Heat Conduction Equation:

Assumption:

Steady 2-D heat conduction

$$\frac{\partial^2 T}{\partial \varkappa^2} + \frac{\partial^2 T}{\partial y^2} = 0 \qquad \text{-------} \; (1)$$

Apply Central Differential Approximation

$$T_{i+1} = T_i + \Delta x \frac{\partial T}{\partial x} + \frac{(\Delta x)^2}{2!}\frac{\partial^2 T}{\partial x^2} + \frac{(\Delta x)^3}{3!}\frac{\partial^3 T}{\partial x^3} + \frac{(\Delta x)^4}{4!}\frac{\partial^4 T}{\partial x^4} + \cdots \text{Hot} \; \text{----------} \; (2)$$

$$T_{i-1} = T_i - \Delta x \frac{\partial T}{\partial x} + \frac{(\Delta x)^2}{2!}\frac{\partial^2 T}{\partial x^2} - \frac{(\Delta x)^3}{3!}\frac{\partial^3 T}{\partial x^3} + \frac{(\Delta x)^4}{4!}\frac{\partial^4 T}{\partial x^4} - \cdots \text{Hot} \; \text{-----------} \; (3)$$

(ADD Equation 1 & 2)

$$T_{i+1} + T_{i-1} = 2T_i + (\Delta x)^2 \frac{\partial^2 T}{\partial x^2} + \frac{2(\Delta x)^4}{4!}\frac{\partial^4 T}{\partial x^4} + \; \ldots \quad \text{Hot} \; \text{---------}(4)$$

$$\frac{\partial^2 T}{\partial^2 x} = \frac{T_{i+1} - 2T_i + T_{i-1}}{(\Delta x)^2} + \frac{2(\Delta x)^2}{4!}\frac{\partial^4 T}{\partial x^4} + \cdots \text{Hot}$$

$$\frac{\partial^2 T}{\partial x^2} = \frac{T_{i+1,j} - 2T_{i,j} + T_{i-1,j}}{(\Delta x^2)} \quad \text{---------}(A)$$

Similarly, apply CDA $\frac{\partial^2 T}{\partial y^2}$

$$\frac{\partial^2 T}{\partial y^2} = \frac{T_{i,j+1} - 2T_{i,j} + T_{i,j-1}}{(\Delta y^2)} \quad \text{------}(B)$$

Substitute A & B in 1,

$$\frac{T_{i+1,j} - 2T_{i,j} + T_{i-1,j}}{(\Delta x^2)} + \frac{T_{i,j+1} - 2T_{i,j} + T_{i,j-1}}{(\Delta y^2)} = 0$$

Rearrange it,

$$2\left(\frac{1}{\Delta x^2} + \frac{1}{\Delta y^2}\right) T_{i,j} = \frac{1}{\Delta x^2} T_{i+1,j} + \frac{1}{\Delta x^2} T_{i-1,j} + \frac{1}{\Delta y^2} T_{i,j+1} + \frac{1}{\Delta y^2} T_{i,j-1}$$

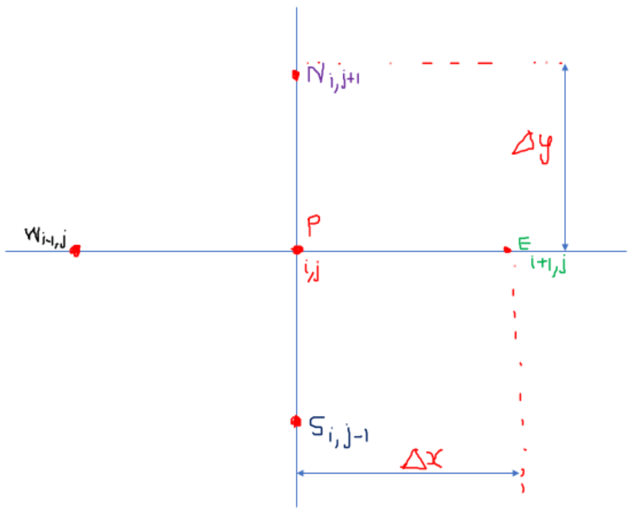
$$E = \frac{1}{\Delta x^2} \; ;$$

$$w = \frac{1}{\Delta x^2};$$

$$N = \frac{1}{\Delta y^2};$$

$$S = \frac{1}{\Delta y^2};$$

$$P = 2\left(\frac{1}{\Delta x^2} + \frac{1}{\Delta y^2}\right)$$



$$P\,T_{i,j} = E\,T_{i+1,j} + W\,T_{i-1,j} + N\,T_{i,j+1} + S\,T_{i,j-1}$$

$$T_{i,j} = \frac{1}{P}\left[\,E\,T_{i+1,j} + W\,T_{i-1,j} + N\,T_{i,j+1} + S\,T_{i,j-1}\,\right]$$

$$\text{Error} = \sqrt{\frac{\sum_{\substack{interior\\ grid\,points}} \left(T_{i,j}^{k+1} - T_{i,j}^{k}\right)^2}{Total\ number\ of\ interior\ points}}$$

K+1 -> Current $T_{i,j}$

K -> Previous $T_{i,j}$