

Tutorial for STA2002

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Chapter 1

Prerequisites

Probability and Statistics I(STA2001) is the prerequisite, which mainly includes the following contents,

- Some usual distributions, like Binomial, Poisson, Normal, Exponential, Gamma, and Chi-square distributions (Relationships among some univariate distributions(Song, 2005));
- Basic terminologies, e.g., independence, expectation, variation, correlation (coefficient), Bayes, and etc;
- Large number theorem, like Central Limit Theorem(CLT).

Chapter 2

Tutorial 1

2.1 Q1

- Moment-generating function $M(t)$ of a random variable X defined in D that has a density function $f(x)$.

$$M(t) = \mathbb{E}(e^{tx}) = \int_D e^{tx} f(x) dx \quad (2.1)$$

$$\mathbb{E}(X^s) = M^{(s)}(0) \quad (2.2)$$

- Relationship between $\bar{X} = \frac{1}{n} \sum_{i=1}^n X_i$ and $S^2 = \frac{1}{n-1} \sum_{i=1}^n (X_i - \bar{X})^2$, independent.
- How to derive a quantity following t distribution from a norm population.

$$T = \frac{\frac{\bar{X} - \mu}{\sigma/\sqrt{n}}}{\sqrt{\frac{(n-1)S^2}{\sigma^2}/(n-1)}} = \frac{\bar{X} - \mu}{S/\sqrt{n}} \quad (2.3)$$

- The t distribution is symmetric, i.e., $t_q(n) = -t_{1-q}(n)$, $q \in (0, 1)$. For example,

```
qt(0.025, 8, lower.tail = F)
```

```
## [1] 2.306004
```

```
-qt(1 - 0.025, 8, lower.tail = F)
```

```
## [1] 2.306004
```

- Properties of F distribution.

2.2 Q2

- Standardize a norm distribution $X \in \mathcal{N}(\mu, \sigma^2)$, i.e., $\frac{X-\mu}{\sigma} \in \mathcal{N}(0, 1)$.
- The distribution of \bar{X} and S^2 .

2.3 Q3

- Central Limit Theorem(CLT)

Theorem 2.1. (*Central Limit Theorem*) Let X_1, \dots, X_n be independent, identically distributed (i.i.d.) random variables with finite expectation μ , and positive, finite variance σ^2 , and set $S_n = X_1 + X_2 + \dots + X_n$, $n \geq 1$. Then

$$\frac{\bar{S}_n - n\mu}{\sigma\sqrt{n}} \xrightarrow{L} N(0, 1) \text{ as } n \rightarrow \infty.$$

- The relationship between Binomial distribution and Poisson distribution
- Aware the power of CLT.

Bibliography

Song, W. T. (2005). Relationships among some univariate distributions. *IIE Transactions*, 37(7):651–656.