Stream Function Equation:

Assumption:

Steady 2-D in compressible and Inviscid flow

$$\frac{\partial^2\psi}{\partial \varkappa^2} + \frac{\partial^2\psi}{\partial y^2} = 0 \qquad \text{------- (1)}$$

where $\psi \rightarrow stream\ function$

Apply Central Differential Approximation

$$\psi_{i+1} = \psi_i + \Delta x\frac{\partial\psi}{\partial x} + \frac{(\Delta x)^2}{2!}\frac{\partial^2\psi}{\partial x^2} + \frac{(\Delta x)^3}{3!}\frac{\partial^3\psi}{\partial x^3} + \frac{(\Delta x)^4}{4!}\frac{\partial^4\psi}{\partial x^4} + \cdots \text{ Hot ---------- (2)}$$

$$\psi_{i-1} = \psi_i - \Delta x\frac{\partial\psi}{\partial x} + \frac{(\Delta x)^2}{2!}\frac{\partial^2\psi}{\partial x^2} - \frac{(\Delta x)^3}{3!}\frac{\partial^3\psi}{\partial x^3} + \frac{(\Delta x)^4}{4!}\frac{\partial^4\psi}{\partial x^4} - \cdots \text{ Hot ----------- (3)}$$

(ADD Equation 1 & 2)

$$\Psi_{i+1} + \psi_{i-1} = 2\psi_i + (\Delta x)^2\frac{\partial^2\psi}{\partial x^2} + \frac{2(\Delta x)^4}{4!}\frac{\partial^4\psi}{\partial x^4} + \ \ldots \quad \text{Hot ---------(4)}$$

$$\frac{\partial^2\psi}{\partial^2 x} = \frac{\psi_{i+1} - 2\psi_i + \psi_{i-1}}{(\Delta x)^2} + \frac{2(\Delta{,}c)^2}{4!}\frac{\partial^4\psi}{\partial x^4} + \cdots \text{ Hot}$$

$$\frac{\partial^2\psi}{\partial x^2} = \frac{\Psi_{i+1,j} - 2\psi_{i,j} + \psi_{i-1,j}}{(\Delta x^2)} \quad \text{---------(A)}$$

Similarly, apply CDA $\frac{\partial^2\psi}{\partial y^2}$

$$\frac{\partial^2\psi}{\partial y^2} = \frac{\Psi_{i,j+1} - 2\psi_{i,j} + \psi_{i,j-1}}{(\Delta y^2)} \quad \text{------(B)}$$

Substitute A & B in 1,

$$\frac{\Psi_{i+1,j} - 2\psi_{i,j} + \psi_{i-1,j}}{(\Delta x^2)} + \frac{\Psi_{i,j+1} - 2\psi_{i,j} + \psi_{i,j-1}}{(\Delta y^2)} = 0$$

Rearrange it,

$$2\left(\frac{1}{\Delta x^2} + \frac{1}{\Delta y^2}\right)\psi_{i,j} = \frac{1}{\Delta x^2}\psi_{i+1,j} + \frac{1}{\Delta x^2}\psi_{i-1,j} + \frac{1}{\Delta y^2}\psi_{i,j+1} + \frac{1}{\Delta y^2}\psi_{i,j-1}$$

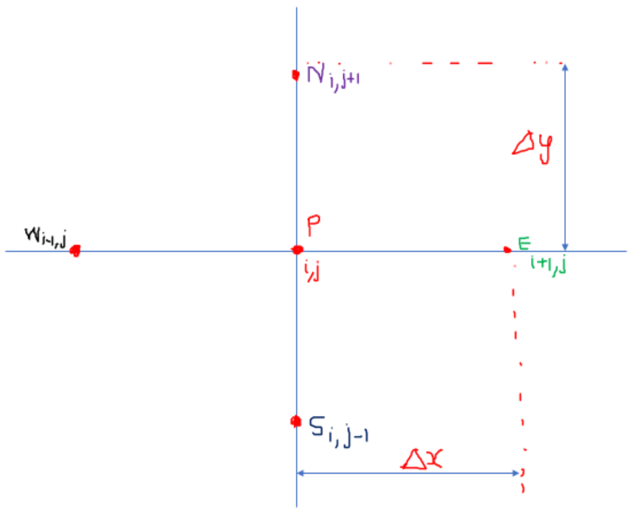
$$E = \frac{1}{\Delta x^2} \; ;$$

$$w = \frac{1}{\Delta x^2};$$

$$N = \frac{1}{\Delta y^2};$$

$$S = \frac{1}{\Delta y^2};$$

$$P = 2\left(\frac{1}{\Delta x^2} + \frac{1}{\Delta y^2}\right)$$



$$P\,\psi_{i,j} = E\,\psi_{i+1,j} + W\,\psi_{i-1,j} + N\,\psi_{i,j+1} + S\,\psi_{i,j-1}$$

$$\psi_{i,j} = \frac{1}{P}\left[\,E\,\psi_{i+1,j} + W\,\psi_{i-1,j} + N\,\psi_{i,j+1} + S\,\psi_{i,j-1}\,\right]$$

$$\text{Error} = \sqrt{\frac{\sum_{\substack{interior \\ grid\ points}} \left(\psi_{i,j}^{k+1} - \psi_{i,j}^{k}\right)^2}{Total\ number\ of\ interior\ points}}$$

K+1 -> Current $\psi_{i,j}$

K -> Previous $\psi_{i,j}$