INDIAN INSTITUTE OF TECHNOLOGY, INDORE

DEPARMENT OF ASTRONOMY, ASTROPHYSICS AND SPACE ENGINEERING (DAASE)

AA 608 - ASTROSTATISTICS

ASSIGNMENT 2 - MARKHOV CHAIN MONTE CARLO

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Aim of the assignment:

- To write an MCMC code to estimate cosmological parameters h, which is the hubble parameter and Ω_m, which is the matter density parameter, from the supernova dataset, assuming the Universe is flat and the errors are a gaussian.
- Use the burn-in process to reject unimportant data and plot a scatter data of the samples.
- To compare the theoretical values of distance modulus as a function of redshift with the observed one and plot a graph.
- To show how the acceptance probability changes as you change the size of the proposal distribution from very small (say 0.001) to very large (say 100).

Density parameter Ω: The density parameter in cosmology is a number that describes how much stuff (matter and energy) there is in the universe compared to how much stuff there would need to be for the universe to stop expanding. The value of the density parameter tells us about the ultimate fate of the universe.
If the density parameter is less than 1, the universe will keep expanding forever. If it's greater than 1, the universe will eventually stop expanding and start collapsing in on itself, an event called "the big crush". If it's exactly 1, the universe is "flat" and will keep expanding but at a slower and slower rate over time.
The density parameter is broken down to three sub-components, which are **matter density parameter, radiation density parameter and dark energy density parameter.** We will be dealing with matter density parameter for this assignment.

The matter density parameter Ω_m: This represents the contribution of matter (both ordinary matter and dark matter) to the overall density of the universe. It is the ratio of the actual density of matter in the universe to the critical density.

Hubble parameter 'h': The Hubble parameter is a measurement of the rate at which the universe is expanding. The Hubble parameter is an important cosmological parameter as it helps us understand the evolution of the universe over time. By measuring the Hubble parameter at different points in cosmic history, we can learn how the universe has expanded and changed over billions of years.

Monte carlo method:

The method is named after the famous Monte Carlo Casino in Monaco, which is known for its games of chance. Here, **a problem is broken down into a large**

number of random trials, where each trial represents a possible outcome. These trials are then simulated using random numbers to generate possible outcomes. By running a large number of these simulations, statisticians and researchers can obtain an estimate of the probability distribution of the outcomes.
The distribution of the results will depend on the specific problem being solved and the underlying probability distribution being used for the simulation.
As the number of samples used in the Monte Carlo method increases, however, the distribution of the results is expected to converge to a normal distribution due to the Central Limit Theorem. This means that **for a large number of simulations, the results are likely to be distributed in a bell-shaped curve that approximates a Gaussian distribution.**

Pros of the Monte carlo method:

- It can be applied to a wide range of problems, even those with complex or poorly understood underlying probability distributions.
- It can be computationally efficient, particularly for problems with many variables or complex interactions
- it can provide probabilistic estimates of results, which can be useful in situations where there is uncertainty or variability in the problem being studied.

Drawbacks of Monte carlo method:

- may not be suitable for problems that require an exact or analytical solution. While the Monte Carlo method can provide accurate estimates of results, it is ultimately a statistical method that relies on probability distributions and random sampling.

Analytical methods, such as differential equations or linear algebra, may be better suited for problems that require exact solutions.
Overall, The Monte Carlo method may not provide an exact solution, but it can often provide a good approximation with a reasonable amount of computational resources.

Markhov chain:

The Markov chain method is **a mathematical model used to describe a system that changes over time in a probabilistic way, where the probability**

of transitioning from one state to another depends only on the current state and not on any previous states.
The Markov chain method is based on the assumption that the probability of transitioning from one state to another only depends on the current state and not on any previous states. Therefore, to predict the future behavior of the system, we only need to know the current state.

Pros of using Markhov chain method:

- They can be used to model a wide range of phenomena with minimal assumptions and computational resources.
- Can be used to model complex systems with many variables and dependencies.

Drawbacks of Markhov chain method:

- Can give perfect results ONLY if the system being modeled has a sequential nature, and the future state depends only on the current state.
- Markov chains assume that the probability distribution of states is constant over time, which may not always be the case in real-world systems.
- The behavior of a Markov chain can be highly dependent on the initial conditions, meaning that small changes in the starting state can result in very different outcomes.

Markhov chain method can be used in many fields:

- Used to model the behavior of biological systems, such as the spread of disease through a population or the growth of a bacterial colony.
- In physics, Markov chain methods are used to model the behavior of physical systems, such as the motion of particles in a gas or the random walk of a molecule.

The merge of Markhov Chain and Monte Carlo:

Estimation of a complex probability distribution is very challenging. To be more elaborate, a probability distribution can be considered complex if it has high dimensionality, nonlinear relationships, non-Gaussian properties, a lack of analytical

solution, or dependencies.
This is where the Markov chain Monte Carlo (MCMC) method comes to the rescue. The Markov chain and Monte Carlo methods were combined to form the Markov chain Monte Carlo (MCMC) method because **they both have properties that make them well-suited for estimating complex probability distributions.** The Markov chain method is used to model the evolution of a system over time, where the probability of moving from one state to another depends only on the current state of the system. The Monte Carlo method, on the other hand, is a statistical technique used to generate random samples from a probability distribution.
By combining these two methods, the MCMC method allows us to estimate the properties of a complex probability distribution by simulating the evolution of a Markov chain that samples from the distribution. The Markov chain method ensures that the samples are correlated in a way that reflects the underlying distribution, while the Monte Carlo method ensures that the samples are randomly generated from the distribution.

The effectiveness of the MCMC method has been demonstrated in many fields, including physics, astronomy, statistics, and machine learning. In astronomy, for example, the MCMC method is used to estimate the parameters of cosmological models and to simulate the evolution of stars and galaxies. We will be dealing with such a problem in this assignment. In machine learning, the MCMC method is used to estimate the parameters of Bayesian models and to train neural networks.

Disadvantages of MCMC:

- MCMC requires a large number of iterations to achieve accurate results. This can be computationally expensive, especially for high-dimensional problems.
- It requires careful tuning of the proposal distribution to ensure efficient sampling. This can be a time-consuming process and requires expertise in the problem domain.
- It can be sensitive to the starting point of the Markov chain. If the chain starts in a low probability region of the distribution, it may take a long time to converge or fail to converge altogether.
- It provides asymptotic results, which means that the method works well for large sample sizes but may not be accurate for small sample sizes.

Results and discussion:

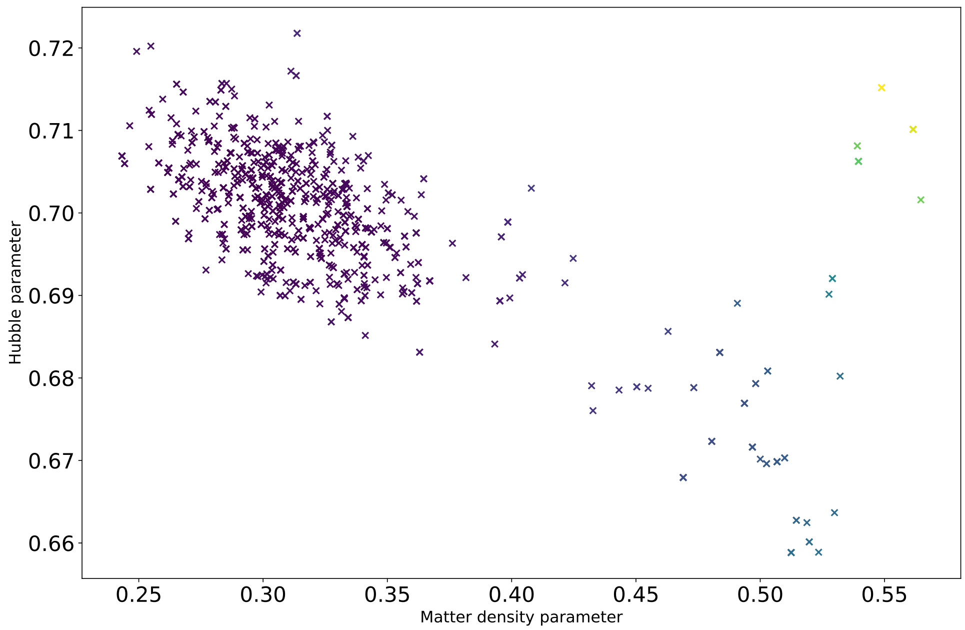


Figure 1: Scatter plot of the samples

At the end, we can say that as we did the sampling of the values earlier, we find that most of the values are crowding between $\Omega_{\mathrm{m}} = 0.3$ and $\mathrm{h} = 0.7$.
As we do the burn-in process, we get the more clear and zoomed in scattering plot of matter density and hubble parameters.

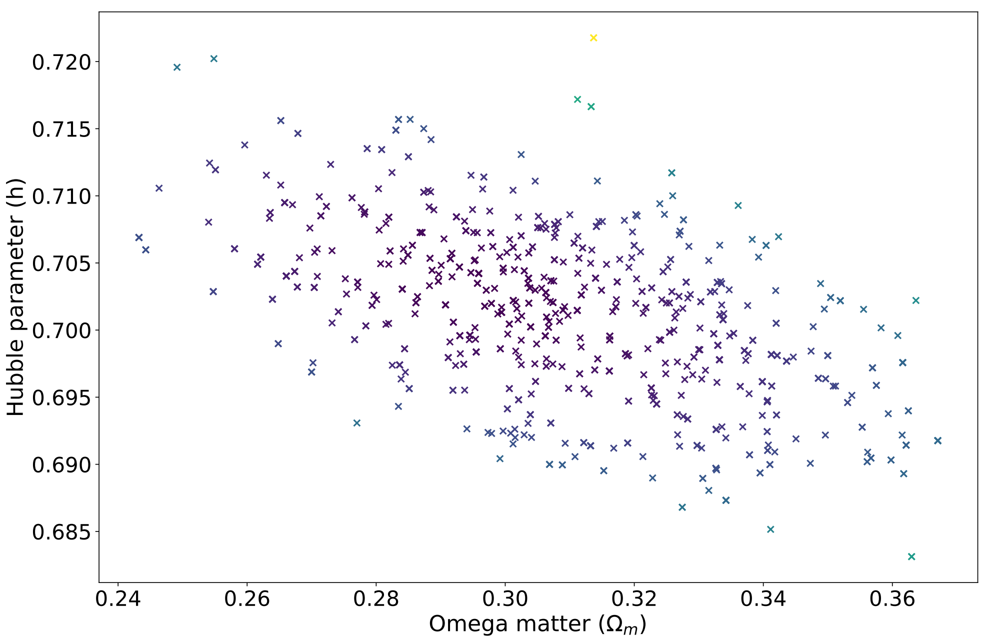


Figure 2: Scatter plot after the burn-in process

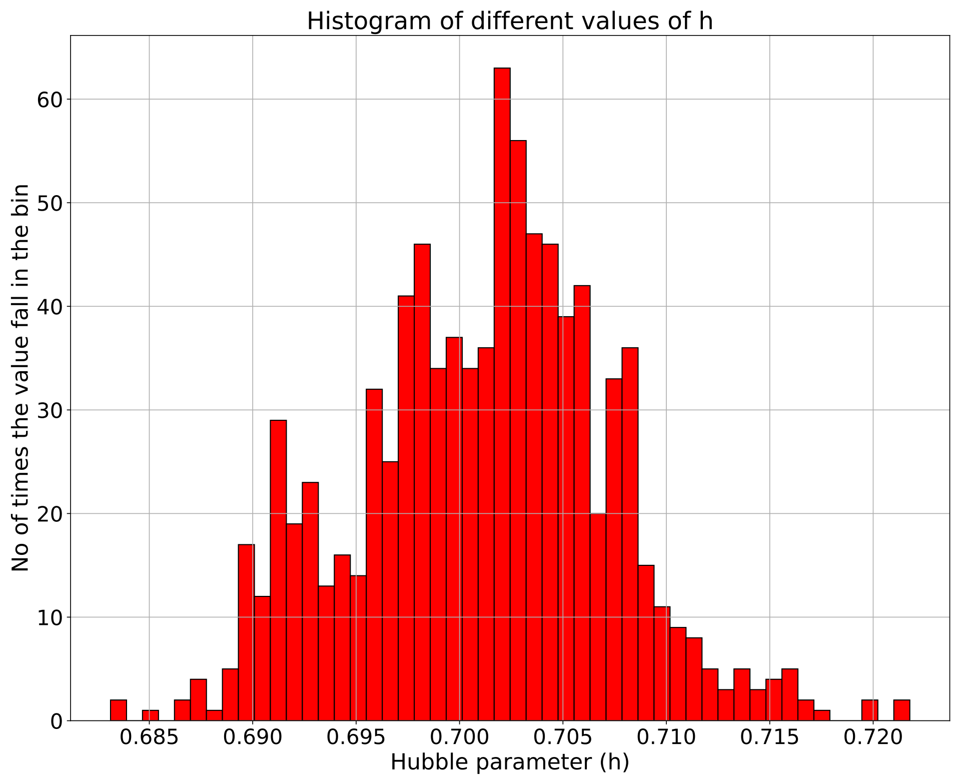


Figure 3: Histogram of different hubble parameter values

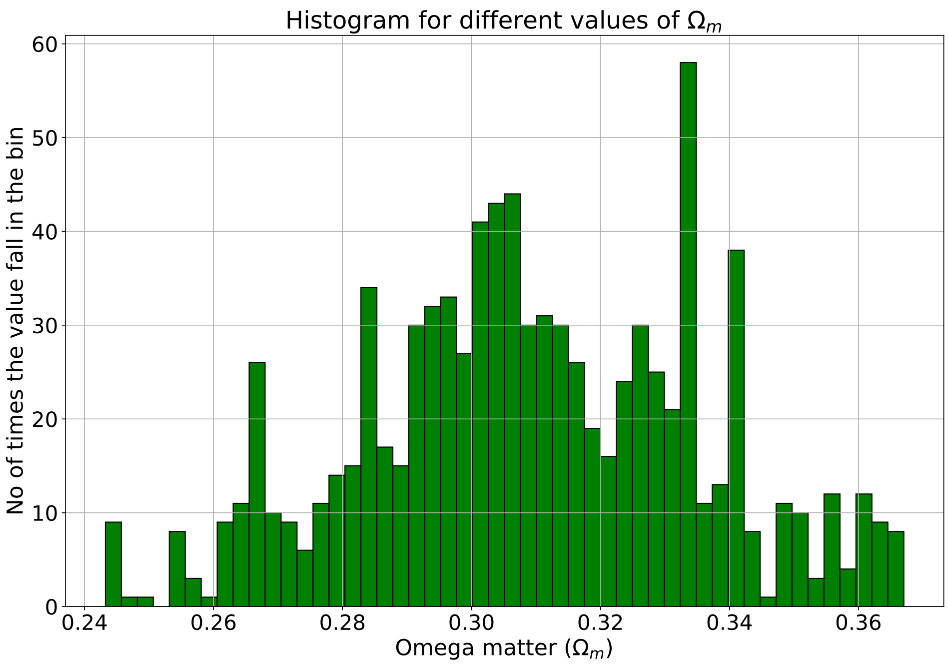
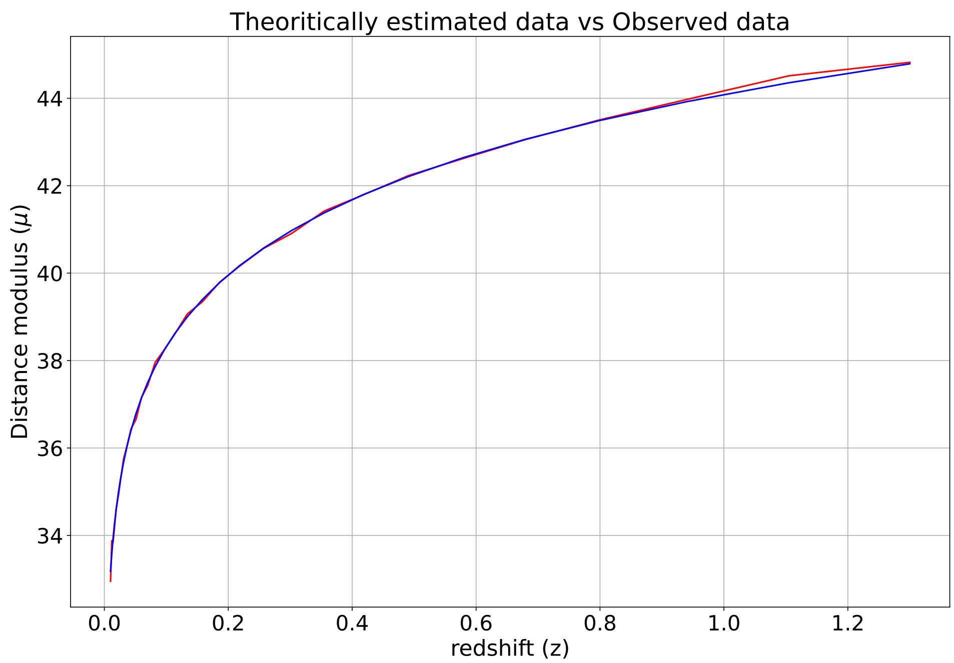


Figure 4: Histogram of different matter density parameter values


Theoritically estimated data vs Observed data
Distance modulus (μ)
redshift (z)
34
36
38
40
42
44
0.0
0.2
0.4
0.6
0.8
1.0
1.2