# Indian Institute of Space science and Technology

Valiamala, Thiruvananthapuram-695547, Kerala

**Computational Astrophysics - ESA614 / ESA414**

**Master of Science in Astronomy & Astrophysics**

**End Semester Examination – Forenoon Session**

*28 December 2020* *Total Marks: 20* *Time: 9:30AM - 12:30PM*

***Instructions: Write all the formulae and methods used in numerical coding to the answer sheet along with the results in sequential order in a single pdf file. Upload the codes and the pdf file as a single zip or tar.gz attachment.***

1. Consider the differential equation describing the motion of a simple pendulum:

$$\frac{d^2\theta}{dt^2} = -\frac{g}{L} sin\theta,$$

subject to the initial conditions: $\theta(t = 0) = \theta_0$ and $\frac{d\theta}{dt}|_{t=0} = 0$. Assume the length of the pendulum arm is 10 cm.

i. Write this equation as two coupled first order differential equations. [1]

ii Solve the equations [i] using simple Euler method for $\theta_0 = 10^\circ$. You may choose the step size $\Delta t = 0.1$ and extend the solution upto five periods. Plot $\theta$ vs time. [2]

iii. Did you notice any problem in the solution [ii]? If yes, explain the reason for the problem. Estimate the average period of the pendulum (*Note: do not use a visual estimation from the plot and clearly write the method of estimation of time period*). [2]

iv. Solve equations [ii] using RK4 method for $\theta_0 = 10^\circ$ and plot the solution. Estimate the average time period for $\theta_0 = 10^\circ$, $45^\circ$, $90^\circ$, $135^\circ$, $170^\circ$ and tabulate the results. [3]

v. The general expression for the time period of the pendulum is given as $T = 4\sqrt{\frac{L}{g}}K(sin\ \frac{\theta_0}{2})$, where $K(x) = \int_0^{\pi/2} \frac{dz}{\sqrt{1-x^2\, sin^2 z}}$. Integrate the function $K(x)$ for different value of $\theta_0$ to estimate the time period and compare the results with [iv] (*Note: Indicate the method used for numerical integration*). [3]

2. Consider a physical process govern by an exponential probability distribution

$$f(x) = \lambda e^{-\lambda x}; \quad x \geq 0$$

i. Assume $n$ random values $x_1, x_2, ..., x_n$ following $f(x)$ and write down the likelihood function. Derive an analytical expression of the maximum likelihood estimator ($\hat{\lambda}$) for $\lambda$. [2]

ii. Write a Monte Carlo code to generate 1000 samples with $\lambda = 1$ and $n = 10$ following the distribution $f(x)$. Estimate $\hat{\lambda}$ for each sample and plot a histogram. Also, calculate the mean ($\lambda_m$) and variance of $\hat{\lambda}$. (*Note: clearly express the method used to generate the random variables*) [5]

iii. Redo [ii] for different values of n=2, 5, 10, 20, 30 and plot $\lambda_m$ vs n. [2]