

## CAP-Lab-7

### Estimation of parameter using Maximum Likelihood

The angular distribution of electron scattering experiments show a probability distribution function of the form:  $p(\cos \theta) = N(1 + \alpha \cos^2 \theta)$ , where  $\theta$  is the angle between incident and scattered direction.

An experiment measures scattering events and finds the following values of  $\cos\theta$ :

-.999, -.983, -.956, -.946, -.933, -.925, -.916, -.910,  
-.881, -.739, -.734, -.717, -.715, -.675, -.665, -.649,  
-.621, -.537, -.522, -.508, -.499, -.471, -.460, -.419,  
-.403, -.311, -.305, -.281, -.170, -.162, -.063, 0.214,  
0.438, .444, .508, .586, .638, .677, .721, .730,  
0.438, .444, .508, .586, .638, .677, .721, .730,  
0.768, .785, .790, .793, .877, .896, .931, .938,  
0.948, 0.999

We wish to calculate the value of  $\alpha$  using the Maximum Likelihood Method.

- (a) Calculate N from normalized probability (notice that it depends on  $\alpha$ )
- (b) Write down the Likelihood Function ( $\mathcal{L}$ ) for this problem.
- (c) Make a plot of the relative Likelihood Function ( $\mathcal{L}/\mathcal{L}_{\max}$ ) vs  $\alpha$ . Use this plot to find a guess value of  $\alpha$ . Maximizes the Likelihood Function to estimate  $\alpha$ . *(Note: You can use log scale if you wish. You are encouraged to be innovative for the maximization method)*
- (d) Make a histogram of the observed data and over plot the probability distribution for the best estimated parameters.