Student Information

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Answer 1

a.

$$
\begin{aligned}
K &= \{s, q_0, q_1, q_2, h\} \\
\Sigma &= \{a, b, \sqcup, \triangleright\} \\
s &= s \\
H &= \{h\}
\end{aligned}
$$

and $\delta =$

q	σ	$\delta(q,\sigma)$	q	σ	$\delta(q,\sigma)$
s	Σ	$(q_0, \rightarrow)$	q_2	a	$(q_2, \leftarrow)$
q_0	a	$(q_1, \sqcup)$	q_2	b	$(q_2, \leftarrow)$
q_0	b	$(q_2, \sqcup)$	q_2	$\sqcup$	(h, b)
q_0	$\sqcup$	$(h, \sqcup)$	s	$\triangleright$	$(s, \rightarrow)$
q_1	a	$(q_1, \leftarrow)$	q_0	$\triangleright$	$(q_0, \rightarrow)$
q_1	b	$(q_1, \leftarrow)$	q_1	$\triangleright$	$(q_1, \rightarrow)$
q_1	$\sqcup$	(h, a)	q_2	$\triangleright$	$(q_2, \rightarrow)$

b.

(i)

$$
\begin{aligned}
(s, \triangleright \sqcup \sqcup b\underline{a}b) &\vdash_M (q_0, \triangleright \sqcup \sqcup ba\underline{b}) \\
&\vdash_M (q_2, \triangleright \sqcup \sqcup ba\underline{\sqcup}) \\
&\vdash_M (q_2s, \triangleright \sqcup \sqcup b\underline{a}\sqcup) \\
&\vdash_M (q_2s, \triangleright \sqcup \sqcup \underline{b}a\sqcup) \\
&\vdash_M (q_2s, \triangleright \sqcup \underline{\sqcup}ba\sqcup) \\
&\vdash_M (h, \triangleright \sqcup \underline{b}ba\sqcup)
\end{aligned}
$$

(ii)

$$
\begin{aligned}
(s, \triangleright a\underline{a}a) &\vdash_M (q_0, \triangleright aa\underline{a}) \\
&\vdash_M (q_1, \triangleright aa\underline{\sqcup}) \\
&\vdash_M (q_1, \triangleright a\underline{a}\sqcup) \\
&\vdash_M (q_1, \triangleright \underline{a}a\sqcup) \\
&\vdash_M (q_1, \underline{\triangleright} aa\sqcup) \\
&\vdash_M (q_1, \triangleright \underline{a}a\sqcup)
\end{aligned}
$$

After this, machine will never stop and will repeat last two configurations.

(iii) $(s, \triangleright\underline{a} \sqcup bb)$ $\quad (q_0, \triangleright a\underline{\sqcup}bb)$
$(h, \triangleright a\underline{\sqcup}bb)$

Answer 2

$$(\triangleright\underline{\sqcup}babc) \vdash (\triangleright \sqcup \underline{b}abc) \vdash^4 (\triangleright \sqcup babc\underline{\sqcup}) \vdash (\triangleright \sqcup bab\underline{c}) \vdash^2 (\triangleright \sqcup babc\underline{\sqcup}) \vdash (\triangleright \sqcup babc \sqcup \underline{b})$$
$$\vdash (\triangleright \sqcup babc \sqcup \underline{c}) \vdash^2 (\triangleright \sqcup babc \sqcup c\underline{\sqcup}) \vdash (\triangleright \sqcup babc \sqcup c \sqcup \underline{b})$$

Machine takes a step to the right. It looks at the symbol on its head.(Let it be variable a) It goes right until it finds a blank symbol. It takes a step back to the left and checks if the symbol there is a c. If it is not a c it halts, else(let this c be called variable b) it takes a step to the right, writes blank symbol there, takes a step to the right. It then writes the a there, and overwrites it with b. Takes a step to right, writes blank symbol, takes one step to right and finally writes a.

Answer 3

a.

$$\{w : w \in \{a, b\}^*\}$$

b.

Let $N_x(s)$ be the number of x symbols in string s.

$$f(w) = a^n b^m \text{ where } w \in \{a, b\}^*, N_a(w) = n, N_b(w) = m$$

Answer 4

$$\begin{tikzcd}
> R^{1,2} \arrow[r, "a^1 \neq \sqcup"] \arrow[dr, "\sqcup^1"] & a^2 \arrow[l, bend right] & \\
& L_{\sqcup}^{1,2} R^{1,2} \arrow[r, "a^2 \neq \sqcup"] \arrow[d, "\sqcup^2"] & a^1 a^1 \arrow[l, bend right] \\
& L_{\sqcup}^{2} c^1 R^{1,2} \arrow[r, "a^2 \neq \sqcup"] & a^1 \arrow[l, bend right]
\end{tikzcd}$$

Answer 5

$$
\begin{tikzcd}
> R^{1,2} \arrow[r, "a^1 \neq \sqcup"] \arrow[dr, "\sqcup^1"] & a^2 \arrow[l, bend right] & & \\
& L_{\sqcup}^{1,2} R^{1,2} \arrow[r, "a^2 \neq \sqcup"] & a^1 R^1 a^1 \sqcup^2 R^2_{\sqcup} L^2 \arrow[r, "a^2 \neq \sqcup"] & a^1 \arrow[ll, bend right]
\end{tikzcd}
$$

Answer 6

a.

$$\delta \;:\; \big((K-H)\times(\Sigma-\{\triangleright\})\mapsto K\times((\Sigma-\{\triangleright\})\cup\{\downarrow\})\big)\cup\big((K-H)\times\{\triangleright\}\mapsto K\times(\Sigma-\{\triangleright\})\big)$$

b.

To preserve determinism, there should only be one transition possible at a given time and configuration. So in order to add e-transitions to TM, we should consider a different approach.

For e-transitions to be achieved, a state p's all transitions must have the same right hand side, i.e. $(p,a)\mapsto(q,X)$ for all p where $q\in K$ and X is any action. Only then, we can replace all these transitions with only $(p,e)\mapsto(q,X)$.

After this flexibility provided, our Σ from the left hand side of δ mapping should be changed with $\Sigma\cup\{e\}$.

c.

Let (q_1, f_1w_1) and (q_2, f_2w_2) be configurations of the machine. Then

$$(q_1, f_1w_1)\vdash(q_2, f_2w_2)$$

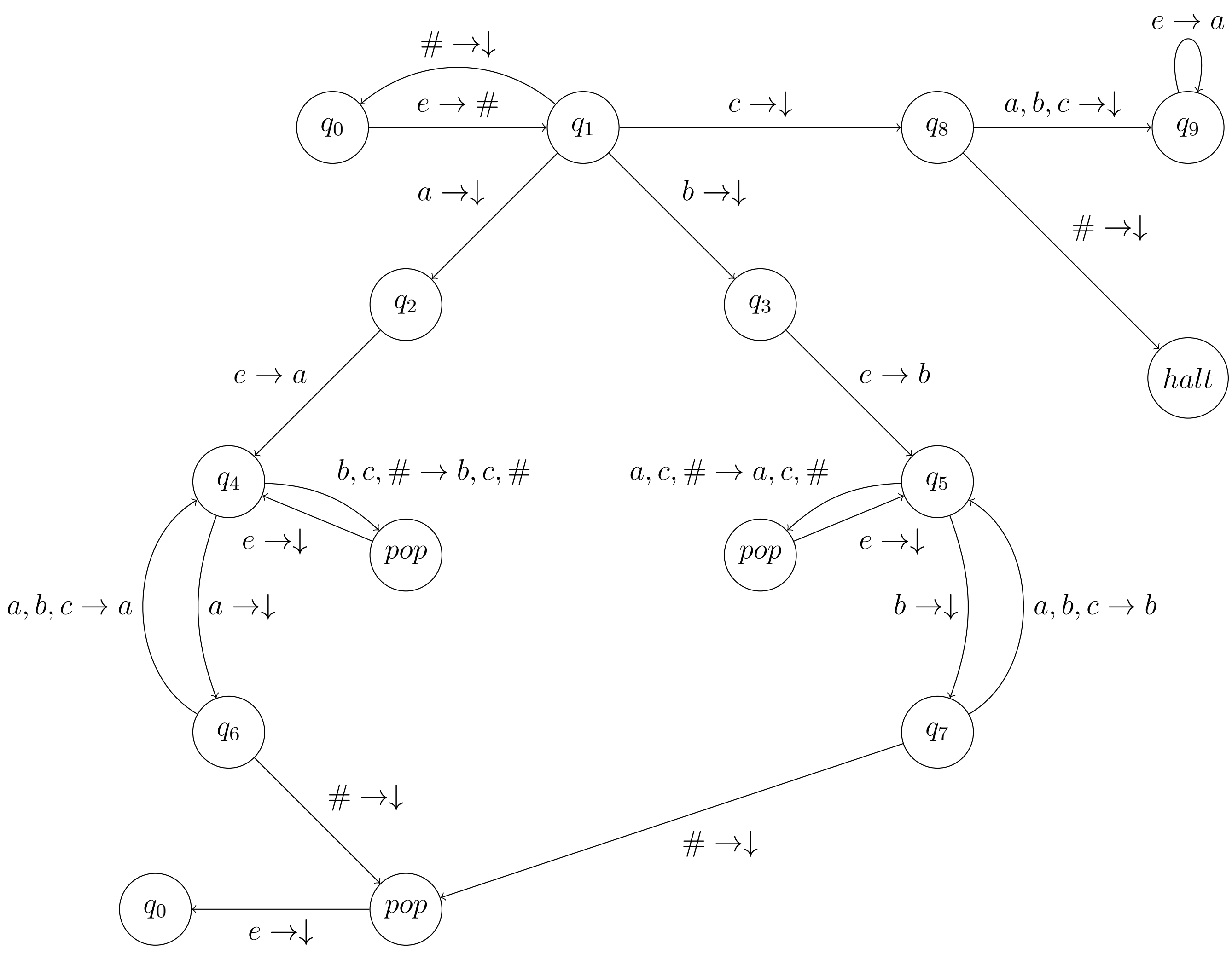
if and only if, for some $X\in\Sigma\cup\{\downarrow\}$, $\delta(q_1,f_1)=(q_2,X)$, and either,

- $X\in\Sigma, f_1=f_2, w_2=w_1X$
- $X=\downarrow, w_1=f_2w_2$.

d.

$$M = \{K, \{a, b, c, \triangleright, \#\}, \delta, q_0, \{halt\}\}$$

- $Y \to X$ means that if front is Y then do X.
- $b, c, \# \to b, c, \#$ means that push the symbol that you have seen. $(b, c, \#)$
- *pop* state which q_6 and q_7 transites to, goes to q_0 and pops one element whatever the front is, i.e. q_0's are the same state. (Hard to draw.)



Answer 7

a.

An insert-delete TM is a quintuple $(K, \Sigma, \delta, s, H)$ where

- K is a finite set of states,
- Σ is an alphabet contaning the left end symbol $\triangleright$,
- s $\in K$ is the initial state,
- $H \subseteq K$ is the set of halting states,
- δ is the transition function where it is defined as

$\delta\ :\ (K - H) \times (\Sigma \cup \{e\} - \{\triangleright\})^2 \mapsto K \times (\Sigma \cup \{e, \uparrow\} - \{\triangleright\})^2 \cup () - ((K - H) \times \{e\}^2 \mapsto K \times \{e\}^2)$

b.

The configuration for an insert-delete TM is a member of $K \times \triangleright\Sigma^*$. We can only work with the first symbol after $\triangleright$ and the last so no need to specify.

c.

Let $(q_1, f_1w_1r_1)$ and $(q_2, f_2w_2r_2)$ be configurations of the machine. Then $(q_1, f_1w_1r_1) \vdash (q_2, f_2w_2r_2)$ iff., for some $a, b \in \Sigma \cup \{\uparrow\}$, $\delta(q_1, l_1, r_1) = (q_2, a, b)$ and after a, b actions to a_1, r_1, the tape became $f_2w_2r_2$. If a or $b \in \Sigma$ then machine inserts at that position, otherwise if a or $b =\uparrow$ it deletes from that position.

d.

Can an insert-delete TM be obtained from a TM? We do only finite number of operations in one step so it can be obtained. We can think of insert-delete TM as a two headed single tape TM such that one head is at the beginning and other at the end of the string. Allowing this TM to have exact functionality as insert-delete TM shows that insert-delete TM can be obtained from a conventional TM.

Can a TM be obtained from an insert-delete TM? Since we know where the end of the string is, we can have unrestricted access to a specific memory place by marking techniques and state configurations. So since we have infinite amount of memory and unrestricted access to it, we can do whatever turing machines can do.

Answer 8

$G = (V, \Sigma, R, S)$ where

$$V = \{S, a, L, R, A, P, H, T\}$$
$$\Sigma = \{a\}$$

$$
\begin{aligned}
R = \{ & S \to LAPR \\
\\
& HA \to aACH \\
& Ha \to aH \\
& HC \to aCH \\
& HR \to AAPR \\
\\
& aP \to Pa \\
& AP \to PA \\
& CP \to PC \\
& LP \to LH \\
& PR \to T \\
\\
& aT \to Ta \\
& AT \to Ta \\
& CT \to Ta \\
& LT \to \epsilon \}.
\end{aligned}
$$

Answer 9

Let L_A be L_1L_2 so that $L = L_A \cap L_3$. We know that L_1 and L_2 are recursively enumerable so there are some TM's M_1 and M_2's that semidecides them respectively.

In order to build a TM M_A that semidecides L_A we can use the high level definition as following:

On input word w:

- Nondeterministically split the word w as w_1w_2.
- Run M_1 on w_1.
- Run M_2 on w_2.
- If both M_1 and M_2 halts(accepts), halt(accept).

Also, since we know L_3 is recursively enumerable, there is a TM M_3 that semidecides it. We now can construct a TM M that semidecides L with the high level definition as following:

On input word w:

- Run M_A on w.
- If M_A halts(accepts), run M_3 on w.
- If M_3 halts(accepts), halt(accept).

It is now proved by defined semideciding TM M that L is recursively enumerable.