CA_LAB

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1 Coupled Linear ODE (RK-4 Method)

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1.1 Importing Libraries

[1]:
```
from rk4 import rk4
import numpy as np
from matplotlib import pyplot as plt
plt.rcdefaults()
plt.style.use('seaborn-ticks')
#plt.style.use('dark_background')
plt.style.use('seaborn-dark-palette')
#plt.style.use('bmh')
plt.rcParams.update({'font.size': 12})
```

2 RK4

[2]:
```
def rk4_cp(x , x_0 , y_0 , z_0, fy , fz , tol=1e-5):

    def rk4_next_val(f,x_0 , y_0 , h):
        f0 = f(x_0,y_0)
        f1 = f(x_0+h/2 ,  y_0+(h/2)*f0)
        f2 = f(x_0+h/2 , y_0+(h/2)*f1)
        f3 = f(x_0+h , y_0+h*f2)
        y_next = y_0+(h/6)*(f0+2*f1+2*f2+f3)
        return y_next

    def calc(h):
        n = int(abs((x_0-x)/h))
        x_next,y_next ,z_next = x_0, y_0 , z_0
        for i in range(n):
            y_next = rk4_next_val(fy(z_next) , x_next , y_next , h)
            z_next = rk4_next_val(fz(y_next) , x_next , z_next , h)
            x_next += h
        return y_next , z_next
```

```
    if(abs((x-x_0))<1e-14):
        return (y_0 , z_0)
    else:
        h = (x-x_0)/2
        prev = calc(h)
        h = h/2
        nxt = calc(h)
        err1 = abs((prev[0]-nxt[0])/(prev[0]))
        err2 = abs((prev[1]-nxt[1])/(prev[1]))
        i = 0

        while(err1>tol or err2>tol):
            i+=1
            h = h/2
            prev = nxt
            nxt = calc(h)
            err1 = abs((prev[0]-nxt[0])/(prev[0]))
            err2 = abs((prev[1]-nxt[1])/(prev[1]))
        return nxt
```

2.1 Defining Constants

all in CGS Units

[4]:
```
kb = 1.38e-16 #Boltzman's constant
me = 9.1e-28 # Mass of electron
mp = 1.6e-24 #mass of proton
c = 2.99792458e10 # speed of light
rg = 3e6 # Swarchchild radius for M = 10 M_sun
```

2.2 Defining Functions

2.2.1 Coulomb Coupling Γ_{ep}

[5]:
```
def gamma(n , tp, te ):
    val = (3.2e-12)*(kb/mp)*(n**2)*(tp-te)*(((me)/(te**3))**0.5)
    return val
```

2.2.2 Bremsstrahlung cooling Λ_e

[6]:
```
def lmd(n,te):
    val = (1.4e-27)*(n**2)*(te**0.5)
    return val
```

2.2.3 Number Density

[7]:
```
def nx(m_dot ,  x):
    denom = 2*np.pi*mp*(rg**2)*c*(x**(3/2))
    val = m_dot/denom
    return val
```

2.2.4 Derivative of T_p w.r.t x

$\frac{dT_p}{dx}$

[8]:
```
def f_tp_wrap(m_dot):
    def f_tp(te):
        def f_in(x, tp):
            n = nx(m_dot,x)
            g = gamma(n , tp , te)
            term1 = ((4*np.pi*mp*rg**3)/(3*kb*m_dot))*g*(x**2)
            term2 = tp*((3*x-4)/(3*x*(x-1)))
            #print(term1,term2)
            val = term1 - term2
            return val
        return f_in
    return f_tp
```

2.2.5 Derivative of T_e w.r.t x

$\frac{dT_e}{dx}$

[9]:
```
def f_te_wrap(m_dot):
    def f_te(tp):
        def f_in(x, te):
            #print(te)
            n = nx(m_dot,x)
            g = gamma(n , tp , te)
            l = lmd(n,te)
            term1 = ((4*np.pi*mp*rg**3)/(3*kb*m_dot))*(g-l)*(x**2)
            term2 = te*((3*x-4)/(3*x*(x-1)))
            #print(term1 , term2)
            val = -term2 -term1
            return val
        return f_in
    return f_te
```

2.3 Problem (c)

Initial Conditions > $x_0 = 10^3$, $Te_0 = 10^8 K$, $Tp_0 = 10^8 K$, $\dot{m} = 10^{17} gm/cc$

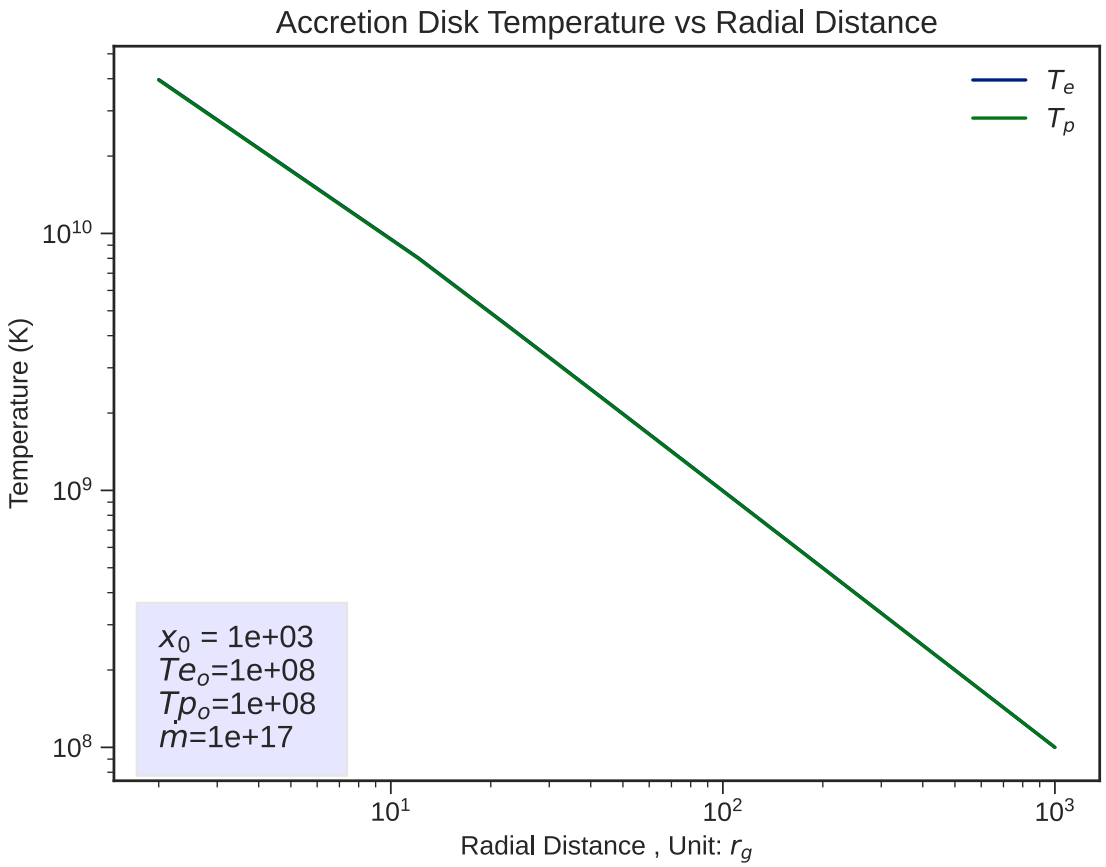
[10]:
```
te_0 = 1e8
tp_0 = 1e8
x_0 = 1e3
m_dot_0 = 1e17
x_range = np.linspace(2,1e3 , 100)
t_q1 = []
for x in x_range:
    temp = rk4_cp(float(x),x_0,te_0,tp_0,f_tp_wrap(m_dot_0),f_te_wrap(m_dot_0)␣
 ↪, tol=1e-5)
    print('x:{:.2f} ,  T_p: {:.2f} , T_e : {:.2f}'.format(x,temp[0],temp[1]))
    t_q1.append(list(temp))
#data = rk4_cp(x , x_0 , te_0 , tp_0 , f_tp_wrap(m_dot_0) , f_te_wrap(m_dot_0))
```

```
x:2.00 ,  T_p: 39693279993.54 , T_e : 39513453316.68
x:12.08 ,  T_p: 8044271128.46 , T_e : 8010524490.09
x:22.16 ,  T_p: 4444324764.43 , T_e : 4426540395.23
x:32.24 ,  T_p: 3069752199.03 , T_e : 3057932335.87
x:42.32 ,  T_p: 2344510951.94 , T_e : 2335783869.21
x:52.40 ,  T_p: 1896440872.94 , T_e : 1889595909.94
x:62.48 ,  T_p: 1592148827.60 , T_e : 1586564701.83
x:72.57 ,  T_p: 1372001729.73 , T_e : 1367318351.68
x:82.65 ,  T_p: 1205337702.33 , T_e : 1201328234.15
x:92.73 ,  T_p: 1074778305.91 , T_e : 1071290891.91
x:102.81 ,  T_p: 969738270.10 , T_e : 966666420.64
x:112.89 ,  T_p: 883401765.74 , T_e : 880668017.71
x:122.97 ,  T_p: 811177782.24 , T_e : 808731943.25
x:133.05 ,  T_p: 749874119.68 , T_e : 747662925.08
x:143.13 ,  T_p: 697185388.81 , T_e : 695174020.86
x:153.21 ,  T_p: 651414857.93 , T_e : 649575537.13
x:163.29 ,  T_p: 611283908.94 , T_e : 609594132.30
x:173.37 ,  T_p: 575810721.22 , T_e : 574252017.19
x:183.45 ,  T_p: 544228878.38 , T_e : 542785907.06
x:193.54 ,  T_p: 515931367.54 , T_e : 514591257.60
x:203.62 ,  T_p: 490431185.90 , T_e : 489183037.33
x:213.70 ,  T_p: 467333079.32 , T_e : 466167584.80
x:223.78 ,  T_p: 446312908.09 , T_e : 445222060.68
x:233.86 ,  T_p: 427102341.78 , T_e : 426079206.26
x:243.94 ,  T_p: 409477343.60 , T_e : 408515875.81
x:254.02 ,  T_p: 393249393.15 , T_e : 392344295.57
x:264.10 ,  T_p: 378258716.79 , T_e : 377405322.06
x:274.18 ,  T_p: 364369009.48 , T_e : 363563185.66
x:284.26 ,  T_p: 351463278.43 , T_e : 350701351.29
x:294.34 ,  T_p: 339440539.77 , T_e : 338719228.64
x:304.42 ,  T_p: 328213170.94 , T_e : 327529535.25
x:314.51 ,  T_p: 317704771.55 , T_e : 317056166.19
x:324.59 ,  T_p: 307846904.33 , T_e : 307233979.95
x:334.67 ,  T_p: 298583838.56 , T_e : 298001201.95
```

x:344.75 , T_p: 289861966.24 , T_e : 289307668.67
x:354.83 , T_p: 281635204.31 , T_e : 281107471.34
x:364.91 , T_p: 273862557.28 , T_e : 273359768.97
x:374.99 , T_p: 266507434.03 , T_e : 266028107.83
x:385.07 , T_p: 259537071.79 , T_e : 259079847.74
x:395.15 , T_p: 252922048.22 , T_e : 252485676.06
x:405.23 , T_p: 246635866.29 , T_e : 246219194.20
x:415.31 , T_p: 240654599.49 , T_e : 240256564.26
x:425.39 , T_p: 234956587.54 , T_e : 234576205.89
x:435.47 , T_p: 229522174.28 , T_e : 229158535.19
x:445.56 , T_p: 224333481.10 , T_e : 223985739.11
x:455.64 , T_p: 219374210.46 , T_e : 219041579.62
x:465.72 , T_p: 214629474.91 , T_e : 214311223.49
x:475.80 , T_p: 210085647.79 , T_e : 209781093.50
x:485.88 , T_p: 205730232.55 , T_e : 205438738.31
x:495.96 , T_p: 201551747.96 , T_e : 201272718.08
x:506.04 , T_p: 197538662.74 , T_e : 197273469.55
x:516.12 , T_p: 193683223.74 , T_e : 193429294.05
x:526.20 , T_p: 189975407.22 , T_e : 189732257.15
x:536.28 , T_p: 186406893.52 , T_e : 186174068.37
x:546.36 , T_p: 182969976.58 , T_e : 182747048.62
x:556.44 , T_p: 179657508.47 , T_e : 179444074.88
x:566.53 , T_p: 176462849.69 , T_e : 176258530.74
x:576.61 , T_p: 173379824.79 , T_e : 173184262.16
x:586.69 , T_p: 170402682.54 , T_e : 170215537.78
x:596.77 , T_p: 167526060.08 , T_e : 167347013.20
x:606.85 , T_p: 164744950.70 , T_e : 164573698.93
x:616.93 , T_p: 162054674.77 , T_e : 161890931.34
x:627.01 , T_p: 159450853.42 , T_e : 159294346.54
x:637.09 , T_p: 156929384.82 , T_e : 156779856.64
x:647.17 , T_p: 154485684.30 , T_e : 154344367.87
x:657.25 , T_p: 152117671.82 , T_e : 151982748.69
x:667.33 , T_p: 149821159.61 , T_e : 149692414.72
x:677.41 , T_p: 147592957.16 , T_e : 147470185.91
x:687.49 , T_p: 145430061.03 , T_e : 145313068.59
x:697.58 , T_p: 143329641.29 , T_e : 143218242.04
x:707.66 , T_p: 141289029.17 , T_e : 141183046.14
x:717.74 , T_p: 139305705.79 , T_e : 139204970.16
x:727.82 , T_p: 137377291.75 , T_e : 137281642.37
x:737.90 , T_p: 135501537.62 , T_e : 135410820.55
x:747.98 , T_p: 133675677.88 , T_e : 133591021.90
x:758.06 , T_p: 131899030.97 , T_e : 131818901.59
x:768.14 , T_p: 130168987.60 , T_e : 130093257.74
x:778.22 , T_p: 128483737.47 , T_e : 128412285.50
x:788.30 , T_p: 126841562.87 , T_e : 126774272.29
x:798.38 , T_p: 125240832.78 , T_e : 125177591.97
x:808.46 , T_p: 123679997.52 , T_e : 123620699.46
x:818.55 , T_p: 122156987.63 , T_e : 122102723.10

```
x:828.63 ,  T_p: 120671665.31 , T_e : 120620998.73
x:838.71 ,  T_p: 119222022.82 , T_e : 119174870.73
x:848.79 ,  T_p: 117806789.41 , T_e : 117763071.66
x:858.87 ,  T_p: 116424753.95 , T_e : 116384393.51
x:868.95 ,  T_p: 115074761.50 , T_e : 115037684.33
x:879.03 ,  T_p: 113755221.27 , T_e : 113722334.87
x:889.11 ,  T_p: 112466142.15 , T_e : 112436232.49
x:899.19 ,  T_p: 111205938.40 , T_e : 111178957.06
x:909.27 ,  T_p: 109973120.49 , T_e : 109950081.15
x:919.35 ,  T_p: 108767945.91 , T_e : 108747510.47
x:929.43 ,  T_p: 107588875.08 , T_e : 107571031.55
x:939.52 ,  T_p: 106434614.19 , T_e : 106420259.95
x:949.60 ,  T_p: 105305409.38 , T_e : 105293339.06
x:959.68 ,  T_p: 104199471.51 , T_e : 104190519.93
x:969.76 ,  T_p: 103116587.95 , T_e : 103110537.49
x:979.84 ,  T_p: 102056167.78 , T_e : 102052566.65
x:989.92 ,  T_p: 101017811.09 , T_e : 101015648.66
x:1000.00 ,  T_p: 100000000.00 , T_e : 100000000.00
```

[12]:
```
te_0 = 1e8
tp_0 = 1e8
x_0 = 1e3
m_dot_0 = 1e17
t_q1 = np.asarray(t_q1)
te_q1 = t_q1[:,0]
tp_q1 = t_q1[:,1]
fig = plt.figure(figsize=(8,6))
ax = fig.add_subplot(111)
ax.loglog(x_range, te_q1)
ax.loglog(x_range,tp_q1)
ax.set_xlabel('Radial Distance , Unit: $r_g$')
ax.set_ylabel('Temperature (K)')
ax.set_title('Accretion Disk Temperature vs Radial Distance')
ax.text(2,1e8 , '$x_0$ = {:.0e}\n$Te_o$={:.0e}\n$Tp_o$={:.0e} \n$\dot m$={}'.
 ↪format(x_0,te_0,tp_0,m_dot_0) , bbox = {'facecolor':'blue' , 'alpha':0.1,␣
 ↪'pad':10 } , fontsize = 14)
plt.legend(['$T_e$','$T_p$'])
plt.show()
```



2.4 Problem (c)

Initial Conditions > $x_0 = 10^3$ $Te_0 = 10^8 K$ $Tp_0 = 5 \times 10^8 K$ $\dot{m} = 10^{17} gm/cc$

[15]:
```
te_0 = 1e8
tp_0 = 5e8
x_0 = 1e3
m_dot_0 = 1e17
x_range = np.linspace(2,1e3 , 100)
t_q2 = []
#print('X , $T_p$ , ')
for x in x_range:
    temp = rk4_cp(x,x_0,te_0,tp_0,f_tp_wrap(m_dot_0),f_te_wrap(m_dot_0) , tol =␣
↪1e-5)
    print('x:{:.2f} ,  T_p: {:.2f} , T_e : {:.2f}'.format(x,temp[0],temp[1]))
    t_q2.append(list(temp))
#data = rk4_cp(x , x_0 , te_0 , tp_0 , f_tp_wrap(m_dot_0) , f_te_wrap(m_dot_0))
```

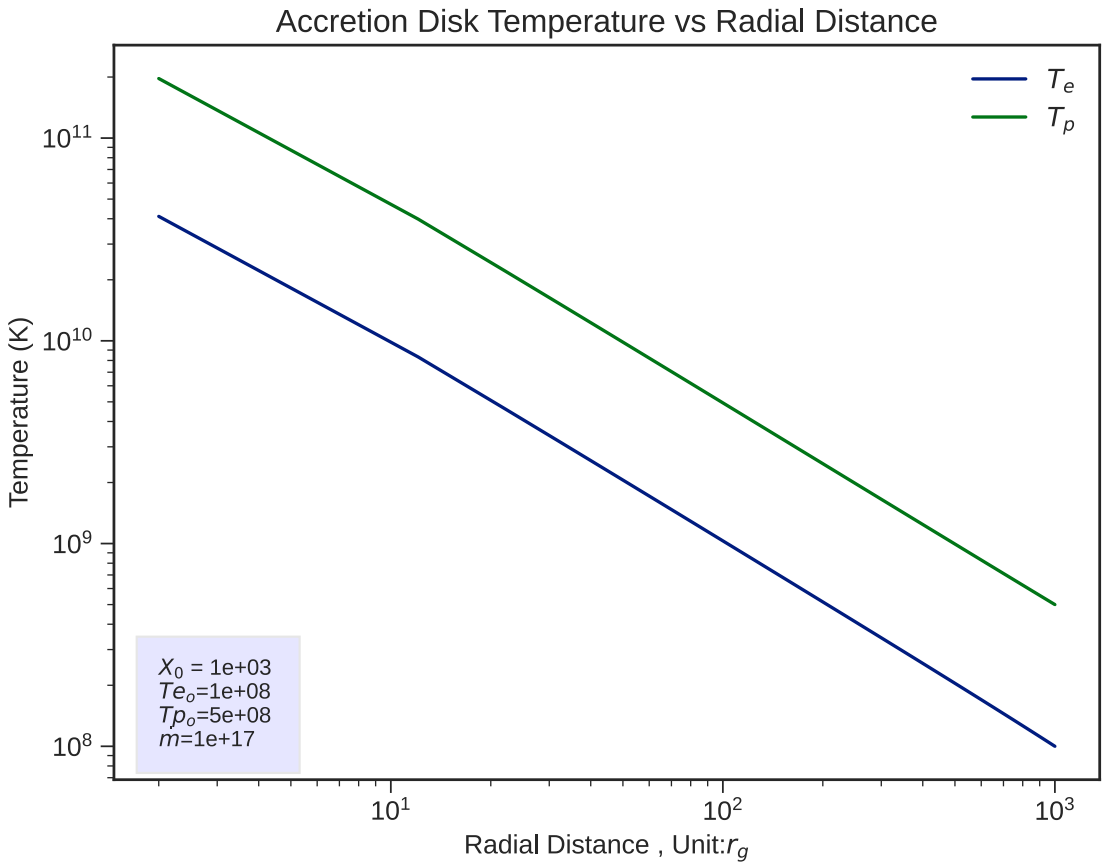
```
x:2.00 ,  T_p: 41099734419.50 , T_e : 196666395274.85
x:12.08 ,  T_p: 8328943049.92 , T_e : 39863000296.28
```

```
x:22.16 ,  T_p: 4601299360.18 , T_e : 22025895685.00
x:32.24 ,  T_p: 3177897142.35 , T_e : 15214874197.91
x:42.32 ,  T_p: 2426858166.52 , T_e : 11621209999.66
x:52.40 ,  T_p: 1962825167.21 , T_e : 9400925318.68
x:62.48 ,  T_p: 1647673899.08 , T_e : 7893071862.37
x:72.57 ,  T_p: 1419655803.42 , T_e : 6802169602.41
x:82.65 ,  T_p: 1247021030.06 , T_e : 5976287837.92
x:92.73 ,  T_p: 1111774348.81 , T_e : 5329314722.80
x:102.81 ,  T_p: 1002954700.29 , T_e : 4808798637.84
x:112.89 ,  T_p: 913504217.56 , T_e : 4380965772.94
x:122.97 ,  T_p: 838672875.41 , T_e : 4023085985.44
x:133.05 ,  T_p: 775146481.67 , T_e : 3719300009.64
x:143.13 ,  T_p: 720542268.10 , T_e : 3458205718.77
x:153.21 ,  T_p: 673103160.01 , T_e : 3231395115.92
x:163.29 ,  T_p: 631505050.29 , T_e : 3032532082.66
x:173.37 ,  T_p: 594731175.65 , T_e : 2856751230.62
x:183.45 ,  T_p: 561987832.99 , T_e : 2700254578.04
x:193.54 ,  T_p: 532646389.60 , T_e : 2560034050.49
x:203.62 ,  T_p: 506202490.28 , T_e : 2433676274.12
x:213.70 ,  T_p: 482246791.85 , T_e : 2319222531.17
x:223.78 ,  T_p: 460445853.26 , T_e : 2215069009.62
x:233.86 ,  T_p: 440517089.00 , T_e : 2119880774.92
x:243.94 ,  T_p: 422230906.24 , T_e : 2032550279.82
x:254.02 ,  T_p: 405392051.02 , T_e : 1952143274.83
x:264.10 ,  T_p: 389835040.30 , T_e : 1877867967.27
x:274.18 ,  T_p: 375418696.03 , T_e : 1809048865.68
x:284.26 ,  T_p: 362021840.79 , T_e : 1745106182.72
x:294.34 ,  T_p: 349539876.98 , T_e : 1685539465.75
x:304.42 ,  T_p: 337882044.87 , T_e : 1629914476.03
x:314.51 ,  T_p: 326969207.34 , T_e : 1577852588.61
x:324.59 ,  T_p: 316732047.06 , T_e : 1529022165.31
x:334.67 ,  T_p: 307109589.11 , T_e : 1483131485.36
x:344.75 ,  T_p: 298047982.51 , T_e : 1439922915.04
x:354.83 ,  T_p: 289499489.33 , T_e : 1399168070.27
x:364.91 ,  T_p: 281421641.13 , T_e : 1360663780.18
x:374.99 ,  T_p: 273776531.35 , T_e : 1324228701.26
x:385.07 ,  T_p: 266530218.74 , T_e : 1289700462.74
x:395.15 ,  T_p: 259652222.07 , T_e : 1256933248.65
x:405.23 ,  T_p: 253115090.11 , T_e : 1225795740.42
x:415.31 ,  T_p: 246894034.26 , T_e : 1196169358.87
x:425.39 ,  T_p: 240967783.24 , T_e : 1167948042.36
x:435.47 ,  T_p: 235313560.61 , T_e : 1141031723.07
x:445.56 ,  T_p: 229914088.04 , T_e : 1115333165.12
x:455.64 ,  T_p: 224752479.05 , T_e : 1090771597.52
x:465.72 ,  T_p: 219813307.88 , T_e : 1067273237.59
x:475.80 ,  T_p: 215082454.85 , T_e : 1044770551.17
x:485.88 ,  T_p: 210546970.99 , T_e : 1023201604.88
x:495.96 ,  T_p: 206194959.15 , T_e : 1002509497.31
```

x:506.04 , T_p: 202015469.32 , T_e : 982641858.12
x:516.12 , T_p: 197998406.21 , T_e : 963550405.89
x:526.20 , T_p: 194134447.42 , T_e : 945190556.64
x:536.28 , T_p: 190414970.89 , T_e : 927521076.60
x:546.36 , T_p: 186831990.31 , T_e : 910503773.28
x:556.44 , T_p: 183378097.57 , T_e : 894103220.19
x:566.53 , T_p: 180046411.39 , T_e : 878286510.88
x:576.61 , T_p: 176831416.21 , T_e : 863024007.56
x:586.69 , T_p: 173725322.40 , T_e : 848285203.60
x:596.77 , T_p: 170723518.35 , T_e : 834044557.13
x:606.85 , T_p: 167820814.34 , T_e : 820277253.89
x:616.93 , T_p: 165012359.55 , T_e : 806960100.64
x:627.01 , T_p: 162293614.85 , T_e : 794071394.89
x:637.09 , T_p: 159660328.12 , T_e : 781590807.11
x:647.17 , T_p: 157108511.97 , T_e : 769499273.75
x:657.25 , T_p: 154634423.41 , T_e : 757778900.29
x:667.33 , T_p: 152234545.41 , T_e : 746412872.92
x:677.41 , T_p: 149905570.10 , T_e : 735385378.23
x:687.49 , T_p: 147644383.48 , T_e : 724681529.84
x:697.58 , T_p: 145448769.74 , T_e : 714288085.69
x:707.66 , T_p: 143314466.57 , T_e : 704190185.66
x:717.74 , T_p: 141239642.39 , T_e : 694376197.59
x:727.82 , T_p: 139221829.79 , T_e : 684834328.08
x:737.90 , T_p: 137258696.06 , T_e : 675553427.93
x:747.98 , T_p: 135348034.14 , T_e : 666522948.74
x:758.06 , T_p: 133487754.28 , T_e : 657732902.97
x:768.14 , T_p: 131675876.29 , T_e : 649173827.17
x:778.22 , T_p: 129910522.53 , T_e : 640836748.02
x:788.30 , T_p: 128190518.30 , T_e : 632713813.10
x:798.38 , T_p: 126512893.18 , T_e : 624795542.57
x:808.46 , T_p: 124876715.79 , T_e : 617074993.17
x:818.55 , T_p: 123280457.57 , T_e : 609544861.42
x:828.63 , T_p: 121722664.25 , T_e : 602198199.10
x:838.71 , T_p: 120201951.41 , T_e : 595028391.95
x:848.79 , T_p: 118717565.70 , T_e : 588029757.75
x:858.87 , T_p: 117267040.08 , T_e : 581194968.31
x:868.95 , T_p: 115849827.79 , T_e : 574519011.40
x:879.03 , T_p: 114464784.90 , T_e : 567996423.35
x:889.11 , T_p: 113111392.05 , T_e : 561622617.99
x:899.19 , T_p: 111787355.40 , T_e : 555391236.34
x:909.27 , T_p: 110492367.68 , T_e : 549298277.73
x:919.35 , T_p: 109225475.21 , T_e : 543339189.07
x:929.43 , T_p: 107986203.97 , T_e : 537510098.96
x:939.52 , T_p: 106772684.38 , T_e : 531805736.26
x:949.60 , T_p: 105584658.25 , T_e : 526222750.75
x:959.68 , T_p: 104421323.86 , T_e : 520757317.33
x:969.76 , T_p: 103281912.82 , T_e : 515405769.92
x:979.84 , T_p: 102165688.31 , T_e : 510164593.29

```
x:989.92 ,  T_p: 101071943.49 , T_e : 505030415.35
x:1000.00 ,  T_p: 100000000.00 , T_e : 500000000.00
```

[17]:
```
te_0 = 1e8
tp_0 = 5e8
x_0 = 1e3
m_dot_0 = 1e17
t_q2 = np.asarray(t_q2)
te_q2 = t_q2[:,0]
tp_q2 = t_q2[:,1]
fig = plt.figure(figsize=(8,6))
ax = fig.add_subplot(111)
ax.loglog(x_range, te_q2)
ax.loglog(x_range,tp_q2)
ax.set_xlabel('Radial Distance , Unit:$r_g$')
ax.set_ylabel('Temperature (K)')
ax.set_title('Accretion Disk Temperature vs Radial Distance')
ax.text(2,1e8 , '$X_0$ = {:.0e}\n$Te_o$={:.0e}\n$Tp_o$={:.0e} \n$\dot m$={}'.
 ↪format(x_0,te_0,tp_0,m_dot_0) , bbox = {'facecolor':'blue' , 'alpha':0.1,␣
 ↪'pad':10 } , fontsize = 10)
plt.legend(['$T_e$','$T_p$'])
plt.show()
```



2.5 Problem (c)

Initial Conditions

$$x_0 = 10^3 \ , Te_0 = 10^8 K \ , Tp_0 = 10^8 K \ , \dot{m} = 10^{19} gm/cc$$

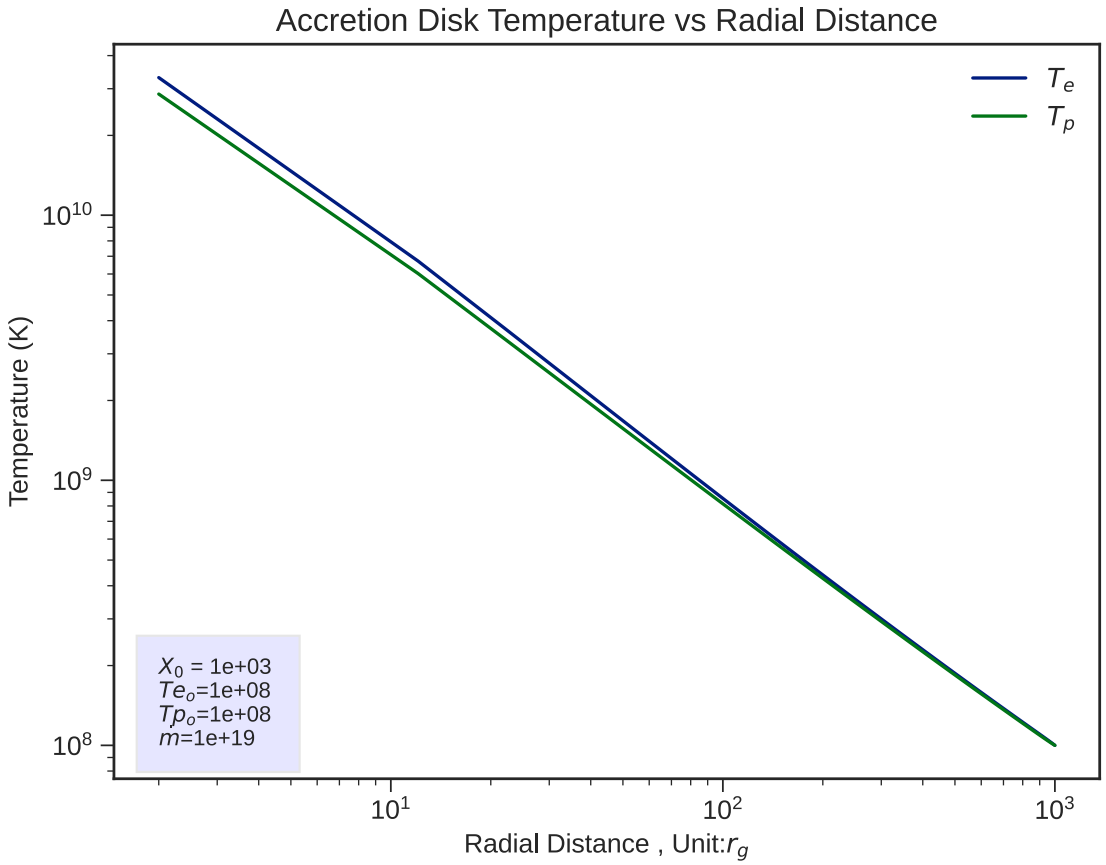
[18]:
```
te_0 = 1e8
tp_0 = 1e8
x_0 = 1e3
m_dot_0 = 1e19
x_range = np.linspace(2,1e3 , 100)
t_q3 = []

for x in x_range:
    temp = rk4_cp(float(x),x_0,te_0,tp_0,f_tp_wrap(m_dot_0),f_te_wrap(m_dot_0)␣
 ↪, tol = 1e-5)
    print('x:{:.2f} ,  T_p: {:.2f} , T_e : {:.2f}'.format(x,temp[0],temp[1]))
    t_q3.append(list(temp))

#data = rk4_cp(2.0 , x_0 , te_0 , tp_0 , f_tp_wrap(m_dot_0) ,␣
 ↪f_te_wrap(m_dot_0) , tol=1e-2)
#print(data)
```

```
x:2.00 ,  T_p: 33046472980.86 , T_e : 28647794323.50
x:12.08 ,  T_p: 6713327149.33 , T_e : 6023213391.39
x:22.16 ,  T_p: 3719713250.06 , T_e : 3392917304.86
x:32.24 ,  T_p: 2577133572.48 , T_e : 2377134991.36
x:42.32 ,  T_p: 1974428603.27 , T_e : 1836505639.92
x:52.40 ,  T_p: 1602103190.55 , T_e : 1500043463.01
x:62.48 ,  T_p: 1349238440.90 , T_e : 1270100391.27
x:72.57 ,  T_p: 1166268721.16 , T_e : 1102810214.35
x:82.65 ,  T_p: 1027715467.87 , T_e : 975522967.10
x:92.73 ,  T_p: 919141596.64 , T_e : 875352028.36
x:102.81 ,  T_p: 831755388.12 , T_e : 794420430.20
x:112.89 ,  T_p: 759896994.49 , T_e : 727639581.56
x:122.97 ,  T_p: 699754829.53 , T_e : 671577539.77
x:133.05 ,  T_p: 648678772.55 , T_e : 623823763.19
x:143.13 ,  T_p: 604755295.57 , T_e : 582649117.35
x:153.21 ,  T_p: 566576199.11 , T_e : 546772467.23
x:163.29 ,  T_p: 533080374.48 , T_e : 515225998.45
x:173.37 ,  T_p: 503453063.02 , T_e : 487264800.82
x:183.45 ,  T_p: 477058277.36 , T_e : 462306084.09
x:193.54 ,  T_p: 453392311.22 , T_e : 439887275.07
x:203.62 ,  T_p: 432051040.45 , T_e : 419636486.38
x:213.70 ,  T_p: 412706466.69 , T_e : 401251291.79
x:223.78 ,  T_p: 395089593.51 , T_e : 384483202.92
```

```
x:233.86 ,  T_p: 378977725.84 , T_e : 369126135.31
x:243.94 ,  T_p: 364183293.41 , T_e : 355009449.97
x:254.02 ,  T_p: 350553191.42 , T_e : 341984243.20
x:264.10 ,  T_p: 337953011.27 , T_e : 329929057.32
x:274.18 ,  T_p: 326269588.08 , T_e : 318738492.07
x:284.26 ,  T_p: 315405778.16 , T_e : 308321938.99
x:294.34 ,  T_p: 305277718.99 , T_e : 298601069.34
x:304.42 ,  T_p: 295812633.07 , T_e : 289507817.73
x:314.51 ,  T_p: 286947053.70 , T_e : 280982751.02
x:324.59 ,  T_p: 278625381.08 , T_e : 272973739.03
x:334.67 ,  T_p: 270798699.06 , T_e : 265434863.95
x:344.75 ,  T_p: 263423799.35 , T_e : 258325519.71
x:354.83 ,  T_p: 256462371.82 , T_e : 251609663.88
x:364.91 ,  T_p: 249880328.99 , T_e : 245255192.69
x:374.99 ,  T_p: 243647239.20 , T_e : 239233416.13
x:385.07 ,  T_p: 237735848.91 , T_e : 233518615.00
x:395.15 ,  T_p: 232121677.90 , T_e : 228087665.20
x:405.23 ,  T_p: 226781686.80 , T_e : 222920827.55
x:415.31 ,  T_p: 221697996.57 , T_e : 217996969.19
x:425.39 ,  T_p: 216851518.81 , T_e : 213300189.09
x:435.47 ,  T_p: 212225884.14 , T_e : 208814969.34
x:445.56 ,  T_p: 207806199.85 , T_e : 204527180.43
x:455.64 ,  T_p: 203578886.87 , T_e : 200423929.04
x:465.72 ,  T_p: 199531537.85 , T_e : 196493425.40
x:475.80 ,  T_p: 195652793.28 , T_e : 192724867.41
x:485.88 ,  T_p: 191932233.01 , T_e : 189108339.16
x:495.96 ,  T_p: 188360280.97 , T_e : 185634721.59
x:506.04 ,  T_p: 184928121.33 , T_e : 182295613.90
x:516.12 ,  T_p: 181627624.33 , T_e : 179083263.97
x:526.20 ,  T_p: 178451280.78 , T_e : 175990506.85
x:536.28 ,  T_p: 175392143.84 , T_e : 173010710.02
x:546.36 ,  T_p: 172443777.26 , T_e : 170137724.74
x:556.44 ,  T_p: 169600209.25 , T_e : 167365842.63
x:566.53 ,  T_p: 166855891.23 , T_e : 164689756.84
x:576.61 ,  T_p: 164205660.97 , T_e : 162104527.27
x:586.69 ,  T_p: 161644039.83 , T_e : 159606342.97
x:596.77 ,  T_p: 159167919.38 , T_e : 157189276.95
x:606.85 ,  T_p: 156772401.06 , T_e : 154850152.80
x:616.93 ,  T_p: 154453568.54 , T_e : 152585214.78
x:627.01 ,  T_p: 152207756.13 , T_e : 150390945.86
x:637.09 ,  T_p: 150031528.92 , T_e : 148264049.03
x:647.17 ,  T_p: 147921664.89 , T_e : 146201430.32
x:657.25 ,  T_p: 145875138.58 , T_e : 144200183.43
x:667.33 ,  T_p: 143889106.25 , T_e : 142257575.71
x:677.41 ,  T_p: 141960892.38 , T_e : 140371035.40
x:687.49 ,  T_p: 140087977.24 , T_e : 138538140.04
x:697.58 ,  T_p: 138267985.66 , T_e : 136756605.83
x:707.66 ,  T_p: 136498676.62 , T_e : 135024278.02
```

```
x:717.74 ,  T_p: 134777933.70 , T_e : 133339122.00
x:727.82 ,  T_p: 133103756.27 , T_e : 131699215.34
x:737.90 ,  T_p: 131474251.40 , T_e : 130102740.42
x:747.98 ,  T_p: 129887123.05 , T_e : 128548603.27
x:758.06 ,  T_p: 128341710.77 , T_e : 127033886.30
x:768.14 ,  T_p: 126835863.19 , T_e : 125557715.50
x:778.22 ,  T_p: 125368046.75 , T_e : 124118632.32
x:788.30 ,  T_p: 123936801.61 , T_e : 122715256.15
x:798.38 ,  T_p: 122540735.61 , T_e : 121346280.86
x:808.46 ,  T_p: 121178518.15 , T_e : 120010472.17
x:818.55 ,  T_p: 119848873.83 , T_e : 118706665.76
x:828.63 ,  T_p: 118550575.75 , T_e : 117433766.53
x:838.71 ,  T_p: 117282438.03 , T_e : 116190749.00
x:848.79 ,  T_p: 116043307.33 , T_e : 114976659.33
x:858.87 ,  T_p: 114831632.46 , T_e : 113791159.19
x:868.95 ,  T_p: 113647174.61 , T_e : 112632321.57
x:879.03 ,  T_p: 112488345.44 , T_e : 111500035.42
x:889.11 ,  T_p: 111353995.40 , T_e : 110393703.53
x:899.19 ,  T_p: 110242925.77 , T_e : 109312855.86
x:909.27 ,  T_p: 109153856.02 , T_e : 108257178.03
x:919.35 ,  T_p: 108084992.79 , T_e : 107227057.36
x:929.43 ,  T_p: 107035591.88 , T_e : 106221522.81
x:939.52 ,  T_p: 106003350.38 , T_e : 105241620.75
x:949.60 ,  T_p: 104985672.51 , T_e : 104288736.09
x:959.68 ,  T_p: 103980752.56 , T_e : 103363116.63
x:969.76 ,  T_p: 102984350.00 , T_e : 102468055.55
x:979.84 ,  T_p: 101992679.99 , T_e : 101606008.64
x:989.92 ,  T_p: 101000365.02 , T_e : 100781187.26
x:1000.00 ,  T_p: 100000000.00 , T_e : 100000000.00
```

[19]:
```
t_q3 = np.asarray(t_q3)
te_q3 = t_q3[:,0]
tp_q3 = t_q3[:,1]
fig = plt.figure(figsize=(8,6))
ax = fig.add_subplot(111)
ax.loglog(x_range, te_q3)
ax.loglog(x_range,tp_q3)
ax.set_xlabel('Radial Distance , Unit:$r_g$')
ax.set_ylabel('Temperature (K)')
ax.set_title('Accretion Disk Temperature vs Radial Distance')
ax.text(2,1e8 , '$X_0$ = {:.0e}\n$Te_o$={:.0e}\n$Tp_o$={:.0e} \n$\dot m$={}'.
 ↪format(x_0,te_0,tp_0,m_dot_0) , bbox = {'facecolor':'blue' , 'alpha':0.1,␣
 ↪'pad':10 } , fontsize = 10)
plt.legend(['$T_e$','$T_p$'])
plt.show()
```


Accretion Disk Temperature vs Radial Distance
T_e
T_p
10^{10}
10^{9}
10^{8}
Temperature (K)
X_0 = 1e+03
Te_o=1e+08
Tp_o=1e+08
m=1e+19
10^{1}
10^{2}
10^{3}
Radial Distance , Unit:r_g