

Special Topics in CFD

DAY 5

Solving Diffusion Equation

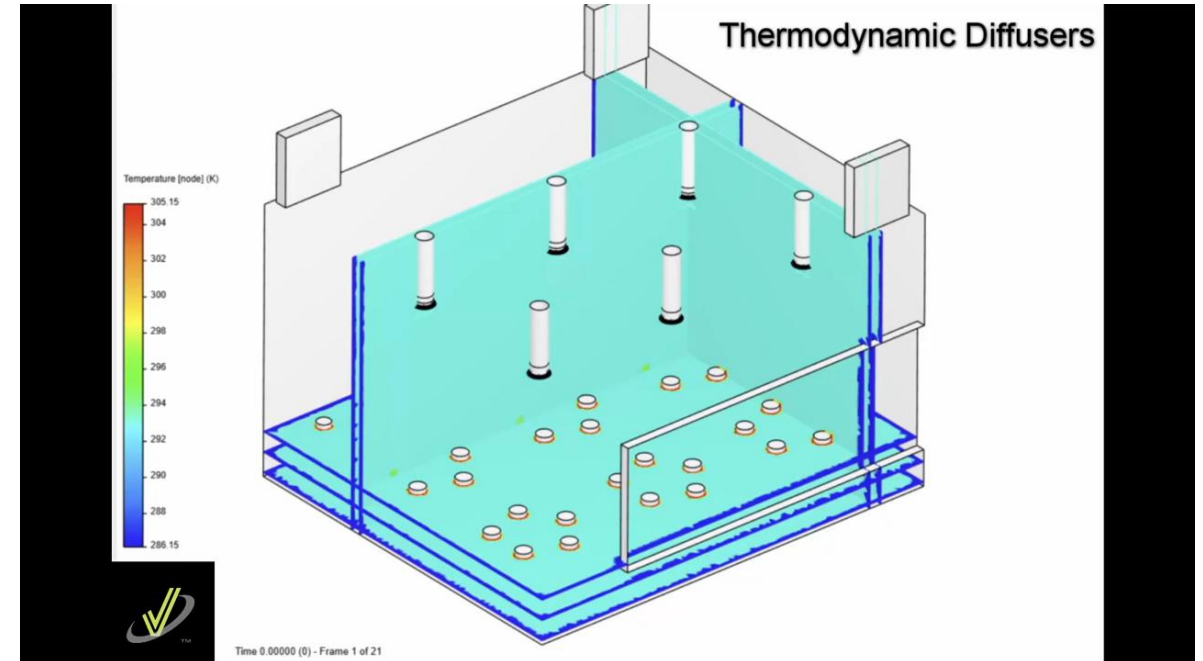
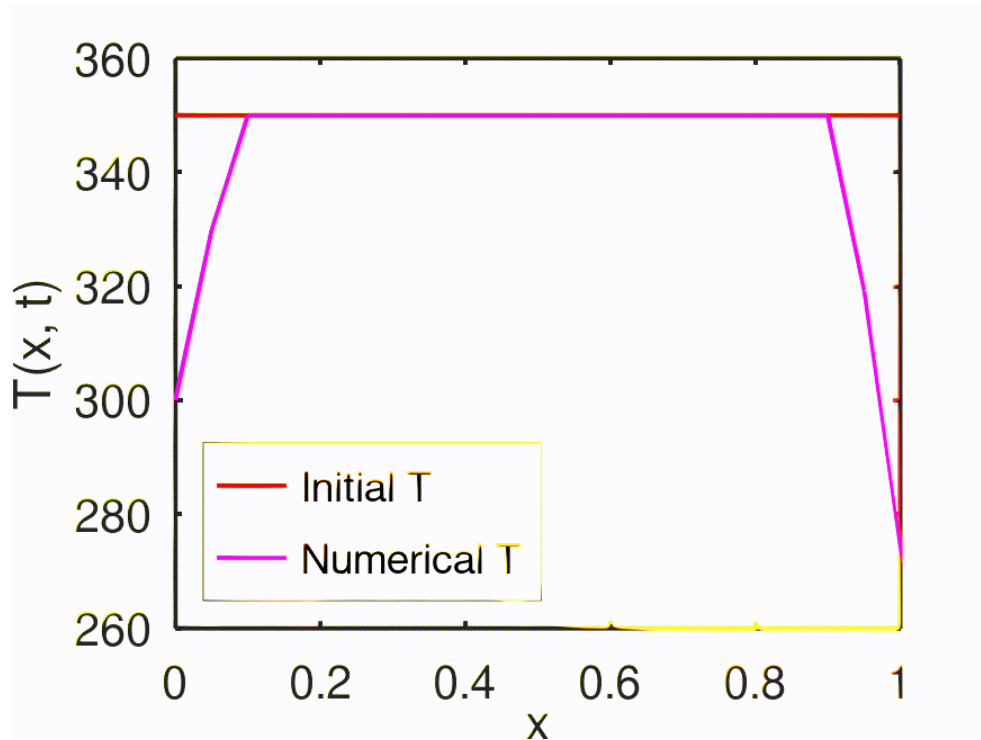
Kumaresh

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Diffusion

1. Fick's law describes the movement of particles from a region of high concentration to a region of lower concentration.
2. Diffusion generally represents the transport of a fluid property (momentum) due to fluctuating motions that are not captured by the bulk motion that is represented by the continuum velocity eqn.



Thermodynamic diffusers supply heated air with a downward jet, the difference in air density causes hot air to rise in the first minute after starting the system.

Numerical Solution to Diffusion Equation

$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2}$$

$$\frac{T_i^{n+1} - T_i^n}{\Delta t} = \alpha \left(\frac{\partial^2 T}{\partial x^2} \right)_i^n$$

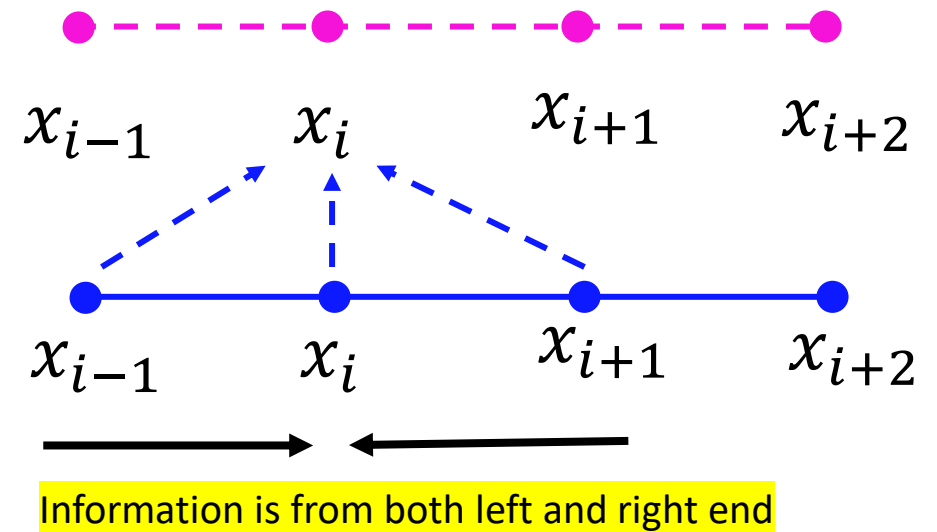
$$\left(\frac{d^2 T}{dx^2} \right)_i = \frac{T(x_{i+1}) - 2T(x_i) + T(x_{i-1}))}{\Delta x_i^2} + O(\Delta x_i^2)$$

$$\frac{T_i^{n+1} - T_i^n}{\Delta t} = \alpha \frac{T_{i+1}^n - 2T_i^n + T_{i-1}^n}{\Delta x^2}$$

$$T_i^{n+1} = T_i^n + \Delta t \alpha \frac{T_{i+1}^n - 2T_i^n + T_{i-1}^n}{\Delta x^2}$$

Time level: $n + 1$

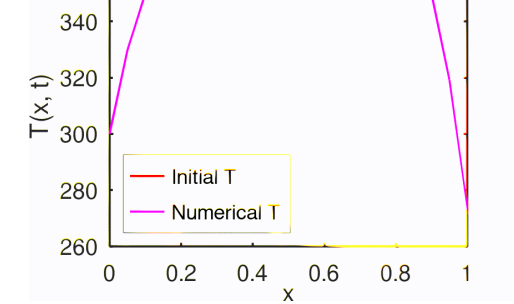
Time level: n



$$u_i^{n+1} = u_i^n - c\Delta t \left(\frac{\partial u}{\partial x} \right)_i^n \rightarrow \left(\frac{\partial u}{\partial x} \right)_i^n \approx \frac{u_{i+1}^n - u_{i-1}^n}{2\Delta x_i}$$

Central difference

Exercise – 5 (Let's solve the Diffusion Equation)



$$T_i^{n+1} = T_i^n + \Delta t \alpha \frac{T_{i+1}^n - 2T_i^n + T_{i-1}^n}{\Delta x^2}$$

`a7_solve_diffusion.m`

1. Resolve the diffusion equation with $\alpha = 1$, $dt = 0.001$, $dx = 0.05$, Dirichlet boundaries ($T_{\text{left}} = 300\text{K}$, $T_{\text{right}} = 273\text{K}$), and write the above numerical solution to extract the results.
2. Change the right boundary condition from Dirichlet to Neumann. Explain about it in few words.
3. Analyze for different time steps (dt) 0.1 and 0.01 and give your comments. Hint: Von Neumann stability analysis.
4. Learning debug skills – fix breakpoints, run, and understand the codes. Explain about it in few words.