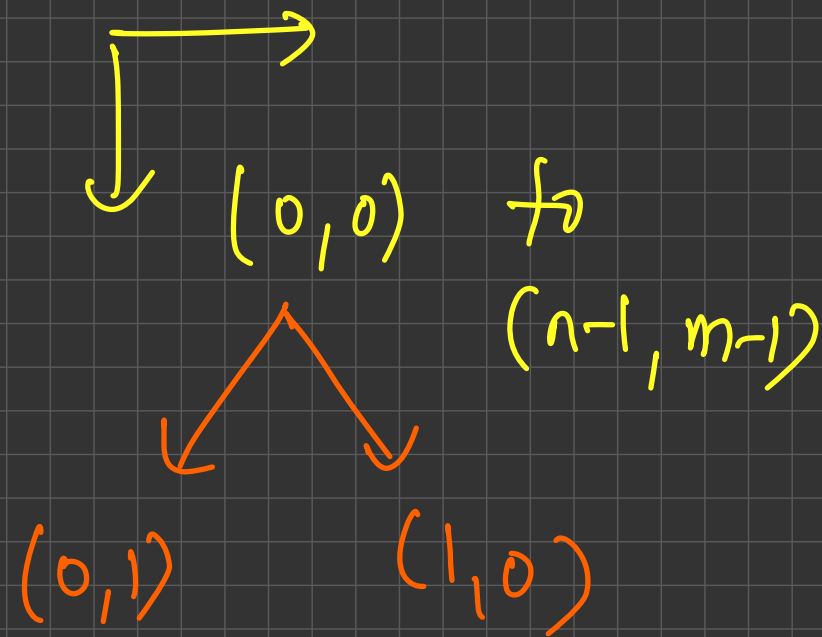
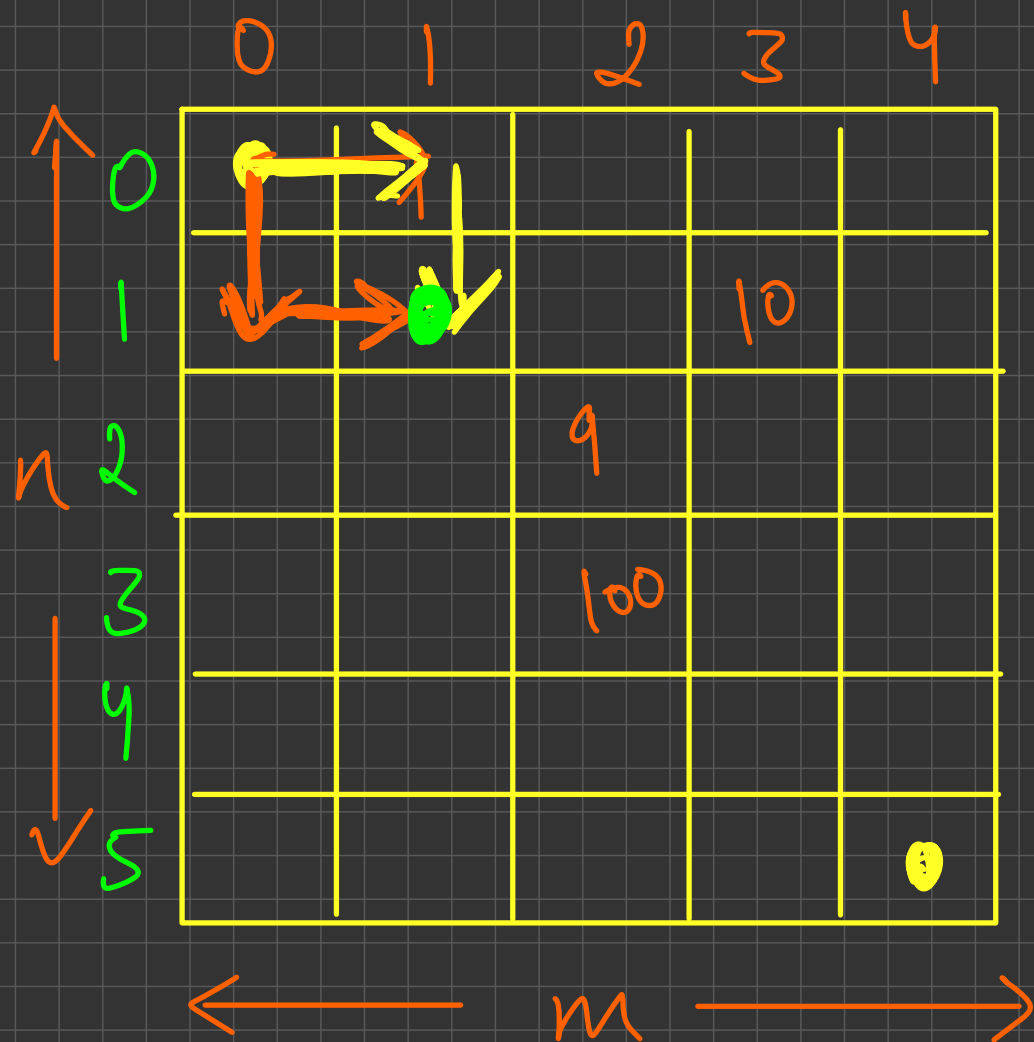


$n \times m$

(i, j)



min sum path
from $(0,0)$
to $(n-1, m-1)$

100

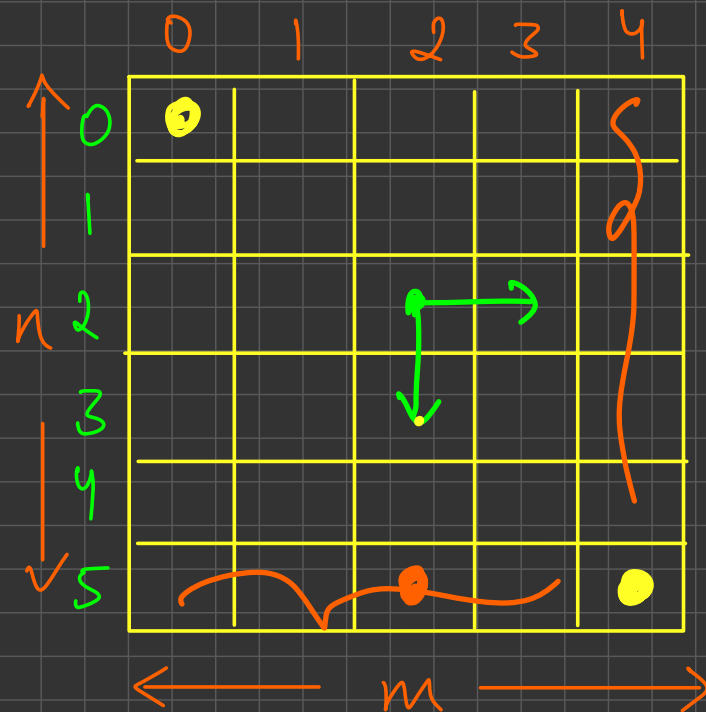
$(1,0)$

$(0,1)$

200

$(n-1, m-1)$

$(n-1, m-1)$



state

$dp(i)(j) = \min$ sum path from

transition (i,j) to $(n-1, m-1)$

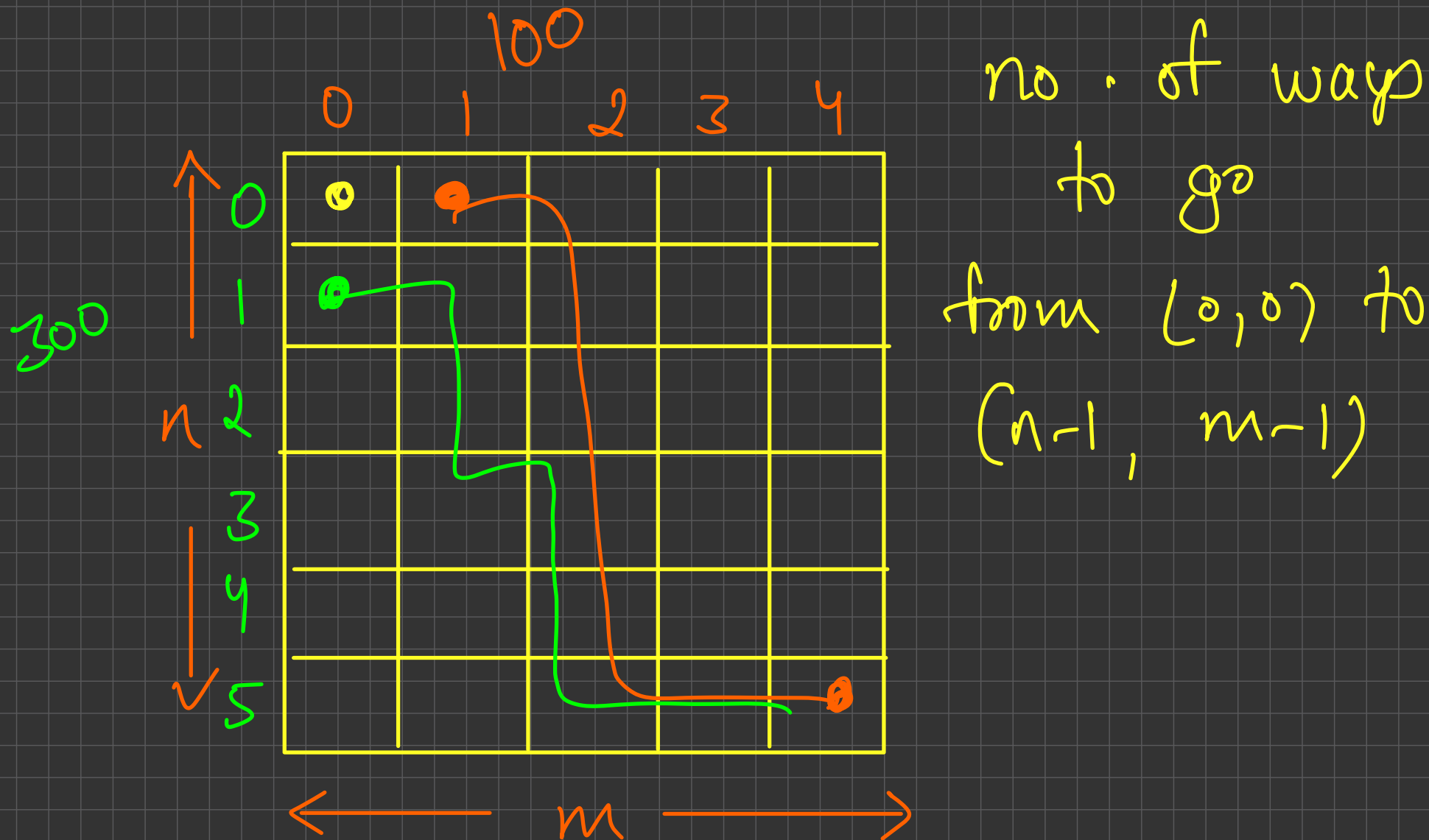
$$dp(i)(j) = \min \left\{ \begin{array}{l} dp(i+1)(j) \\ dp(i)(j+1) \end{array} \right\} + \underline{\underline{grid(i)(j)}}$$

Base Case

$$dp[n-1][m-1] = grid[n-1][m-1]$$

Final subproblem

$$dp[0][0] = \text{min sum path from} \\ (0,0) \text{ to } (n-1, m-1)$$



$dp(i)(j)$ = no. of ways to go from
 state (i, j) to $(n-1, m-1)$

$$dp[i][j] = dp[i+1][j] + dp[i][j+1]$$

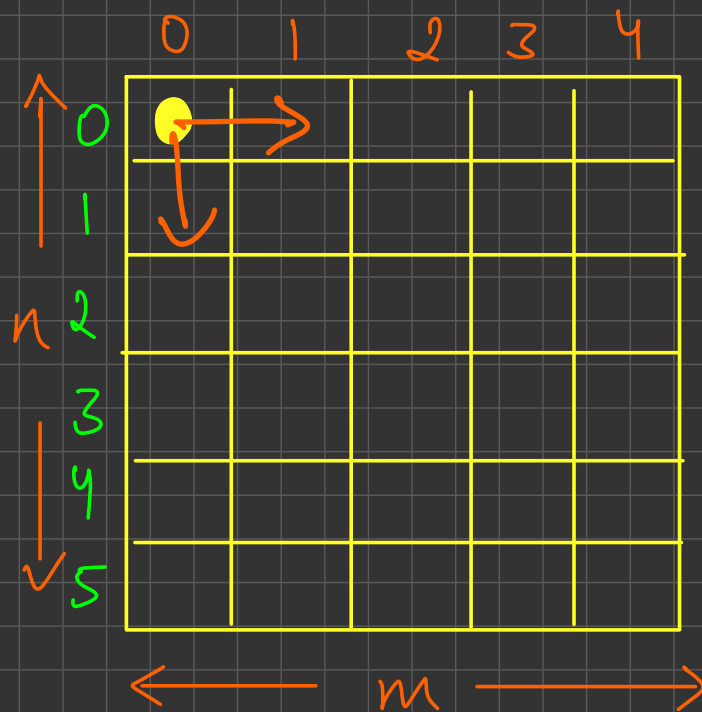
transition

$$dp[n-1][m-1] = 1$$

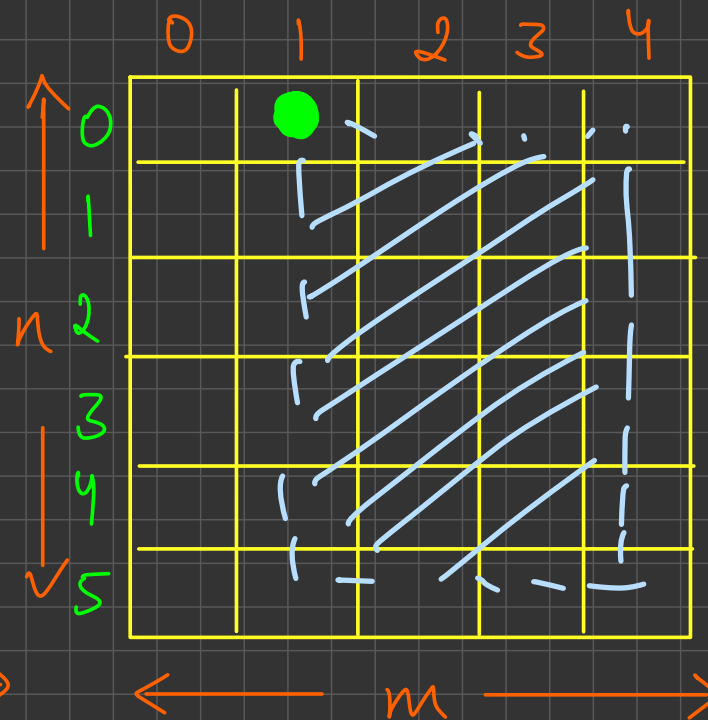
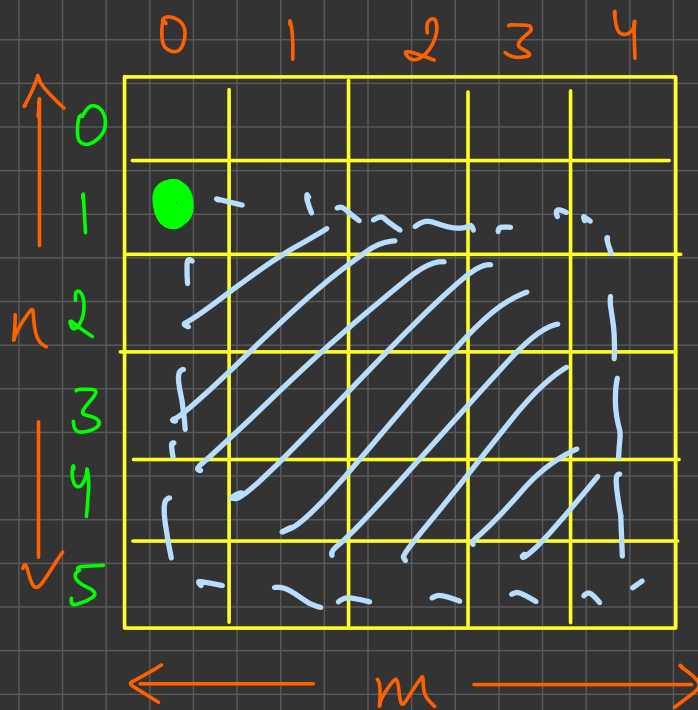
Base case

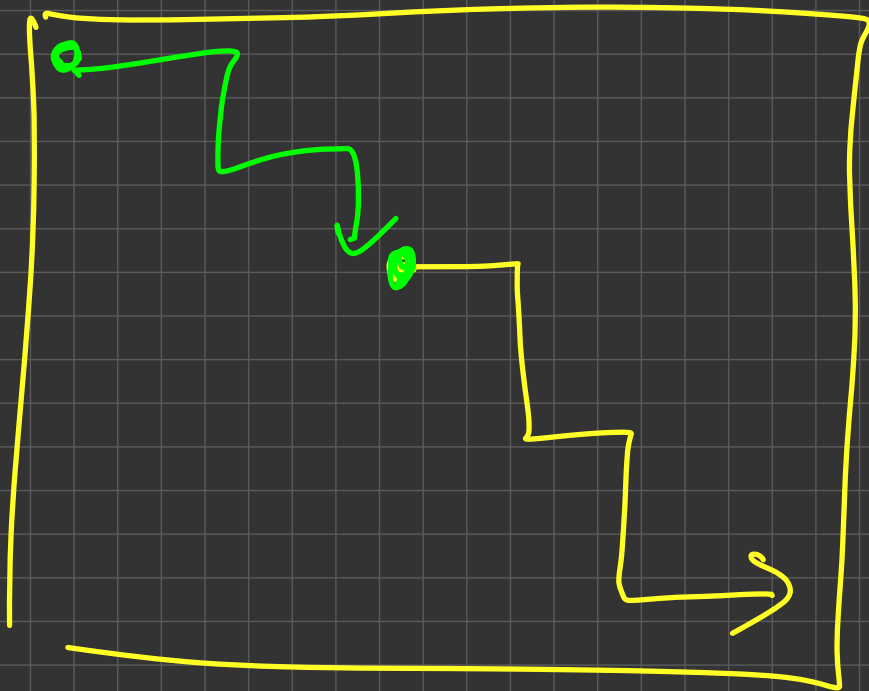
final subproblem:

$$dp[0][0]$$



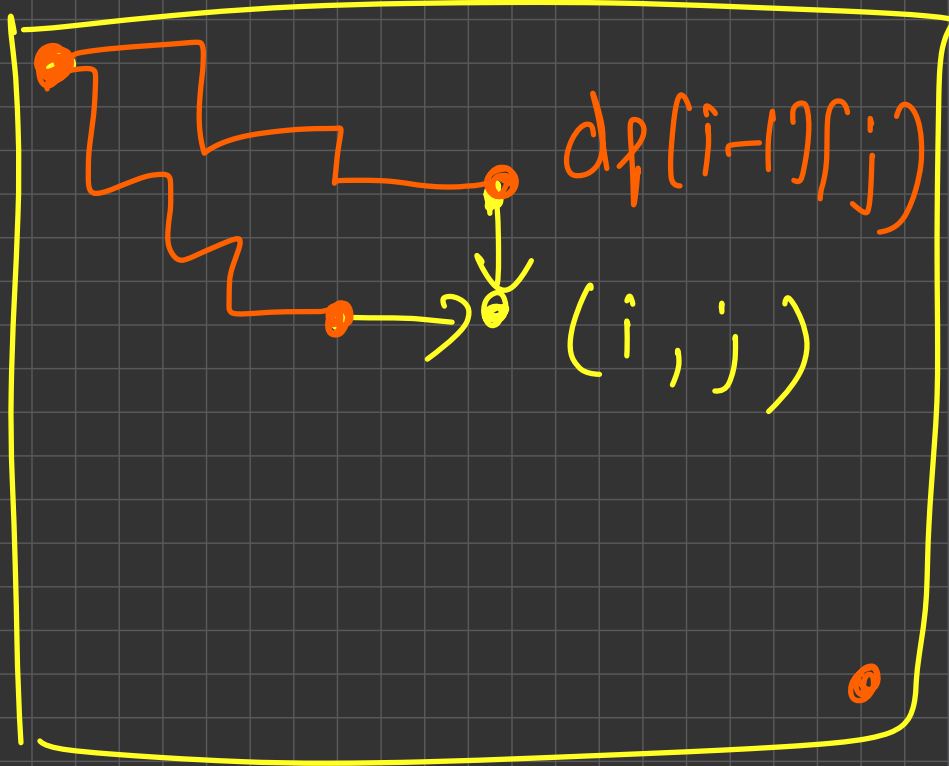
Subproblem?





find out min
path from $(0, 0)$
to $(n-1, m-1)$

$dp(i, j) = \text{min sum path from}$
 $(0, 0)$ to (i, j)



$$dp(i)(j) = \min \left[\begin{array}{l} dp(i-1)(j) \\ dp(i)(j-1) \end{array} \right] + grid(i)(j)$$

Base case: $dp(0)(0) = \overline{grid(0)(0)}$

Final subproblem: $dp[n-1][m-1]$

① { max. min / no. of ways }

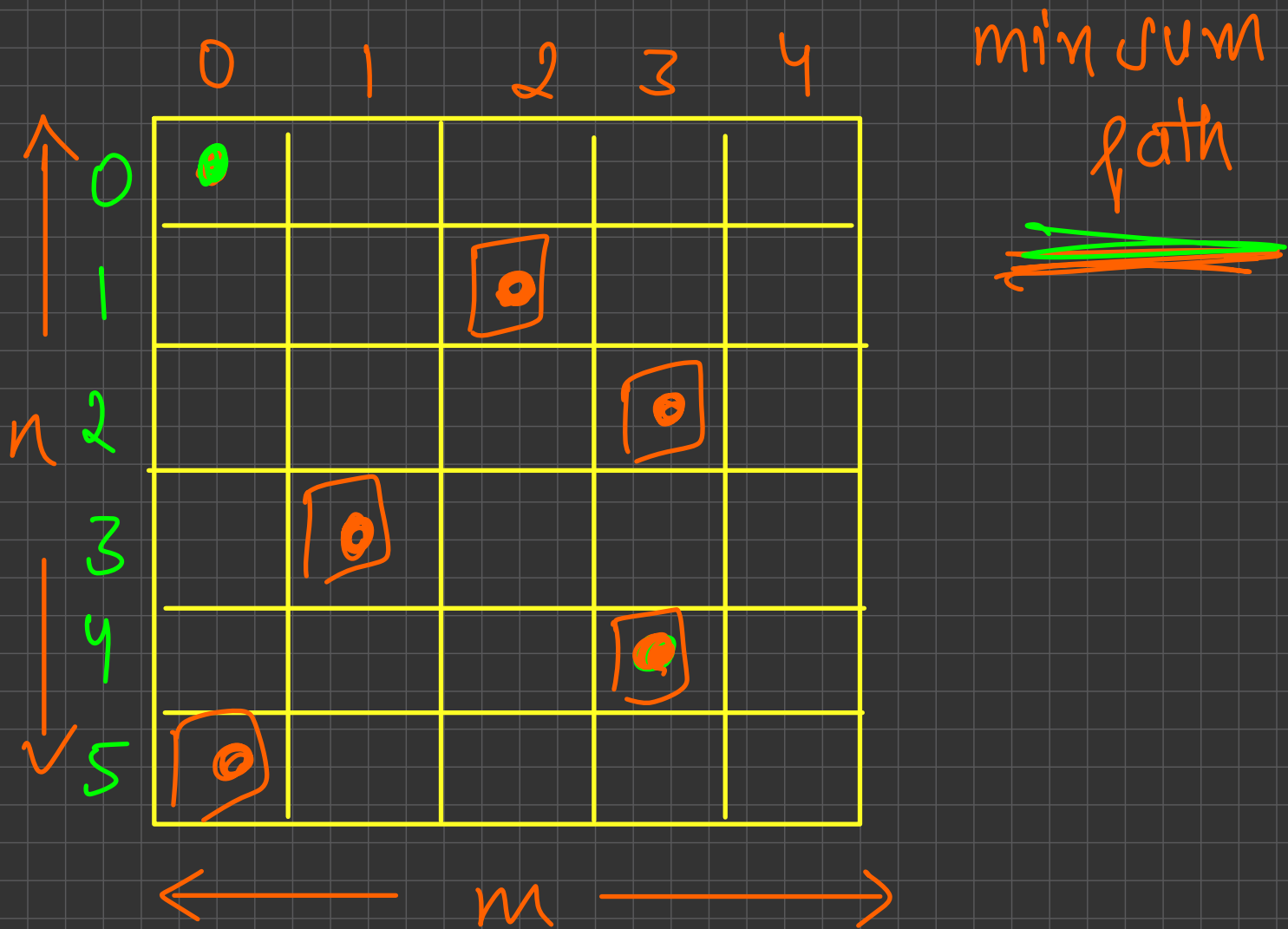
② avoiding obstacles

no. of ways

③ including 1 checkpoint

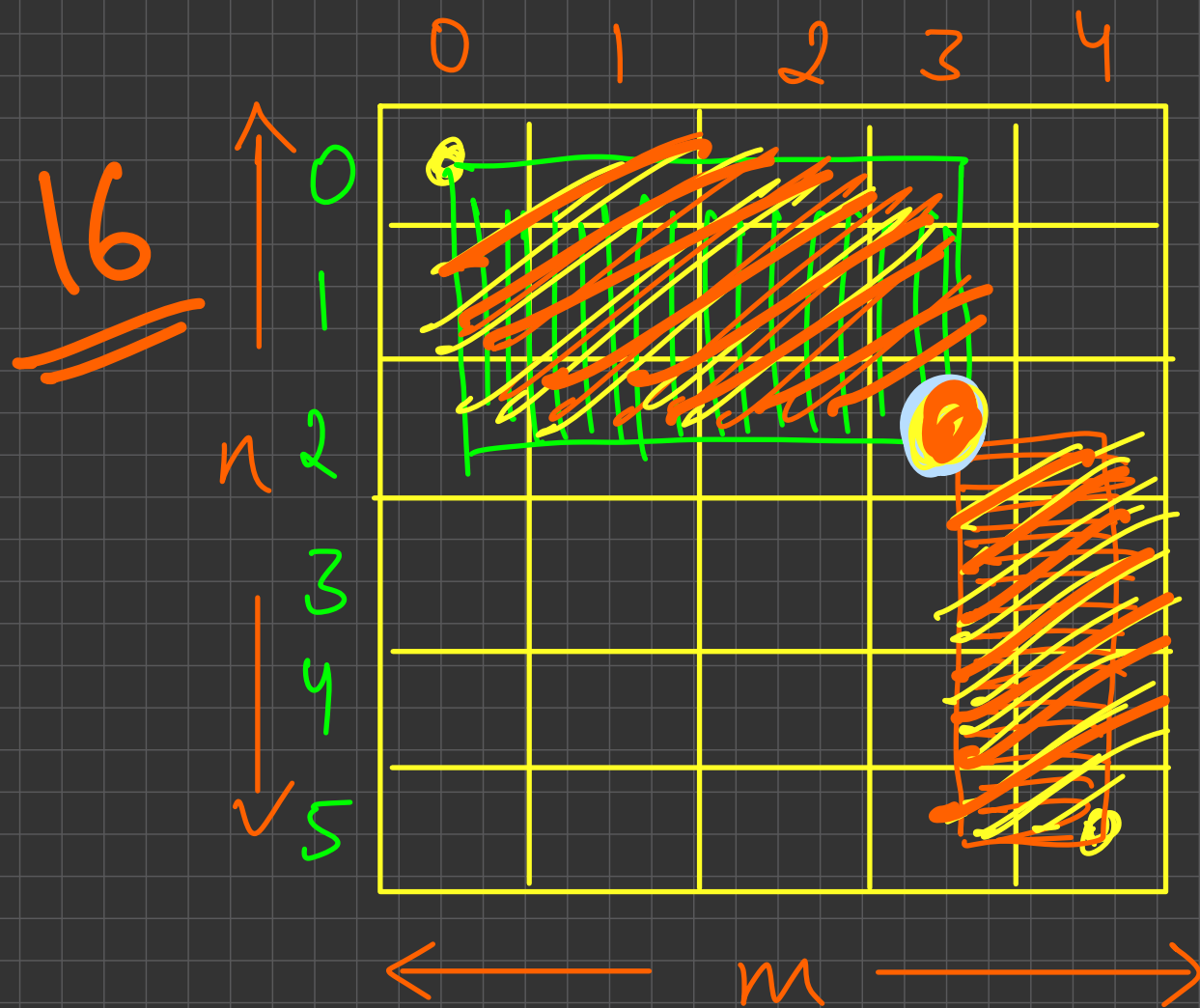
④ without dp (not covered in this lecture)

⑤ multiple checkpoints



$dp[i][j] = \text{min sum path to}$
 go from (i,j) to $(n-1, m-1)$

$dp(i,j) = \infty$ if (i,j) is on
obstacle



min / =
 max / =
 no. of ways

3

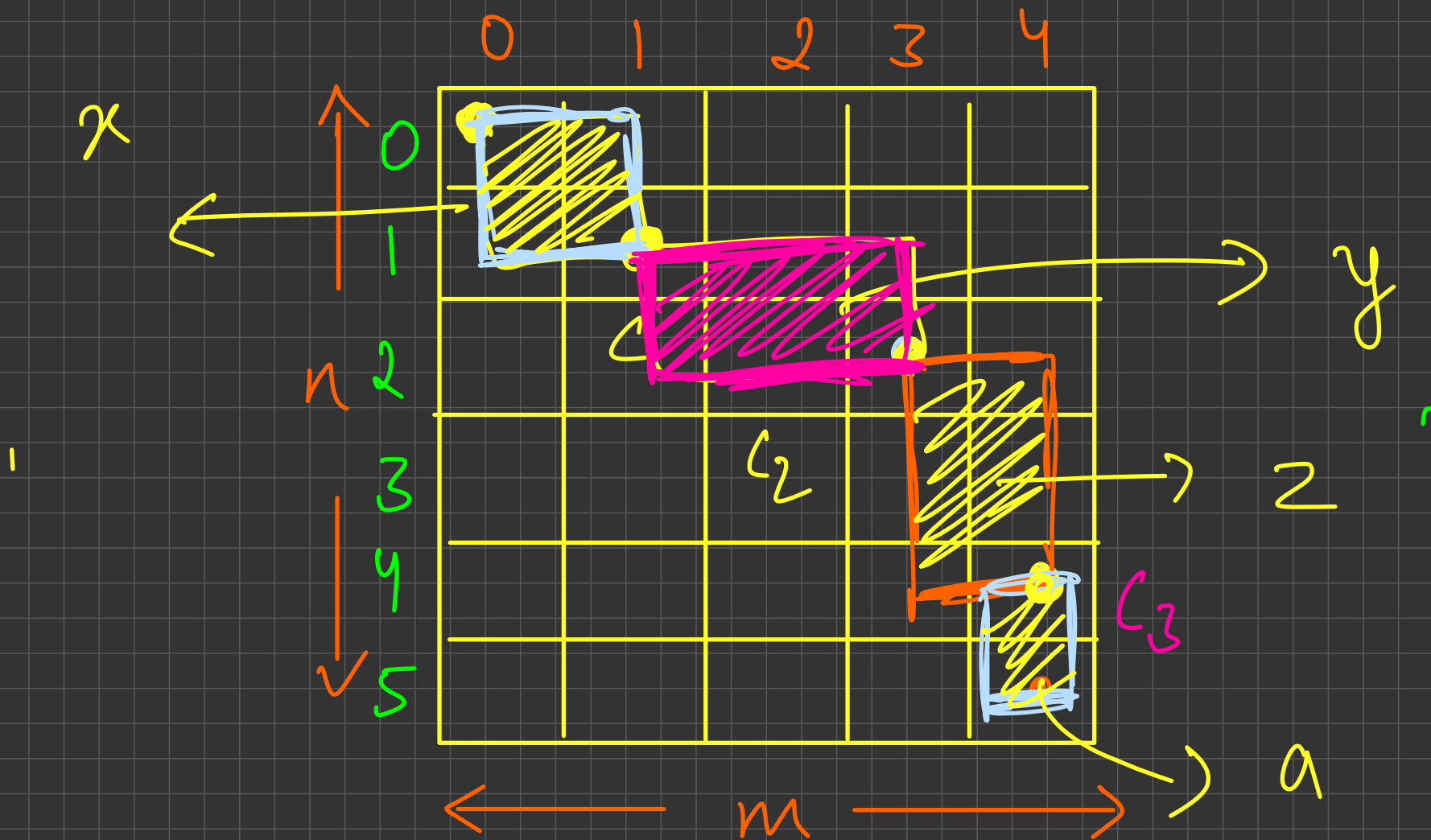
$(0,0)$ to (i,j) x
 (i,j) to $(n-1, m-1)$ y

ans for $(i, j) = x + y - \text{grid}[i][j]$

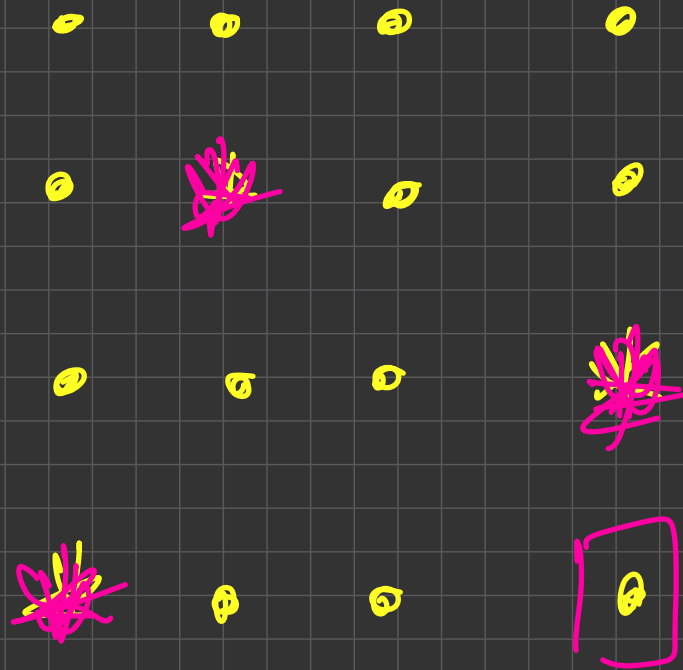
$\text{dp1}[i][j]$ = min sum path from
 $(0, 0)$ to (i, j)

$\text{dp2}[i][j]$ = min sum path from
 (i, j) to $(n-1, m-1)$

Final ans = $\text{dp1}[c_i][c_j] + \text{dp2}[c_i][c_j] - \text{grid}[c_i][c_j]$



(x, y, z, a)



$dp(i, j)$ = no. of ways to go
 from (i, j) to
 state $(n-1, m-1)$

$$\frac{n \times m}{n \times n}$$

transition

$$\underline{dp(i)(j)} = \underline{dp(i+1)(j)} + \underline{dp(i)(j+1)}$$

$$dp(n-1)(n-1) \begin{cases} 1 & \text{O/W} \\ 0 & \text{obstacle} \end{cases}$$

Base case

$dp(0)(0)$ find subproblem

time complexity: $O(n^2)$

space complexity: $O(n^2)$

