Indian Institute of Space science and Technology

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Computational Astrophysics - ESA614 / ESA414

Master of Science in Astronomy & Astrophysics

Quiz-II Examination

23 November 2020 *Total Marks: 20* *Time: 2:00 - 4:30 PM*

Instructions: Write all the formulae and methods used in numerical coding to the answer sheet along with the results in sequential order in a single pdf file. Also attach the code.

1. Stars are luminous due to nuclear reactions at the stellar interior. The stellar core can be assumed in thermal equilibrium following a Maxwellian distribution of particles which is proportional to the function: $f_M = e^{-E/kT}$, where E is kinetic energy of the nucleons, k is Boltzmann constant and T is temperature of the core. The nuclear reactions happen through quantum mechanical tunneling with a probability proportional to: $f_t = e^{-b/\sqrt{E}}$, where b is a constant. The nuclear reaction rate is proportional to the product $P(E,T) = f_M \times f_t$.

Given an experimental data set "QIIdata" with the flowing description of columns:
column[1]: energy E in keV
column[2]: particle probability distribution
column[3]: error in the estimation of particle distribution
column[4]: estimated tunneling probability
column[5]: error in estimated tunneling probability
Answer the following questions:

i. Fit the data in column[2] by the function f_M and estimate the temperature (T in Kelvin) of the stellar core. Also, quote the error in your estimation. [5]

ii Fit the data in column[4] with the function f_t and estimate the parameter b with uncertainty. [5]

iii. Numerically calculate the first derivative of the product of column[2] and column[4]. Plot the first derivative vs E. What is the value of E (say E_0), where first derivative is zero? [3]

iv. Estimate the second derivative of the product of column[2] and column[4] at E_0. Comment on whether the product has a maximum/minimum at E_0 and interpret your findings. [2]

v. Integrate the product of column[2] and column[4] in the energy range given in the data file. Compare this with the area under the curve due to $P(E,T)$ in the entire energy range. [5]

Appendix:

Numerical formula to calculate 1^{st} and 2^{nd} derivative of $y = f(x)$

$$y'_i = \frac{f_{i+1} - f_{i-1}}{2\Delta x}$$

$$y''_i = \frac{f_{i+1} - 2f_i + f_{i-1}}{(\Delta x)^2}$$