Computational Astrophysics - ESA614 / ESA414

Master of Science in Astronomy & Astrophysics

Lab on ODE

In this lab session, we are going to solve two coupled differential equations for calculating the distribution of electron (T_e) and protons (T_p) temperatures in an accretion disc. The differential equations are given as:

$$\frac{dT_p}{dx} + \frac{T_p(3x-4)}{3x(x-1)} - \frac{4\pi m_p r_g^3}{3k_B \dot{m}} x^2 \Gamma_{ep} = 0, \tag{1}$$

$$\frac{dT_e}{dx} + \frac{T_e(3x-4)}{3x(x-1)} + \frac{4\pi m_p r_g^3}{3k_B \dot{m}} x^2 (\Gamma_{ep} - \Lambda_e) = 0, \tag{2}$$

where x is radial distance measured in units of Schwarzschild radius ($r_g = 3 \times 10^5 \frac{M}{M_\odot}$), k_B is Boltzmann constant, n is number density, $\dot{m}$ is accretion rate, m_e and m_p are mass of electron and proton respectively. The two source term, namely Coulomb coupling (Γ_{ep}) and bremsstrahlung cooling (Λ_e) terms in cgs units are given as:

$$\Gamma_{ep} = 3.2 \times 10^{-12} \frac{k_B}{m_p} n^2 (T_p - T_e) \sqrt{\frac{m_e}{T_e^3}}$$

$$\Lambda_e = 1.4 \times 10^{-27} n^2 \sqrt{T_e}$$

The number density can be calculated from the mass conservation equation

$$n(x) = \frac{\dot{m}}{2\pi m_p r_g^2 c x^{3/2}},$$

where c is speed of light in vacuum.

(a) Solve equations (1) and (2) simultaneously using RK4 method for $M = 10M_\odot$ and x in the range [2,10^3] with the initial parameters at $x_o = 10^3$ as $T_e(x_o) = T_p(x_o) = 10^8 K$ and $\dot{m} = 10^{17}$ gm/cc.

(b) Plot T_e and T_p as a function of x.

(c) Redo both (a) and (b) for another initial condition at $x_o = 10^3$ as $T_e(x_o) = 10^8 K, T_p(x_o) = 5 \times 10^8 K$ and $\dot{m} = 10^{17}$ gm/cc.

(d) Redo both (a) and (b) for another initial condition at $x_o = 10^3$ as $T_e(x_o) = 10^8 K, T_p(x_o) = 10^8 K$ and $\dot{m} = 10^{19}$ gm/cc.

Fundamental Constants:
$k_B = 1.38 \times 10^{-16}$ erg/K
$m_e = 9.1 \times 10^{-28}$ gm
$m_p = 1.6 \times 10^{-24}$ gm