#Q1

$eq := diff(u(x), x\$2) - diff(u(x), x) - 6 \cdot u(x) = -18 \cdot \sin(x);$

$$eq := \frac{d^2}{dx^2}\, u(x) - \frac{d}{dx}\, u(x) - 6\, u(x) = -18 \sin(x) \quad (1)$$

#a) bounded sol = Find a particular solution u(x) that doesn't go to $\pm\infty$ *as* $x \to \pm\infty$.

$sol1 := dsolve(eq, u(x));$

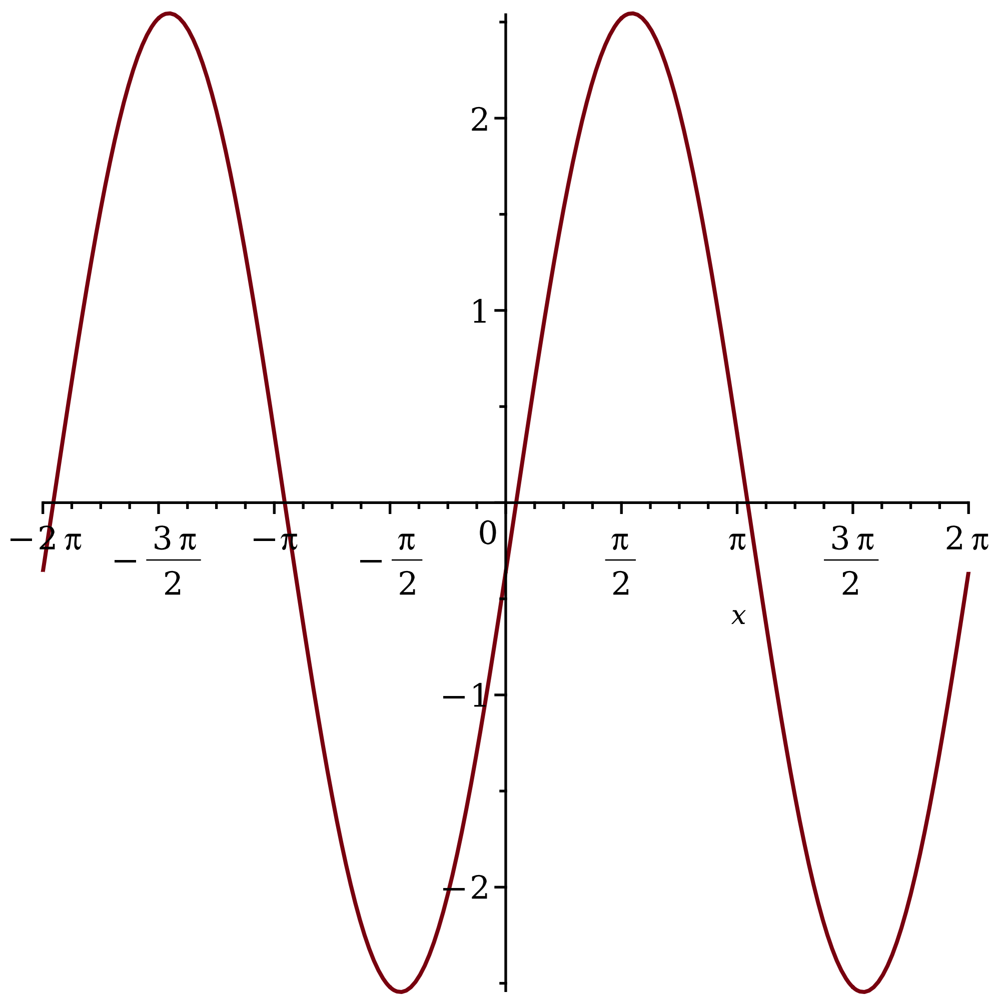
$$sol1 := u(x) = e^{-2x}\, c_2 + e^{3x}\, c_1 - \frac{9 \cos(x)}{25} + \frac{63 \sin(x)}{25} \quad (2)$$

$ps := subs(\{c_1 = 0, c_2 = 0\}, sol1);$

$$ps := u(x) = -\frac{9 \cos(x)}{25} + \frac{63 \sin(x)}{25} \quad (3)$$

#b)[-2·Pi,2·Pi]

$plot(rhs(ps), x = -2 \cdot \mathrm{Pi}..2 \cdot \mathrm{Pi});$



#c)

$evalf\left(subs\left(x = \mathrm{Pi}\cdot\sqrt{6}, rhs(ps)\right)\right);$

$$2.431453171 \tag{4}$$

#d)

$evalf\left(subs\left(x = \mathrm{Pi}\cdot\sqrt{6}, diff(-(9*\cos(x))/25 + (63*\sin(x))/25, x)\right)\right);$

$$0.7536812850 \tag{5}$$

#Q2---

#x'=y,y'=-9x-6y,x(0)=1,y(0)=-2

$with(linalg) : with(VectorCalculus) :$

#a)

$A := Matrix([[0, 1], [-9,-6]]);$

$$A := \begin{bmatrix} 0 & 1 \\ -9 & -6 \end{bmatrix} \tag{6}$$

$det(A);$

$$9 \tag{7}$$

#b)

$eigenvalues(A);$

$$-3, -3 \tag{8}$$

#c)

$MatrixExponential(t\cdot A);$

$$MatrixExponential\left(\begin{bmatrix} 0 & t \\ -9\,t & -6\,t \end{bmatrix}\right) \tag{9}$$

#d)

#x'=y,y'=-9x-6y,x(0)=1,y(0)=-2

$eq1 := diff(x(t), t) = y(t);$

$$eq1 := \frac{\mathrm{d}}{\mathrm{d}t}\, x(t) = y(t) \tag{10}$$

$eq2 := diff(y(t), t) = -9\cdot x(t) - 6\cdot y(t);$

$$eq2 := \frac{\mathrm{d}}{\mathrm{d}t}\, y(t) = -9\,x(t) - 6\,y(t) \tag{11}$$

$ic := x(0) = 1, y(0) = -2;$

$$ic := x(0) = 1, y(0) = -2 \quad (12)$$

$dsolve(\{eq1, eq2, ic\}, [x(t), y(t)]);$

$$\left\{x(t) = -\frac{e^{-3t}(-9t-9)}{9}, y(t) = e^{-3t}(-3t-2)\right\} \quad (13)$$

#e)

$Jm := Jacobian([y,-9\cdot x - 6y], [x, y]);$

$$Jm := \begin{bmatrix} 0 & 1 \\ -9 & -6 \end{bmatrix} \quad (14)$$

$eigenvalues(Jm);$

$$-3, -3 \quad (15)$$

#stable node

#f) eigevanls are real and negative

#Q3--

#a)

$solve(\{-x + y + x^2 = 0, y - 2\cdot x\cdot y = 0\});$

$$\{x = 0, y = 0\}, \{x = 1, y = 0\}, \left\{x = \frac{1}{2}, y = \frac{1}{4}\right\} \quad (16)$$

#b)

$Jm1 := Jacobian([-x + y + x^2, y - 2\cdot x\cdot y], [x, y]);$

$$Jm1 := \begin{bmatrix} 2x-1 & 1 \\ -2y & -2x+1 \end{bmatrix} \quad (17)$$

#c)

$J1 := subs([x = 1, y = 0], Jm1);$

$$J1 := \begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix} \quad (18)$$

#d)

$eigenvalues(J1);$

$$1, -1 \quad (19)$$

#e)

#real part !=0 => hyperbolic

#f),g)

#saddle point,unstable

restart;

with(DEtools) :

$sys := [diff(x(t), t) = -x(t) + y(t) + x(t)^2, \mathrm{diff}(\mathrm{y}(\mathrm{t}), \mathrm{t}) = \mathrm{y}(\mathrm{t}) - 2 \cdot \mathrm{x}(\mathrm{t}) \cdot \mathrm{y}(\mathrm{t})];$

$$sys := \left[\frac{\mathrm{d}}{\mathrm{d}t} x(t) = x(t)^2 - x(t) + y(t), \frac{\mathrm{d}}{\mathrm{d}t} y(t) = y(t) - 2\, x(t)\, y(t) \right] \quad (20)$$

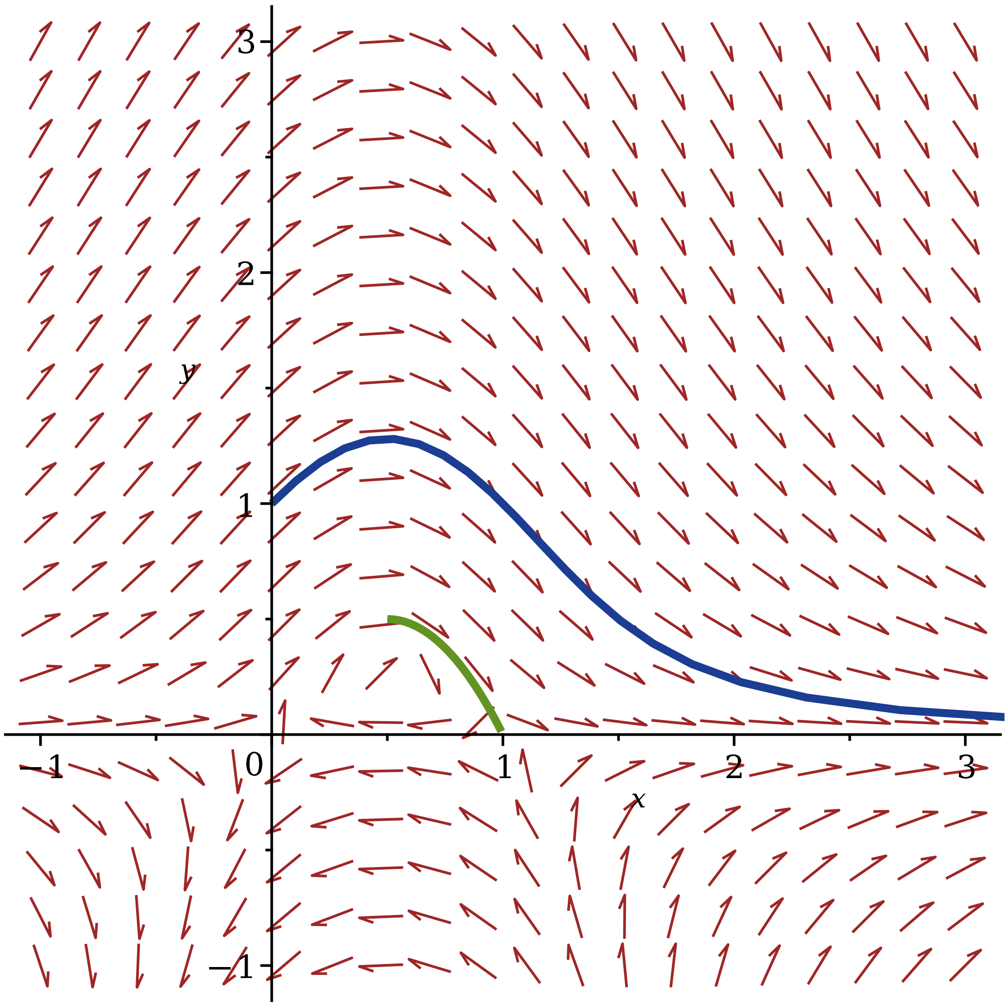
$IC1 := [x(0) = 0, y(0) = 1];$

$$IC1 := [x(0) = 0, y(0) = 1] \quad (21)$$

$IC2 := [x(0) = 0.5, y(0) = 0.5];$

$$IC2 := [x(0) = 0.5, y(0) = 0.5] \quad (22)$$

$DEplot(sys, [x(t), y(t)], t = 0 .. 5, [IC1, IC2], x = -1 .. 3, y = -1 .. 3,);$



#Q4---------------------------------- — — — — — — — — — — — — — — — — —

#y'=$y^2 - x^2 + 2$

$xx := 0; y := 1$

$$xx := 0$$

$$y := 1 \tag{23}$$

$h := 0.02;$

$$h := 0.02 \tag{24}$$

$f := (x, y) \to y^2 - x^2 + 2;$

$$f := (x, y) \mapsto y^2 - x^2 + 2 \tag{25}$$

phi := $dsolve(\{diff(y(x), x) = f(x, y(x)), y(0) = 1\});$

Error, (in dsolve) required an indication of the solving variables for the given system

restart

restart;

$f := (x, y) \rightarrow y^2 - x^2 + 2;$

$$f := (x, y) \mapsto y^2 - x^2 + 2 \qquad (26)$$

$h := 0.02;\ x := 1;\ y := 0;$

$$h := 0.02$$

$$x := 1$$

$$y := 0 \qquad (27)$$

#in $[1, 2]$
#length: 2-1 = 1
#h = 0.02
#number of steps = (interval length) : step size = 1 : 0.02 = 50 steps

for i **from** 1 **to** 50 **do** $y := y + \frac{h}{2} \cdot f(x, y) + \frac{h}{2} \cdot f(x + h, y + h \cdot f(x, y))$: $\mathrm{psi}(i) := y$: $x := x + h$: *print*(x, y);**od**:

1.02, 0.01960000000
1.04, 0.03839889575
1.06, 0.05639389879
1.08, 0.07358044006
1.10, 0.08995223147
1.12, 0.1055013234
1.14, 0.1202181590
1.16, 0.1340916265
1.18, 0.1471091104
1.20, 0.1592565428
1.22, 0.1705184555
1.24, 0.1808780338
1.26, 0.1903171739
1.28, 0.1988165436
1.30, 0.2063556480
1.32, 0.2129129013
1.34, 0.2184657056
1.36, 0.2229905376

1.38, 0.2264630438
1.40, 0.2288581460
1.42, 0.2301501576
1.44, 0.2303129114
1.46, 0.2293199001
1.48, 0.2271444298
1.50, 0.2237597871
1.52, 0.2191394212
1.54, 0.2132571394
1.56, 0.2060873179
1.58, 0.1976051268
1.60, 0.1877867692
1.62, 0.1766097326
1.64, 0.1640530536
1.66, 0.1500975920
1.68, 0.1347263145
1.70, 0.1179245840
1.72, 0.09968045314
1.74, 0.07998495801
1.76, 0.05883240897
1.78, 0.03622067424
1.80, 0.01215145227
1.82, −0.01336947204
1.84, −0.04033199200
1.86, −0.06872146971
1.88, −0.09851855771
1.90, −0.1296990565
1.92, −0.1622338132
1.94, −0.1960886651
1.96, −0.2312244323
1.98, −0.2675969636
2.00, −0.3051572363 **(28)**