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In data mining, machine learning, we trying to find function f that maps input x to output y.

$$y = f(x) \tag{1}$$

Such a function is assumed to be *stationary*, i.e., distribution generating the data is fixed (but unknown). But, real-life data is

- Non-stationary
- Evolving

Need methods which can detect and adapt to changes in the underlying function. Underlying function is called *concept*. Also known as: **Change detection, co-variate shift, dataset shift etc.** Note: cf. Noise vs concept drift

An example

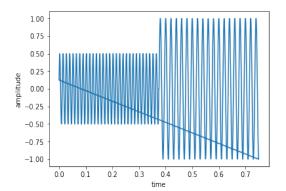


Figure: Concept drift in signals

Causes of Concept drift

A drift can occur in many ways. Let us look at the Bayesian decision formula used in naive Bayes.

$$p(c/D) = \frac{P(D/c)p(c)}{p(D)}$$

 $posterior \propto likelihood \times prior$

Where D and c denote the data and class labels. A change can then be attributed due to:

- Class prior p(c) may change over time
- likelihood p(D/c) might change
- posterior p(c/D) can change.

Three causes of concept drift

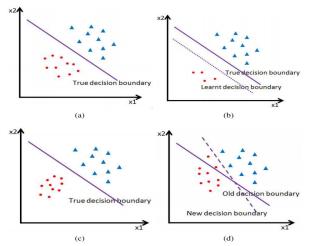


Fig. 2. Illustration of three concept drift types. (a) Original distribution. (b) P(y) drift. (c) $p(\mathbf{x}|y)$ drift. (d) $P(y|\mathbf{x})$ drift.

Figure: Causes of concept drift [Wang et al., 2018]



Types of concept drifts

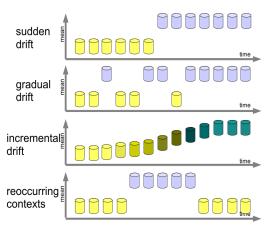


Figure: Types of concept drifts

An example could be changing gear.



Handling concept drift

Concept drift can be handled in many ways. Mostly, they are characterized in the following ways:

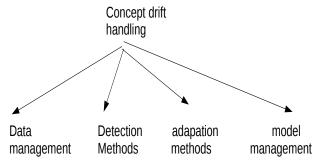


Figure: Methods of handling concept drift

Data Management

These technique characterize the data stored in memory for handling concept drift. They can be further classified as :

- Full Memory- maintain sufficient statistics via weighting examples and learn model on that.
- Partial Memory-uses recent examples to adapt the learner
 - Fixed window (aka gradual forgetting)
 - Adaptive window (abrupt forgetting)

Detection Methods

These methods characterize the techniques and mechanism of drift detection. Advantages:

- Can identify the location of the change
- Quantification of change

Two approaches:

- Monitoring evolution of performance measures over time
- Monitoring distributions over 2 different time-windows-A reference window and most recent example window

An example of the first approach is by Joachims et al. [Klinkenberg and Joachims, 2000] and second approach is given in [Kifer et al., 2004]

Example of detection method: CUSUM Algorithm

CUSUM (CUmulative SUM) algorithm is a change detection algorithm. It monitors the cumulative sum of log-likelihood ratio to detect a change.

- Consider a sequence of independent random variables x_t with pdf $p_{\theta}(x)$.
- **Our** parameter is θ which before change is θ_0 and after change is θ_1 .
- \bullet Assume θ_0 is known.
- The log-likelihood ratio is defined by

$$s_t = \log \frac{p_{\theta_1}(x)}{p_{\theta_0}(x)} \tag{2}$$

Example of detection method: CUSUM Algorithm

Formally, let S_t be the current cumsum of log-likelihood ratio and m_t the current min value of S_t , the CUSUM compares this difference with a threshold.

$$g_t = S_t - m_t \ge \delta$$

where

$$S_t = \sum_{i=0}^{t} s_i$$

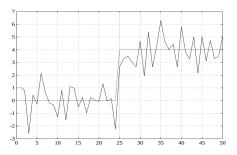
$$s_i = \log \frac{p_{\theta_1}(x)}{p_{\theta_0}(x)}$$

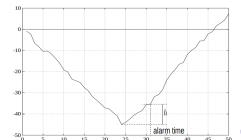
$$m_t = \min_{0 \le i \le t} S_j$$

So the change point is:

$$t_a = \min\{t : S_t \ge m_t + \delta\}$$

Typical behavior of the log-likelihood ratio S_k corresponding to a change in the mean of a Gaussian sequence with constant variance.







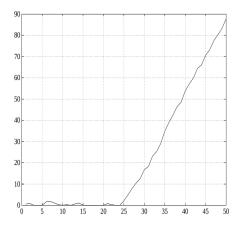


Figure: Behavior of CUSUM g_t

Adaptive methods

These methods characterise the adeptness of the decision model. Two approaches:

- Blind methods: Change decision model at regular intervals irrespective of whether a change has occurred or not. Examples includes weighting and time windows of fixed size.
- Informed methods: It includes methods which alter the model only after a change has been detected.

Decision model management

These techniques characterizes the number of decision model that needs to be kept in-memory. Key assumption:

Data comes from multiple distributions

Intuitively, we maintain a separate model each time a change is detected. What happens when the number of changes is high? Dynamic weighted majority (DWM) algorithm.

Key Idea: learn a separate model each time a change is detected and keep doing this until memory limit is hit. After that, delete a model based on its performance on the unseen data. In short, you dynamically create and delete model based on their performance. Decision about a new test point is given by majority voting.

Exercise: DWM paper reading and implementation [Kolter and Maloof, 2007]

Illustrative examples: Drift detection Statistical Process Control Algorithm (SPC)

- Performance of drift detection models is evaluated by error rate.
- Error is binomial r.v. with probability p_i and s.d. $s_i = \sqrt{p_i(1-p_i)/i}$.
- Binomial r.v. can be approximated by normal distribution since streams are infinite.
- $(1 \alpha/2)$ confidence interval for p for large enough samples can be approximated by $p_i \pm z * s_i$.
- Thus, SPC maintain two variables p_i and s_i and compares them with p_{\min} and s_{\min} .

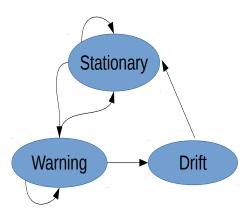


Figure: State transition diagram during concept drift

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Algorithm 8: The SPC Algorithm
  input: Φ: Current decision model
          Sequence of examples: \{\vec{x}_i, y_i\}^n
  begin
      Let \vec{x}_i, y_i be the current example;
      Let \hat{y}_i \leftarrow \Phi(\vec{x}_i);
      Let error_j \leftarrow L(\hat{y}_j, y_j);
      Compute error's mean p_i and variance s_i;
      if p_j + s_j < p_{min} + s_{min} then
          p_{min} \leftarrow p_i;
         s_{min} \leftarrow s_i;
      if p_i + s_i < p_{min} + \beta \times s_{min} then
          /* In-Control
                                                                                       */
          Warning? \leftarrow False:
          Update the current decision model with the example \vec{x}_i, y_i;
      else
          if p_i + s_i < p_{min} + \alpha \times s_{min} then
                                                                                       */
              /* Warning Zone
              if NOT Warning? then
                  buffer \leftarrow \{\vec{x}_i, y_i\};
                  Warning? \leftarrow TRUE;
              else
                \buffer \leftarrow buffer \cup \{\vec{x}_j, y_j\};
          else
              /* Out-Control
              Re-learn a new decision model using the examples in the
              buffer:
               Warning? \leftarrow False;
              Re-start p_{min} and s_{min};
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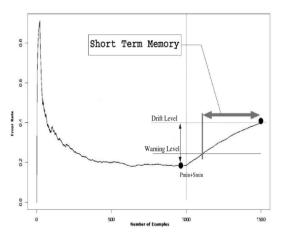


Figure: Dynamically constructed time-window. The vertical line marks the change of concept.

Implementation of SPC

Exercise: implement SPC algorithm and plot error rate (no. of errors divided by no of examples processed)

- SEA
- Intrusion detection
- spam detection
- Usenet

dataset link:

http://www.liaad.up.pt/kdus/products/datasets-for-concept-drift

Evaluation of drift detection methods

Drift detection methods are evaluated on the following metrics:

- Error rate (Number of mistakes made so far)
- Probability of true detection or TPR
- Probability of false alarm or FPR
- Delay in detection
- precision/recall/AUC etc.

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