Question 9 (10 pts)

The following language is given.

$$L = \{1^t 0^m 1^{2^n} : t < 5,\ m \neq n, \text{ and } t, m, n \in \mathbb{N}\}.$$

Prove that L is **not** a regular language by

a. **pumping lemma** for regular languages. (10 pts)

***sol 2*:** Assume that L is a regular language. Define another language L' as follows:

$$\begin{aligned} L' &= \mathcal{L}((e \cup 1 \cup 11 \cup 111 \cup 1111)0^*1^*) \setminus L \\ &= \{1^t 0^n 1^m : t, n, m \in \mathbb{N},\ t < 5,\ m = 2^n \text{ or } m \neq 2^k \text{ for } k \in \mathbb{N}\}. \end{aligned}$$

Since regular languages are closed under complementation (hence set difference), L' is also a regular language. Then, the pumping lemma for regular languages must apply to L'.

Let $n \in \mathbb{N}^+$ be the constant given us by the pumping lemma. Select $w = 0^n 1^{2^n} \in L'$ such that $|w| = n + 2^n \geq n$ for all n.

Concerning each decomposition of w of the form $w = xyz$ under the constraints $|xy| \leq n$ and $y \neq e$, $xy^iz \in L'$ must be true for $i \in \mathbb{N}$. In other words, let p and q be natural numbers such that $x = 0^{n-p-q}$, $y = 0^p$, and $z = 0^q 1^{2^n}$ provided that the inequalities $1 \leq p \leq n$ and $0 \leq q \leq n - p$ simultaneously hold; we can obtain every valid decomposition of w through varying legal p, q values.

Consider the case in which $i = 0$. Then, we get the strings $xy^0z = xz = 0^{n-p-q}0^q 1^{2^n} = 0^{n-p}1^{2^n}$. As $1 \leq p \leq n$ holds, the inequality $0 \leq n - p \leq n - 1$ must be satisfied. Note that for each valid p value, $n > n - p$. Since 2^n is a power of 2 and it is always the case that $n \neq n - p$, $xz \notin L'$.

We have reached a contradiction, so the initial assumption that L' is regular must be refuted. Consequently, L' is not a regular language. In a chain reaction, our first assumption is invalidated as well. Therefore, L is not a regular language.