Root finding

1. Write a code to implement root finding by bisection and Newton-Raphson method. Verify your program by finding the root of $f(x) = \cos(x) - x^3$

2. A planet is moving around the sun in an elliptic Kepler orbit with semi-major axis a, semi-minor axis b, and eccentricity $e = \sqrt{1 - b^2/a^2}$. The planet is orbiting the Sun and was last at its perihelion at t=0. ω=2π/T is its angular frequency and T is the duration of its orbit.

If we define a 2D coordinate system *(x,y)* with origin at center of the ellipse, then the points on the ellipse are described by the equation
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.$$

The location of the planet in the (x,y) coordinate system is given by
$x = a\cos E,\ y = b\sin E,$
where $E = \omega t + e\sin E.$

Earth has an orbital period of 365.25635 days, a semi-major axis $a = 1.496 \times 10^{16}$km, and its orbit has an eccentricity e=0.0167. Compute *(x,y)* for t=91 days, t=182 days and t=273 days. Fractional error in E at the end of your computation should be less than 10^{-7}. How many iterations your program require?

Now, suppose something happened to put earth in a heavily eccentric orbit with e=0.99999. How many iterations does the code take now? See if you can accelerate the convergence.