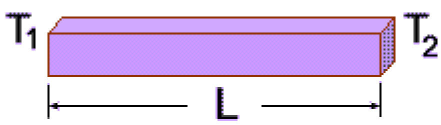
Advanced Computational Fluid Dynamics (AM 6513)



Report on Assignment 1

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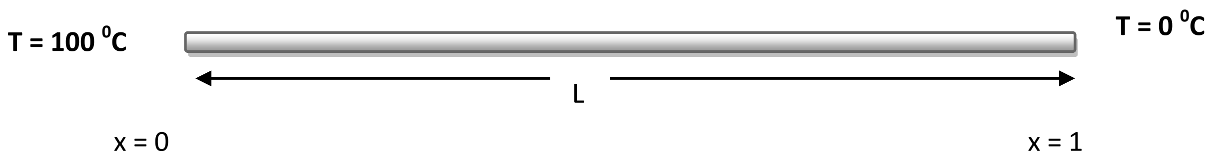
Problem definition:



$\propto = 1\frac{m^2}{s}$

$Length = 1\ m$

$\Delta x = 0.1\ m$

Compute the solution from t = 0 to t = 10 sec in steps of

1. $\Delta t = 0.1$
2. $\Delta t = 0.01$
3. $\Delta t = 0.001$

Plot the results for t = 0, 0.5, 1, 2, 5, 10 sec.

Governing Equation:

$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2}$$

where

- T is the temperature
- t is the time
- α is the thermal diffusivity
- x is the distance

Initial Condition:

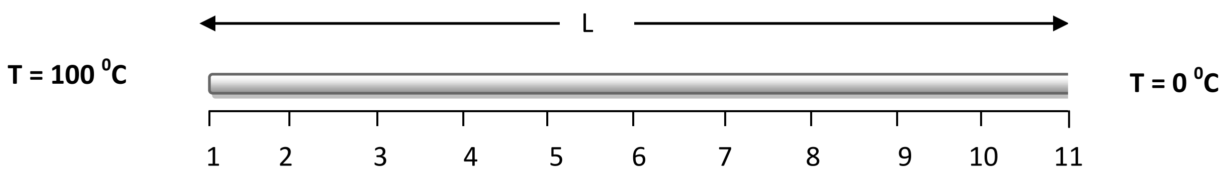
$$T = 0\ ℃\ everywhere$$

Boundary Condition:

$$T_{x=0} = 100℃$$

$$T_{x=1} = 0\ ℃$$

Numerical Formulation:



Using the FTCS (Forward in time and Central in Space) to discretize the governing equation, we get:

$$\frac{T_i^{n+1} - T_i^n}{\Delta t} = \propto \frac{T_{i+1}^n - 2T_i^n + T_{i-1}^n}{\Delta x^2} \qquad for\ i = 2,3,4, \ldots \ldots 10$$

$$T_i^{n+1} = \gamma T_{i+1}^n + (1 - 2\gamma) T_i^n + \gamma T_{i-1}^n \quad for\ i = 2,3,4 \ldots \ldots \ldots 10$$

$$where\ \gamma = \alpha \frac{\Delta t}{\Delta x^2}$$

Imposing the Boundary condition:

$$T_1 = 100\ °C$$
$$T_{11} = 0°C$$

Algorithm:

1. Initialize Δt, Δx, N, α.
2. Initialize T_old[i] = 0 for i = 2,…, N and T_old[1] = 100 (Boundary Condition)
3. Calculate T_new[i] using the above formulation for i = 2… N – 1.
   T_new[1] = 100 ^{0}C and T_old[0] = 0^0C.
4. Swap T_old with T_new.
5. Go to step 3 till time elapsed is less than 10 s.

Input data:

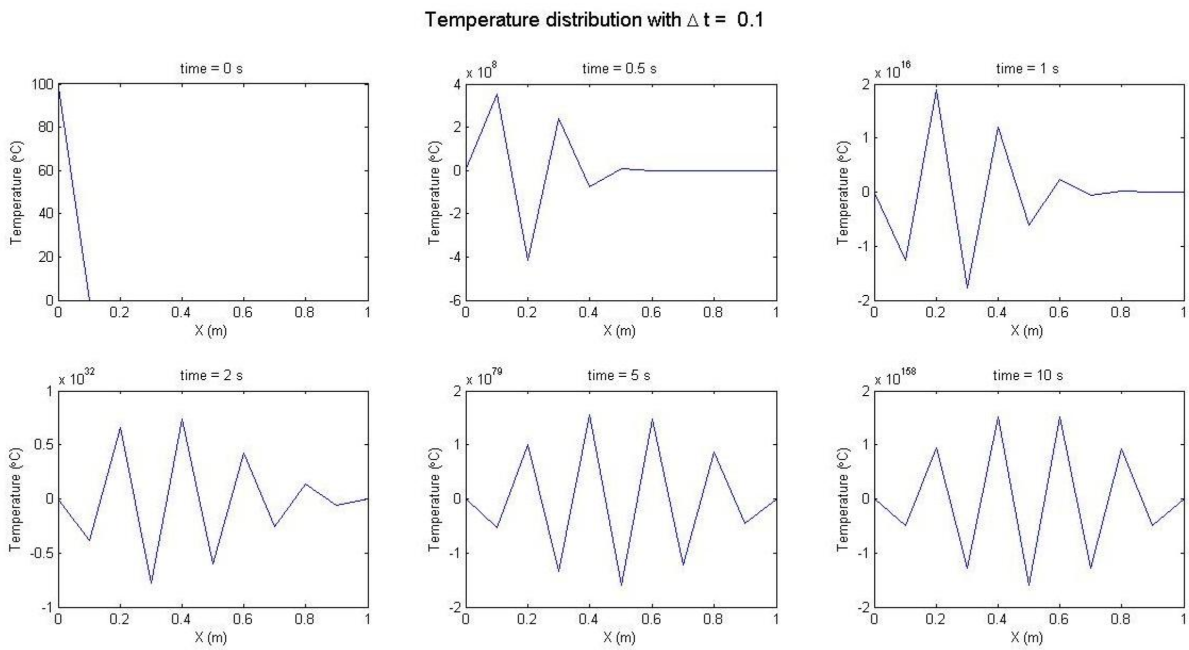
	A	B	C	D	E
1	Δ t	α	*Length*	Δx	ϒ (calculated)
2	0.1	1	1	0.1	10
3	0.01				1
4	0.001				0.1
5					

Results:

- **Case 1:**

  $\Delta t = 0.1$ s
  ϒ = 10

  Since ϒ > 0.5, therefore, we see oscillations in this case.

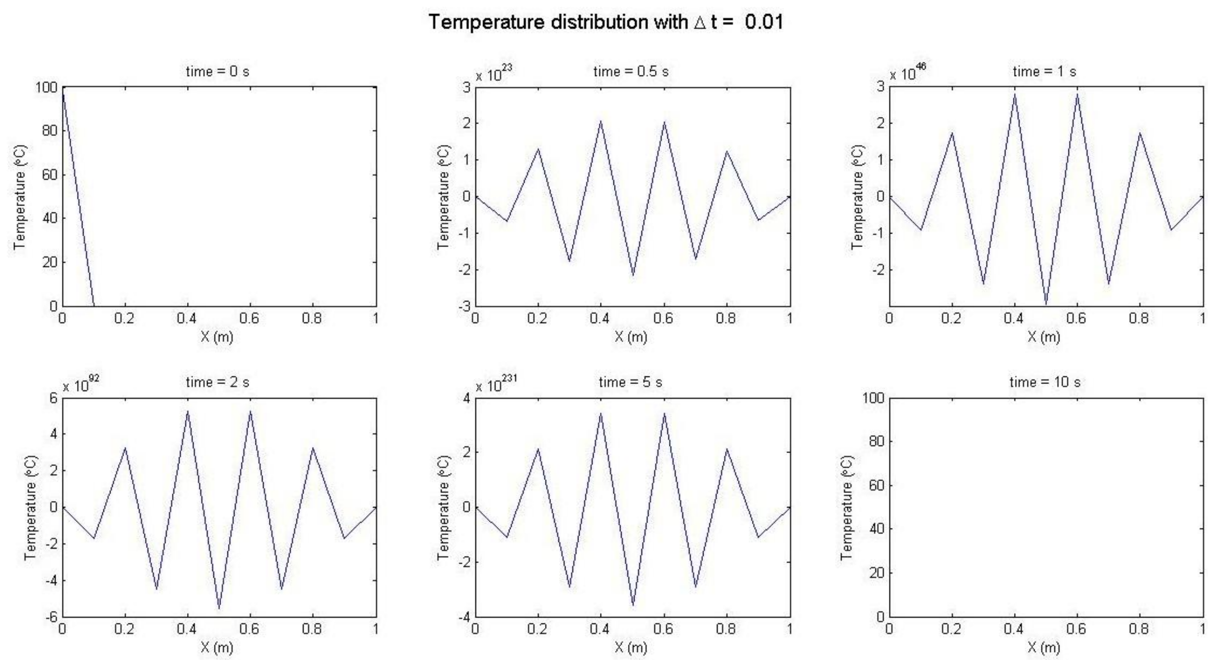


- ## Case 2:

Δt = 0.01 s
Υ = 1

Since $\Upsilon > 0.5$, therefore, we see oscillations in this case.

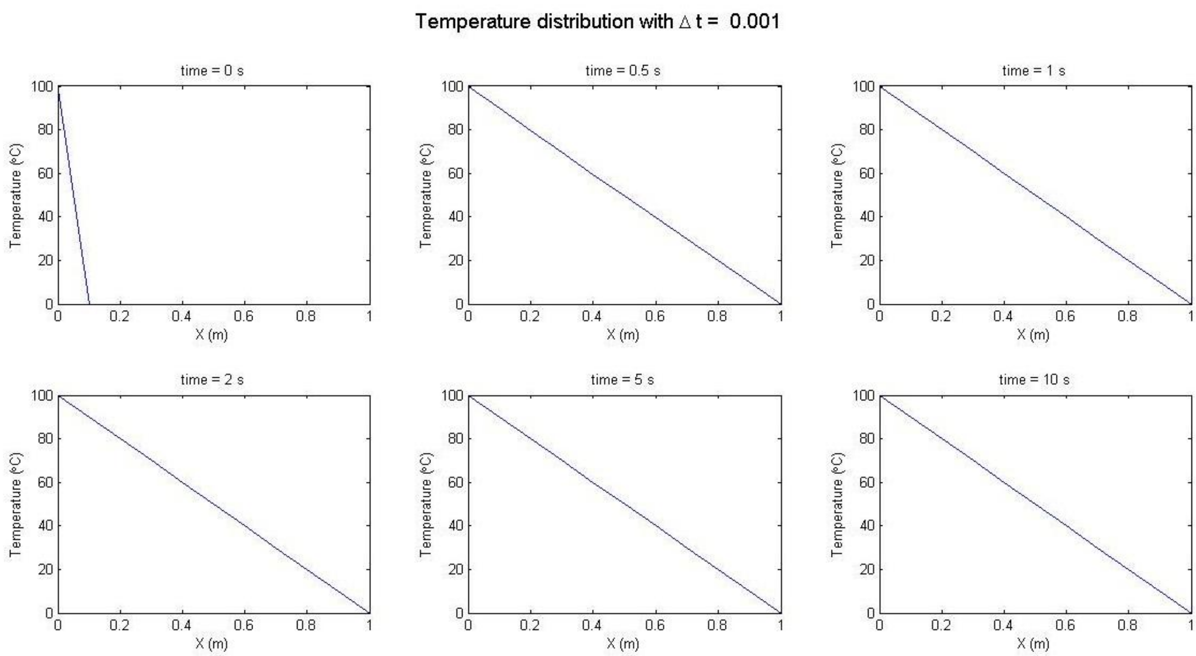
At time = 10 s, the temperature began to oscillate between - ∞ (Minus Infinity) and ∞ (Infinity). So we are not able to see any curve.



- **Case 3:**

  Δt = 0.001 s
  ϒ = 0.1

  Since ϒ < 0.5, therefore, we do not see oscillations in this case and steady state is attained.



Appendix

Matlab Code:

```
close all;
clear all;
clc;

%% Details
% Author Name: Kumar Saurabh
% Roll No. MA14M004

%% Reading the data from input file
Time_interval = xlsread('inputMA14M004_assign1.xlsx','Sheet1','A2:A4');
alpha = xlsread('inputMA14M004_assign1.xlsx','Sheet1','B2');
len = xlsread('inputMA14M004_assign1.xlsx','Sheet1','C2');
delta_x = xlsread('inputMA14M004_assign1.xlsx','Sheet1','D2');
for i = 1:3

    %% Initializing the input
    delta_t = Time_interval(i);
    X = 0:delta_x:len;
    time = 0;
    figure(i);

    %% Calculations
    x_size = (len/delta_x) + 1;
    gamma = alpha*delta_t/(delta_x^2);
    T_old = zeros(x_size,1);
    T_old(1) = 100; %Boundary Condition

    %% At time t = 0 s
    T_0 = T_old;
    subplot(2,3,1);
    plot(X,T_0);
    xlabel('X (m)');
    ylabel('Temperature (\circC)');
    title('time = 0 s');
    %% Calculations for various times
    while(time < 10)
        T_new = zeros(x_size,1);
        T_new(1) = 100;
        time = time + delta_t;

        for j = 2:(x_size - 1)
            T_new(j) = gamma*T_old(j+1) + (1 - (2*gamma))*T_old(j) +
gamma*T_old(j - 1);
        end

        %% At time t = 0.5 s
        if(abs(time - 0.5)<(10^(-6)))
            T_point5 = T_new;
            subplot(2,3,2);
```

```
            plot(X,T_point5);
            title('time = 0.5 s');
            xlabel('X (m)');
            ylabel('Temperature (\circC)');
        end

        %% At time t = 1 s
        if(abs(time - 1) < (10^(-6)))
            T_1 = T_new;
            subplot(2,3,3);
            plot(X,T_1);
            title('time = 1 s');
            xlabel('X (m)');
            ylabel('Temperature (\circC)');
        end

        %% At time t = 2 s
        if(abs(time - 2.000) < (10^(-6)))
            T_2 = T_new;
            subplot(2,3,4);
            plot(X,T_2);
            title('time = 2 s');
            xlabel('X (m)');
            ylabel('Temperature (\circC)');
        end

        %% At time t = 5 s
        if(abs(time - 5.000) < (10^(-6)))
            T_5 = T_new;
            subplot(2,3,5);
            plot(X,T_5);
            title('time = 5 s');
            xlabel('X (m)');
            ylabel('Temperature (\circC)');
        end

        %% At time t = 10 s
        if(abs(time - 10.000) < (10^(-6)))
            T_10 = T_new;
            subplot(2,3,6);
            plot(X,T_10);
            title('time = 10 s');
            xlabel('X (m)');
            ylabel('Temperature (\circC)');
        end

        %% Updation of Temperature
        T_old = T_new;


    end

    suptitle(['Temperature distribution with {\Delta t} =
',num2str(delta_t)])
end
```