Middle East Technical University Department of Computer Engineering

CENG 280

Formal Languages and Abstract Machines

Spring 2018-2019

Take Home Exam 2 - Solutions

Due date: April 26, 2019, Friday, 23:55

1 Context-Free Grammars (20 pts)

a. Formally construct CFGs that generate each of the following languages. (10 pts)

(i) $\{a^i\, ba^j\, ba^{i+j} \in \{a,b\}^* \,|\, i,j \in \mathbb{N}\}$

$G = (\{a, b, S, S_1, S_2\}, \{a, b\}, R, S)$, where,

$$R = \{S \to S_1, \quad S_1 \to aS_1a \,|\, bS_2, \quad S_2 \to aS_2a \,|\, b\}$$

generates the given language.

(ii) $\{w \in \{a,b\}^* \,|$ the first, last and middle character of w are equal, $|w| > 3$ and is odd$\}$.

$G = (\{a, b, S, A, B\}, \{a, b\}, R, S)$, where,

$$S \to aAa \,|\, bBb, \quad A \to aAa \,|\, aAb \,|\, bAa \,|\, bAb \,|\, a, \quad B \to aBa \,|\, aBb \,|\, bBa \,|\, bBb \,|\, b$$

generates the given language.

(iii) $\{a^i\, b^j\, c^k \in \{a,b,c\}^* \,|\, i,j,k \geq 0$ and $i \neq j$ or $j \neq k\}$

The language is the union of $L_1 = \{a^i\, b^j\, c^k \,|\, i,j,k \in \mathbb{N}, i < j\}$, $L_2 = \{a^i\, b^j\, c^k \,|\, i,j,k \in \mathbb{N}, i > j\}$, $L_3 = \{a^i\, b^j\, c^k \,|\, i,j,k \in \mathbb{N}, j < k\}$, and $L_4 = \{a^i\, b^j\, c^k \,|\, i,j,k \in \mathbb{N}, k > j\}$. Define $V_i = \{a, b, c, S_i, F_i, A, C_i\}$ for $i = 1,2,3,4$, $\Sigma = \{a, b, c\}$, and

$$R_1 = \{S_1 \to F_1C_1, \quad C_1 \to C_1c \,|\, c, \quad F_1 \to aF_1b \,|\, F_1b \,|\, b\}$$

$$R_2 = \{S_2 \to F_2C_2, \quad C_2 \to C_2c \,|\, c, \quad F_2 \to aF_2b \,|\, aF_2 \,|\, a\}$$

$$R_3 = \{S_3 \to C_3F_3, \quad C_3 \to C_3a \,|\, a, \quad F_3 \to bF_3c \,|\, F_3c \,|\, c\}$$

$$R_4 = \{S_4 \to C_4F_4, \quad C_4 \to C_4a \,|\, a, \quad F_4 \to bF_4c \,|\, bF_4 \,|\, b\}$$

Then each $G_i = (V_i, \Sigma, R_i, S_i)$ generates L_i where $i = 1, 2, 3, 4$. Constructed grammar $G = (V, \Sigma, R, S)$ where $V = \bigcup_{i=1}^{4} V_i \cup \{S\}$ and $R = \bigcup_{i=1}^{4} R_i \cup \{S \to S_1 \,|\, S_2 \,|\, S_3 \,|\, S_4\}$ generates the language.

(iv) $\{w_1cw_2c\ldots cw_kccw_j^R \in \{a,b,c\}^* \,|\, k \geq 1,\ 1 \leq j \leq k,\ w_i \in \{a,b\}^+ \text{ for } i = 1, \ldots, k\}$

$G = (\{a, b, c, S, W, W_1, W_2\}, \{a, b, c\}, R, S)$ where,

$$
\begin{aligned}
R = \{ & \\
& S \to aW_1 \,|\, bW_2 \,|\, W, \\
& W_1 \to aW_1 \,|\, bW_1 \,|\, cW \,|\, cS, \\
& W \to aWa \,|\, bWb \,|\, acW_2ca \,|\, bcW_2cb, \\
& W_2 \to aW_2 \,|\, bW_2 \,|\, acW_2 \,|\, bcW_2 \,|\, ac \,|\, bc \,|\, e \\
\} &
\end{aligned}
$$

b. Consider the CFG $G = (V, \Sigma, R, S)$, where (10 pts)

$$
\begin{aligned}
V &= \{a, b, S, A, B\}, \\
\Sigma &= \{a, b\}, \\
R &= \{S \to aB \,|\, bA, \\
&\quad A \to a \,|\, aS \,|\, BAA, \\
&\quad B \to b \,|\, bS \,|\, ABB\}.
\end{aligned}
$$

Prove that $L(G)$ is the set of all nonempty strings in $\{a, b\}$ that have equal numbers of occurrences of a and b.

Define a function $f : V^+ \to \mathbb{Z}$ such that

$$
\begin{aligned}
& f(a) = 1,\ f(b) = -1 \\
& f(A) = 1,\ f(B) = -1\, f(S) = 0 \\
& \text{if } w = w_1w_2\ldots w_n, n \geq 1 \text{ then } f(w) = f(w_1) + f(w_2) + \cdots + f(w_k).
\end{aligned}
$$

Suppose that $x \Rightarrow^* y$, then we show that $f(x) = f(y)$.
If $x \Rightarrow y$, then $x = uFv$ and $y = u\alpha v$ where $F \to \alpha \in R$. From the definition of f, it follows that $f(x) = f(u) + f(F) + f(v)$ and $f(y) = f(u) + f(\alpha) + f(v)$. Then showing $f(F) = f(\alpha)$ where $F \to \alpha \in R$ is sufficient. The following shows this.

$$
\begin{aligned}
& f(S) = f(aB) = f(bA) = 0 \\
& f(A) = f(a) = f(aS) = f(BAA) = 1 \\
& f(B) = f(b) = f(bS) = f(ABB) = -1
\end{aligned}
$$

This implies if $x \Rightarrow y$ then $f(x) = f(y)$, which in turn implies if $x \Rightarrow^* y$ then $f(x) = f(y)$.

Now we show that any $x \in L(G)$ has equal number of as and bs.

Since $S \Rightarrow^* x$,

$$f(x) = \sum_{i=1}^{n} f(x_i) = f(S) = 0$$

by the above proven claim. Since each x_i is either a or b, it follows that x has equal number of as and bs.

It remains to prove that G produces *all* balanced strings. Claim: if $f(x) = 0$, then $S \Rightarrow^* x$, if $f(x) = 1$, then $A \Rightarrow^* x$, and if $f(x) = -1$, then $B \Rightarrow^* x$. We prove this by induction on the length of x. Base cases are $S \Rightarrow aB \Rightarrow ab$, $S \Rightarrow bA \Rightarrow ba$, $A \Rightarrow a$, $B \Rightarrow b$. Assume this holds for strings of length $n - 1$.

First assume $|x| = n$, n even. (Note that we are interested with xs that satisfy $f(x) = 0$ only, and since n is even, such x represents all balanced strings.) Then $x = ay$ or $x = by$. For the former (analogous steps apply for the latter) we have $B \Rightarrow^* y$ since $f(y) = -1$, $|y| = n - 1$. But since $S \Rightarrow aB \Rightarrow^* ay = x$ this string is generated by G.

Assume now $|x| = n$, for n odd. Then $f(x) = 1$ or $f(x) = -1$. For the former (analogous steps apply for the latter, again) there are two cases to consider. Then $x = ay$ or $x = by$. **(i)** If $x = ay$, $f(y) = 0$ and by induction hypothesis we have $S \Rightarrow^* y$. Therefore, $A \Rightarrow aS \Rightarrow^* ay = x$. **(ii)** If $x = by$, $f(y) = 2$. Then y can be written as $y = uv$ where $f(u) = f(v) = 1$. Then the induction hypothesis applies for u and v, $A \Rightarrow^* u$, $A \Rightarrow^* v$. Therefore $A \Rightarrow BAA \Rightarrow bAA \Rightarrow^* buA \Rightarrow^* buv$.

Since we have shown that if $S \Rightarrow^* x$ then x has equal number of as and bs, and if x has equal number of as and bs then $S \Rightarrow^* x$ we are done.

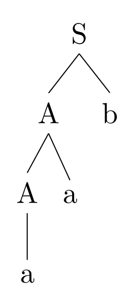
2 Parse Trees and Derivations (10 pts)

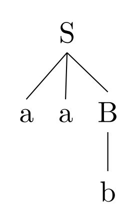
Consider the CFG $G = (V, \Sigma, R, S)$ where

$$V = \{a, b, S, A, B\},\ \Sigma = \{a, b\},\ R = \{S \to Ab \,|\, aaB,\ A \to a \,|\, Aa,\ B \to b\}.$$

a. Find the string s generated by the grammar that has two leftmost derivations. Give these derivations and corresponding derivation trees. (4 pts)

$s = aab$ is the only string causing the ambiguity of given grammar. Two distinct derivations of it are: $S \Rightarrow aaB \Rightarrow aab$ and $S \Rightarrow Ab \Rightarrow Aab \Rightarrow aab$. Corresponding parse trees are,



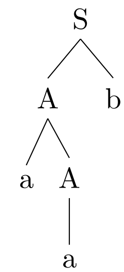


b. Find an equivalent unambiguous context-free grammar. (3 pts)

$G = (\{a, b, S, A\}, \{a, b\}, R, S)$ where $R = \{S \to Ab, \quad A \to aA \mid a\}$.

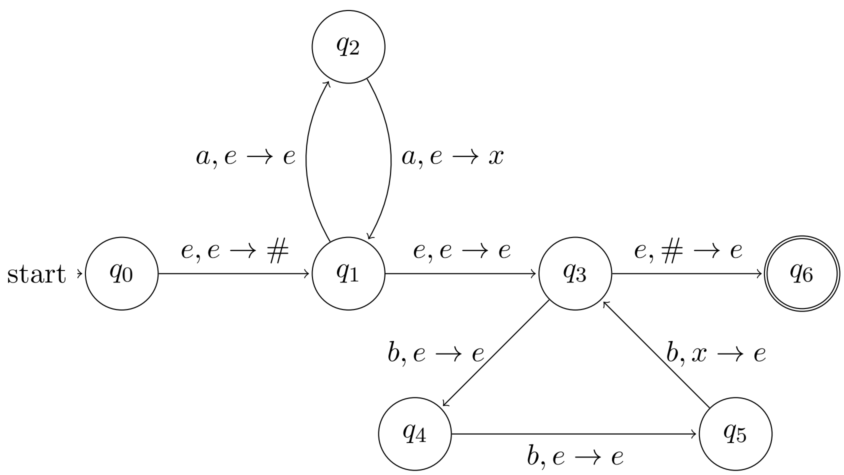
c. Give the unique leftmost derivation and derivation tree for the string s generated from the unambiguous grammar acquired in **b**. (3 pts)

$S \Rightarrow Ab \Rightarrow aAb \Rightarrow aab$, and the parse tree is:

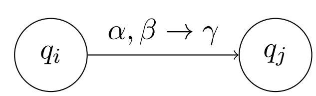


3 Pushdown Automata (15 pts)

a. Find the language generated by the PDA given below (5 pts)

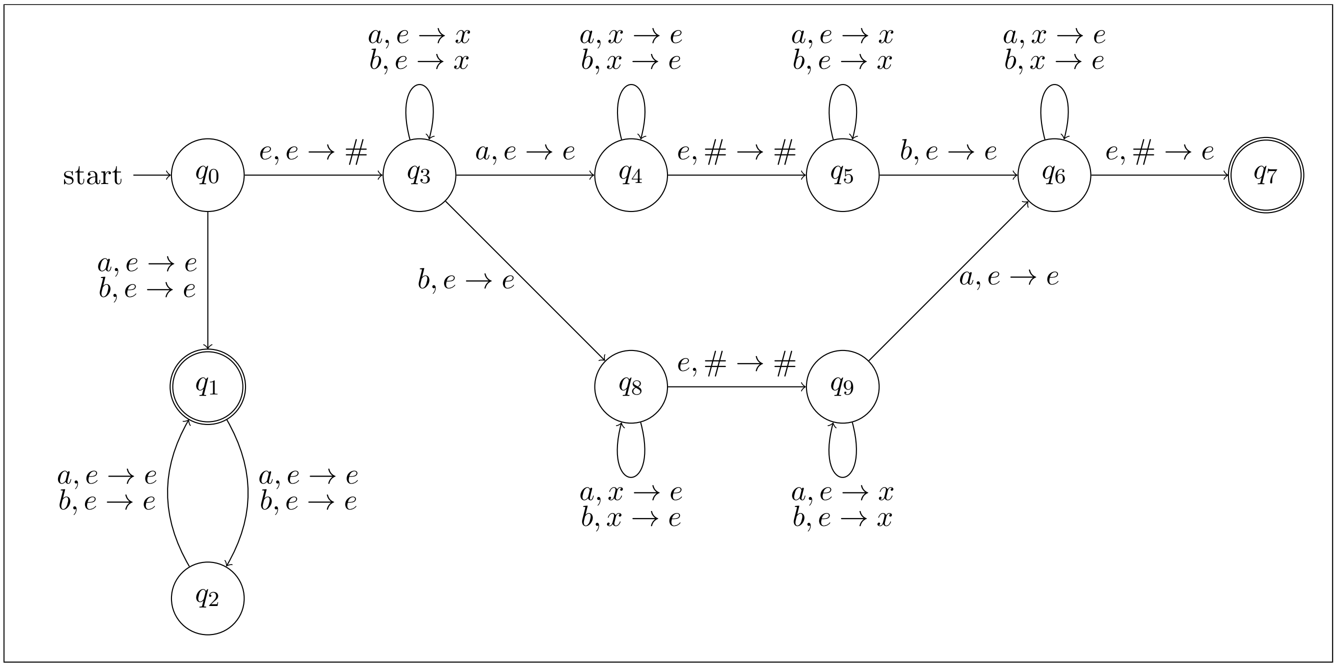


where the transition $((q_i, \alpha, \beta), (q_j, \gamma))$ is represented as:



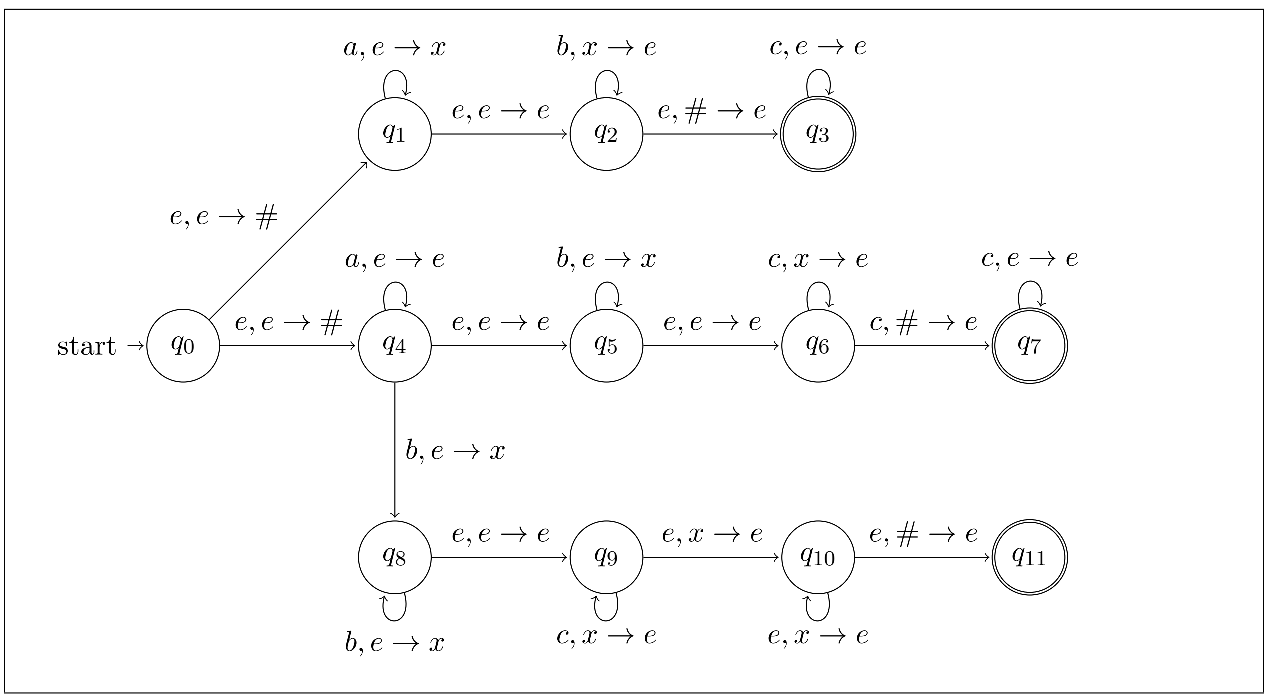
$$L = \{a^{2n}b^{3n} \in \{a, b\}^* \mid n \in \mathbb{N}\}$$

b. Design a PDA that generates the complement of the language $\{ww \in \{a, b\}^* \mid w \in \{a, b\}^*\}$. (5 pts)



c.

(i) Design a PDA that generates the language $\{a^i\, b^j\, c^k \mid i, j, k \geq 0, \text{ and } i = j \text{ or } j \neq k\}$. (5 pts)



(ii) Show that $aabcc \in L(M)$ and $bac \notin L(M)$ by tracing M on these strings. (5 pts)

State	Unread Input	Stack	Transition
q_0	abbcc	e	-
q_1	abbcc	#	$e, e \to \#$
q_1	bbcc	x#	$a, e \to x$
q_2	bbcc	x#	$e, e \to e$
q_2	bcc	#	$b, x \to e$
q_3	bcc	#	$e, \# \to e$
			Rejects.

State	Unread Input	Stack	Transition
q_0	aabcc	e	-
q_4	aabcc	#	$e, e \to \#$
q_4	abcc	#	$a, e \to e$
q_4	bcc	#	$a, e \to e$
q_5	bcc	x#	$e, e \to e$
q_5	cc	x#	$b, e \to x$
q_6	cc	x#	$e, e \to e$
q_6	c	#	$c, x \to e$
q_7	e	e	$c, \# \to e$
			Accepts.

4 PDA and CFGs (15 pts)

a. Consider the CFG $G = (V, \Sigma, R, E)$, where (5 pts)

$$\begin{aligned}
V &= \{a, +, \times, (,), E, T, F\},\\
\Sigma &= \{a, +, \times, (,)\},\\
R &= \{E \to E + T \mid T,\\
&\quad\; T \to T \times F \mid F,\\
&\quad\; F \to (E) \mid a\}.
\end{aligned}$$

Convert G to an equivalent PDA.

Apply the construction given in Lemma 3.4.1 of the textbook to build the equivalent PDA $M = (\{p, q\}, \Sigma, V, \Delta, p, \{q\})$, where,

$$\begin{array}{lll}
\Delta = \{ & & \\
& ((p, e, e), (q, E)), & \\
& ((q, e, E), (q, E + T)), & ((q, e, E), (q, T)), \\
& ((q, e, T), (q, T \times F)), & ((q, e, T), (q, F)), \\
& ((q, e, F), (q, (E))), & ((q, e, F), (q, a)), \\
& ((q, a, a), (q, e)), & , \\
& ((q, +, +), (q, e)), & ((q, \times, \times), (q, e)), \\
& ((q, (, (), (q, e)), & ((q,),)), (q, e))\}.
\end{array}$$

b. Show that if $M = (K, \Sigma, \Gamma, \Delta, s, F)$ is a PDA, then there is another PDA $M' = (K', \Sigma, \Gamma, \Delta', s, F)$ such that $L(M') = L(M)$ and for all $((q_i, u, \beta), (q_j, \gamma)) \in \Delta'$, $|\beta| + |\gamma| \leq 1$.

Apply the first two steps of converting a given PDA to a simple PDA to ensure that $|\beta| \leq 1$ and $|\gamma| \leq 1$. Then we have to only deal with transitions that pushes and pops at one step. We can easily divide this process into two by replacing each $((q_i, u, \beta), (q_j, \gamma))$ by $((q_i, u, \beta), (q_k, e))$ (pop) and $((q_k, u, e), (q_j, \gamma))$ (push) where q_k is a new state whose only incoming transition is from q_i and only outgoing transition is to q_j.

5 Closure Properties and Pumping Theorem (20 pts)

a. Use closure properties for CFLs to prove that the following languages are context-free.

(i) $\{a^m\, b^{m+n}\, a^n \in \{a,b\}^* \mid m, n \in \mathbb{N}\}$ (5 pts)

This is the concatenation of the CFL $L = \{a^n\, b^n \in \{a,b\}^* \mid m \in \mathbb{N}\}$ with $\{b^n\, a^n \in \{a,b\}^* \mid m \in \mathbb{N}\}$. Since CFLs are closed under concatenation, given language is also a CFL.

(ii) $\{a,b\}^* - L$, where $L = \{babaabaaab \ldots ba^{n-1}ba^n \in \{a,b\}^* \mid n \geq 1\}$ (5 pts)

Call the resulting language L_2. The words to be excluded from L_2, i.e., the words in L

- do not start with a
- do not end with b
- do not have consecutive bs
- have each of their substrings that has exactly three bs, and starting and ending with b, in the form $\{ba^nba^m \in \{a,b\}^* \mid n, m \in \mathbb{N} \text{ and } m = n+1\}$

In view of these, we can write L_2 as

$$L_2 = a\Sigma^* \cup \Sigma^* b \cup \Sigma^* bb \Sigma^* \cup \Sigma^* \{ba^nba^m \mid n+1 \neq m\}\Sigma^*$$

Since each of these languages are context-free, their union is also context-free.

b. Use Pumping Theorem for CFLs to show that following languages are not context-free.

(i) $\{a^m b^n \in \{a,b\}^* \mid m, n \in \mathbb{N} \text{ and } m \leq n^2\}$ (5 pts)

Assume that the given language is context-free. Consider the string $w = a^{k^2} b^k \in L$ where k is the pumping length. Then $w = uvxyz$ such that $|vy| > 0$ and $|vxy| \leq k$. When v (or(y), including both as and bs, is pumped, the acquired string does not belong to the language as it will mix as and bs. Hence it is either in a^{k^2} or b^k.
Case 1 v is in a^{k^2} and y is in b^k
$v = a^m$, $b = a^n$ and $0 < m+n \leq k$. **(i)** If $n \geq 1$, $uv^0xy^0z = a^{k^2-m}b^{k-n}$. Then $k^2 - m \leq (k-n)^2$. However, $(k-n)^2 \leq (k-1)^2 = k^2 - 2k + 1 \leq k^2 - m$, where the last step follows from $m \leq k$. **(ii)** If $n = 0$, $uv^2xy^2z = a^{k^2+m}b^k$. Then $k^2 + m \leq k^2$ which is impossible.
Case 2 v and y are in a^{k^2}
When pumped up, this produces $a^{k^2+m}b^k$ where $m = |u| + |y| > 0$. This implies $k^2 + m \leq k^2$ which is impossible.
Case 3 v and y are in b^k
When pumped down, this produces $a^{k^2}b^{k-m}$ where $m = |u| + |y| > 0$. This implies $k^2 \leq (k-m)^2 = k^2 - 2m + 1$ which is impossible.

(ii) $\{www \in \{a,b\}^* \mid w \in \{a,b\}^*\}$ (5 pts)

Assume that the given language is context-free. Consider the string $w = a^k b a^k b a^k b \in L = \{www \in \{a,b\}^* \mid w \in \{a,b\}^*\}$ where k is the pumping length. Then $w = uvxyz$ such that $|vy| > 0$ and $|vxy| \leq k$, thus, v and y cannot contain more than one b. Assume v contains a b or (not exclusive) y contains a b. Then uv^2xy^2z contains four bs or five bs. Since neither four nor five is divisible by three, this string is not in L. Now assume $v = a^n$, $y = a^m$, v comes before first b and y comes after the first b. Then $uv^2xy^2z = a^{k+n}ba^{k+m}ba^k$ which is not in L. The other cases that are not considered are derivatives of this last case, and identical reasoning applies. Since we reached a contradiction, using Pumping Theorem, our first assumption must be wrong, and the language is not context-free.

6 CNF and CYK (14 pts)

a. Consider the CFG $G = (V, \Sigma, R, S)$, where (6 pts)

$$\begin{aligned} V &= \{a, b, c, S, A, B, C\}, \\ \Sigma &= \{a, b, c\}, \\ R &= \{S \rightarrow aAB \mid bBA \\ &\quad\; A \rightarrow BS \mid C \\ &\quad\; B \rightarrow bA \\ &\quad\; C \rightarrow c \mid e\}. \end{aligned}$$

Convert G into an equivalent CFG in Chomsky normal form.

First step is to reduce long production rules. Introduce $F_1 \rightarrow AB$ and $F_2 \rightarrow BA$ to acquire

$\boldsymbol{S \rightarrow aF_1 \mid bF_2}$
$A \rightarrow BS \mid C$
$B \rightarrow bA$
$C \rightarrow c \mid e$
$\boldsymbol{F_1 \rightarrow AB}$
$\boldsymbol{F_2 \rightarrow BA}$.

Next, we eliminate e-rules. The set of erasable nonterminals is $\mathcal{E} = \{C, A\}$. This elimination yields,

$S \rightarrow aF_1 \mid bF_2$
$A \rightarrow BS \mid C$
$\boldsymbol{B} \rightarrow bA \mid \boldsymbol{b}$
$\boldsymbol{C} \rightarrow c$
$\boldsymbol{F_1} \rightarrow AB \mid \boldsymbol{B}$
$\boldsymbol{F_2} \rightarrow BA \mid \boldsymbol{B}$.

The only remaining violations are short rules. We build sets $\mathcal{D}(v)$ for $v \in V$ as

$$\mathcal{D}(a) = \{a\}, \quad \mathcal{D}(b) = \{b\}, \quad \mathcal{D}(S) = \{S\}, \quad \mathcal{D}(A) = \{A, C, c\}, \quad \mathcal{D}(B) = \{B, b\},$$

$$\mathcal{D}(C) = \{C, c\}, \quad \mathcal{D}(F_1) = \{F_1, B, b\}, \quad \mathcal{D}(F_2) = \{F_2, B, b\}.$$

Now we remove short rules and replace each rule of the form $A \to BC$ with all possible rules of the form $A \to B'C'$ where $B' \in \mathcal{D}(B)$ and $C' \in \mathcal{D}(C)$.

$\boldsymbol{S} \to aF_1 \,|\, bF_2 \,|\, \boldsymbol{aB} \,|\, \boldsymbol{ab} \,|\, \boldsymbol{bB} \,|\, \boldsymbol{bb}$

$\boldsymbol{A} \to BS \,|\, \boldsymbol{bS}$

$\boldsymbol{B} \to bA \,|\, \boldsymbol{bC} \,|\, \boldsymbol{bc}$

$\boldsymbol{F_1} \to AB \,|\, \boldsymbol{CB} \,|\, \boldsymbol{cB} \,|\, \boldsymbol{Ab} \,|\, \boldsymbol{Cb} \,|\, \boldsymbol{cb}$

$\boldsymbol{F_2} \to BA \,|\, \boldsymbol{bA} \,|\, \boldsymbol{BC} \,|\, \boldsymbol{bC} \,|\, \boldsymbol{Bc} \,|\, \boldsymbol{bc}$.

Since $C \to c$ was a short rule, it is removed, and we can safely refine this grammar by removing any rules of the form $U \to VC$ or $U \to CV$. Then the final grammar is

$S \to aF_1 \,|\, bF_2 \,|\, aB \,|\, ab \,|\, bB \,|\, bb$

$A \to BS \,|\, bS$

$B \to bA \,|\, bc$

$F_1 \to AB \,|\, cB \,|\, Ab \,|\, cb$

$F_2 \to BA \,|\, bA \,|\, Bc \,|\, bc$.

b. Using CYK decide whether the following strings belong to $L(G)$. (8 pts)

(i) $w_1 = babcb$

				$\{b\}$	1
			$\{c\}$	$\{F_1\}$	2
		$\{b\}$	$\{B, F_2\}$	$\emptyset$	3
	$\{a\}$	$\emptyset$	{S}	$\emptyset$	4
$\{b\}$	$\emptyset$	$\emptyset$	{A}	$\{F_1\}$	5
1	2	3	4	5	

Since the cell $N[5,5] = \{F_1\}$, this string can be produced starting from F_1. But since F_1 is not the starting state of the given PDA, this string is rejected.

(ii) $w_2 = acbbab$

					$\{b\}$	1
				$\{a\}$	$\{S\}$	2
			$\{b\}$	$\emptyset$	$\{A\}$	3
		$\{b\}$	$\emptyset$	$\emptyset$	$\{B\}$	4
	$\{c\}$	$\{F_1\}$	$\emptyset$	$\emptyset$	$\{F_1\}$	5
$\{a\}$	$\emptyset$	$\{S\}$	$\emptyset$	$\emptyset$	$\{S\}$	6
1	2	3	4	5	6	

We have the starting state S at $N[6,6]$. From this we conclude that the grammar accepts the string $acbbab$.

answer to the old version

1) Add a new start state

$\boldsymbol{S_0 \to S}$
$S \to aAB \mid bBA$
$A \to Ba \mid C$
$B \to bA \mid ABA$
$C \to c \mid e$

2) Remove e productions ($A \to e$)

a) Eliminate $C \to e$

$S_0 \to S$
$S \to aAB \mid bBA$
$\boldsymbol{A} \to Ba \mid C \mid \boldsymbol{e}$
$B \to bA \mid ABA$
$\boldsymbol{C} \to c$

b) Eliminate $A \to e$

$S_0 \to S$
$\boldsymbol{S} \to aAB \mid \boldsymbol{aB} \mid bBA \mid \boldsymbol{bB}$
$\boldsymbol{A} \to Ba \mid C$
$\boldsymbol{B} \to bA \mid ABA \mid \boldsymbol{BA} \mid \boldsymbol{AB} \mid \boldsymbol{B}$
$C \to c$

3) Remove unit productions ($A \to B$)

a) Eliminate $B \to B$

$S_0 \to S$
$S \to aAB \mid aB \mid bBA \mid bB$
$A \to Ba \mid C$
$\boldsymbol{B} \to bA \mid ABA \mid BA \mid AB$
$C \to c$

b) Eliminate $A \to C$

$S_0 \to S$
$S \to aAB \mid aB \mid bBA \mid bB$
$\boldsymbol{A} \to Ba \mid \boldsymbol{c}$
$B \to bA \mid ABA \mid BA \mid AB$
$C \to c$

3) c) Eliminate $S_0 \to S$

$\boldsymbol{S_0 \to aAB \mid aB \mid bBA \mid bB}$
$S \to aAB \mid aB \mid bBA \mid bB$
$A \to Ba \mid c$
$B \to bA \mid ABA \mid BA \mid AB$
$C \to c$

4) Replace long productions ($A \to \alpha\beta\gamma$)

a) Introduce $F_1 \to BA$

$\boldsymbol{S_0} \to aAB \mid aB \mid \boldsymbol{bF_1} \mid bB$
$\boldsymbol{S} \to aAB \mid aB \mid \boldsymbol{bF_1} \mid bB$
$A \to Ba \mid c$
$\boldsymbol{B} \to bA \mid A\boldsymbol{F_1} \mid BA \mid AB$
$C \to c$
$\boldsymbol{F_1 \to BA}$

b) Introduce $F_2 \to AB$

$\boldsymbol{S_0} \to \boldsymbol{aF_2} \mid aB \mid bF_1 \mid bB$
$\boldsymbol{S} \to \boldsymbol{aF_2} \mid aB \mid bF_1 \mid bB$
$A \to Ba \mid c$
$B \to bA \mid AF_1 \mid BA \mid AB$
$C \to c$
$F_1 \to BA$
$\boldsymbol{F_2 \to AB}$

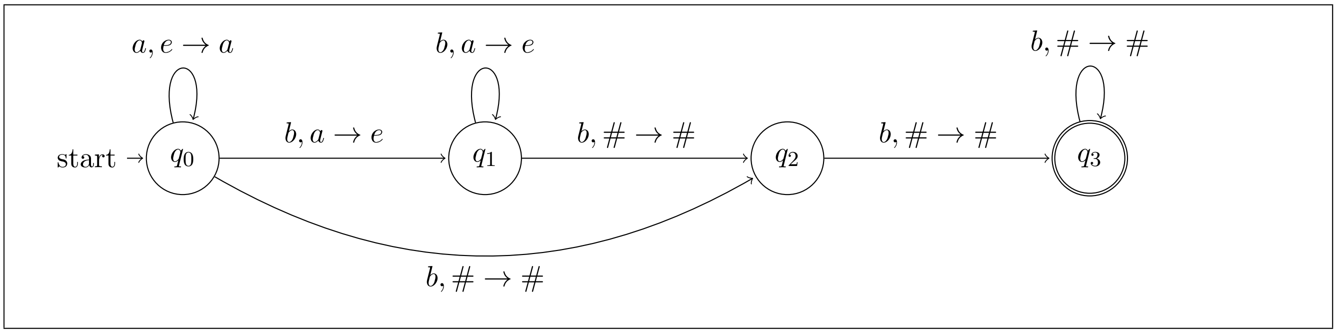
5) Move terminals to unit productions

$\boldsymbol{S_0 \to T_1F_2 \mid T_1B \mid T_2F_1 \mid T_2B}$
$\boldsymbol{S \to T_1F_2 \mid T_1B \mid T_2F_1 \mid T_2B}$
$\boldsymbol{A \to BT_1} \mid c$
$\boldsymbol{B \to T_2A} \mid AF_1 \mid BA \mid AB$
$C \to c$
$F_1 \to BA$
$\boldsymbol{F_2 \to AB}$
$\boldsymbol{T_1 \to a}$
$\boldsymbol{T_2 \to b}$

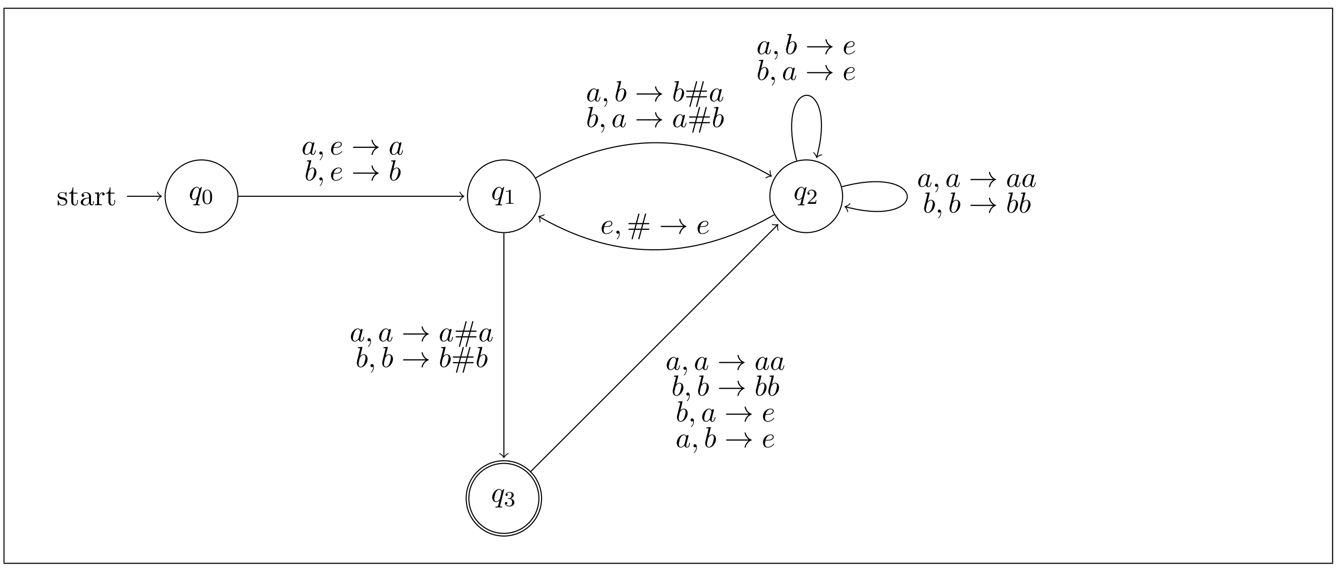
7 Deterministic Pushdown Automata (not graded)

Construct a DPDA that generates the given languages.

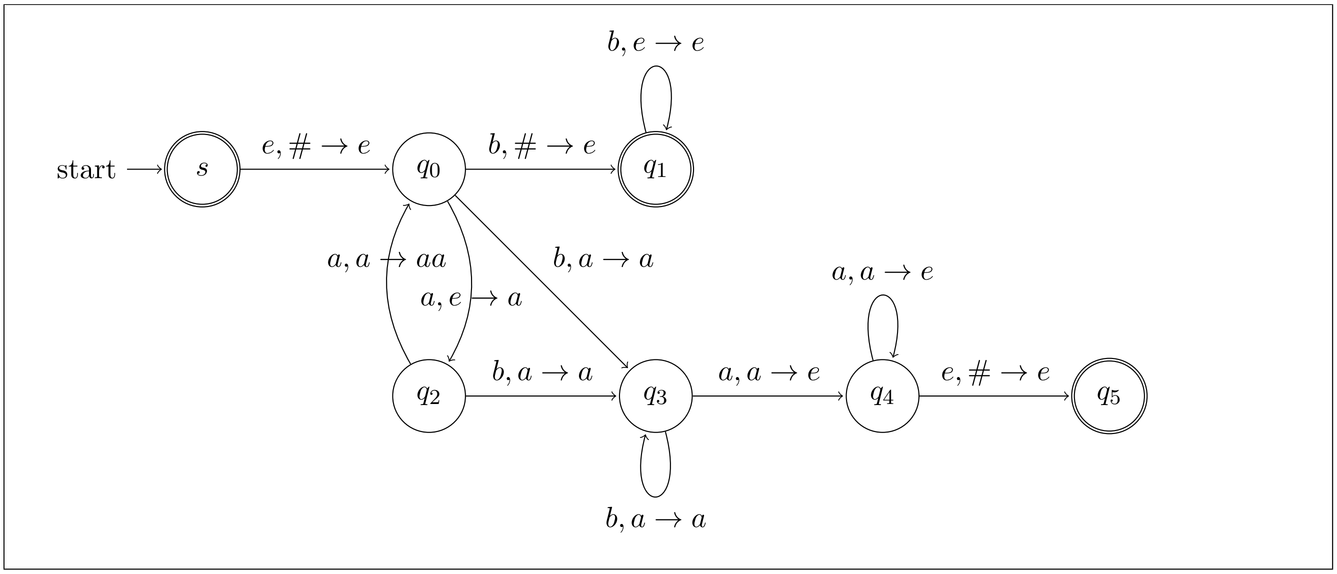
a. $\{a^n b^m \in \{a, b\}^* \mid n, m \in \mathbb{N} \text{ and } m \geq n + 2\}$



b. $\{w \in \{a, b\}^* \mid w$ starts and ends with the same symbol and have the same number of as and bs $\}$



c. $\{a^n b^m a^n \in \{a, b\}^* \mid m, n \in \mathbb{N}\}$


start
s
$e, \# \to e$
q_0
$b, \# \to e$
q_1
$b, e \to e$
$a, a \to aa$
$a, e \to a$
$b, a \to a$
$a, a \to e$
q_2
$b, a \to a$
q_3
$a, a \to e$
q_4
$e, \# \to e$
q_5
$b, a \to a$