Preface

Markov chains are the simplest mathematical models for random phenomena evolving in time. Their simple structure makes it possible to say a great deal about their behaviour. At the same time, the class of Markov chains is rich enough to serve in many applications. This makes Markov chains the first and most important examples of random processes. Indeed, the whole of the mathematical study of random processes can be regarded as a generalization in one way or another of the theory of Markov chains.

This book is an account of the elementary theory of Markov chains, with applications. It was conceived as a text for advanced undergraduates or master's level students, and is developed from a course taught to undergraduates for several years. There are no strict prerequisites but it is envisaged that the reader will have taken a course in elementary probability. In particular, measure theory is not a prerequisite.

The first half of the book is based on lecture notes for the undergraduate course. Illustrative examples introduce many of the key ideas. Careful proofs are given throughout. There is a selection of exercises, which forms the basis of classwork done by the students, and which has been tested over several years. Chapter 1 deals with the theory of discrete-time Markov chains, and is the basis of all that follows. You must begin here. The material is quite straightforward and the ideas introduced permeate the whole book. The basic pattern of Chapter 1 is repeated in Chapter 3 for continuous-time chains, making it easy to follow the development by analogy. In between, Chapter 2 explains how to set up the theory of continuous-time chains, beginning with simple examples such as the Poisson process and chains with finite state space.

The second half of the book comprises three independent chapters intended to complement the first half. In some sections the style is a little more demanding. Chapter 4 introduces, in the context of elementary Markov chains, some of the ideas crucial to the advanced study of Markov processes, such as martingales, potentials, electrical networks and Brownian motion. Chapter 5 is devoted to applications, for example to population growth, mathematical genetics, queues and networks of queues, Markov decision processes and Monte Carlo simulation. Chapter 6 is an appendix to the main text, where we explain some of the basic notions of probability and measure used in the rest of the book and give careful proofs of the few points where measure theory is really needed.

The following paragraph is directed primarily at an instructor and assumes some familiarity with the subject. Overall, the book is more focused on the Markovian context than most other books dealing with the elementary theory of stochastic processes. I believe that this restriction in scope is desirable for the greater coherence and depth it allows. The treatment of discrete-time chains in Chapter 1 includes the calculation of transition probabilities, hitting probabilities, expected hitting times and invariant distributions. Also treated are recurrence and transience, convergence to equilibrium, reversibility, and the ergodic theorem for long-run averages. All the results are proved, exploiting to the full the probabilistic viewpoint. For example, we use excursions and the strong Markov property to obtain conditions for recurrence and transience, and convergence to equilibrium is proved by the coupling method. In Chapters 2 and 3 we proceed via the jump chain/holding time construction to treat all right-continuous, minimal continuous-time chains, and establish analogues of all the main results obtained for discrete time. No conditions of uniformly bounded rates are needed. The student has the option to take Chapter 3 first, to study the *properties* of continuous-time chains before the technically more demanding *construction*. We have left measure theory in the background, but the proofs are intended to be rigorous, or very easily made rigorous, when considered in measure-theoretic terms. Some further details are given in Chapter 6.

It is a pleasure to acknowledge the work of colleagues from which I have benefitted in preparing this book. The course on which it is based has evolved over many years and under many hands – I inherited parts of it from Martin Barlow and Chris Rogers. In recent years it has been given by Doug Kennedy and Colin Sparrow. Richard Gibbens, Geoffrey Grimmett, Frank Kelly and Gareth Roberts gave expert advice at various stages. Meena Lakshmanan, Violet Lo and David Rose pointed out many typos and ambiguities. Brian Ripley and David Williams made constructive suggestions for improvement of an early version.

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Cambridge, 1996 James Norris