Chapter 4
Construction of geometric figures

4.1 Angles revision

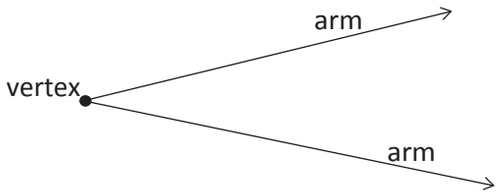
small angle

big angle

When two lines point in different directions, we say they are **at an angle** to each other. If the directions are almost the same, we say the **angle** between them is small. If the directions are very different, we say the angle between them is big.

Words we use to describe angles:

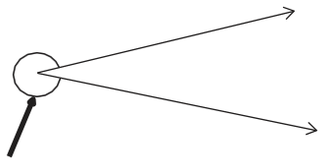
- **Arms of the angle:** the two lines that are at an angle to each other
- **The vertex:** the point where the two arms meet
- **Vertices:** plural of "vertex"



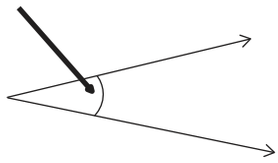
Symbols to describe angles:

Arrowheads on the lines mean that the lines keep on going. The length of an angle's arms does not change the size of the angle. Whether the arms are long or short, the angle size stays the same.

There are **two angles at a vertex** so it is important to show which one we are talking about.

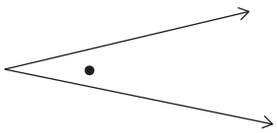


The arc shows where the angle is

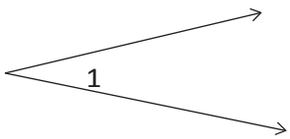


Labelling angles:

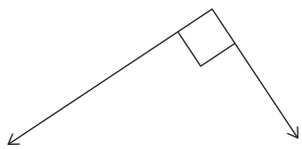
There are many different ways to label angles. Look at the examples below:



Using a dot or a star

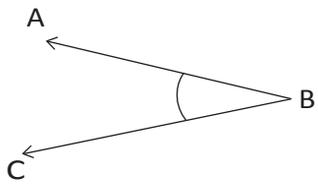


Angle 1



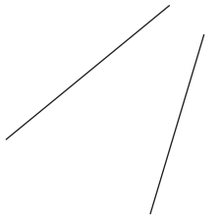
Right angle (90°)

You can name the angle on the right in different ways: you can say $A\hat{B}C$ or $C\hat{B}A$ or just $\hat{B}$. The "hat" on the letter shows where the angle is.



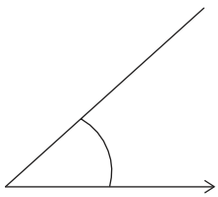
revision: seeing angles and describing angles

1. Look at the drawing on the right.
   (a) Are these lines at an angle to each other? Do the lines have to meet to be at an angle?
   (b) Copy the lines. Use a pencil and your ruler to draw the lines a bit longer so they meet. Did you change the angle between the lines when you extended them?

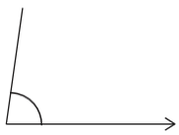


2. Arrange the angles from biggest to smallest. Just write the letters (a) to (f) in the correct order.

(a)

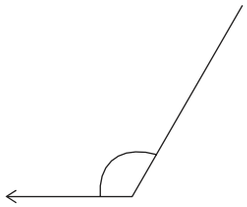


(b)

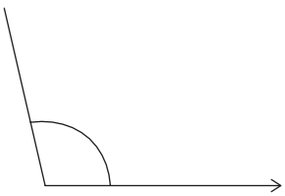


(c)

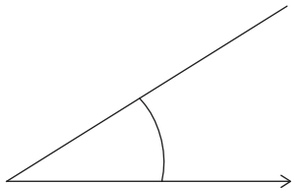
(d)



(e)

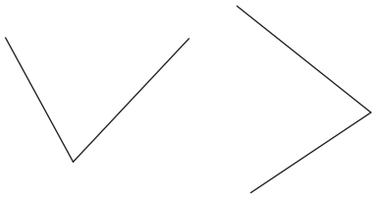


(f)

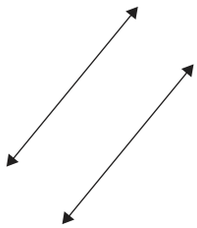


3. How can you check that an angle is a right angle without using any special mathematics equipment? (*Hint*: Think about where you can find right angles around you.)

4. Are these two angles the same size? Describe how you found your answer. (*Hint*: A piece of scrap paper may help!)

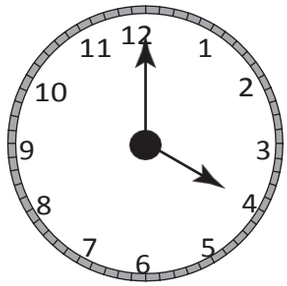


5. Two lines are drawn by holding down a ruler and drawing lines on both sides. What can you say about the two lines?



6. Look at the analogue clock face on the next page. The minute hand and the hour hand make an angle. Focus on the smaller angle for now.

(a) Explain why the angle between the hands at 8 o' clock is the same size as the angle at 4 o' clock.
(b) Compare the angle at 2 o' clock with the angle at 4 o' clock. What do you notice? Why is thisso?
(c) Is the angle at 3 o' clock the same as the angle at a quarter past 12? Explain.



7. When you open the cover of a hardcover book you can make different angles. Can you think of at least five other situations in everyday life where objects are turned through angles? Say what the arms and the vertices are in each of your examples.

4.2 The degree: a unit for measuring angles

Imagine if we didn't have units for measuring length. *How would tailors make clothes to the right size without a tape measure? How could an architect design a safe and beautiful house without a ruler? How could we lay out a professional soccer field without being able to measure accurately in metres?*

We need units and measuring instruments in many situations. You know that we use metres, centimetres, kilometres, millimetres, etc. for measuring lengths.

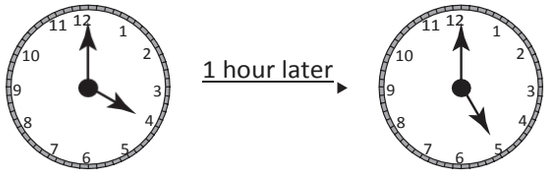
We should also have units for measuring angles. The units we use for measuring angles are very ancient. No one today is completely sure why, but our ancestors decided many thousands of years ago that a revolution should be divided into 360 equal parts. We call these parts degrees. The symbol for a degree is °.

some familiar angles in degrees

1. Copy and complete the table by filling in the size of each angle described.

Angle (in words)	Angle (degrees)
right angle	90°
straight angle	
revolution	360°
half a right angle	
a third of a right angle	
a quarter of a right angle	22,5°
half a straight angle	
three quarters of a revolution	
a third of a revolution	

2. Look at the clock shown. How many degrees does:
   (a) the minute hand move in an hour?
   (b) the hour hand move in an hour?

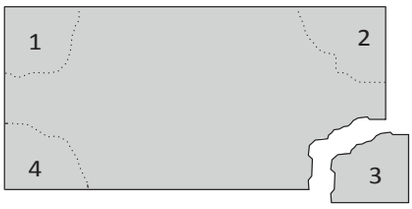


3. In Grade 6 you learnt that angles are classified into types. Copy and complete the table. The first one has been done as an example for you.

Angle	Size of the angle	Sketch of the angle
Acute angle	Between 0° and 90°	
Right angle		
Obtuse angle		
Straight angle		
Reflex angle		
Revolution		

comparing angles using a4 paper

You need a sheet of A4 paper. At the corners you have four right angles. Number them and tear the corners off as shown in the diagram. Do not make them too small.



Now use your right angles to investigate the following situations:

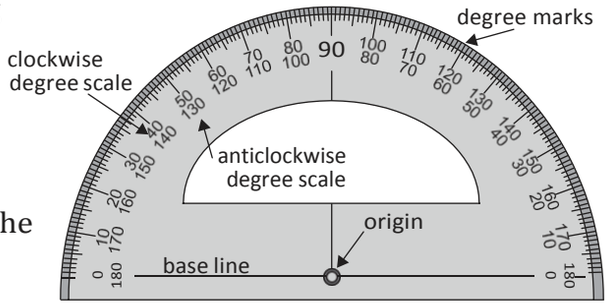
1. Show that a straight angle is two right angles.
   You can sketch what you have done in your book.
2. Show that a revolution is four right angles.
   You can sketch what you have done in your book.
3. Create a right angle using three of your corners.
   You can sketch what you have done in your book.
4. Describe how you can use one of your corners to check if an angle is acute, right or obtuse.
5. (a) Fold corner 1 so that you can use it to measure 45°.
   (b) Fold corner 2 so that you can use it to measure 30°.
   (c) Fold corner 3 so that you can use it to measure 22,5°.
   (d) Which is bigger: a right angle *or* half a right angle + a third of a right angle + a quarter of a right angle? Can you do a calculation to show that?

Important: Keep your folded pieces of paper for the next lesson!

4.3 Using the protractor

We have a special instrument for measuring angles. It is called a **protractor**. Look at the picture of a typical protractor with its important parts labelled.

Protractors can be big or small but they all measure angles in exactly the same way. The size of the protractor makes no difference to an angle's size.



measuring some familiar angles

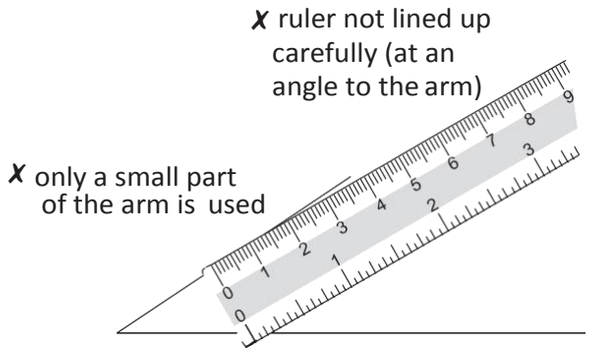
You need the four folded angles from the previous activity on page 60. If you didn't do that activity, then go back now and follow the instructions in question 5.

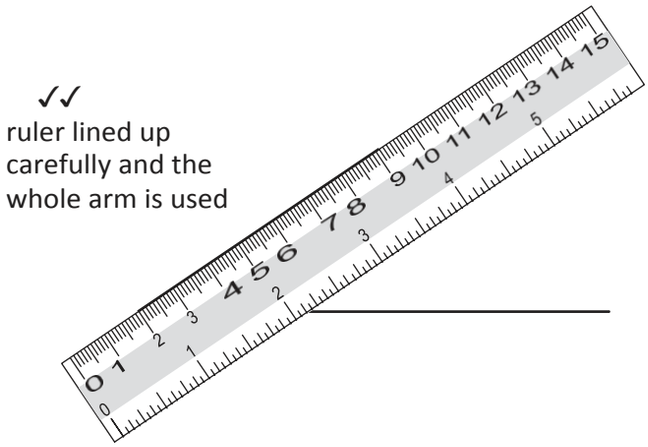
1. In a group of three or four, use your protractor to measure the angles that you made: 90°; 45°; 30° and 22,5°.
2. Did you measure the correct sized angle? If not, then ask yourself the following questions:
   - Did you put the vertex of the angle at the origin of the protractor?
   - Is the bottom arm of your angle lined up with the base line?
   - Did you fold your corners correctly?

how to use a protractor to measure an angle

Step 1: Are the angle arms long enough?

The angle arms must be a little longer than the distance from the origin of the protractor to its edge. If they are too short, use a sharp pencil and a ruler to make them longer. Be careful to line the ruler up with the arm.





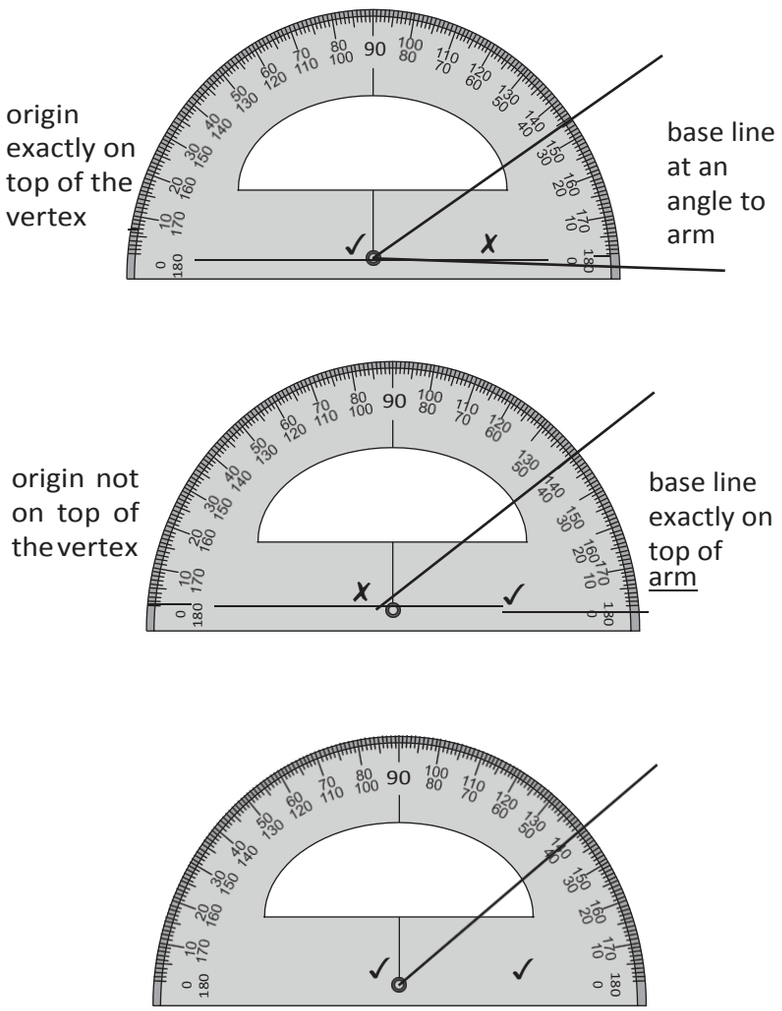
Now you are ready to start measuring your angle.

Step 2: Line up the angle and your protractor

Place your protractor on top of the angle. Make sure of the following:

- the origin is exactly on the vertex of the angle, and
- the base line is exactly on top of one of the arms of the angle.

Keep adjusting the position of the protractor until the origin and the base line are exactly lined up.

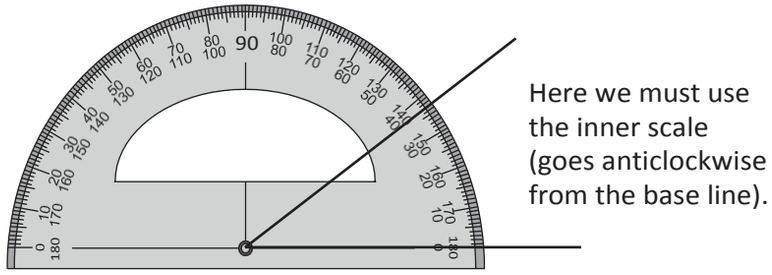


origin exactly on top of the vertex and base line exactly on top ofarm

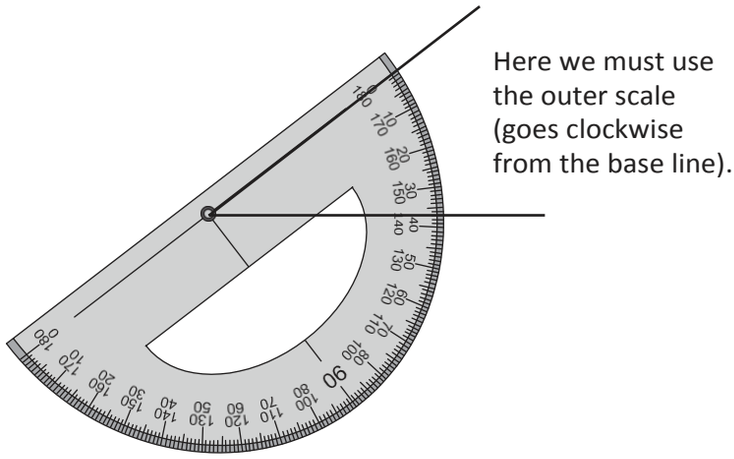
Once your protractor is in the correct place, keep a finger on the protractor to stop it from moving. If it moves… start again! You are now ready to make a measurement.

Step 3: Measure the angle

A protractor gives a clockwise degree scale and an anticlockwise degree scale. You choose the correct scale by finding the one that starts with 0° on the angle arm. Look at where the other angle arm passes under the degree scale. That is where your measurement is.



You can also place the protractor on the angle using the other arm. Then the correct position looks like this:

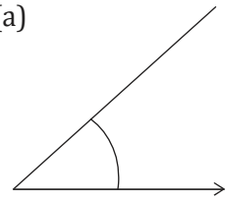


The angle in the pictures above is 37°. Do you agree? Do you see that there are two ways to measure an angle?

practise measuring with a protractor

1. Measure the angles and copy and complete the table on the next page. If you need to, copy the drawing and extend the arms.

(a)



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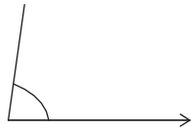
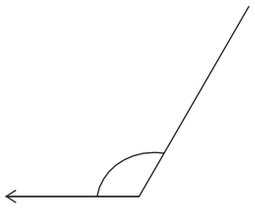
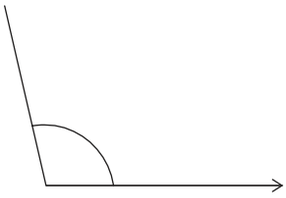


(c)

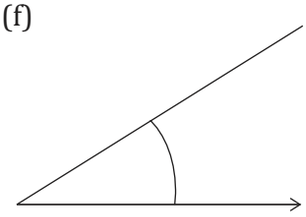
(d)



(e)



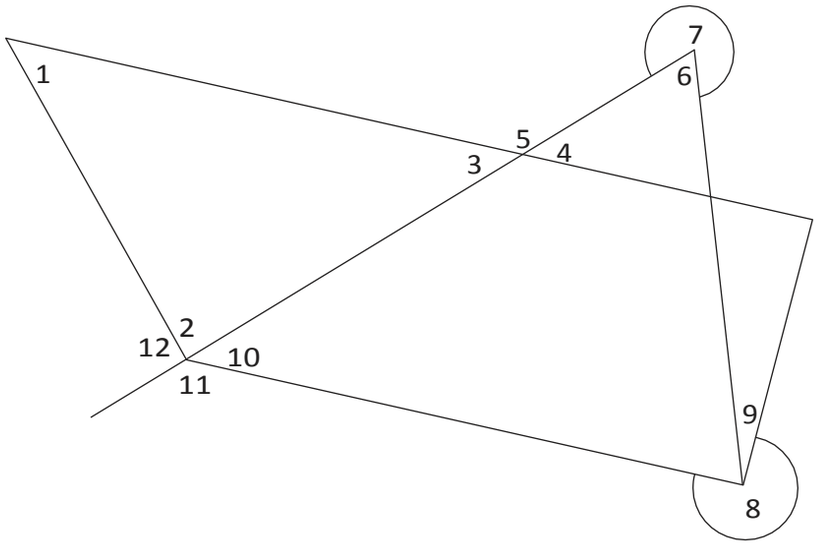
(f)



Angle	(a)	(b)	(c)	(d)	(e)	(f)
Angle size in degrees						

2. Copy the table below. Measure all the numbered angles in the following figure. Some angles can be measured directly, others not. Your protractor cannot measure reflex angles like angles 7 and 8. So you will have to make a plan!

Angle	Size
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	



3. Write a short note for yourself about measuring reflex angles.

some things to think about

Look at your answers in question 2 above.

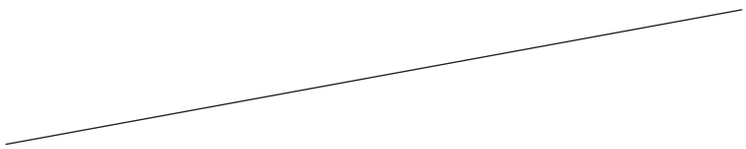
1. How do angles 3 and 4 compare?
2. What about angles 6 and 7?
3. What about angles 4 and 5?
4. There are some interesting ideas here. Try to do some further investigation and show your teacher what you discover.

4.4 Using a protractor to construct angles

constructing angles to a given line

Work together with a partner on this activity. You need your protractor, a sharp pencil and a straight ruler.

1. Your first challenge is to copy the line below and construct a line at exactly right angles to it. Begin by choosing a point on the line. You must mark this point clearly and neatly with a small dot. Then use your understanding of a protractor to draw a 90° angle.



2. Now copy Steps 2 to 4 and fill in the missing words.

   Step 1: Choose a point anywhere on the line. Make a small mark on the line. (You don't always have a choice here. Sometimes you must use a specific point on the line.)

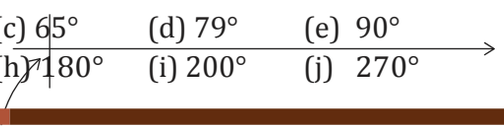
   Step 2: Place the protractor with its _____ on the line and its origin exactly on top of the _____

   Step 3: Make a small, clear mark at the _____

   Step 4: Use a ruler to line up the two _____ and draw a straight line that passes exactly through them.

3. Copy the line below and use it to construct the angles listed below. The line below will be one arm of the angles you are going to construct. The vertex for each of your angles is the point labelled O where the tiny vertical line cuts the long horizontal one. Your angles must be measured *anticlockwise* from the line.

   (a) 23° (b) 45° (c) 65° (d) 79° (e) 90°
   (f) 121° (g) 154° (h) 180° (i) 200° (j) 270°
   (k) 300°



angle direction

The line you have been given below is called a **reference line**.

Mathematicians usually measure angles **anticlockwise** from the reference line.

O

4. Copy the line on the right. Then at each end, draw lines at an angle of 60° to form a triangle. What sort of triangle is this?

5. Copy and complete the quadrilateral below. The angle at P must be 52° and the one at Q, must be 23°.

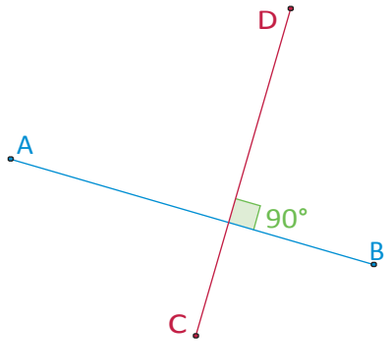
4.5 Parallel and perpendicular lines

Perpendicular lines meet each other at an angle of 90°.

The sketch shows two perpendicular lines.

We say: AB is perpendicular to DC.

We write: AB ⊥DC

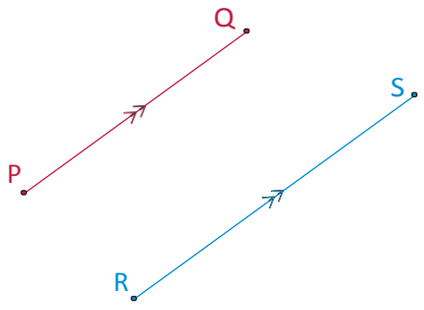


Parallel lines never meet each other. They are an equal distance apart. They have the same direction.

The sketch shows two parallel lines.
We say: PQ is parallel to RS.

We write: PQ //RS

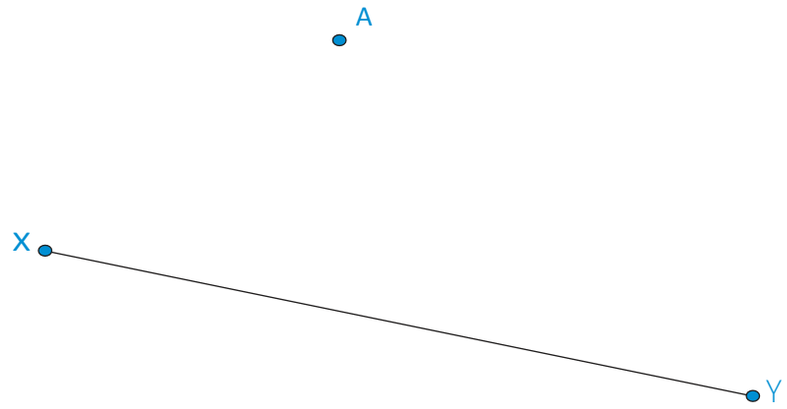
The arrows on the middle of the lines show that the lines are parallel to each other.



constructing perpendicular and parallel lines

When constructing parallel lines, remember that the lines always stay the same distance apart. Follow the steps on page 67 to draw perpendicular and parallel lines using a protractor and a ruler.

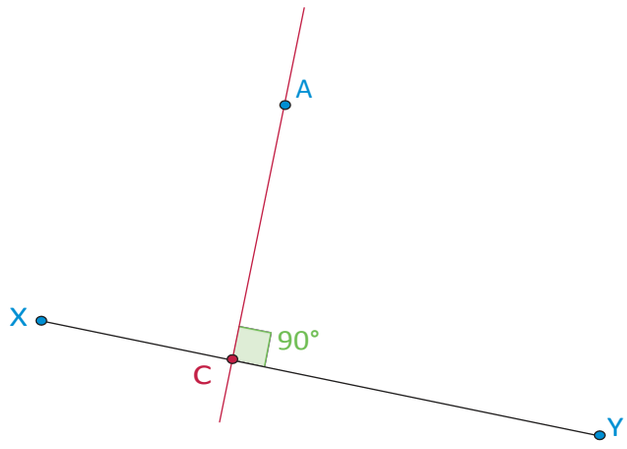
1. We want to draw a line that is parallel to XY and that passes through point A.



Step 1: Draw a perpendicular line between A and XY.

Copy the line XY above. Use your protractor to draw a line that goes through A and is at 90° to XY. Labelthepoint Cwhereyour new line touches XY.

Look at the sketch on the right if you get stuck.

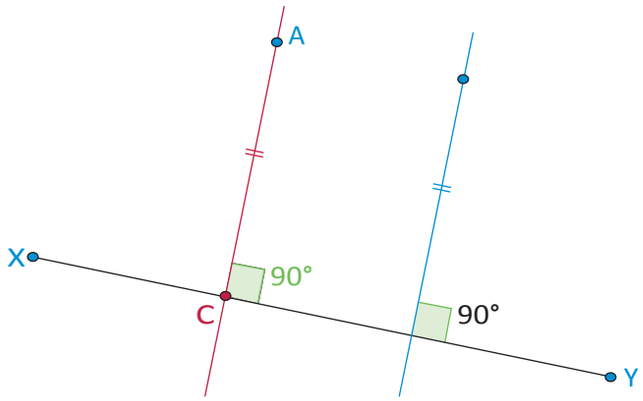


Step 2: Measure the perpendicular distance between the point and the line.

Write down the length of AC.

Step 3: Draw a point that is the same distance from the line.

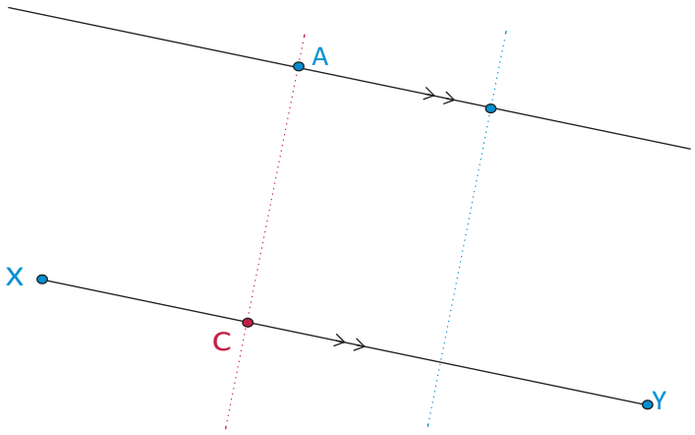
Then draw another line that is perpendicular to line XY. Mark off the same length as AC on that line. The sketch below shows what you must do.



Step 4: Draw the parallel line.

Join A with the new point that is an equal distance away from XY.

You now have a parallel line.



2. Practise constructing perpendicular and parallel lines using a protractor and a ruler.

4.6 Circles are very special figures

And now for something slightly different ... let us have a look at **circles**.

a circle with string

You may need to work with a partner here. You need two sharp pencils and a short length of string, an A4 sheet of paper and a ruler.

1. Tie the string to both pencils with double knots. The knots must be firm but not tight. The string must swing easily around the pencils without falling off. Once you have tied your string, the distance between the pencils when the string is tight should not be more than 8 cm.
2. Your partner must hold one pencil *vertically* with its point near the centre of the sheet of paper.
3. Now carefully move the tip of the other pencil around the middle one, drawing as you go. Try to keep the string *stretched* and the pencil *vertical* as you draw.
   If you have been careful, you will have drawn a circle (well, hopefully something pretty close to a circle). You can swop now so your partner also has a turn drawing while you hold the centre pencil.
4. Mark three points on the circle line. Measure the distance between the point and the centre of the circle for each. If you have a circle you should find that the distances are the same.

Circles are special for many reasons. The most important reason is the following:

The distance from the centre of a circle to the edge is the same in any direction.

This distance is called the **radius**.

We pronounce this: "ray-dee-us".

The plural of **radius** is **radii**.

We pronounce this: "ray-dee-eye".

think about it

Can you think of any other figure where the distance between the centre and the edge is constant in all directions?

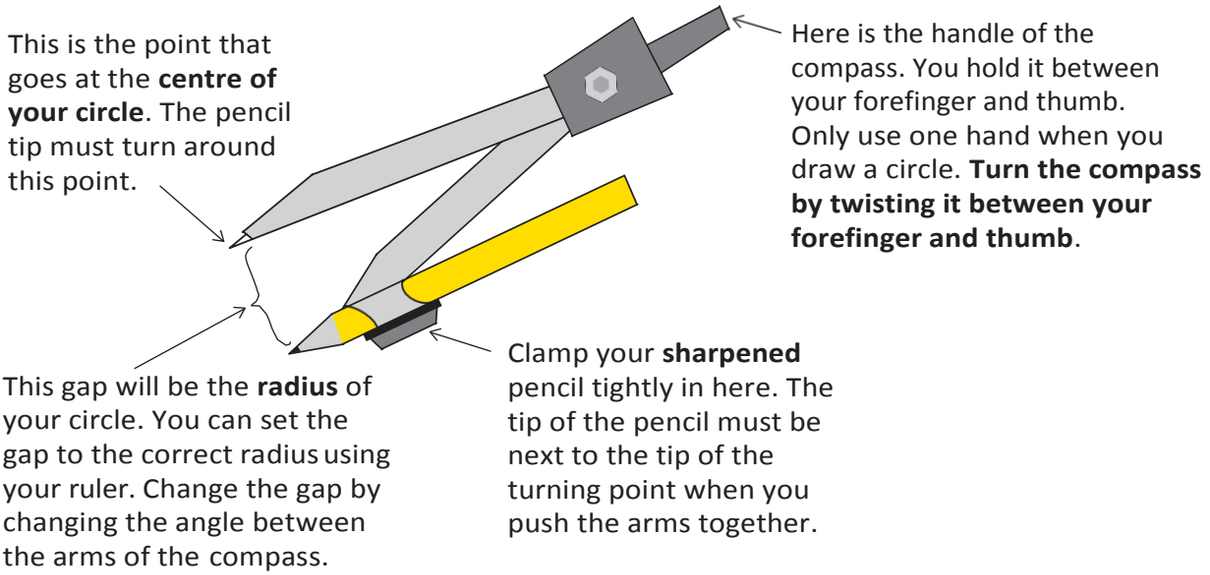
- A square?
- A hexagon?
- What about an oval shape (ellipse)?

do some investigation to see what you can find.

Do you agree that the two pencils and string are not a good way to draw circles? The string is stretchy. It is difficult to change the radius. And, the drawing pencil can wander off course and make a spiral or a wobbly curve. We need something better.

4.7 Using the compass

We need a special instrument for drawing circles. It must have a pointy tip, like the centre pencil. It must also have a drawing tip, like the pencil you moved. If you can set the distance between these two tips, you can draw circles of any radius. This instrument is called a **pair of compasses**, or often just a **compass**.

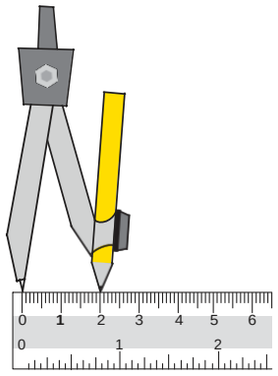


constructing circles with a compass

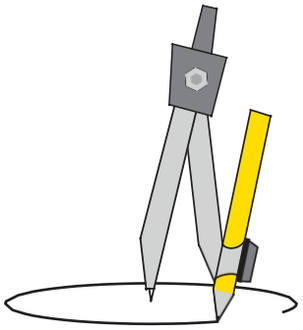
1. You will see a point labelled A below. Follow the steps below and on the next page to draw a circle with a radius of 2 cm. The centre must be at A.

   Step 1: Place the pointed tip on the zero line of your ruler. Carefully widen the angle between the arms. Move the pencil tip until it is exactly at 2 cm. Make sure that the pointed tip is still on zero. Be careful not to change the gap once it is set to 2 cm.

   Step 2: Mark point A in the centre of your page. Gently push the pointed tip into point A. Push just deep enough into the paper to keep it in place. This will be the centre of your circle.



Step 1



Step 4

A

   Step 3: Hold the handle between the forefinger and thumb of your writing hand. Keep your other hand out of the way. Use only one hand when you draw a circle with a compass.

   Step 4: Twist the handle between your thumb and forefinger. If you are right-handed, it is easiest to twist the compass clockwise. If you are left-handed, turn the compass anticlockwise. Let the pencil tip *drag* over the paper. Don't push down too hard on the pencil. Rather, push down lightly on the pointed arm as you draw. The pencil tip must move smoothly and easily.

2. Then draw concentric circles at centre A with radii of 3 cm, 4 cm, 5 cm and 6 cm. Set the gap carefully each time. Write the radius on the edge of each circle.

concentric circles have the same midpoint.

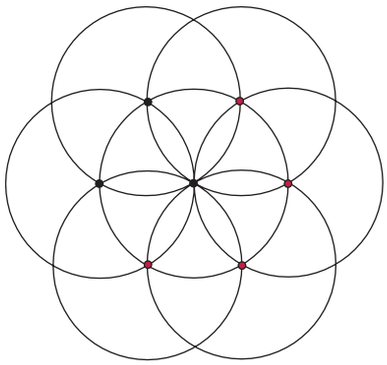
Learning to use a compass is like learning to ride a bicycle. It takes co-ordination and practice. Don't be embarrassed if it goes wrong. With practice you will get very good at it. If your circles end up being all wobbly lines, just begin again!

Here are some tips for drawing circles:

- If your circles are turning into spirals it is because the arms of your compass have moved. Check their width again against a ruler.
- If the arms of your compass won't stay in the position you set them at, it is because the nut at the hinge below the handle is loose. Ask your teacher to help you if you can't tighten it yourself.
- If you can't do the twist, imagine you have a small piece of soft clay between your thumb and forefinger and you are trying to roll it into a small strip. The twist for turning your compass uses the same type of sliding movement. Let the compass hang from your hand in the air and twist the handle. Then try it on scrap paper a few times until you can turn the compass easily.

circles on circles

It's time to have some fun with the compass while getting better at using it. Follow these instructions to draw the beautiful pattern shown on the right in your exercise book.



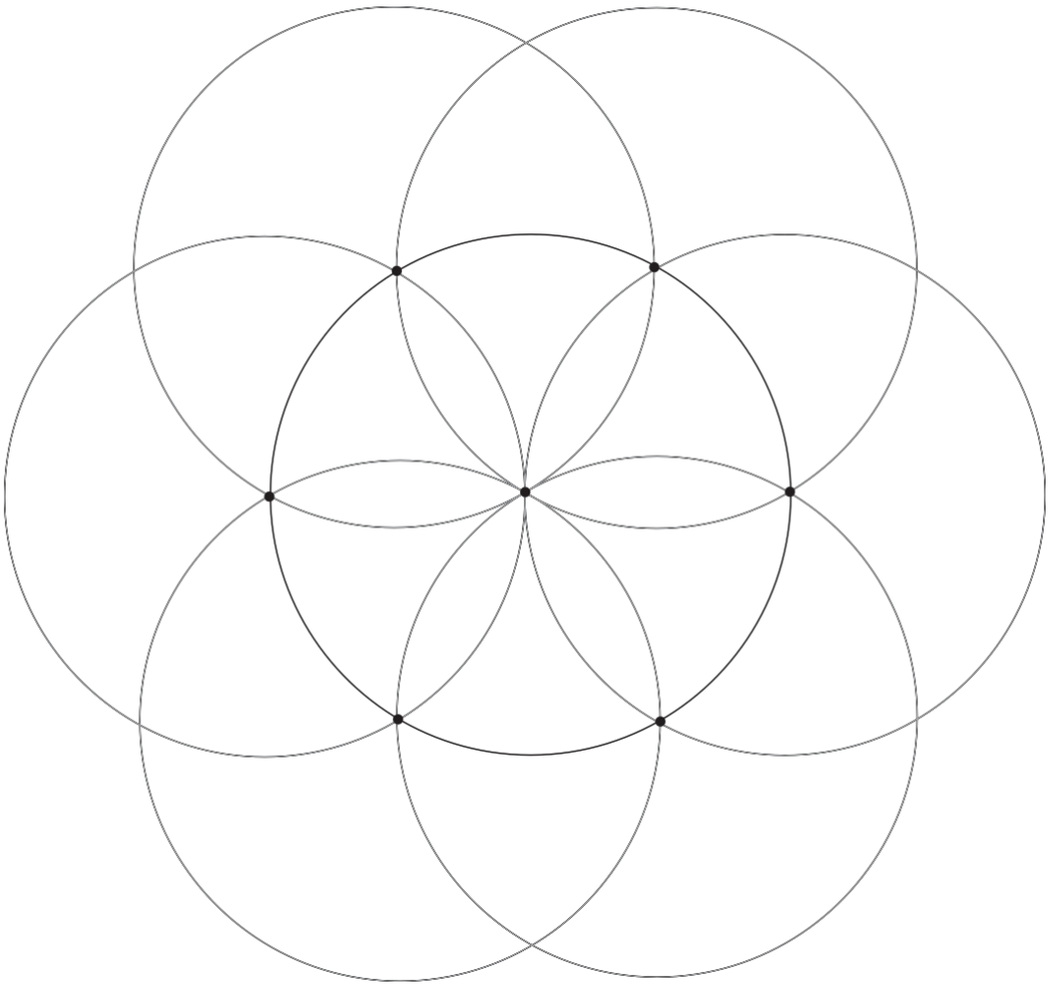
1. Make sure your pencil is sharp; then place it in the compass.
2. Set the radius to 4 cm. Draw a circle at the centre of your page. Important: your radius must stay the same for the whole activity.
3. Put your compass point anywhere on the circle edge. Draw another circle. This circle should pass through the centre of your first circle (they have the same radius).
4. Your second circle cuts the first circle at two points. Choose one of these points. Place your compass point at this point. Draw another circle of radius 4 cm.
5. Repeat step 3 with your third circle, fourth circle etc. You should end up with six circles on your first circle. That is, seven circles in total.
6. Decorate it as you please. (You can decorate your pattern further by adding more circles or joining points with straight lines, and so on. See what patterns and shapes you can discover among all the circles.)

4.8 Using circles to draw other figures

geometric figures hiding in the circles

Below is a set of seven circles like the one you drew. Sit with a partner and try to find hidden polygons.

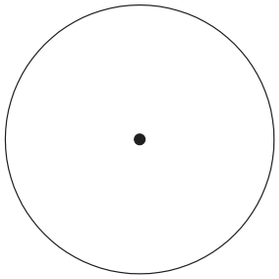
You will find these polygons by joining the points where the circles cut each other. The points will be the vertices of the polygons. Look carefully. There are triangles, quadrilaterals, pentagons and hexagons. When you can see them, neatly and carefully rule in their sides with a pencil. If there is not enough space on the set of circles below, redraw the circles on a separate piece of paper and show the figures there. If you wish, you can measure the angles at each vertex and the lengths of the sides.



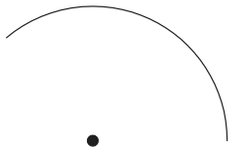
Arcs of circles

We do not have to draw whole circles to construct figures. We are only really interested in the points where the circles cross each other, so we could just draw arcs where they cross. Next year, you will use arcs in your geometric constructions.

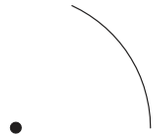
An **arc** is a small part of a circle. We use the term **circumference** when we refer to the distance around a circle or around any other curved shape.



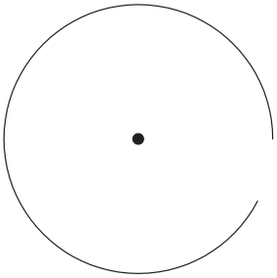
This is a full circle.



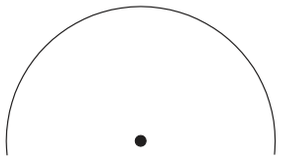
This is only part of a circle. A part of a circle like this is called an arc.



This arc is smaller.



This arc is almost a full circle.



This arc is just more than a semi-circle.

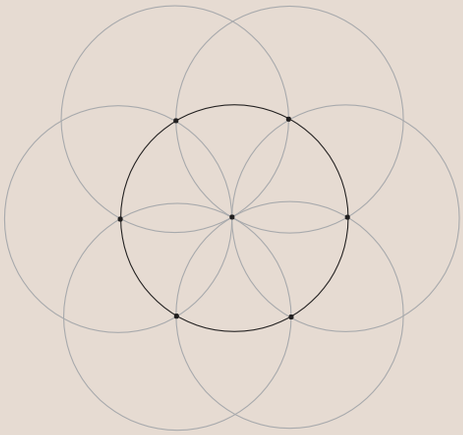
This arc is a quarter circle.

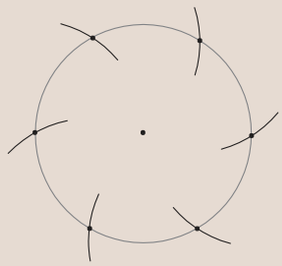
Do the following in your exercise book:

1. Draw an arc using a radius of 3 cm.
2. Draw an arc bigger than a quarter circle, using a radius of 5 cm.
3. Draw an arc smaller than a quarter circle, using a radius of 5 cm.

enrichment

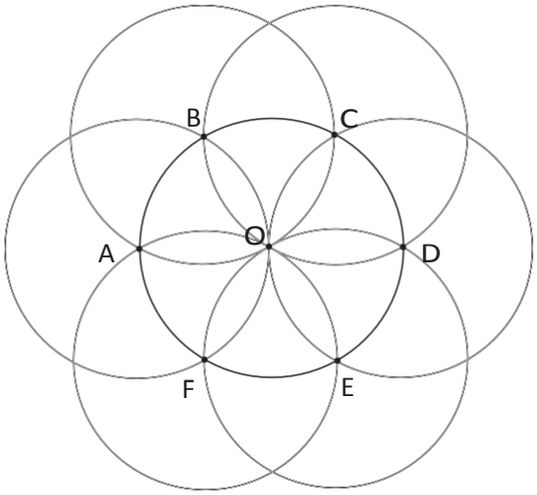
Once you have finished the work in section 4.8, experiment with drawing only the arcs that you need in various constructions. Here is an example to show how to construct a regular hexagon with only arcs:





familiar figures in the seven-circle pattern

For this activity you need five seven-circle sets like the ones drawn in the previous two activities. Start by drawing these on blank pieces of paper. Don't make your radius bigger than 4 cm. Number your sets figure 2 to figure 6. Label each figure as shown on the right.



1. Copy the figure above right by following the instructions below.
   - Figure 1: Draw lines connecting AB, BC, CD, … up to FA.
   - Figure 2: Draw lines connecting A, O and B.
   - Figure 3: Draw lines connecting B, F and D.
   - Figure 4: Draw lines connecting BC, CE, EF and FB.
   - Figure 5: Draw lines connecting CD, DE, EF and FC.
   - Figure 6: Draw lines connecting AB, BC, CE and EA.
2. Copy and complete the table below. It shows the name of each figure and its properties.
   Figure 1 (on the right) has been done as an example.

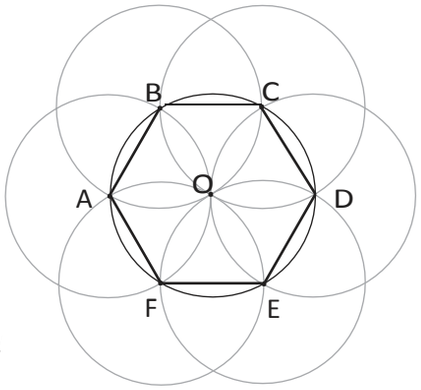


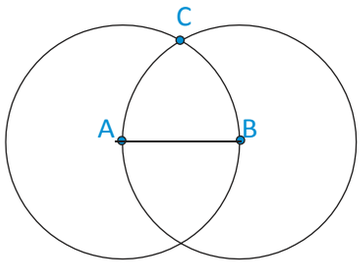
Figure 1

Figure	Name of figure	Properties
1	Regular hexagon	six-sided figure. All the sides are equal. All the interior angles are equal.
2		
3		
4		
5		
6		

construct some more figures

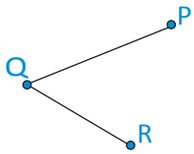
Read the instructions carefully and follow them exactly.

1. (a) Draw a line in your exercise book. The line should be between 3 cm and 6 cm long. Draw it in the middle of your page.
   (b) Label the ends A and B.
   (c) Place the point of your compass at point A. Carefully set the radius of your compass to the distance between A and B.
   (d) Draw a circle with the compass point at A.
   (e) Draw another circle with the compass point at B without changing the radius width.
   (f) The circles cross at two points. Choose one of these points. Label it C. Check that you are on the right track by comparing your sketch to the one on the right.



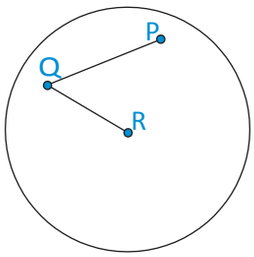
   (g) Carefully rule the lines AC and BC.
   (h) What sort of figure is ABC? Check this by measuring angles. Why do you think this happened?

2. (a) Draw two lines PQ and QR in your exercise book.
   - The lines meet and form an angle at Q.
   - You can make your angle anysize.
   - Make your line lengths different.
   - Do not make your lines longer than 6 cm each.

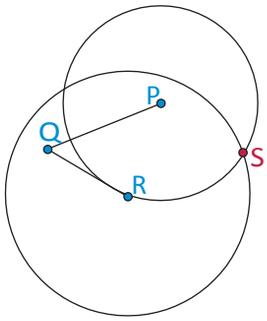


Step (a)

   (b) Place your compass point at point Q. Set the radius of your compass to the distance QP. Place the compass point at R. Draw a circle.
   (c) Place the compass point back at Q. Set the radius to the length QR. Place the compass point at P. Draw a circle.
   (d) The two circles cross at two points. Decide which point will be the vertex of a parallelogram. Call this point S.
   (e) Join the lines SP and SR. Is PQRS a parallelogram?



Step (b)



Steps (c) and (d)

something to think about
Why does this method form a parallelogram?

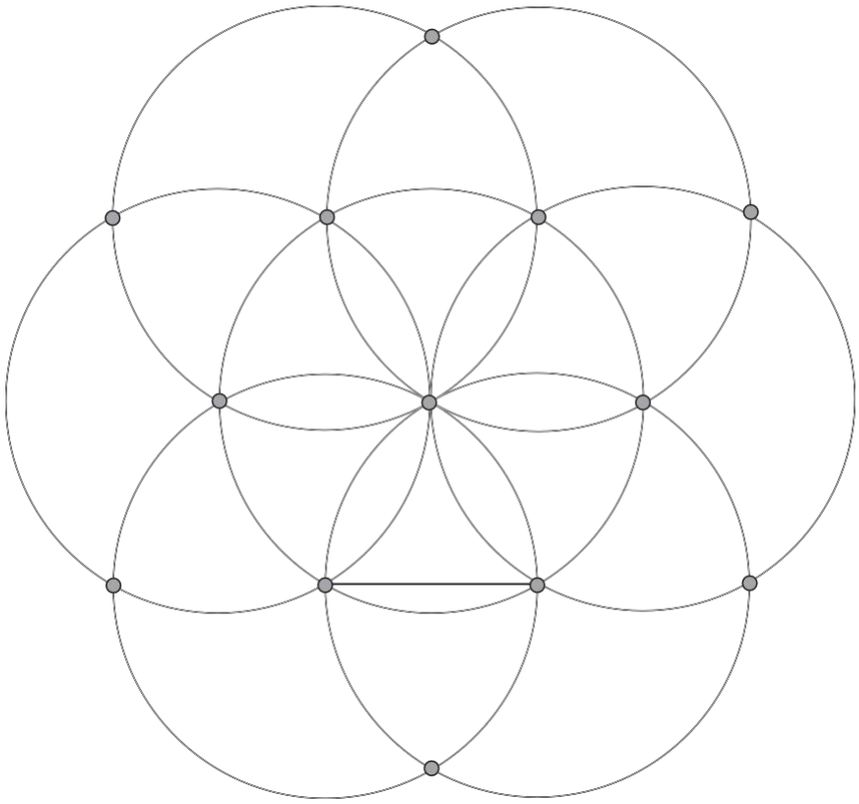
4.9 Parallel and perpendicular lines with circles

parallel and perpendicular

1. Revision: Copy and complete these definitions.
   (a) When one line is parallel to another line, the lines ...
   (b) When one line is perpendicular to another line, the lines ...

2. Copy the seven-circle figure below. The intersection points have been marked. A line segment has been drawn in. Use a ruler and pencil to join pairs of points so that the lines are:
   (a) parallel to the line segment
   (b) perpendicular to the line segment.

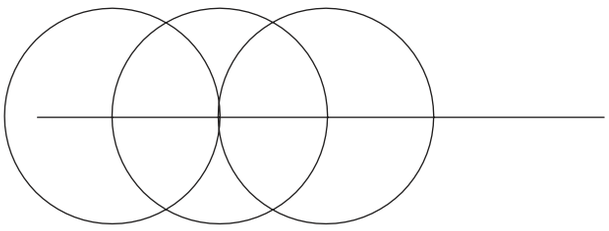
When two lines (or arcs) cross each other we say they **intersect**.
The **intersection point** is the place where they meet.



You should have drawn seven lines (two parallel and five perpendicular to the line segment).

Compare your lines with a friend's lines. Do you agree?

3. Draw a few circles with the same radius along a line. Start by drawing a line. Then use your compass to draw a circle with the midpoint on the line.



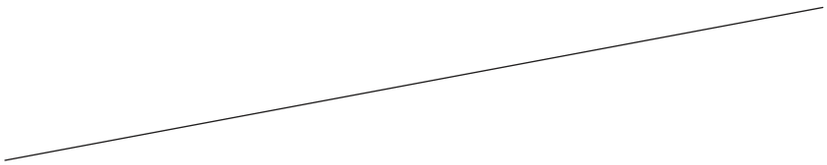
Keep your compass the same width and draw another circle with the centre where the first circle crossed the line. Repeat as many times as you wish. In the example at the bottom of the previous page, only three circles have been drawn.

(a) Can you find that example in the seven-circle figure? Look carefully until you see it.

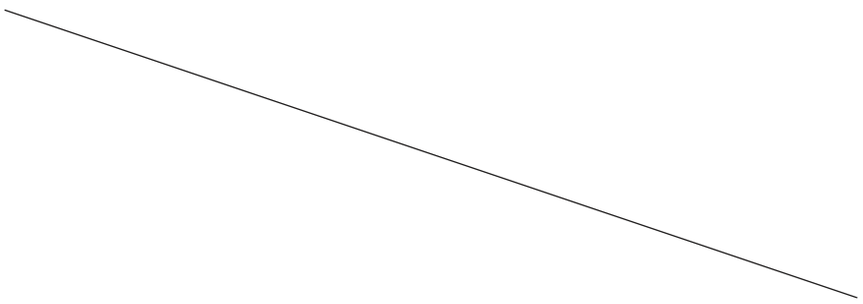
(b) Can you see where you can construct lines that are perpendicular to the given line? Draw them carefully with a pencil and your ruler.

(c) Can you see the two lines that are parallel to the given line? Draw them in too.

4. Copy the line below. Use circles to construct a line that is perpendicular to the line below.



5. Copy the line below. Use circles to construct a line that is parallel to the line below.



extension

1. Write P; as in the example below. Set your compass at a certain distance, for example 3 cm, and investigate points that are the same distance from a fixed point, P.

• P

2. Write two points, A and B, as shown below. Use your compass and investigate all the points