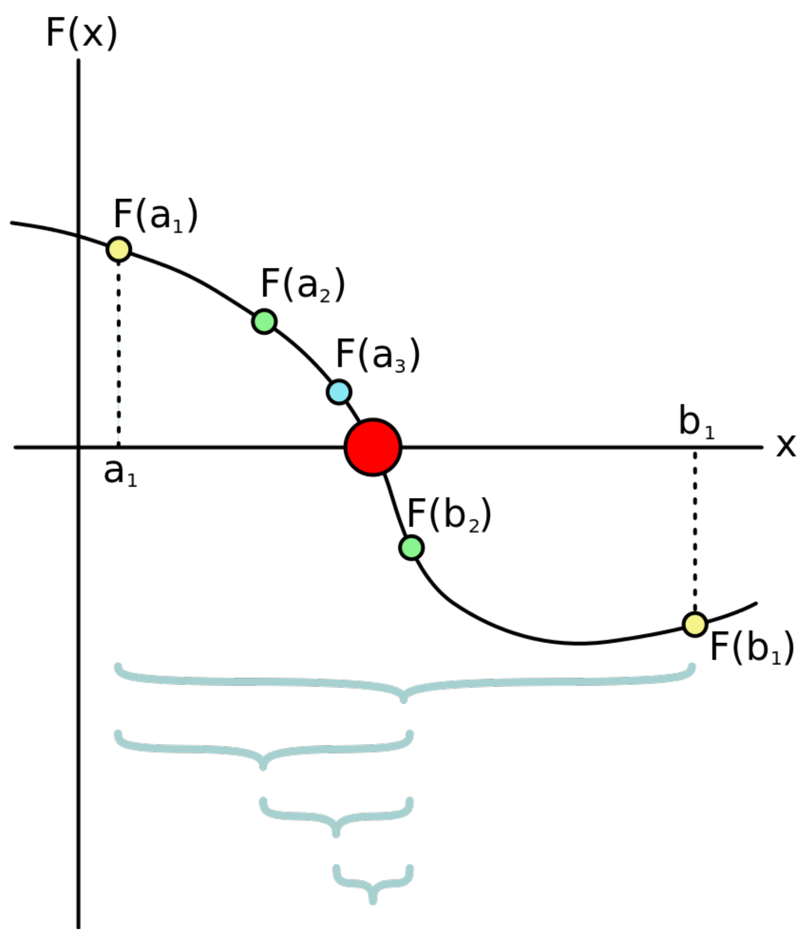
1. Temperature for interstellar grains

There is a random cloud (dust/gas) in the far away ISM. You happened to know the heating and cooling rate from physics text books. How can you apply that to get the temperature of the cloud?

Root finding

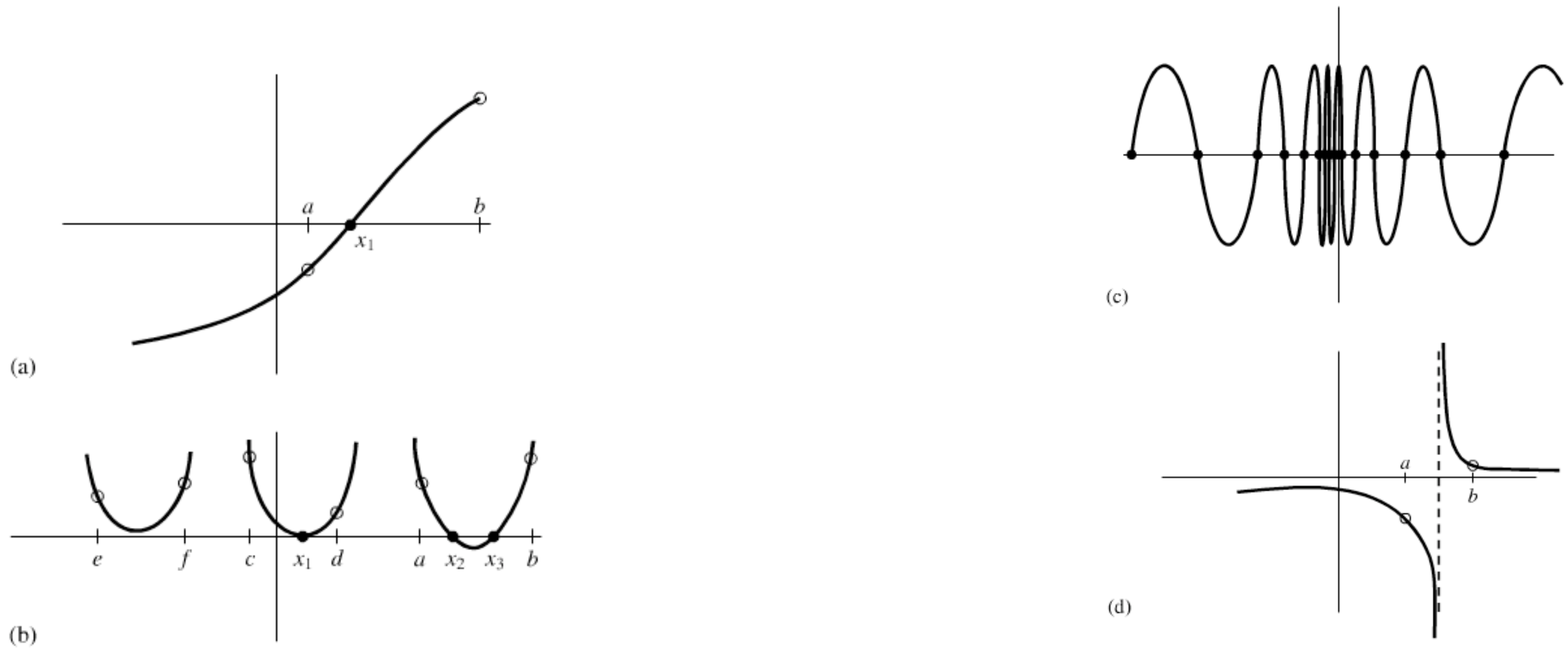


Before going ahead to
find root of f(x) = 0, graph your f(x)

At what tolerance would you want
to quote your numerical value?

- Bisection is a robust algorithm in 1D problems
- Bracketing interval decreases by 2 in every step. How many steps are required to start from ε_0 and reach a tolerance of ε?
- Bisection converges linearly (slow convergence) because successive significant figures are won linearly

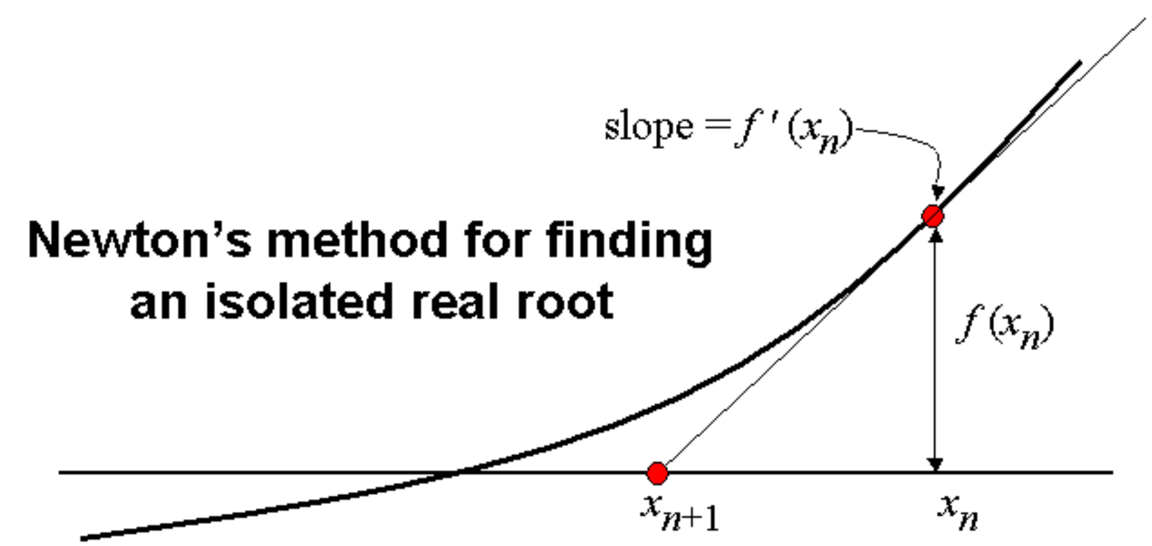
Root finding



(a) bracketed single isolated root
(b) none, double, multiple [there may be a root, but no sign change between the sides. how do we handle this?]
(c) pathological function with many roots — your result is going to heavily depend on your initial value
(d) it is bracketing a singularity, not root!! danger!

Newton-Raphson

- Better than linear convergence
- **[Both the function and the derivative if continuous near the root]** Taylor's series: $f(x) \approx f(a) + (x\text{-}a)\, f'(a) + \ldots$.
- With the derivative $f'(x)$ and the initial guess a, can we get the root, x_0?

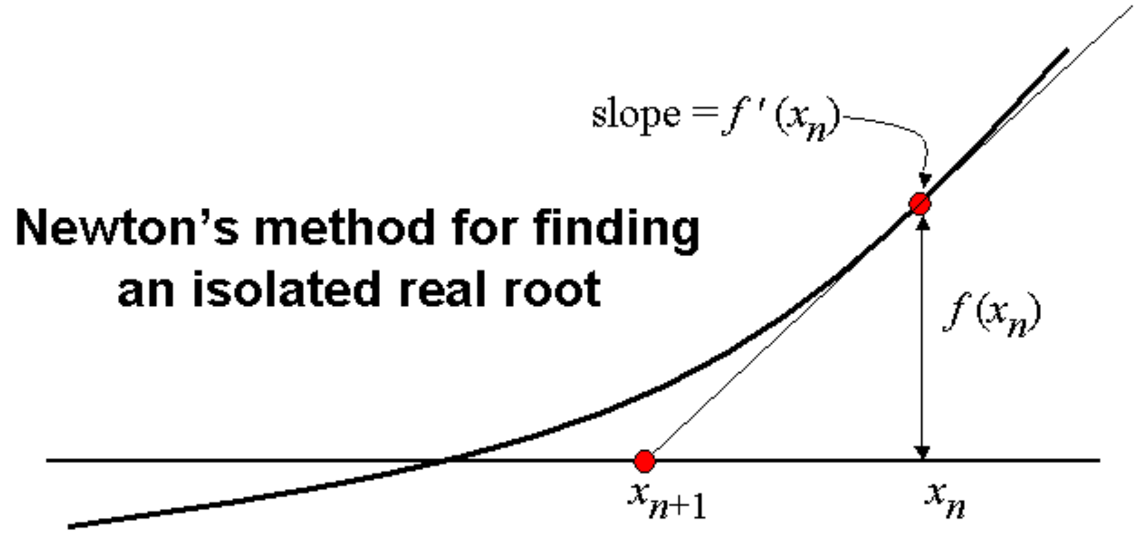


Newton-Raphson dervn

a is the initial guess of the root, which we assume to be near the original root, x.

Taylor series expansion of f(x) gives,

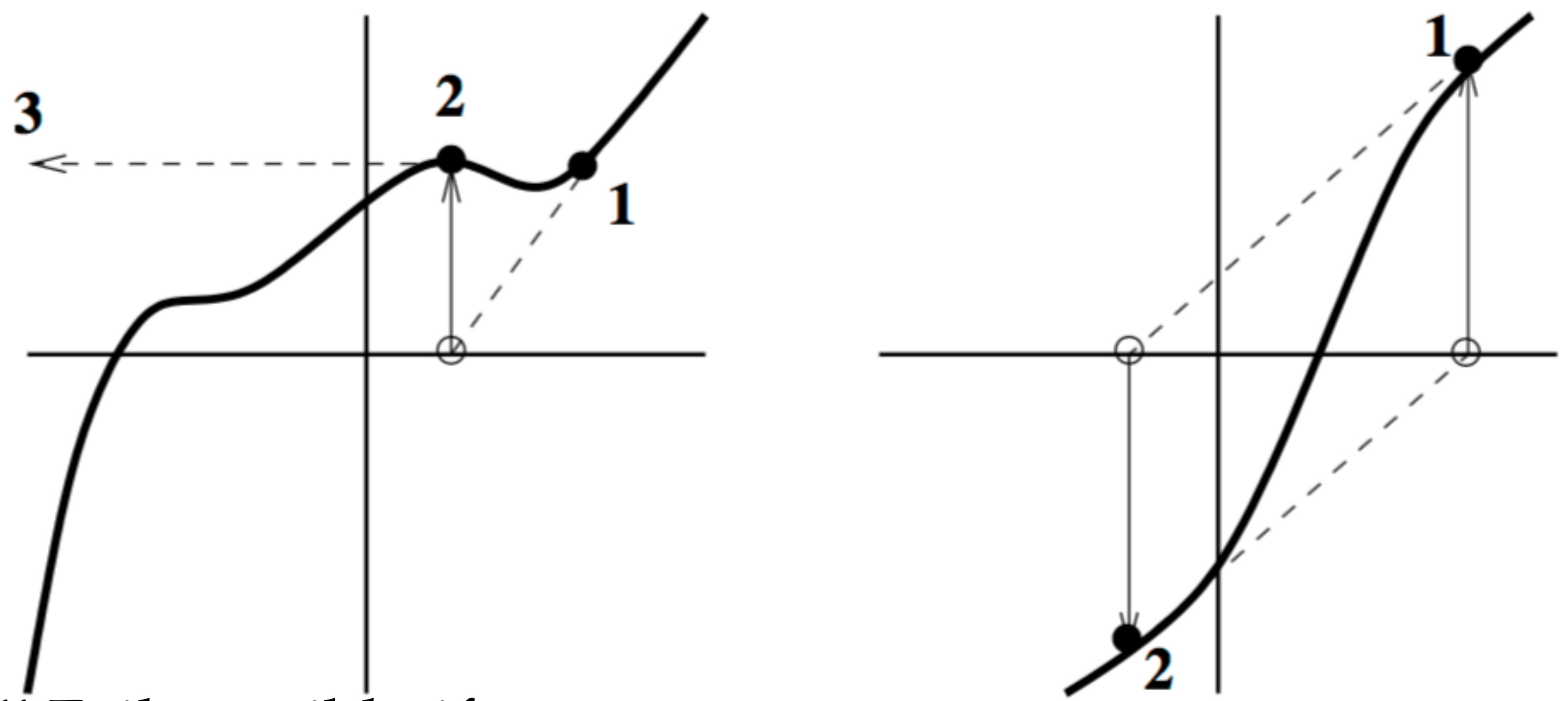
f(x) = f(a) +(x-a) f′(a)
If x is the root, f(x) =0,
==> f(a) = (a-x)f′(a)
ie., f(a)/f′(a) = a-x
therefore, x=a- f(a)/f′(a)
i.e, new guess = initial guess - f(a)/f′(a)



general exprn

$$x_{n+1} = x_n - f(x_n)/f'(x_n)$$

Newton-Raphson



- Only works near the root!!! Fails terribly if you go away from the root

- Then why use? :- quadratic convergence (very fast) :- **Prove this!**

- Best is to polish the bisection root