

$$\frac{\partial}{\partial \tilde{y}}\left[\left(\frac{\partial \tilde{u}}{\partial \tilde{t}}+\tilde{u}\frac{\partial \tilde{u}}{\partial \tilde{x}}+\tilde{v}\frac{\partial \tilde{u}}{\partial \tilde{y}}\right)=-\frac{\partial \tilde{p}}{\partial \tilde{x}}+\frac{1}{\mathrm{Re}}\left(\frac{\partial^2 \tilde{u}}{\partial \tilde{x}^2}+\frac{\partial^2 \tilde{u}}{\partial \tilde{y}^2}\right)\right] \quad (1)$$

- Differentiating Y – momentum Equation with respect to x:

$$\frac{\partial}{\partial \tilde{x}}\left[\left(\frac{\partial \tilde{v}}{\partial \tilde{t}}+\tilde{u}\frac{\partial \tilde{v}}{\partial \tilde{x}}+\tilde{v}\frac{\partial \tilde{v}}{\partial \tilde{y}}\right)=-\frac{\partial \tilde{p}}{\partial \tilde{y}}+\frac{1}{\mathrm{Re}}\left(\frac{\partial^2 \tilde{v}}{\partial \tilde{x}^2}+\frac{\partial^2 \tilde{v}}{\partial \tilde{y}^2}\right)\right] \quad (2)$$

The vorticity Equation

- Subtracting equation (1) from (2)

$$\frac{\partial \tilde{\omega}}{\partial \tilde{t}} + \tilde{u}\frac{\partial \tilde{\omega}}{\partial \tilde{x}} + \tilde{v}\frac{\partial \tilde{\omega}}{\partial \tilde{y}} = \frac{1}{\mathrm{Re}}\left(\frac{\partial^2 \tilde{\omega}}{\partial \tilde{x}^2} + \frac{\partial^2 \tilde{\omega}}{\partial \tilde{y}^2}\right)$$

$$\tilde{\omega} = \frac{\partial \tilde{v}}{\partial \tilde{x}} - \frac{\partial \tilde{u}}{\partial \tilde{y}}$$

Stream Function Equation

- Define the stream function:

$$\tilde{u} = \frac{\partial \tilde{\psi}}{\partial \tilde{y}}, \qquad \tilde{v} = -\frac{\partial \tilde{\psi}}{\partial \tilde{x}}$$

- Which automatically satisfies the continuity equation for incompressible fluid:

$$\frac{\partial \tilde{u}}{\partial \tilde{x}} + \frac{\partial \tilde{v}}{\partial \tilde{y}} = 0$$

$$\frac{\partial}{\partial \tilde{x}} \frac{\partial \tilde{\psi}}{\partial \tilde{y}} - \frac{\partial}{\partial \tilde{y}} \frac{\partial \tilde{\psi}}{\partial \tilde{x}} = 0$$

Stream Function Equation

- Substituting:

$$\tilde{u} = \frac{\partial \tilde{\psi}}{\partial \tilde{y}}; \qquad \tilde{v} = -\frac{\partial \tilde{\psi}}{\partial \tilde{x}}$$

into the definition of vorticity:

$$\tilde{\omega} = \frac{\partial \tilde{v}}{\partial \tilde{x}} - \frac{\partial \tilde{u}}{\partial \tilde{y}}$$

yields

$$\frac{\partial^2 \tilde{\psi}}{\partial \tilde{x}^2} + \frac{\partial^2 \tilde{\psi}}{\partial \tilde{y}^2} = -\omega$$

Navier-Stokes Equation in vorticity - Stream Function form

- Advection/diffusion Equation:

$$\frac{\partial \tilde{\omega}}{\partial \tilde{t}} = -\frac{\partial \tilde{\psi}}{\partial \tilde{y}}\frac{\partial \tilde{\omega}}{\partial \tilde{x}} + \frac{\partial \tilde{\psi}}{\partial \tilde{x}}\frac{\partial \tilde{\omega}}{\partial \tilde{y}} + \frac{1}{\mathrm{Re}}\left(\frac{\partial^2 \tilde{\omega}}{\partial \tilde{x}^2} + \frac{\partial^2 \tilde{\omega}}{\partial \tilde{y}^2}\right)$$

- Elliptic Equation:

$$\frac{\partial^2 \tilde{\psi}}{\partial \tilde{x}^2} + \frac{\partial^2 \tilde{\psi}}{\partial \tilde{y}^2} = -\omega$$

FINITE DIFFERENCE APPROXIMATION

Finite Difference Approximation

$$\frac{\partial\tilde{\omega}}{\partial\tilde{t}} = -\frac{\partial\tilde{\psi}}{\partial\tilde{y}}\frac{\partial\tilde{\omega}}{\partial\tilde{x}} + \frac{\partial\tilde{\psi}}{\partial\tilde{x}}\frac{\partial\tilde{\omega}}{\partial\tilde{y}} + \frac{1}{\mathrm{Re}}\left(\frac{\partial^2\tilde{\omega}}{\partial\tilde{x}^2} + \frac{\partial^2\tilde{\omega}}{\partial\tilde{y}^2}\right)$$

$$\frac{\tilde{\omega}_{i,j}^{n+1} - \tilde{\omega}_{i,j}^{n}}{\Delta\tilde{t}} =$$

$$-\left(\frac{\tilde{\psi}_{i,j+1}^{n} - \tilde{\psi}_{i,j-1}^{n}}{2\Delta\tilde{y}}\right)\left(\frac{\tilde{\omega}_{i+1,j}^{n} - \tilde{\omega}_{i-1,j}^{n}}{2\Delta\tilde{x}}\right) + \left(\frac{\tilde{\psi}_{i+1,j}^{n} - \tilde{\psi}_{i-1,j}^{n}}{2\Delta\tilde{x}}\right)\left(\frac{\tilde{\omega}_{i,j+1}^{n} - \tilde{\omega}_{i,j-1}^{n}}{2\Delta\tilde{y}}\right)$$

$$+\frac{1}{\mathrm{Re}}\left(\frac{\tilde{\omega}_{i+1,j}^{n} - 2\tilde{\omega}_{i,j}^{n} + \tilde{\omega}_{i-1,j}^{n}}{\Delta\tilde{x}^2} + \frac{\tilde{\omega}_{i,j+1}^{n} - 2\tilde{\omega}_{i,j}^{n} + \tilde{\omega}_{i,j-1}^{n}}{\Delta\tilde{y}^2}\right)$$

Vorticity at new time

$$\omega_{i,j}^{n+1} =$$

$$\omega_{i,j}^{n} + \Delta\tilde{t}\left[-\left(\frac{\tilde{\psi}_{i,j+1}^{n} - \tilde{\psi}_{i,j-1}^{n}}{2\Delta\tilde{y}}\right)\left(\frac{\tilde{\omega}_{i+1,j}^{n} - \tilde{\omega}_{i-1,j}^{n}}{2\Delta\tilde{x}}\right) + \left(\frac{\tilde{\psi}_{i+1,j}^{n} - \tilde{\psi}_{i-1,j}^{n}}{2\Delta\tilde{x}}\right)\left(\frac{\tilde{\omega}_{i,j+1}^{n} - \tilde{\omega}_{i,j-1}^{n}}{2\Delta\tilde{y}}\right) + \frac{1}{\mathrm{Re}}\left(\frac{\tilde{\omega}_{i+1,j}^{n} - 2\tilde{\omega}_{i,j}^{n} + \tilde{\omega}_{i-1,j}^{n}}{\Delta\tilde{x}^{2}} + \frac{\tilde{\omega}_{i,j+1}^{n} - 2\tilde{\omega}_{i,j}^{n} + \tilde{\omega}_{i,j-1}^{n}}{\Delta\tilde{y}^{2}}\right)\right]$$

```
for i = 2:nx - 1
    for j = 2:ny - 1
    w(j,i) = vort(j,i) + delta_t*(-((PSI(j+1,i) - PSI(j - 1,i))/(2*dy))*((vort(j,i+1) - vort(j,i - 1))/(2*dx))+...
        +((PSI(j,i+1) - PSI(j,i - 1))/(2*dx))*((vort(j+1,i) - vort(j - 1,i))/(2*dy))+...
        (1/Re)*(((vort(j,i+1)-2*vort(j,i)+vort(j,i-1))/(dx^2)) + (vort(j+1,i)-2*vort(j,i)+vort(j - 1,i))/dy^2));

    end
end
```

Stream Function Equation

- $$\frac{\partial^2 \tilde{\psi}}{\partial \tilde{x}^2} + \frac{\partial^2 \tilde{\psi}}{\partial \tilde{y}^2} = -\tilde{\omega}$$

- Solving the elliptic equation:

$$\frac{\tilde{\psi}^n_{i+1,j} - 2\tilde{\psi}^n_{i,j} + \tilde{\psi}^n_{i-1,j}}{\Delta \tilde{x}^2} + \frac{\tilde{\psi}^n_{i,j+1} - 2\tilde{\psi}^n_{i,j} + \tilde{\psi}^n_{i,j-1}}{\Delta \tilde{y}^2} = -\tilde{\omega}^n_{i,j}$$

- This can be solved using Gauss-Seidel Iteration by Successive Over-relaxation (SOR) or Successive Under-relaxation.

Stream Function Equation

```
while (err > 10^(-6))
    tempPSI = PSI;
    for i = 2:nx - 1
        for j = 2:ny - 1
            PSI(j,i) = relax*(1/(2/dX^2+2/dY^2)*(((PSI(j,i+1)+PSI(j,i-1))/dX^2+...
                (PSI(j+1,i)+PSI(j-1,i))/dY^2) + vort(j,i))) + (1 - relax)*PSI(j,i)
        end
    end
    PSI(2:ny,nx) = 2*PSI(2:ny,(nx - 1)) - PSI(2:ny,(nx - 2));
    err = 0;
    for i = 2:nx - 1
        for j = 2:ny - 1
            err = err+ (abs(PSI(j,i) - tempPSI(j,i))/PSI(j,i));
        end
    end

    n = n+1;
end
```

Limitation on Time Step

- Courant – Friedrichs – Lewy condition (CFL):

$$\left(\frac{u\Delta t}{\Delta x}+\frac{v\Delta t}{\Delta y}\right)\leq 1$$

- Grid Fourier Number:

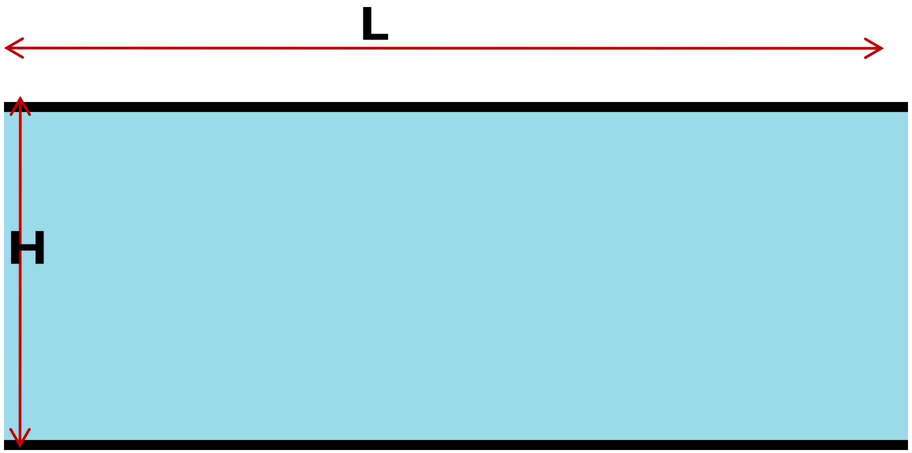
$$\upsilon\Delta t\left(\frac{1}{\Delta x^2}+\frac{1}{\Delta y^2}\right)\leq\frac{1}{2}$$

- $\Delta t < \Delta x$

CHANNEL FLOW

Channel flow parameters

- Re = 100
- Pr = 0.7 (air)
- H = 1m
- L = ???



Entrance length

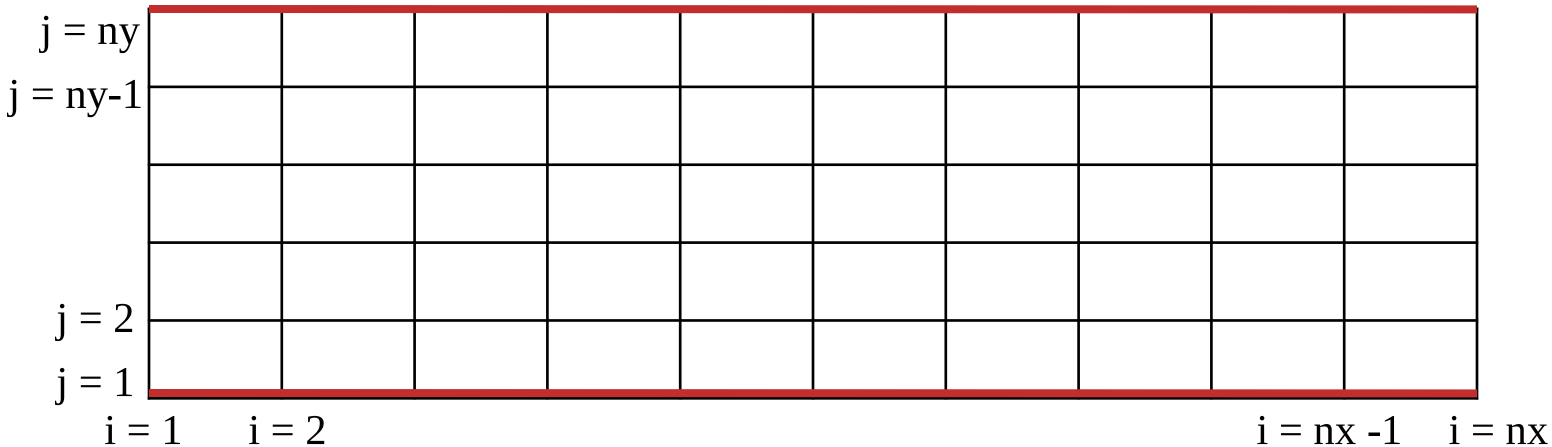
- Hydrodynamic Entrance Length:

$$\frac{L_h}{H} = 0.05 * \mathrm{Re}$$

- Thermal Entrance Length:

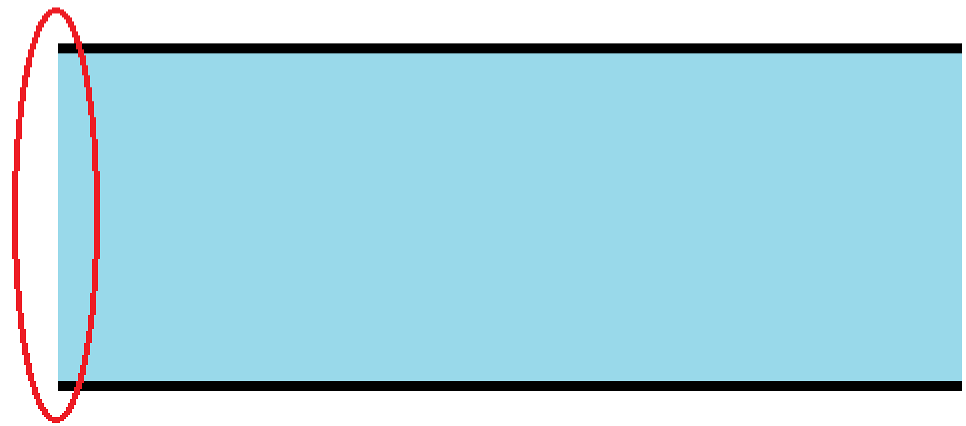
$$\frac{L_t}{H} = 0.05 * \mathrm{Re} * \mathrm{Pr}$$

- L = 14 m


j = ny
j = ny-1
j = 2
j = 1
i = 1
i = 2
i = nx -1
i = nx

Inlet Boundary Condition

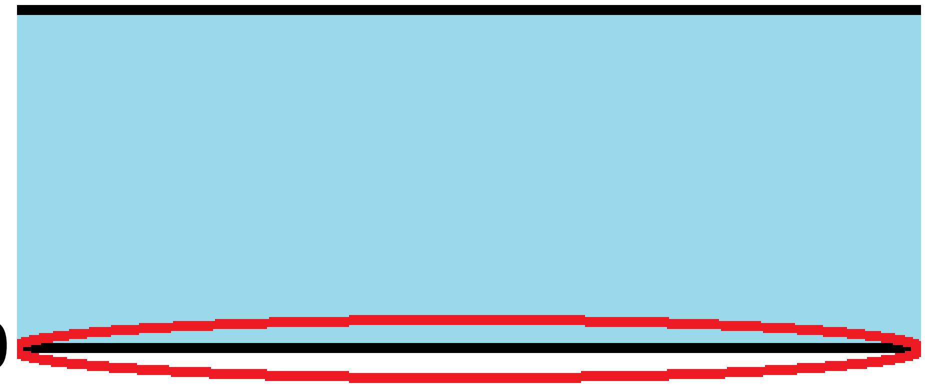
- Assuming uniform velocity:
  - $\tilde{u} = 1$
  - $\tilde{v} = 0$
- $\tilde{\psi}_{inlet} = \int_0^H \tilde{u}(\tilde{y})_{inlet} \, d\tilde{y}$
- $\tilde{\psi}_{inlet}(\tilde{y}) = \tilde{y}$
-
```
for j = 1:ny
    PSI(j,1) = (j-1)*dY; % inlet
end
```
- $\tilde{\omega}_{inlet} = 0$



Outflow Boundary Condition

- $\frac{\partial \tilde{u}}{\partial \tilde{x}} = \frac{\partial \tilde{v}}{\partial \tilde{x}} = 0$
- $\frac{\partial^2 \tilde{\psi}}{\partial \tilde{x}^2} = 0$
- $\tilde{\psi}_{nx,j} = 2 * \tilde{\psi}_{nx-1,j} - \tilde{\psi}_{nx-2,j}$
- ```
  PSI(2:ny,nx) = 2*PSI(2:ny,(nx - 1)) - PSI(2:ny,(nx - 2));
  ```
- $\frac{\partial \tilde{\omega}}{\partial \tilde{x}} = 0$
- $\tilde{\omega}_{nx,j} = \tilde{\omega}_{nx-1,j}$
- ```
  vort(2:ny-1,nx) = vort(2:ny -1,nx - 1);
  ```

Bottom wall : No slip Boundary Condition

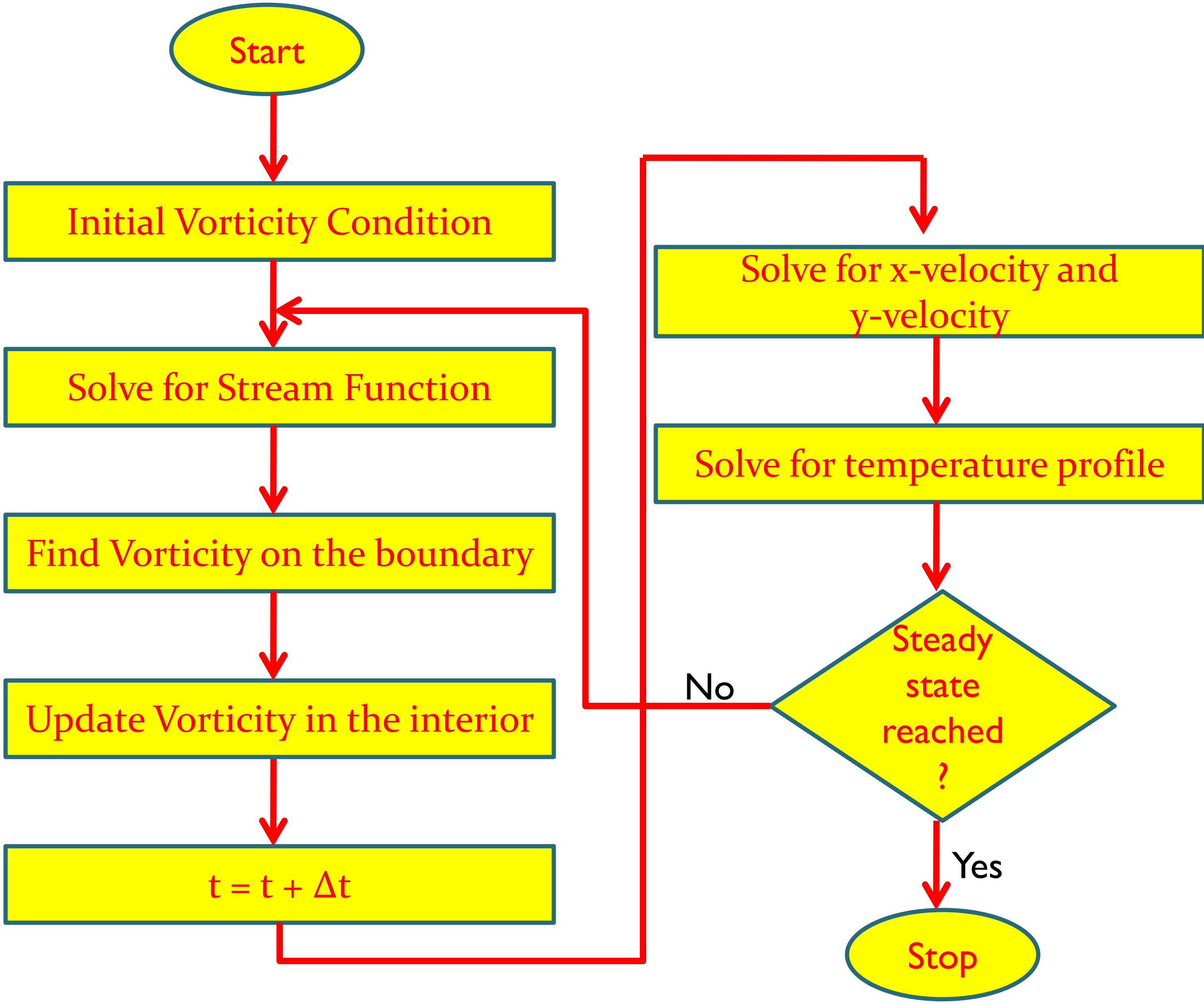


- $\tilde{v} = 0 \Rightarrow \dfrac{\partial \tilde{\psi}}{\partial \tilde{x}} = 0$
- $\tilde{\psi} = constant = 0$
- $\dfrac{\partial^2 \tilde{\psi}}{\partial \tilde{x}^2} + \dfrac{\partial^2 \tilde{\psi}}{\partial \tilde{y}^2} = -\tilde{\omega} \quad \Rightarrow \quad \tilde{\omega}_{wall} = -\dfrac{\partial^2 \tilde{\psi}_{i,j=1}}{\partial \tilde{y}^2}$
- $\tilde{\psi}_{i,j=2} = \tilde{\psi}_{i,j=1} + (\Delta\tilde{y}) \underbrace{\dfrac{\partial \tilde{\psi}_{i,j=1}}{\partial \tilde{y}}}_{0} + \dfrac{\Delta\tilde{y}^2}{2} \dfrac{\partial^2 \tilde{\psi}_{i,j=1}}{\partial \tilde{y}^2} + O(\Delta\tilde{y}^3)$
- $\tilde{\omega}_{wall} = (\underbrace{\tilde{\psi}_{i,j=1}}_{0} - \tilde{\psi}_{i,j=2}) \dfrac{2}{\Delta\tilde{y}^2} + O(\Delta\tilde{y})$
-
```
vort(1,2:nx) = -2.0*(PSI(2,2:nx))/(dy^2); % bottom wall
```

Top wall : No slip Boundary Condition

- $\tilde{v}=0 \Rightarrow \dfrac{\partial \tilde{\psi}}{\partial \tilde{x}}=0$
- $\tilde{\psi}=constant=1$
- $\dfrac{\partial^2 \tilde{\psi}}{\partial \tilde{x}^2}+\dfrac{\partial^2 \tilde{\psi}}{\partial \tilde{y}^2}=-\tilde{\omega} \quad \Rightarrow \quad \tilde{\omega}_{wall}=-\dfrac{\partial^2 \tilde{\psi}_{i,j=ny}}{\partial \tilde{y}^2}$
- $\tilde{\psi}_{i,j=ny-1}=\tilde{\psi}_{i,j=ny}-(\Delta\tilde{y})\cancelto{0}{\dfrac{\partial \tilde{\psi}_{i,j=ny}}{\partial \tilde{y}}}+\dfrac{\Delta\tilde{y}^2}{2}\dfrac{\partial^2 \tilde{\psi}_{i,j=ny}}{\partial \tilde{y}^2}+O(\Delta\tilde{y}^3)$
- $\tilde{\omega}_{wall}=(\tilde{\psi}_{i,j=ny}-\tilde{\psi}_{i,j=ny-1})\dfrac{2}{\Delta\tilde{y}^2}+O(\Delta\tilde{y})$
-
```
vort(ny,2:nx) = 2.0*(PSI(ny,2:nx) - PSI(ny - 1,2:nx))/(dy^2); % top wall
```

FLOW CHART


Start
Initial Vorticity Condition
Solve for Stream Function
Find Vorticity on the boundary
Update Vorticity in the interior
t = t + Δt
Solve for x-velocity and y-velocity
Solve for temperature profile
Steady state reached ?
No
Yes
Stop

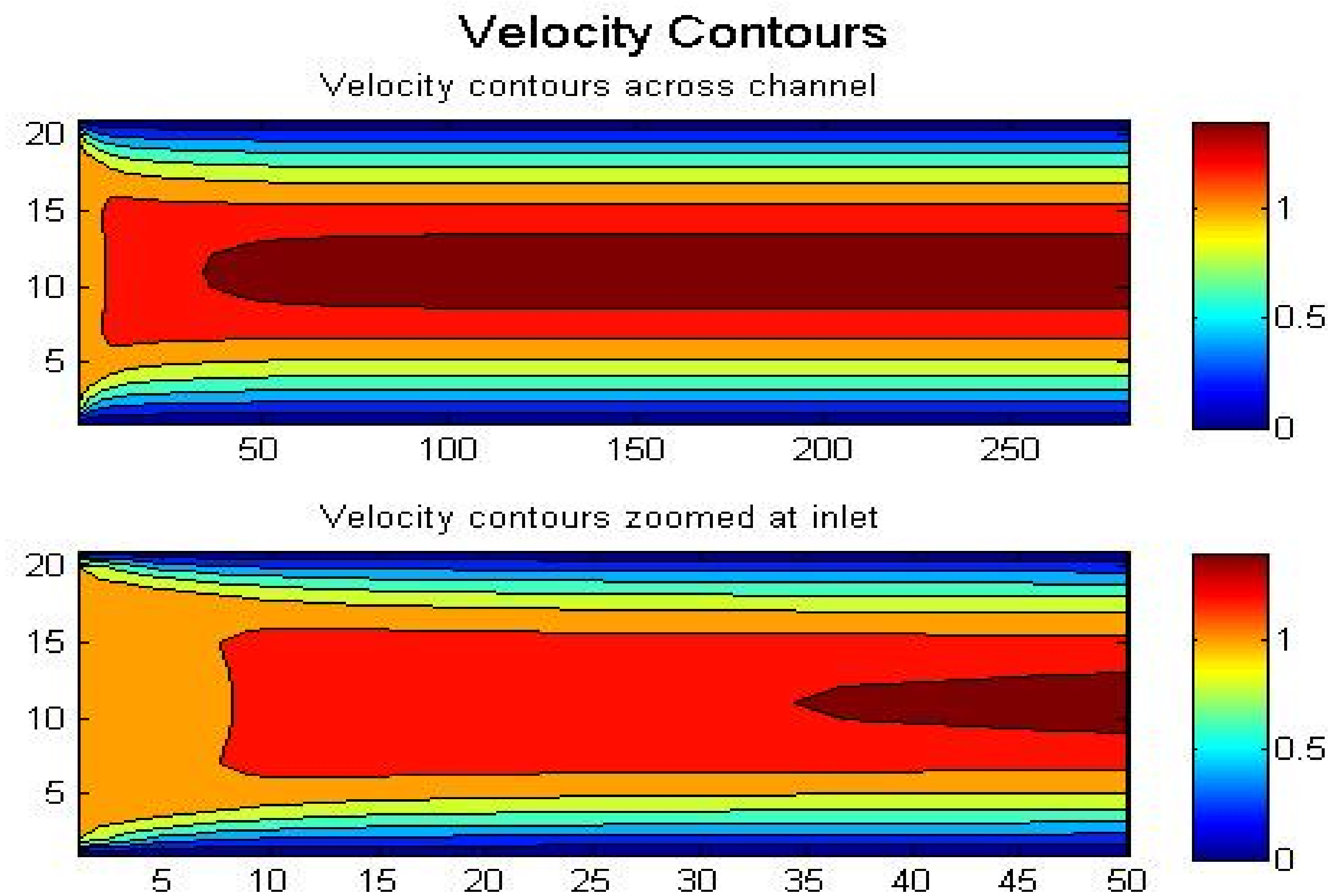
VELOCITY PROFILE

Velocity profile:

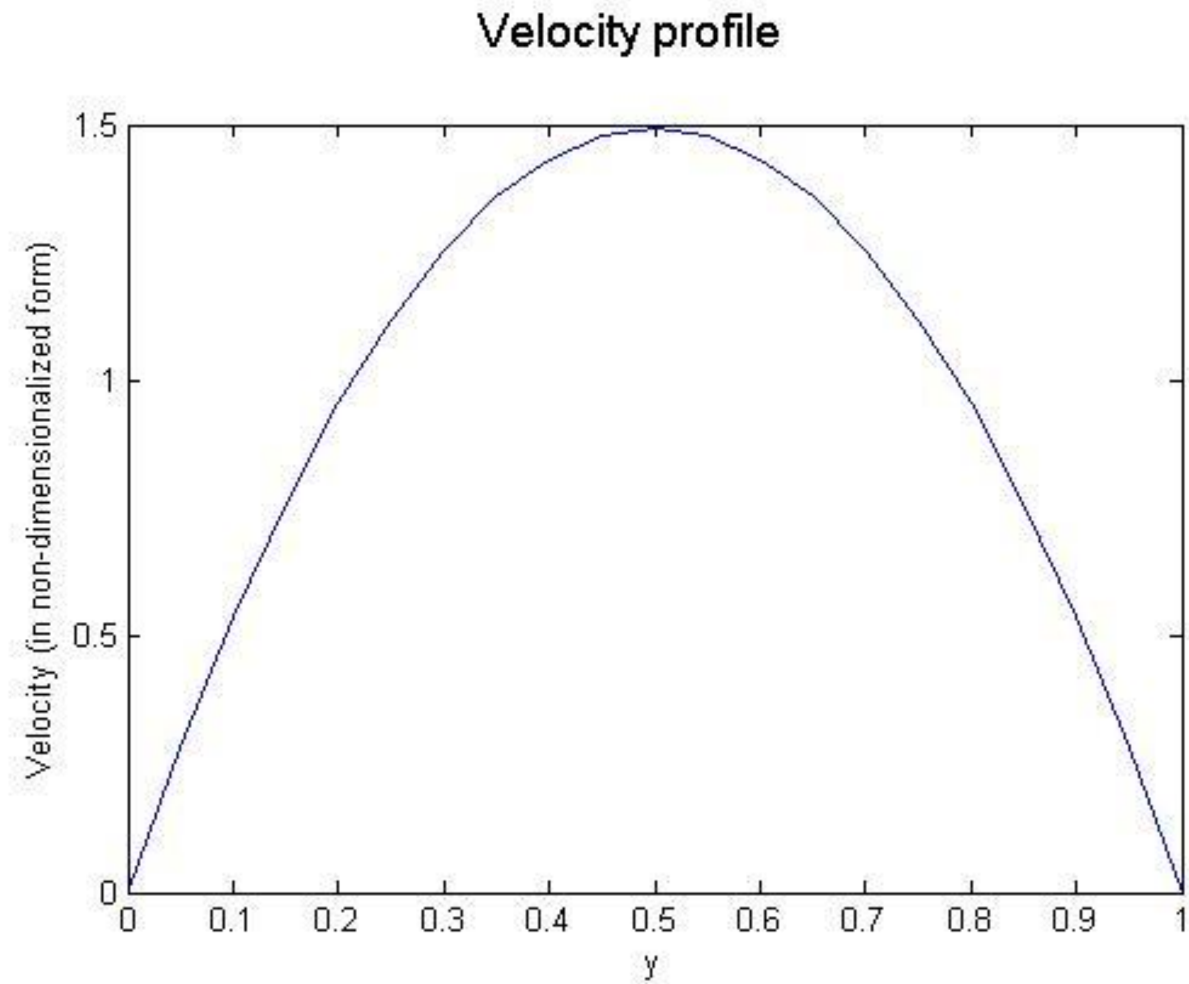
- $$\tilde{u} = \frac{\partial \tilde{\psi}}{\partial \tilde{y}}; \qquad \tilde{v} = -\frac{\partial \tilde{\psi}}{\partial \tilde{x}}$$
- $$\tilde{u}_{i,j} = \frac{\tilde{\psi}_{i,j+1} - \tilde{\psi}_{i,j-1}}{2\Delta \tilde{y}}$$
- $$\tilde{v}_{i,j} = -\frac{\tilde{\psi}_{i+1,j} - \tilde{\psi}_{i-1,j}}{2\Delta \tilde{x}}$$
- ```
  for j = 2:ny-1
      u(j,:) = (psi(j+1,:)-psi(j-1,:))/(2*dy);
  end
  for i = 2:nx-1
      v(:,i) = -(psi(:,i+1)-psi(:,i-1))/(2*dx);
  end
  ```
- Boundary conditions:

```
u(:,1) = 1;                % inlet
v(:,1) = 0;                % inlet
u(1,:) = 0;                % bottom wall
u(ny,:) = 0;               % top wall
u(2:ny,nx) = u(2:ny,nx - 1); % outflow
```
```

Velocity Contour



Velocity Profile near the exit



TEMPERATURE PROFILE

Temperature profile calculation

- $$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right)$$

- Non-dimensionalizing:

- $$\theta = \frac{T - T_\infty}{T_W - T_\infty}$$

- $$\frac{\partial \theta}{\partial \tilde{t}} + \tilde{u}\frac{\partial \theta}{\partial \tilde{x}} + \tilde{v}\frac{\partial \theta}{\partial \tilde{y}} = \frac{1}{\mathrm{Re}^* \mathrm{Pr}}\left(\frac{\partial^2 \theta}{\partial \tilde{x}^2} + \frac{\partial^2 \theta}{\partial \tilde{y}^2}\right)$$

Finite Difference Approximation

- $$\frac{\partial \theta}{\partial \tilde{t}} + \tilde{u}\frac{\partial \theta}{\partial \tilde{x}} + \tilde{v}\frac{\partial \theta}{\partial \tilde{y}} = \frac{1}{\text{Re}^*\,\text{Pr}}\left(\frac{\partial^2 \theta}{\partial \tilde{x}^2} + \frac{\partial^2 \theta}{\partial \tilde{y}^2}\right)$$

- $$\frac{\theta_{i,j}^{n+1} - \theta_{i,j}^{n}}{\Delta \tilde{t}} = -\left(\tilde{u}_{i,j}^{n}\frac{\theta_{i+1,j}^{n} - \theta_{i-1,j}^{n}}{2\Delta \tilde{x}} + \tilde{v}_{i,j}^{n}\frac{\theta_{i,j+1}^{n} - \theta_{i,j-1}^{n}}{2\Delta \tilde{y}}\right) + \frac{1}{\text{Re}^*\,\text{Pr}}\left(\frac{\theta_{i+1,j}^{n} - 2\theta_{i,j}^{n} + \theta_{i-1,j}^{n}}{\Delta \tilde{x}^2} + \frac{\theta_{i,j+1}^{n} - 2\theta_{i,j}^{n} + \theta_{i,j-1}^{n}}{\Delta \tilde{y}^2}\right)$$

Temperature at new time

$$\theta_{i,j}^{n+1} =$$

$$\theta_{i,j}^{n} + \Delta\tilde{t}\left[-\left(\tilde{u}_{i,j}^{n}\frac{\theta_{i+1,j}^{n}-\theta_{i-1,j}^{n}}{2\Delta\tilde{x}} + \tilde{v}_{i,j}^{n}\frac{\theta_{i,j+1}^{n}-\theta_{i,j-1}^{n}}{2\Delta\tilde{y}}\right) + \frac{1}{\mathrm{Re}^{*}\,\mathrm{Pr}}\left(\frac{\theta_{i+1,j}^{n}-2\theta_{i,j}^{n}+\theta_{i-1,j}^{n}}{\Delta\tilde{x}^{2}} + \frac{\theta_{i,j+1}^{n}-2\theta_{i,j}^{n}+\theta_{i,j-1}^{n}}{\Delta\tilde{y}^{2}}\right)\right]$$

```
for i = 2:nx - 1
    for j = 2: ny - 1
        Conv1 = -u(j,i)*(thetha_prime(j,i+1) - thetha_prime(j,i - 1))/(2*dx);
        Conv2 = -v(j,i)*(thetha_prime(j+1,i) - thetha_prime(j-1,i))/(2*dy);
        diff1 = (1/(Re*Pr))*(thetha_prime(j,i-1) + thetha_prime(j,i+1) - 2*thetha_prime(j,i))/dx^2;
        diff2 = (1/(Re*Pr))*(thetha_prime(j+1,i) + thetha_prime(j-1,i) - 2*thetha_prime(j,i))/dy^2;
        thetha(j,i) = thetha_prime(j,i) + delta_t*(Conv1 + Conv2 + diff1 + diff2);
    end
end
```

Boundary Conditions:

- Inlet:

$$\theta = 0$$

- Outflow:

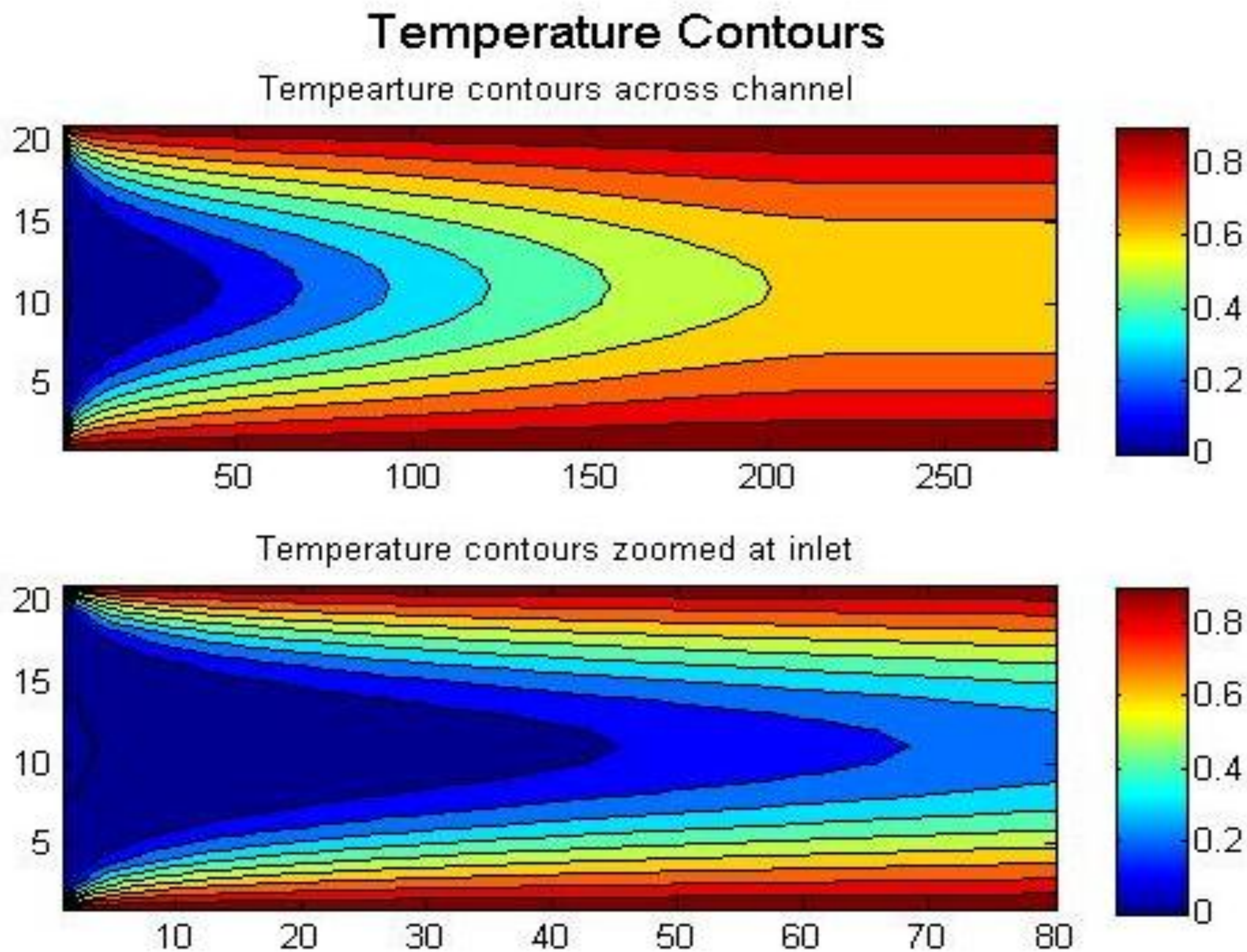
$$\frac{\partial \theta}{\partial \tilde{x}} = 0$$

- Top Wall and Bottom Wall:

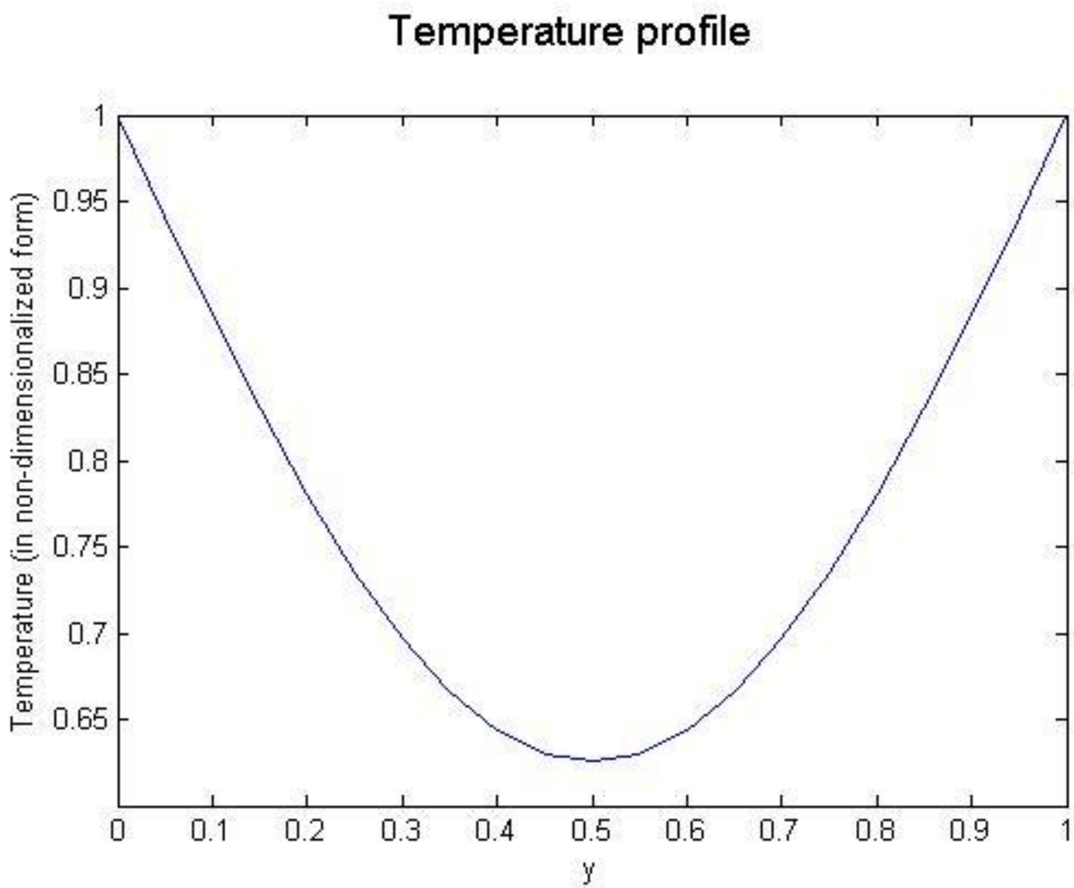
$$\theta = 1$$

```
thetha(1,:) = 1;
thetha(ny,:) = 1;
thetha(2:ny - 1,1) = 0;
thetha(2:ny - 1,nx) = thetha(2:ny - 1,nx -1);
```

Temperature contours



Temperature profile



Nusselt Number

- From Fourier's law of conduction and Newton's law of cooling:

$$-k\frac{\partial T}{\partial y} = h(T - T_\infty)$$

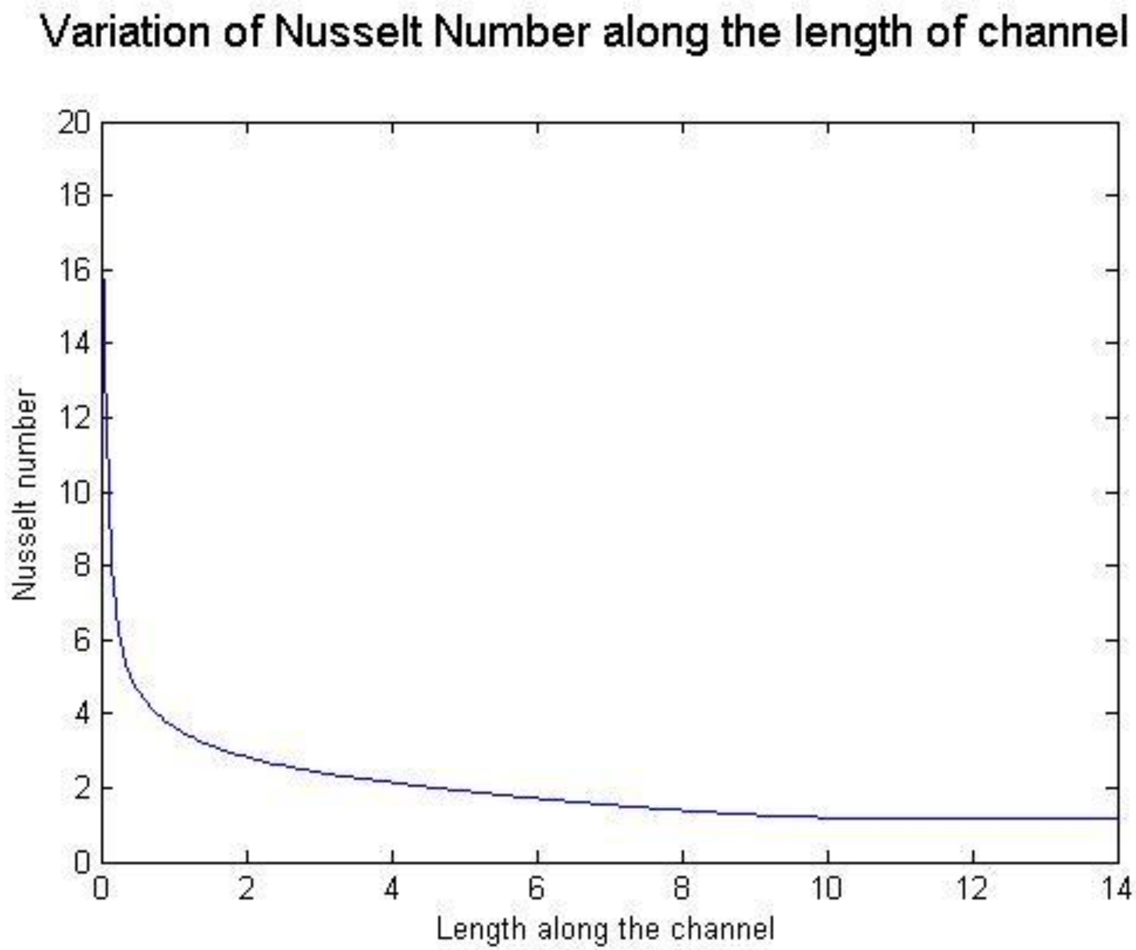
$$\Rightarrow -k\frac{(T_w - T_\infty)\partial\theta}{H\partial\tilde{y}} = h\theta\left(T_W - T_\infty\right)$$

$$\Rightarrow \frac{hH}{k} = Nu = -\frac{1}{\theta}\frac{\partial\theta}{\partial\tilde{y}}$$

$$Nu_{wall} = -\frac{1}{\theta_{wall}}\left(\frac{\partial\theta}{\partial\tilde{y}}\right)$$

$$\Rightarrow Nu_{wall} = -\frac{\partial\theta}{\partial\tilde{y}}$$

Variation of Nusselt Number near the wall



Local Nusselt Number

```
Steady state reached
Local Nusselt Number near the walls = 2.042
```

References:

- **nptel**.ac.in/courses/112104030/pdf/lecture25.pdf
- Fully Developed Pipe and Channel Flows: Indo – German Winter Academy, 2006