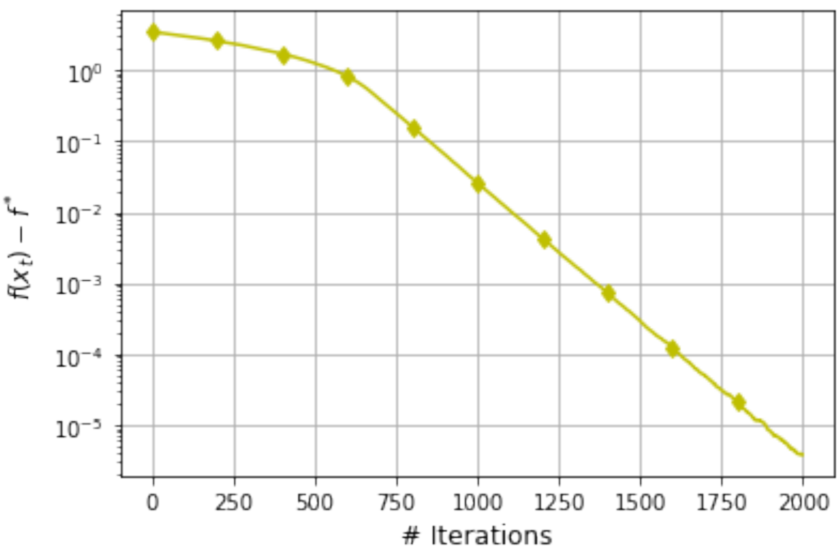
Verifying FedAvg

We first consider the following example to verify that FedAvg works on the example when the stochastic gradients violate relaxed growth condition but satisfy ABC condition. We consider $n = 2$ case, where

$$f_1(x) = f_2(x) = f(x) = \begin{cases} \frac{x^2}{2}, & |x| < 1, \\ |x| - \frac{1}{2}, & \text{Otherwise} \end{cases},$$

the stochastic gradient is set as

$$g_1(x) = g_2(x) = g(x) = \begin{cases} \nabla f(x) + \sqrt{|x|}, & \text{with probability } 1/2 \\ \nabla f(x) - \sqrt{|x|}, & \text{with probability } 1/2 \end{cases}$$



BGD can be invariant while the performance of FedAvg varies

We consider

$$f_1(x) = x^2, \; f_2(x) = \begin{cases} \frac{(x-d)^2}{2}, & |x - d| < 1, \\ |x - d| - \frac{1}{2}, & \text{Otherwise} \end{cases},$$

where larger $|d|$ indicates larger $f^* - \frac{1}{2}(f_1^* + f_2^*)$. To obtain a tight estimate of (ζ^2, ψ^2) in the BGD assumption, we consider the subproblem

$$\min_{\zeta,\psi>0} \zeta^2 + \psi^2, \text{ s.t. } \frac{1}{2}\sum_{i=1}^{2} \|\nabla f_i(x) - \nabla f(x)\|^2 \le \zeta^2 + \psi^2 \|\nabla f(x)\|^2, \forall x \in \mathbb{R}.$$

It suffices to consider three cases:

- $x-d>1$, the constraint becomes

$$\frac{1}{2}(1+4x^2) \leq \zeta^2+\psi^2(x+\frac{1}{2})^2.$$

- $x-d<-1$, the constrain becomes

$$\frac{1}{2}(1+4x^2) \leq \zeta^2+\psi^2(x+\frac{-1}{2})^2.$$

- $|x-d| \leq 1$, the constraint becomes

$$\frac{1}{2}((x-d)^2+4x^2) \leq \zeta^2+\psi^2(x+\frac{x-d}{2})^2.$$

For $|d|>2$, we conclude the above three constraints lead to the same minimal value of $\zeta^2+\psi^2$ for any $x \in \mathbb{R}$. Therefore, the two constants of BGD assumption are invariant. However, the performance of FedAvg varies according to different d's as shown in the following figure.