

$$p(x) = \lambda e^{-\lambda x}$$

$$\begin{aligned} L(\lambda | x) &= \prod_{i=1}^n \lambda e^{-\lambda x_i} \\ &= \lambda^n e^{-\lambda \sum x_i} \end{aligned}$$

$$\log L = n \log \lambda - \lambda \sum x_i$$

$$\Rightarrow \frac{\partial \log L}{\partial \lambda} = \frac{n}{\lambda} - \sum x_i$$

$$\Rightarrow \frac{n}{\hat{\lambda}} = \sum x_i \Rightarrow \boxed{\hat{\lambda} = \frac{n}{\sum x_i}}$$