1. L1 Distance (Required for Scoring)

Given the number of points

$$N = 1500,$$

the total L1 distance between the observed curve points and the predicted parametric curve, computed as

$$L_1 = \sum_{i=1}^{N} \left(|x_i^{pred} - x_i^{obs}| + |y_i^{pred} - y_i^{obs}| \right),$$

is found to be

$$L_1 = 38102.17797216173.$$

The mean and median pointwise L1 errors are:

$$MeanL1perpoint = 25.401452, \qquad MedianL1perpoint = 22.165210.$$

Since the provided `xy_data.csv` contains only (x, y) values and no t values, we reconstructed the parameter vector using a uniform sampling over the given interval:

$$t = linspace(6, 60, N).$$

2. Final Fitted Parameters

The optimal parameters obtained from nonlinear least-squares fitting are:

$$\theta = 0.5163140382\ rad \approx 29.5826^\circ, \qquad M = -0.05, \qquad X = 55.0135530041.$$

These satisfy all given parameter constraints:

$$0^\circ < \theta < 50^\circ \quad (),$$

$$-0.05 < M < 0.05 \quad (),$$

$$0 < X < 100 \quad ().$$

Note: The parameter M reached the lower bound -0.05 during optimization, suggesting the fitting procedure attempts to push M below the given limit, but is restricted by the constraints.

3. Explanation of the Complete Process

Problem Restatement

The parametric curve is given by:

$$x(t) = t\cos\theta - e^{M|t|}\sin(0.3t)\sin\theta + X,$$

$$y(t) = 42 + t\sin\theta + e^{M|t|}\sin(0.3t)\cos\theta.$$

Unknown parameters:

$$\theta, \quad M, \quad X,$$

with constraints:

$$0^\circ < \theta < 50^\circ, \qquad -0.05 < M < 0.05, \qquad 0 < X < 100.$$

Model and Objective Function

Using the parametric equations, predicted values (x_{pred}, y_{pred}) were computed for any parameter vector (θ, M, X).

To estimate the unknowns, we solved a nonlinear least-squares problem minimizing:

$$\mathbf{r}(\theta, M, X) = x_{pred} - x_{obs} y_{pred} - y_{obs},$$

which corresponds to minimizing the L2 norm:

$$\min_{\theta, M, X} \|\mathbf{r}\|_2^2.$$

This yields a smooth and stable optimization landscape and is well supported by scientific libraries.

Optimization Details

- Solver: `scipy.optimize.least_squares` (TRF algorithm)
- Initial guess:
$$\theta = 0.4 \; rad, \quad M = 0, \quad X = 50.$$
- Parameter bounds:
$$\theta \in [0,\ \text{deg2rad}(50)], \quad M \in [-0.05,\ 0.05], \quad X \in [0,\ 100].$$
- Maximum function evaluations increased to ensure convergence.

Result Interpretation

The solver converged to:

$$\theta = 0.5163 \; rad, \qquad M = -0.05, \qquad X = 55.0136.$$

Since M reached its lower bound, the model likely prefers a slightly smaller value of M, but is restricted by the constraint.

L1 Evaluation as Required in the Assignment

Once the parameters were estimated, the final L1 distance was computed exactly as required:

$$L_1 = \sum_{i=1}^{N} \left(|x_i^{pred} - x_i^{obs}| + |y_i^{pred} - y_i^{obs}| \right).$$

This L1 score was obtained using the reconstructed uniform t vector and the finalized parameter values.

Potential Improvements

- If actual t values are available, retraining with true timestamps would significantly reduce error.
- One may directly minimize L1 instead of L2 (e.g., via robust optimization or linear programming), but L2 is smoother and yields good initial solutions.
- Since M hits the bound, relaxing the constraints for diagnostic purposes may help understand model behavior (though not allowed in this assignment).

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1 Introduction