SOLA-VOF (Solution Algorithm – Volume of Fluid)

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Term Paper

- Volume of Fluid (VOF) Method for the Dynamics of Free Boundaries.
- C.W. Hirt and B.D. Nichols
- Journal of Computational Physics, 39, 201-225 (1981).

Volume of Fluid (VOF)

- VOF is a free-surface modelling technique used for tracking and locating the free surface (or fluid interface).
- SOLA – VOF uses the VOF technique to track the free fluid surface.

Representation of Fluid

- **<u>Lagrangian Representation</u>**
  - Each zone of a grid that subdivides the fluid into elements that remains identified with the same fluid element for all the time.
  - The grid moves with the computed velocities.

- **<u>Eulerian Representation</u>**
  - The identity of fluid element is not maintained.
  - The grid remains fixed.

Representation of Fluid

- **Lagrangian Representation**
  - It treats the particle as discrete phase and tracks the pathway of each individual particle.

- **Eulerian Representation**
  - It treats the particle phase as the continuum and develops its conservation equation on a control volume basis.

Fluid Flow

- It is customary to view the fluid in an Eulerian mesh cell as a fluid element on which body and surface force may be computed.
- It is necessary to compute the flow of fluid through the mesh.
- This flow or convective flux calculation requires an averaging of flow properties of all fluid elements that find themselves in a given mesh after some period of time.

Fluid Flow

- Averaging process: Biggest drawback of Eulerian method.
- Convective averaging results in a smoothening of all variation in flow quantities and smearing of surfaces of discontinuity such as free surfaces.
- So, to overcome this loss in resolution some special treatment that recognizes the discontinuity and avoids the averaging across it.

Volume of Fluid Function (F)

$$F = \begin{cases} 1 & cell\ occupied\ by\ the\ fluid \\ 0 & no\ fluid\ in\ the\ cell \\ 0 < F < 1 & free\ surface \end{cases}$$

- Time dependence of F

$$\frac{\partial F}{\partial t} + u\frac{\partial F}{\partial x} + v\frac{\partial F}{\partial y} = 0$$

- Standard finite difference approximation would lead to smearing of F function and interfaces would loose its definition.

SOLA - VOF

- Equations to be solved

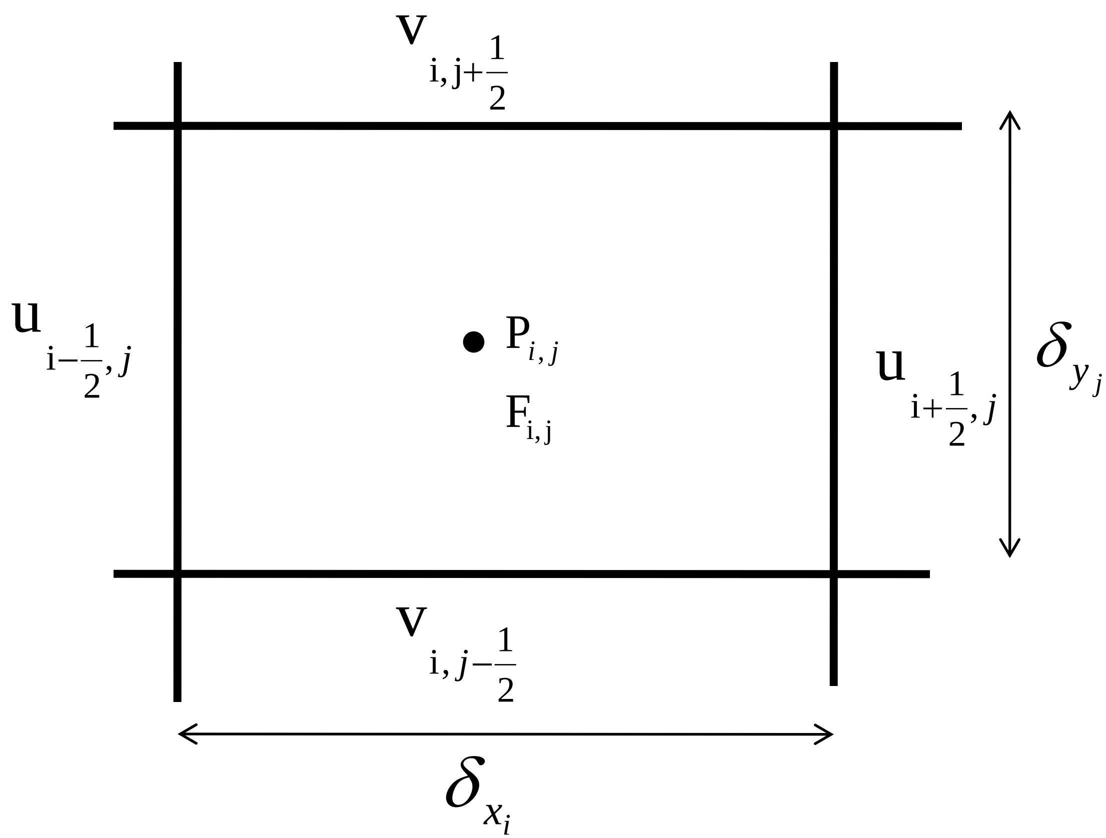
$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+g_x+\upsilon\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)$$

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial y}+g_y+\upsilon\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)$$

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0$$

$$\frac{\partial F}{\partial t}+u\frac{\partial F}{\partial x}+v\frac{\partial F}{\partial y}=0$$

Location of variables



Definitions:

- **Free Surface**: A free surface or interface cell (i,j) is defined as a cell containing a non zero value of F and having at least one neighboring cell (i±1,j) or (i,j±1) that contains a non zero value of F.
- **Empty cell**: Cells with zero F values are empty or contain material of density ρ_C.
- **Filled cells**: Cells with non zero F values and no empty neighbors are treated as cells full of liquid with density ρ_F .

Momentum Equation Approximation

- Finite Difference Approximation of X momentum Equation:

$$u^{n+1}_{i+\frac{1}{2},j} = u^{n}_{i+\frac{1}{2},j} + \delta t\left[-\frac{\left(p^{n+1}_{i+1,j} - p^{n+1}_{i,j}\right)}{\delta \rho x_{i+\frac{1}{2}}} + g_x - FUX - FUY + VISX \right]$$

- Finite Difference Approximation of Y – Momentum Equation

$$v^{n+1}_{i,j+\frac{1}{2}} = v^{n}_{i,j+\frac{1}{2}} + \delta t\left[-\frac{\left(p^{n+1}_{i,j+1} - p^{n+1}_{i,j}\right)}{\delta \rho y_{j+\frac{1}{2}}} + g_y - FVX - FVY + VISY \right]$$

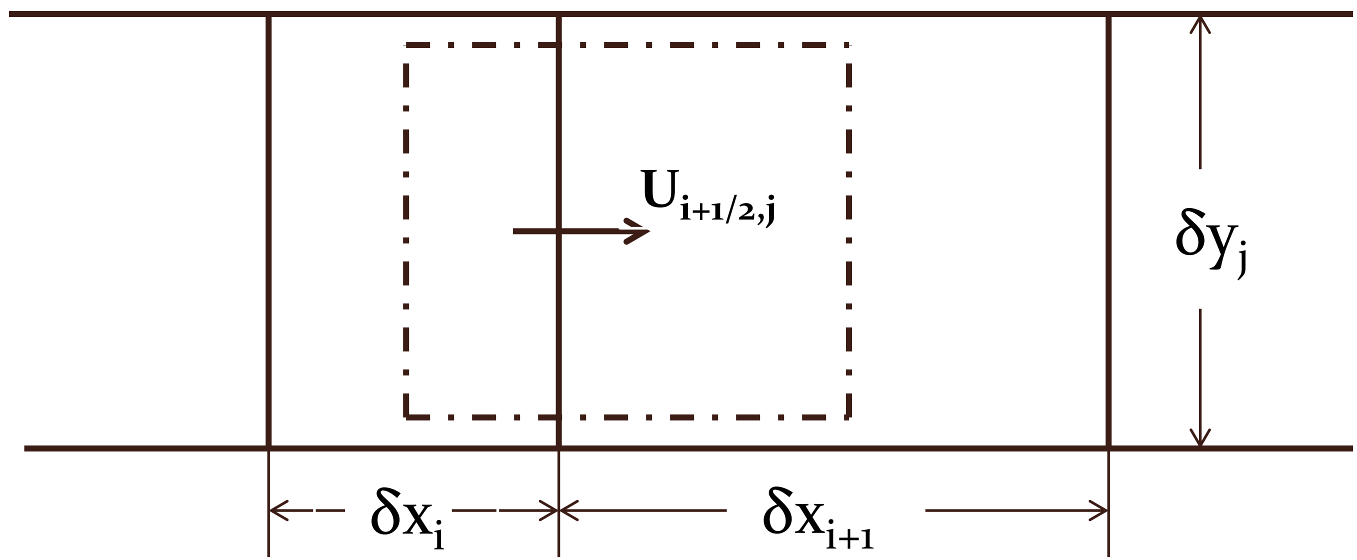
Momentum Equation Approximation

- FUX, FUY: Advective flux of u in the x and y direction respectively.
- FVX, FVY: Advective flux of v in the x and y direction respectively.
- VISX, VISY: Diffusive flux in the x and y direction respectively.

$$\delta\rho x_{i+\frac{1}{2}} = \frac{1}{2}\left\{\left(\rho_C + (\rho_F - \rho_C)F_{i,j}\right)\delta x_{i+1} + \left(\rho_C + (\rho_F - \rho_C)F_{i+1,j}\right)\delta x_i\right\}$$

$$\delta\rho y_{j+\frac{1}{2}} = \frac{1}{2}\left\{\left(\rho_C + (\rho_F - \rho_C)F_{i,j}\right)\delta y_{j+1} + \left(\rho_C + (\rho_F - \rho_C)F_{i,j+1}\right)\delta y_j\right\}$$

Estimation of FUX



Estimation of FUX

- The divergence was preferred, i.e.

  $\nabla u.u$ instead of $u\nabla u$

- It provides a simple way to ensure the conservation of momentum in the finite difference approximation.

$$FUX = \frac{\left[u_{i+1,j} < u_{i+1,j} > -u_{i,j} < u_{i,j} >\right]}{\delta x_{i+\frac{1}{2}}}$$

where:

$$u_{i,j} = \frac{u_{i-\frac{1}{2},j} + u_{i+\frac{1}{2},j}}{2};$$

$$< u_{i,j} >= \begin{matrix} u_{i-\frac{1}{2},j}, \text{ if } u_{i,j} \geq 0 \\ u_{i+\frac{1}{2},j}, \text{ if } u_{i,j} \leq 0 \end{matrix}$$

Advancing F in time

- For an incompressible fluid:

$$\frac{\partial F}{\partial t}+\frac{\partial Fu}{\partial x}+\frac{\partial Fv}{\partial y}=0$$

- When integrated over the computational cell, the changes in F in a cell reduces to fluxes of F across the cell faces.
- The special care must be taken in computing these fluxes to preserve the sharp definition of the free surfaces.
- SOLA – VOF: Donor Acceptor flux approximation.

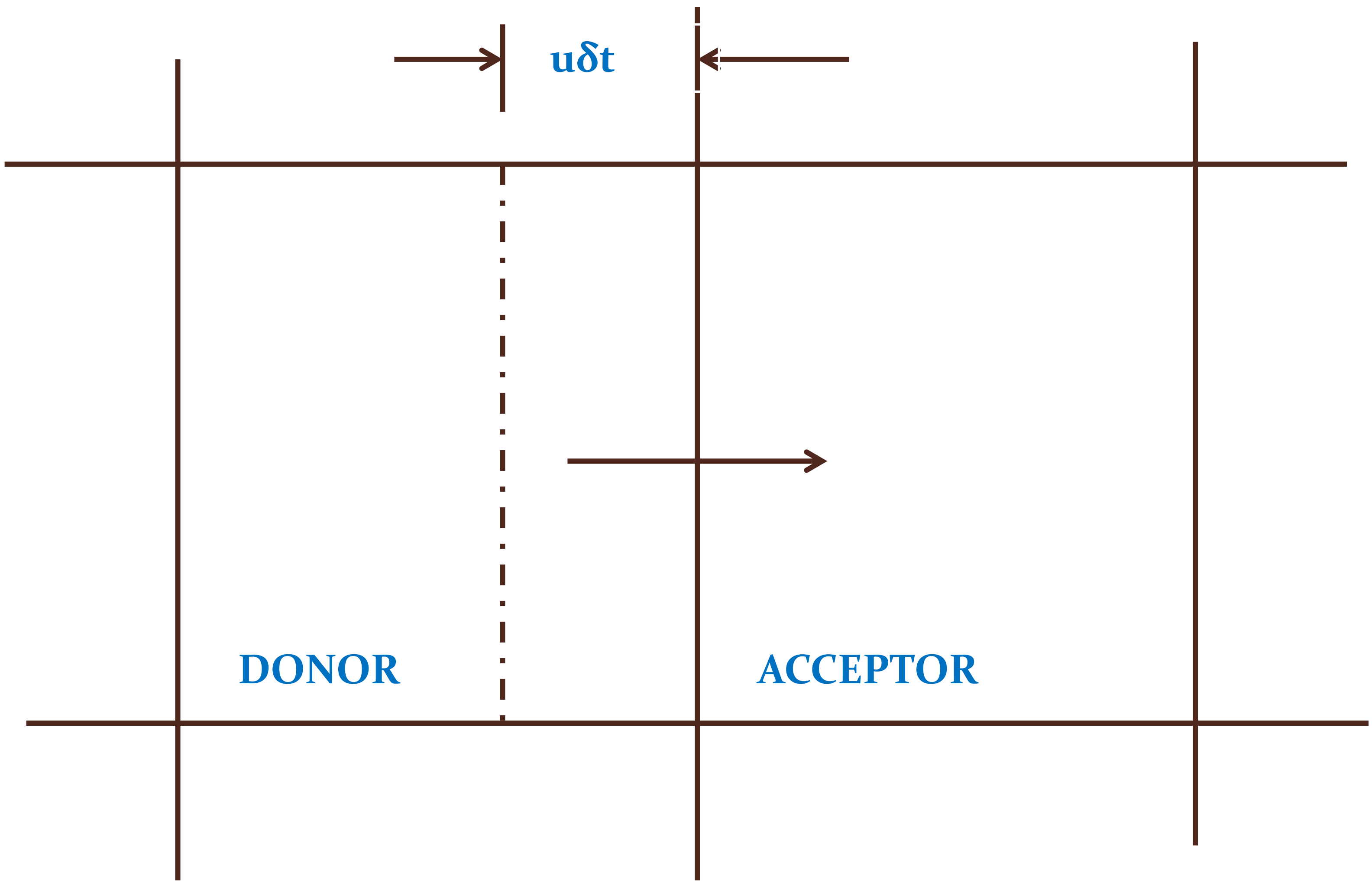
Donor – Acceptor Flux Approximation

- The total flux of fluid volume and void volume crossing the right cell per unit cross sectional area is:

$$V_x = u\delta t$$

- Where u is the normal velocity at the face.
- The sign of u determines the donor and acceptor cell.

Donor Acceptor Arrangement



Donor and Acceptor ell

- Donor cell: Cell loosing fluid.
- Acceptor cell: Cell gaining fluid.

Sign of u	Donor cell	Acceptor cell
Positive	Upstream (the left cell)	Downstream (the right cell)
Negative	Upstream (the right cell)	Downstream (the left cell)

Fluxed amount

- The amount of F fluxed across the cell face in one time step is δ_F times the cross sectional area.

$$\delta F = MIN\left\{F_{AD}\left|V_x\right| + CF, F_D \delta x_D\right\}$$

$$CF = MAX\left\{(1 - F_{AD})\left|V_x\right| - (1.0 - F_D)\delta x_D, 0.0\right\}$$

- F_A : Acceptor cell.
- F_D : Donor cell.
- F_{AD}: Acceptor or donor cell depending on the interface relative to the direction of the flow

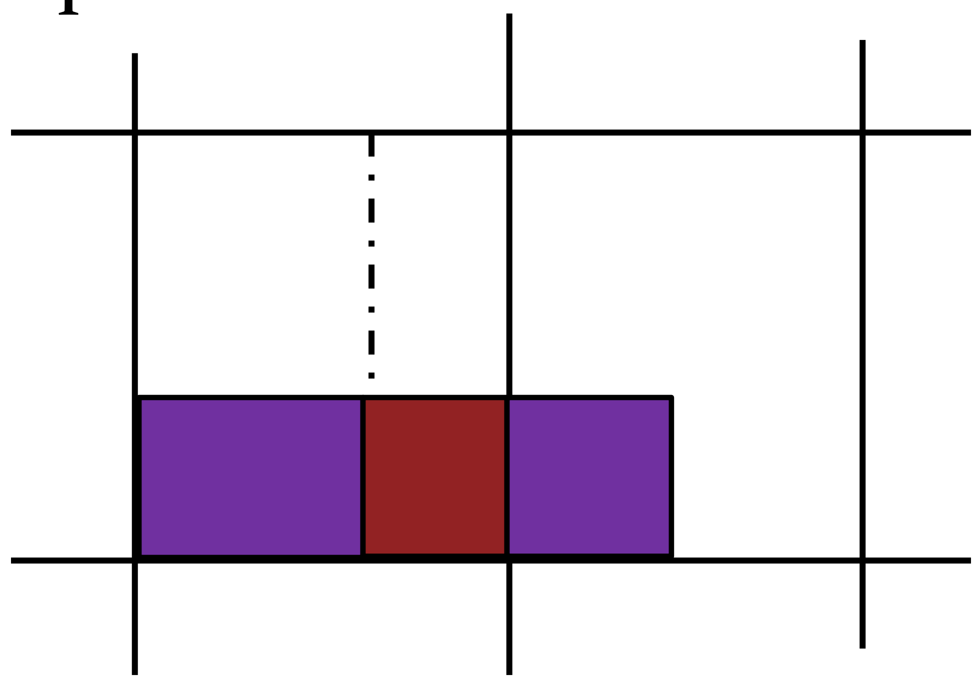
$$\left|V_x\right| < \delta x_D$$

Significance of MIN and MAX

- MIN feature prevents the fluxing of more F from the donor cell than it has to give.
- MAX feature accounts for the additional fluid flux, CF, if the amount of the fluid exceeds the amount available.

$F_{AD} = F_D$

- The flux is an ordinary donor cell value.
- F value in the donor cells used to define the fractional area of the cell face fluxing the liquid.
- CF = 0
- $\delta F = F_D |V_x|$

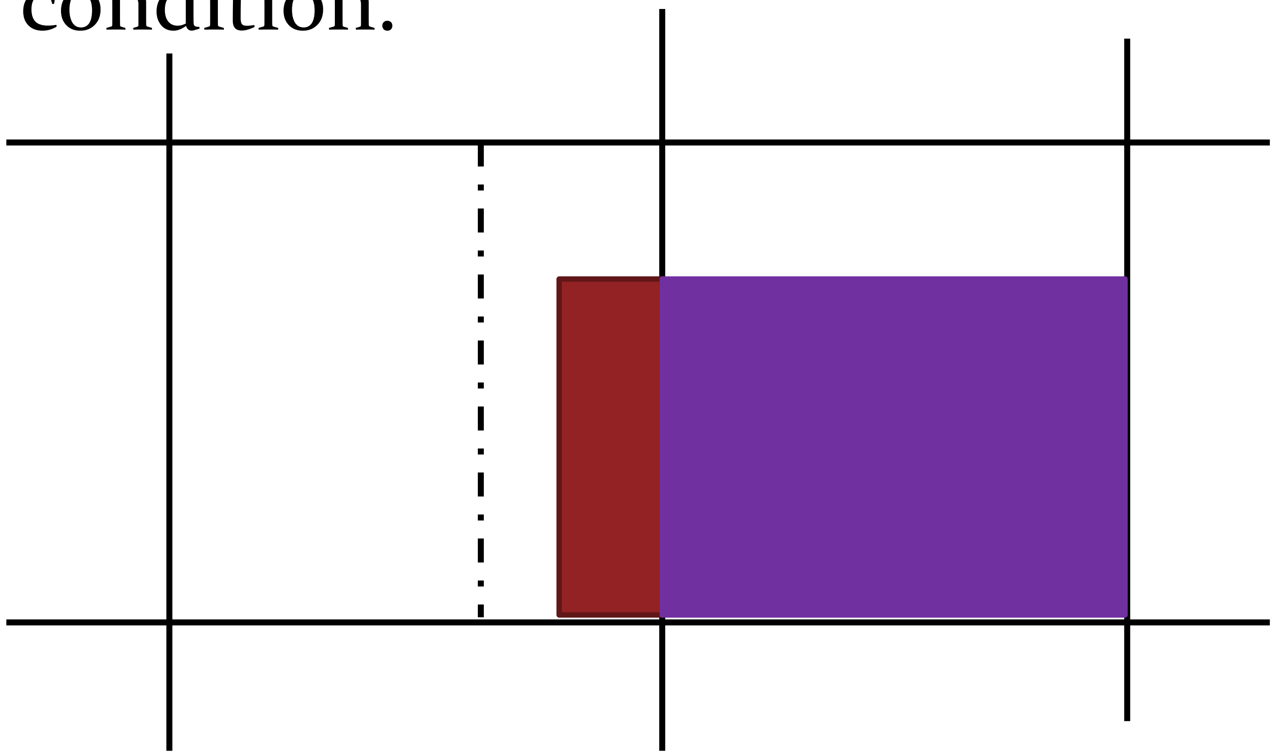


$$F_{AD} = F_A$$

- The value of F in the acceptor cell is used to define the fractional area of the cell face across which the fluid is flowing.

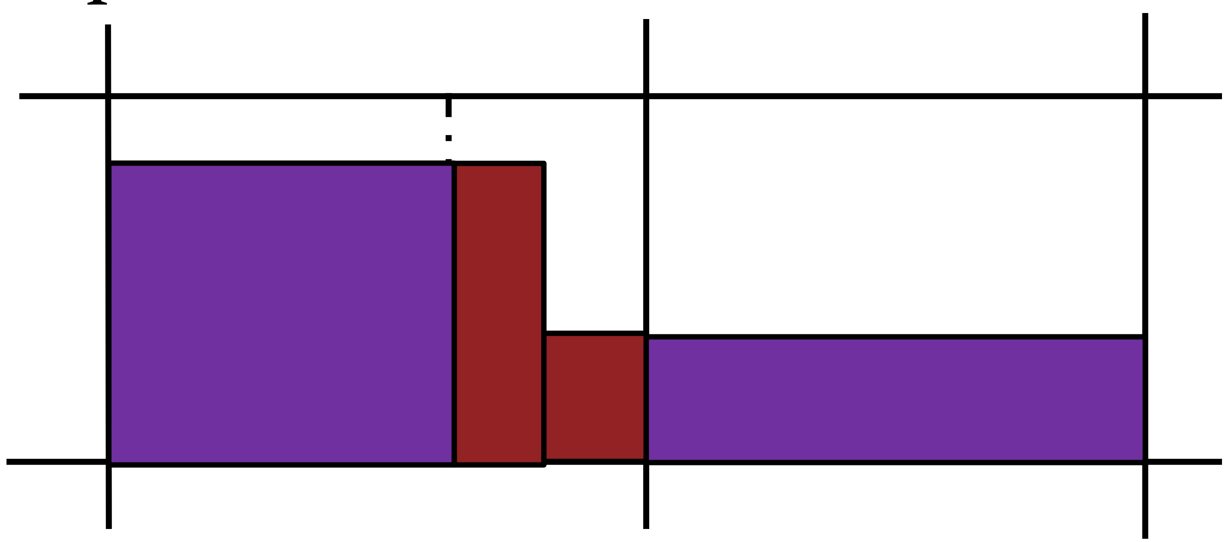
$F_{AD} = F_A$ (Case 1)

- All the fluid in the donor cell is fluxed.
- Example of exercising the MIN condition.



$F_{AD} = F_A$ (Case 2)

- More fluid than $F_A|V_x|$ must be fluxed.
- Example of exercising the MAX condition.
- Extra fluid across the boundary will be equal to CF value.



Determining the interfaces within the cell

$$Y_i = Y(x_i) = F(i, j-1)\delta y_{j-1} + F(i, j)\delta y_j + F(i, j+1)\delta y_{j+1}$$

$$\left(\frac{dY}{dx}\right)_i = \frac{2(Y_{i+1} - Y_{i-1})}{\delta x_{i+1} + 2\delta x_i + \delta x_{i-1}}$$

$$X_j = X(y_j) = F(i-1, j)\delta x_{i-1} + F(i, j)\delta x_i + F(i+1, j)\delta x_{i+1}$$

$$\left(\frac{dX}{dy}\right)_j = \frac{2(X_{j+1} - X_{j-1})}{\delta y_{j+1} + 2\delta y_j + \delta y_{j-1}}$$

$$\frac{dY}{dx} < \frac{dX}{dy} : \text{Surface is more nearly horizontal.}$$

$$\frac{dY}{dx} > \frac{dX}{dy} : \text{Surface is more nearly vertical.}$$

$F_{AD} = F_A/F_D$

- Acceptor cell: When the surface is convected normal to itself.
  - $F_{AD} = F_A$
- Donor cell: When the surface is convected more parallel to itself.
  - $F_{AD} = F_D$
- If the acceptor cell is empty, then the acceptor cell is used to determine the flux regardless of the orientation of the surface.
  - $F_{AD} = F_A$

Bookkeeping adjustments

- Surface cells have values of F lying between 0 and 1.
- However in the numerical solution F values can not be tested against the exact number such as 0 and 1.
- $F > 1 - \varepsilon_F$ (of the order of 10^{-6}).
  - F is set to 1.
- $F < \varepsilon_F$
  - F is set to zero.
  - All the neighboring full cells become surface cells by having their F values reduced from unity by an amount $1.1\ \varepsilon_{F.}$

Numerical Stability

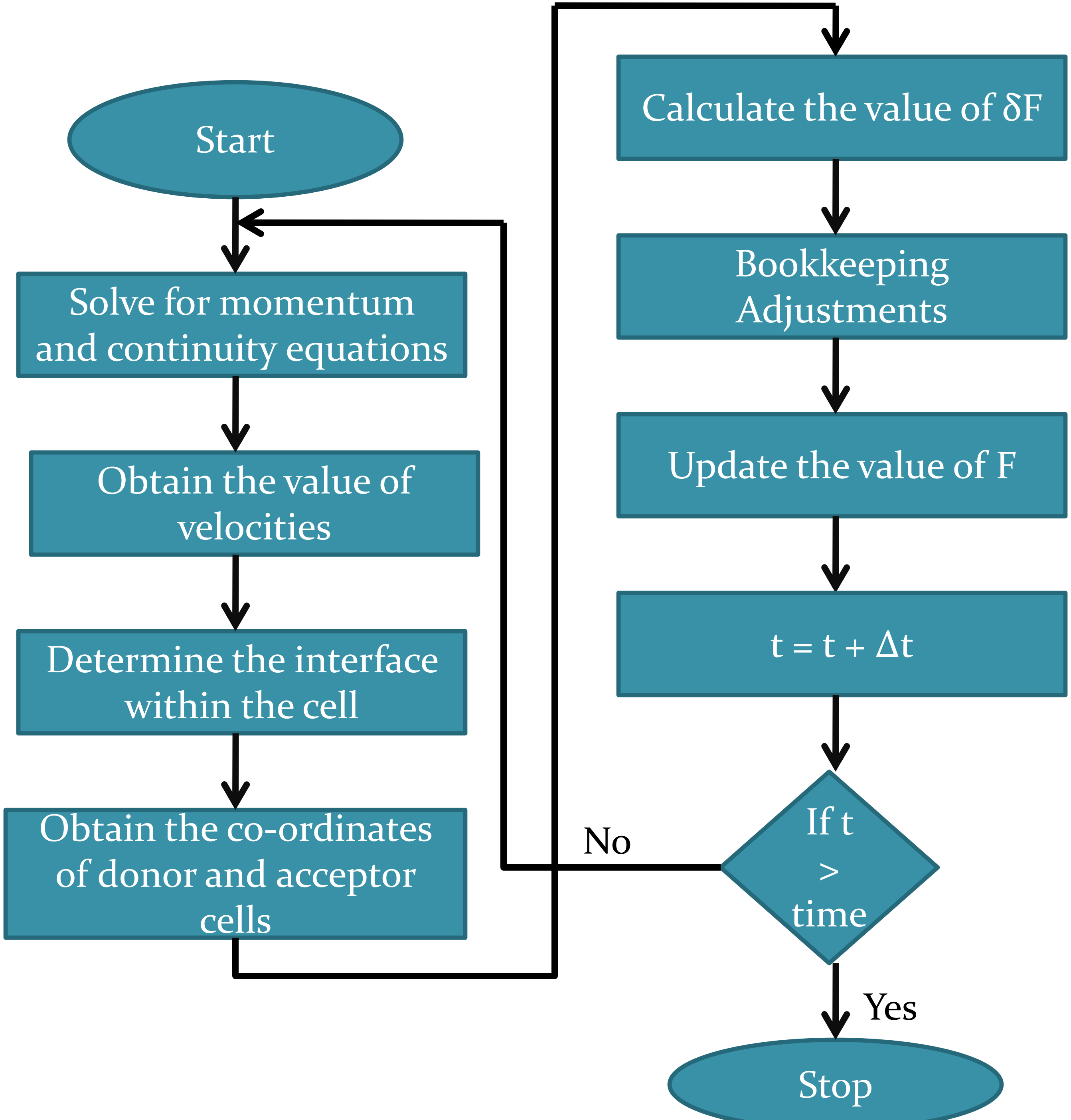
- Courant – Friedrichs – Lewy condition (CFL):

$$\left(\frac{u\delta t}{\delta x}+\frac{v\delta t}{\delta y}\right)\leq 1$$

- Grid Fourier Number:

$$\upsilon\delta t\left(\frac{1}{\delta x^2}+\frac{1}{\delta y^2}\right)\leq\frac{1}{2}$$

FLOW CHART



Start
Solve for momentum and continuity equations
Obtain the value of velocities
Determine the interface within the cell
Obtain the co-ordinates of donor and acceptor cells
Calculate the value of δF
Bookkeeping Adjustments
Update the value of F
t = t + Δt
If t > time
No
Yes
Stop