Computational Astrophysics
Maximum Likelihood --2

PART-A (**already done**)

Last lab, we have estimated the parameters using direct MLE using angular distribution of electron scattering experiments with a probability distribution function : $p(\cos\theta) = N(1 + \alpha \cos^2\theta)$, where θ is the angle between incident and scattered direction.

PART-B

For a better estimation of error in parameter, write a Monte Carlo code to generate events using the normalized probability distribution $p(\cos\theta) = N(1 + \alpha \cos^2\theta)$. Take $\cos\theta$ in [-1,1], and $p(\cos\theta)$ in $[0, N(1+\alpha)]$.

(i) Generate 500 accepted events with $\alpha = 5.5$ and draw a histogram of probability distribution with 20 bins between [-1, 1]. Plot the theoretical probability distribution on top of the simulated histogram. Calculate the mean and variance of the generated events and compare that with theoretical values.

(ii) Redo (i) with 4000 accepted events.

(iii) Calculate the likelihood function ($\mathcal{L}$) for 500 and 4000 accepted events in (i & ii) by changing α between [0, 25]. Value of likelihood function may be extremely small and hence better to calculate $-\log(\mathcal{L})$.

(iv) Plot $-\log(\mathcal{L})$ vs α for (i) and (ii) in the same figure. As they may have very different values, scale $-\log(\mathcal{L})$ with respect to the minimum vale in each case.

(v) Estimate the parameter and its uncertainty. Over plot the scaled $-\log(\mathcal{L})$ in (iv) and result from part-A.

Note: Mentioned the method used to minimize $-\log(\mathcal{L})$.