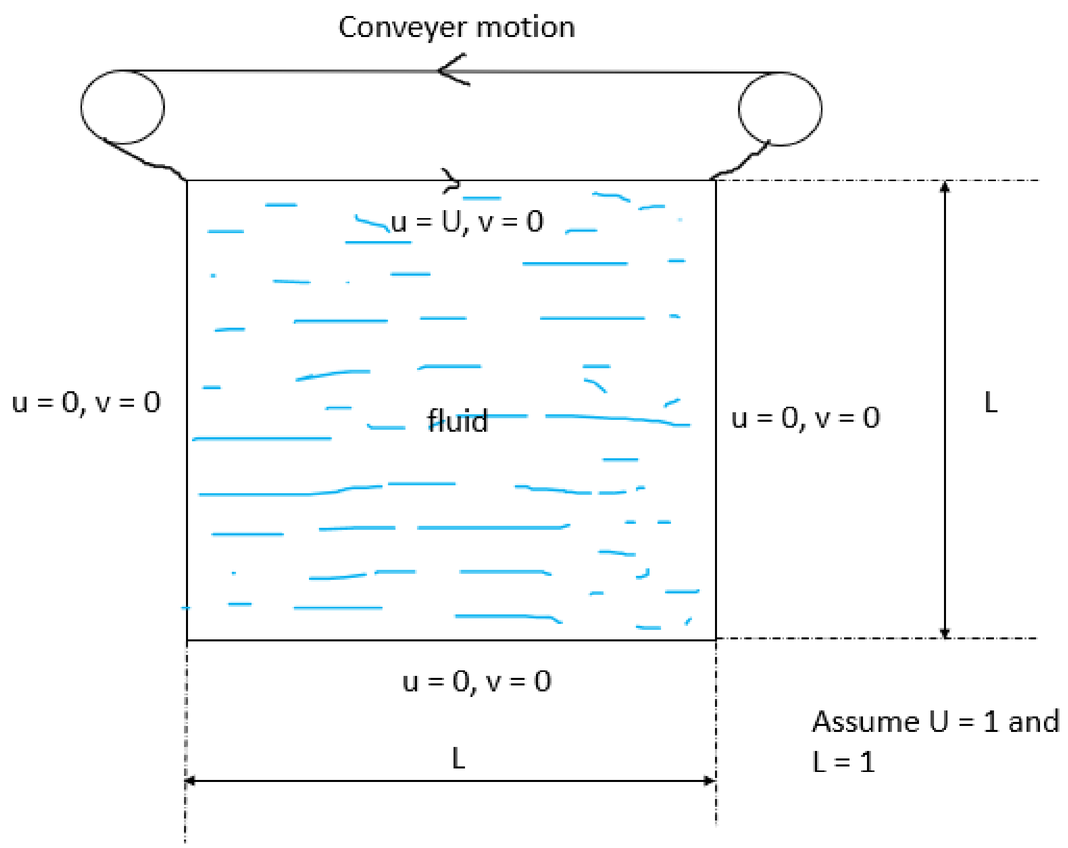
Problem

Lid driven cavity flow



Velocity boundary conditions:

Left wall

u = v = 0

Right wall

u = v = 0

Bottom wall

u = v = 0

Top wall

u = U, v = 0

Boundary conditions for ψ

Left wall

$$u = 0;\ \frac{\partial \psi}{\partial y} = 0\ ;\ \psi = c_1$$

Right wall

$$u = 0;\ \frac{\partial \psi}{\partial y} = 0\ ;\ \psi = c_2$$

Bottom wall

$$v = 0;\ -\frac{\partial \psi}{\partial y} = 0\ ;\ \psi = c_3$$

Top wall

$$v = 0;\ -\frac{\partial \psi}{\partial y} = 0\ ;\ \psi = c_4$$

$$\psi = c\ \ for\ all\ four\ boundries$$

$$\psi = 0\ for\ all\ four\ boundries$$

Boundary condition for ω

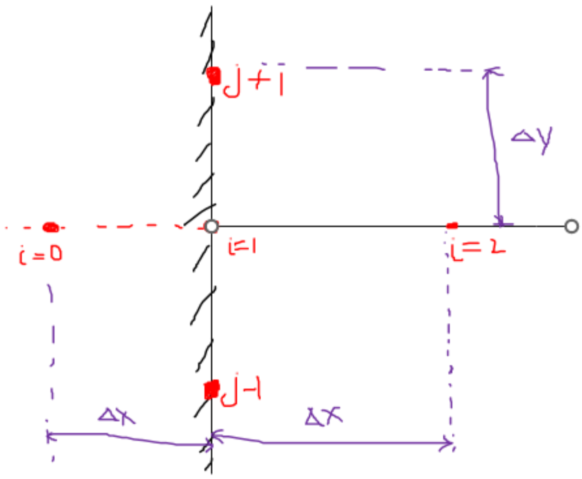
$$\omega = -\left(\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2}\right)$$

Left wall

u = 0; $\frac{\partial \psi}{\partial y} = 0$; $\frac{\partial^2 \psi}{\partial y^2}$ = 0

$$\omega = -\frac{\partial^2 \psi}{\partial x^2}$$

$$\omega_{1,j} = -\frac{\psi_{2,j} - 2\psi_{1,j} + \psi_{0,j}}{(\Delta x)^2}$$



$$v = 0; \; -\frac{\partial \psi}{\partial x} = 0 \;;$$

$$\frac{\psi_{2,j} - \psi_{0,j}}{2\Delta x} = 0$$

$$\psi_{0,j} = \psi_{2,j}$$

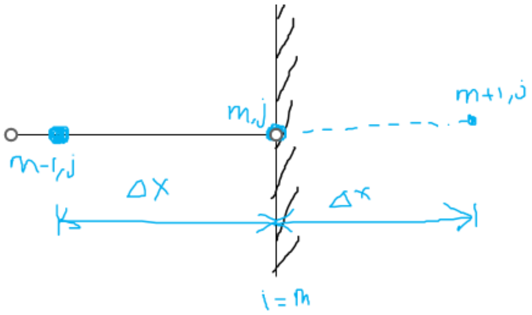
$$\omega_{1,j} = -\frac{\psi_{2,j} - 2\psi_{1,j} + \psi_{2,j}}{(\Delta x)^2}$$

$$\omega_{1,j} = -2\frac{(\psi_{2,j} - \psi_{1,j})}{(\Delta x)^2}$$

Right wall

$$u = 0; \frac{\partial \psi}{\partial y} = 0 \;; \quad \frac{\partial^2 \psi}{\partial y^2} = 0$$

$$\omega = -\frac{\partial^2 \psi}{\partial x^2}$$



$$\omega_{m,j} = -\frac{\psi_{m+1,j} - 2\psi_{m,j} + \psi_{m-1,j}}{(\Delta x)^2}$$

$$v = 0; \; -\frac{\partial \psi}{\partial x} = 0 \;;$$

$$\psi_{m+1,j} = \psi_{m-1,j}$$

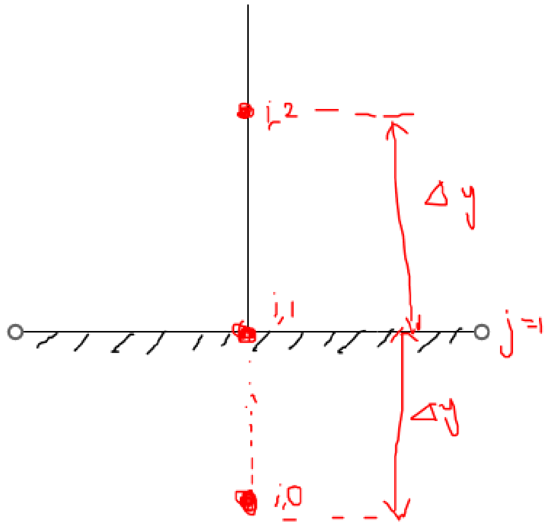
$$\omega_{m,j} = -\frac{\psi_{m-1,j} - 2\psi_{m,j} + \psi_{m-1,j}}{(\Delta x)^2}$$

$$\omega_{m,j} = -\frac{2(\psi_{m-1,j} - \psi_{m,j})}{(\Delta x)^2}$$

Bottom wall

v = 0; $\frac{\partial \psi}{\partial x} = 0$; $\frac{\partial^2 \psi}{\partial x^2}$ = 0

$$\omega = -\frac{\partial^2 \psi}{\partial y^2}$$



$$\omega_{i,1} = -\frac{\psi_{i,2} - 2\psi_{i,1} + \psi_{i,0}}{(\Delta y)^2}$$

u= 0; $\frac{\partial \psi}{\partial y} = 0$;

$$\frac{\psi_{i,2} - \psi_{i,0}}{2\Delta y} = 0$$

$$\psi_{i,0} = \psi_{i,2}$$

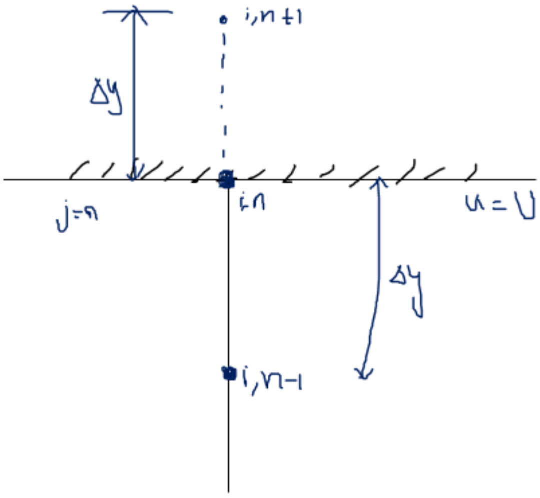
$$\omega_{i,1} = -\frac{\psi_{i,2} - 2\psi_{i,1} + \psi_{i,2}}{(\Delta y)^2}$$

$$\omega_{i,1} = -2\frac{(\psi_{i,2} - \psi_{i,1})}{(\Delta y)^2}$$

Top wall

v = 0; $\frac{\partial \psi}{\partial x} = 0$; $\frac{\partial^2 \psi}{\partial x^2}$ = 0

$$\omega = -\frac{\partial^2 \psi}{\partial y^2}$$



$$\omega_{i,n} = -\frac{\psi_{i,n+1} - 2\psi_{i,n} + \psi_{i,n-1}}{(\Delta y)^2}$$

u = 0; $\frac{\partial \psi}{\partial x} = U$;

$$\frac{\psi_{i.n+1} - \psi_{i,n-1}}{2\Delta y} = U$$

$$\psi_{i.n+1} = \psi_{i,n-1} + 2U\Delta y$$

$$\omega_{i,n} = -\frac{\psi_{i,n-1} - 2\psi_{i,n} + \psi_{i,n-1} + 2U\Delta y}{(\Delta y)^2}$$

$$\omega_{i,n} = -2\frac{(\psi_{i,n-1} - \psi_{i,n} + 2U\Delta y)}{(\Delta y)^2}$$

Results: