Student Information

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Answer 1

Population variance σ's are unknown. We will use Student's t distribution since the sample sizes are normally distributed and small. Assuming variances between old and young populations are equal. s_p is the pooled standard deviation and:

$$s_p = \sqrt{\frac{(n-1)s_X^2 + (m-1)S_Y^2}{n+m-2}} = \sqrt{\frac{(19-1)(0.96)^2 + (15-1)(1.12)^2}{19+15-2}} = \sqrt{\frac{34.1504}{32}} = 1.0672$$

Degree of freedom is $n + m - 2 = 19 + 15 - 2 = 32$.

Part a)

Target confidence interval is 95%, so α is 0.05 and $t_{\alpha/2} = 2.037$. So the margin (let's call it m) is:

$$m = t_{\alpha/2} \cdot s_p \sqrt{\frac{1}{n} + \frac{1}{m}} = 2.037 \cdot 1.0672 \sqrt{\frac{1}{19} + \frac{1}{15}} = 0.7509$$

So the confidence interval is:

$$\left[\overline{X} - \overline{Y} - m, \overline{X} - \overline{Y} + m\right] = \left[3.375 - 2.05 - 0.7509, 3.375 - 2.05 + 0.7509\right] = \left[0.5741, 2.0759\right]$$

Part b)

Target confidence interval is 90%, so α is 0.1 and $t_{\alpha/2} = 1.694$. So the margin (let's call it m) is:

$$m = t_{\alpha/2} \cdot s_p \sqrt{\frac{1}{n} + \frac{1}{m}} = 1.694 \cdot 1.0672 \sqrt{\frac{1}{19} + \frac{1}{15}} = 0.6244$$

So the confidence interval is:

$$\left[\overline{X} - \overline{Y} - m, \overline{X} - \overline{Y} + m\right] = \left[3.375 - 2.05 - 0.6244, 3.375 - 2.05 + 0.6244\right] = \left[0.7006, 1.9494\right]$$

Part c)

We need to establish an interval of $(3, 5]$ in order to answer this. Consider hypothesis' $H_0 : \mu = 3$ and $H_A : \mu > 3$. σ is unknown, population is normally distributed and sample size is small, so let's use *right-tail t-test*. $\alpha = 0.05$, degree of freedom is $19 - 1 = 18$ and $t_\alpha = 1.734$.

$$t = \frac{\overline{X} - \mu_0}{s/\sqrt{n}} = \frac{3.375 - 3}{0.96/\sqrt{19}} = 1.703$$

Since $t < t_\alpha$, it is in the accepting area so we accept the null hypothesis. Thus, we conclude that we cannot say this with a 95% confidence.

Answer 2

Part a)

Null hypothesis is: Mean weight is 20 kg.

$$H_0 : \mu = 20$$

Part b)

Alternate hypothesis is: Mean weight is not 20 kg.

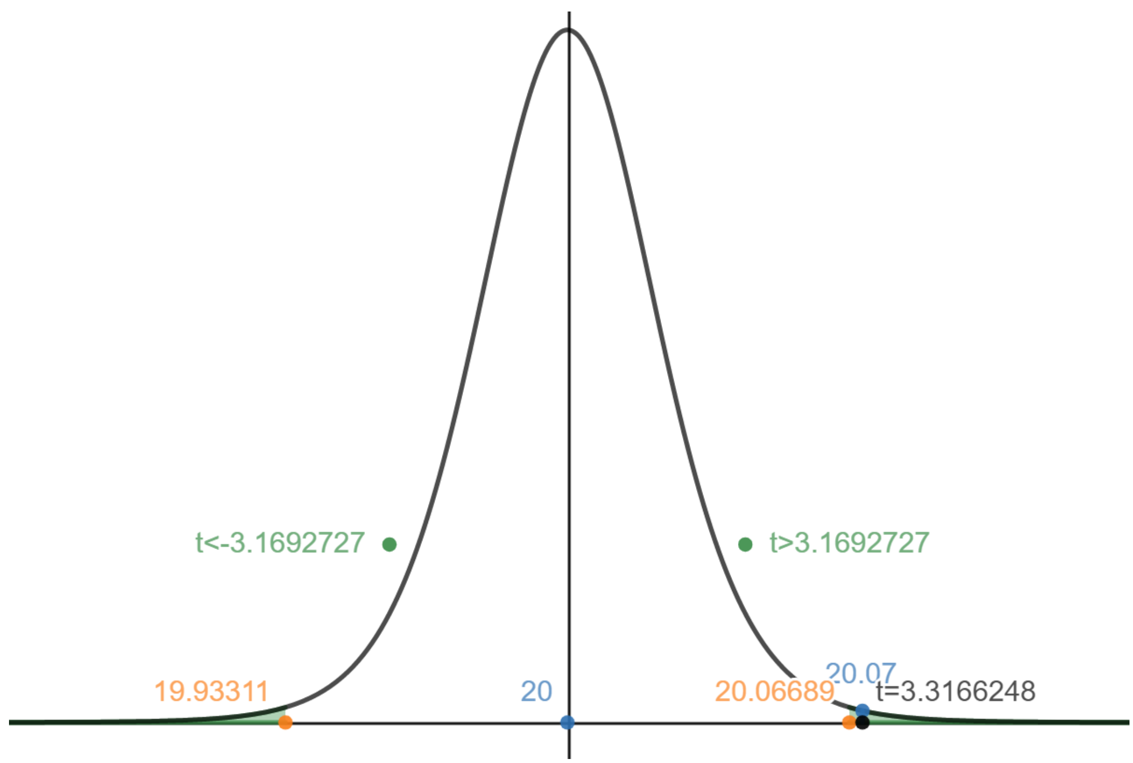
$$H_A : \mu \neq 20$$

Part c)

Since level of significance is 1%, α is 0.01 and degree of freedom is $11 - 1 = 10$. We will use *two-sided t-test* since both short and long bars are unfavourable, so $t_{\alpha/2} = 3.169$.

$$t = \frac{\overline{X} - \mu_0}{s/\sqrt{n}} = \frac{20.07 - 20}{0.07/\sqrt{11}} = 3.317$$

Since $t > t_{\alpha/2}$, it is in the rejection area so we reject the null hypothesis. Thus, we conclude that the line staff should stop production and check the line.



Answer 3

Part a)

Null hypothesis is: Difference of mean durations is 0 minutes.

$$H_0 : \mu_X - \mu_Y = 0$$

Part b)

Alternate hypothesis is: Difference of mean durations is less then 0 minutes. (μ_X is better.)

$$H_A : \mu_X - \mu_Y < 0$$

Part c)

Since level of significance is 5%, α is 0.05. We will use *left-tail Z-test* since knowing if the duration is below 0 is enough, so $z_\alpha = 1.645$.

$$Z = \frac{\overline{X} - \overline{Y} - D}{\sqrt{\frac{\sigma_X^2}{n} + \frac{\sigma_Y^2}{m}}} = \frac{2.8 - 3 - 0}{\sqrt{\frac{1.7^2}{68} + \frac{1.4^2}{68}}} = \frac{-0.2}{0.267} = -0.749$$

Since $Z > -Z_\alpha$, it is in the accepting area so we accept the null hypothesis. Thus, we conclude that the new painkiller does not produce better results.