Artificial Intelligence Week 10: Revision and Sample Problems

Debasis Ganguly
University of Glasgow, Glasgow, UK

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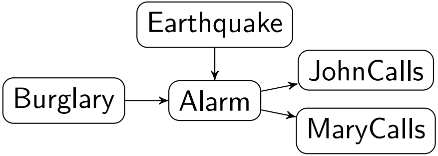
Bayesian Net Numeric Example

Bayesian Network: Burglary–Earthquake–Alarm Example



We want the posterior:

$$P(B \mid J = 1, M = 1)$$

Priors:

$$P(B{=}1) = 0.001, \; P(E{=}1) = 0.002$$

Alarm CPT:

B	E	$P(A{=}1 \mid B, E)$
1	1	0.95
1	0	0.94
0	1	0.29
0	0	0.001

Neighbor call probabilities:

$$P(J{=}1 \mid A{=}1) = 0.9$$

$$P(J{=}1 \mid A{=}0) = 0.05$$

$$P(M{=}1 \mid A{=}1) = 0.7$$

$$P(M{=}1 \mid A{=}0) = 0.01$$

Computing the Posterior

We compute:

$$P(B \mid J, M) = \frac{P(B, J, M)}{P(B, J, M) + P(\neg B, J, M)}.$$

Marginalization over E and A

$$P(b, J, M) = P(b) \sum_e P(e) \sum_a P(a \mid b, e)\, P(J \mid a)\, P(M \mid a).$$

We compute four configurations for $B{=}1$.

$$(1)\ B = 1, E = 1, A = 1: \quad \underbrace{0.001}_{P(B)} \cdot \underbrace{0.002}_{P(E)} \cdot \underbrace{0.95}_{P(A=1|B,E)} \cdot \underbrace{0.9}_{P(J|A)} \cdot \underbrace{0.7}_{P(M|A)}$$

$$= 1.197 \times 10^{-6}$$

$$(2)\ B = 1, E = 1, A = 0: \quad 5 \times 10^{-11}$$

$$(3)\ B = 1, E = 0, A = 1: \quad 5.910156 \times 10^{-4}$$

$$(4)\ B = 1, E = 0, A = 0: \quad 2.994 \times 10^{-8}$$

$$P(B, J, M) = 0.00059224259$$

Denominator: Case $B = 0$

Similarly compute the four terms for $B = 0$:

$$(1)\ \neg B, E = 1, A = 1 : 3.650346 \times 10^{-4}$$
$$(2)\ \neg B, E = 1, A = 0 : 7.0929 \times 10^{-7}$$
$$(3)\ \neg B, E = 0, A = 1 : 6.2811126 \times 10^{-4}$$
$$(4)\ \neg B, E = 0, A = 0 : 4.98002499 \times 10^{-4}$$

$$P(\neg B, J, M) = 0.001491858649$$

Thus the normalizing denominator:

$$P(B, J, M) + P(\neg B, J, M) = 0.002084101239.$$

Final Posterior and Interpretation

$$P(B \mid J = 1, M = 1) = \frac{0.00059224259}{0.002084101239} \approx 0.284.$$

Interpretation:

- Prior burglary probability was 0.1%.
- Receiving calls from both neighbors raises it to about **28%**.
- Because two calls are very strong evidence that the alarm really sounded.
- The alarm is much more likely to sound in case of a burglary than by chance.

Evidence: John calls, Mary did NOT call

We now condition on the observed evidence:

Menti-Quiz (Code: 2875-9119)

Work out how the posterior changes if **Mary did not call**.

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We now condition on the observed evidence:

Menti-Quiz (Code: 2875-9119)

Work out how the posterior changes if **Mary did not call**.

$$J = 1, \qquad M = 0.$$

Compute the posterior

$$P(B \mid J = 1, M = 0) = \frac{P(B, J = 1, M = 0)}{P(B, J = 1, M = 0) + P(\neg B, J = 1, M = 0)}.$$

All CPTs are as before: $P(B) = 0.001$, $P(E) = 0.002$, Alarm CPT and call probabilities unchanged.

Contributions to $P(B, J{=}1, M{=}0)$ (B=1)

$$P(B, J = 1, M = 0) = \sum_{e \in \{0,1\}} \sum_{a \in \{0,1\}} P(B)\, P(e)\, P(a \mid B, e)\, P(J = 1 \mid a)\, P(M = 0 \mid a).$$

Numeric contributions (each term is $P(B)\, P(E)\, P(A \mid B, E)\, P(J \mid A)\, P(M{=}0 \mid A)$):

$$(B = 1, E = 1, A = 1): \quad 0.001 \cdot 0.002 \cdot 0.95 \cdot 0.90 \cdot (1 - 0.70) = 5.130 \times 10^{-7}$$

$$(B = 1, E = 1, A = 0): \quad 0.001 \cdot 0.002 \cdot 0.05 \cdot 0.05 \cdot (1 - 0.01) = 4.950 \times 10^{-9}$$

$$(B = 1, E = 0, A = 1): \quad 0.001 \cdot 0.998 \cdot 0.94 \cdot 0.90 \cdot (1 - 0.70) = 2.532924 \times 10^{-4}$$

$$(B = 1, E = 0, A = 0): \quad 0.001 \cdot 0.998 \cdot 0.06 \cdot 0.05 \cdot (1 - 0.01) = 2.96406 \times 10^{-6}$$

Summing these:

$$P(B, J = 1, M = 0) \approx 0.00025677441.$$

Contributions to $P(\neg B, J{=}1, M{=}0)$ (B=0)

Analogous terms for $B = 0$ (use $P(\neg B) = 0.999$):

$$(\neg B, E = 1, A = 1): \quad 0.999 \cdot 0.002 \cdot \underbrace{0.29}_{P(A|,\neg B,E)} \cdot 0.90 \cdot \underbrace{(1 - 0.70)}_{P(\neg M|A)} = 1.564434 \times 10^{-4}$$

$$(\neg B, E = 1, A = 0): \quad 0.999 \cdot 0.002 \cdot 0.71 \cdot 0.05 \cdot (1 - 0.01) = 7.021971 \times 10^{-5}$$

$$(\neg B, E = 0, A = 1): \quad 0.999 \cdot 0.998 \cdot 0.001 \cdot 0.90 \cdot (1 - 0.70) = 2.6919054 \times 10^{-4}$$

$$(\neg B, E = 0, A = 0): \quad .999 \times .998 \times 0.999 \times .05 \times (1 - .01) = 4.93022 \times 10^{-2}$$

Summing these:

$$P(\neg B, J = 1, M = 0) \approx 0.049798101051.$$

Final result: $P(B \mid J = 1, M = 0)$

Denominator (normalizer):

$$P(J = 1, M = 0) = P(B, J = 1, M = 0) + P(\neg B, J = 1, M = 0) \approx 0.050054875461.$$

Hence

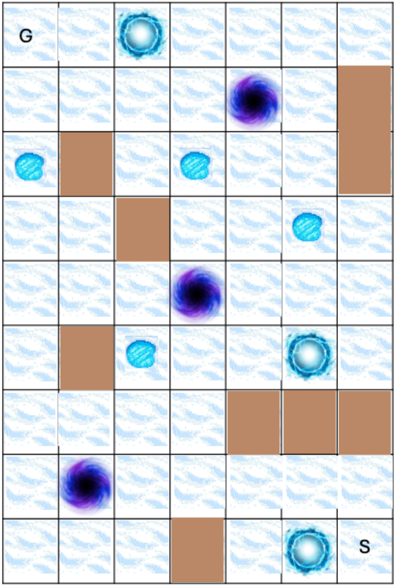
$$P(B \mid J = 1, M = 0) = \frac{0.00025677441}{0.050054875461} \approx 0.00513 \text{ (about } 0.513\%\text{)}.$$

Interpretation:

- Prior $P(B) = 0.1\%$. With John calling but Mary *not* calling, the posterior rises to $\approx$ **0.51%**.
- This is a smaller update than when both neighbors call ($\approx$28.4%), because Mary's silence reduces the likelihood that the alarm truly went off.
- Intuitively: John calling alone is weak evidence, and Mary not calling makes the case for a burglary even less strong.

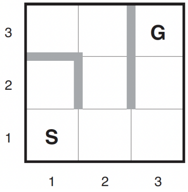
A Quick Recap of the Environments, Agent types and Agent Functions taking the maze with portals on a frozen lake as an example (MWFLP environment of the coursework)

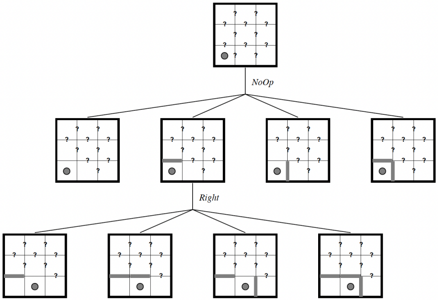
Types of Environments and Agents



- Deterministic or Stochastic? **Stochastic** because actions may not end up in a desired state.
- Completely observable or partially observable? **Partially observable** because the agent can only have percepts from its neighborhood.
- In an unobservable environment an agent is only aware of its current state (may not also aware of its location).
- In the real world, equipping the agent with more knowledge about the information incurs a cost. The extra information may not be worth too much.

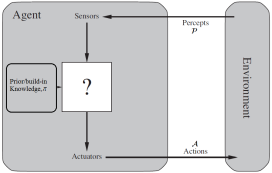
Example of an unobservable environment

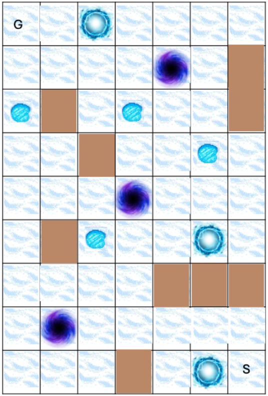




- An agent can keep track of a set of states that it could be in (called **belief states**).
- Often randomising actions from states yields good results (actually used in cost-effective robotic vacuum cleaners).

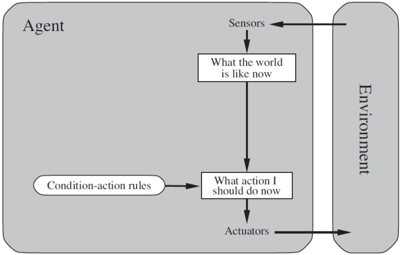
Tabular (Rule-based) Agents

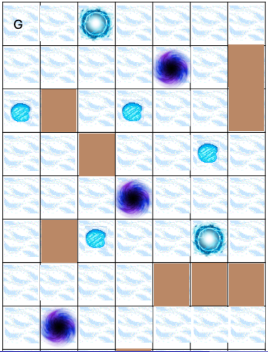




- When very little information is known about the environment, this agent function is most likely not to be rational.

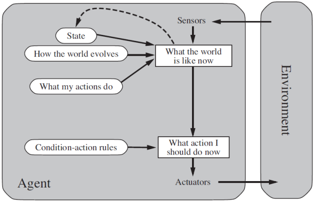
Reflex-based Agents

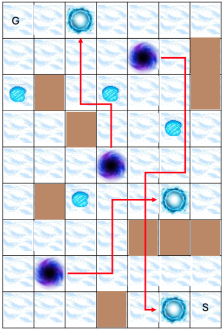




- A reflex-based agent has no information on the state of the environment, such as:
  - Where is it now?
  - How far is the goal?
- What can a reflex-based agent do in the MWFLP environment?
- Is randomisation required to ensure progress?

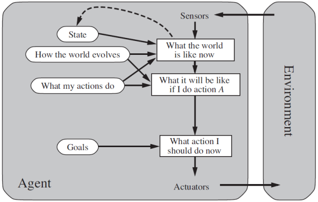
Model-based Agents

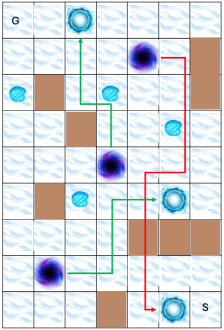




- By **exploring**, a model-based agent tries to discriminate good actions vs. bad actions from the states.
- The utilities of states are not **long-term** and **goal-driven**.
- In the MWFLP environment, the agent can at least know which states are portal entries, which are exits, which are obstacles and so on.

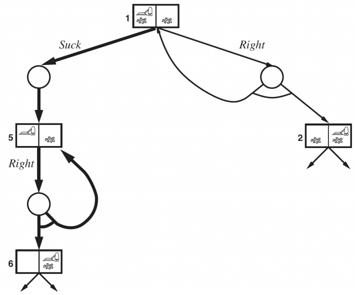
Goal-based Agents





- The agent now knows the coordinates of its locations, which means it can now know how far the goal is (in an ideal environment).
- A* algorithm works optimally in a deterministic environment.
- In the MWFLP environment, the agent can **estimate** which portals are good and which are bad.
- What happens in non-deterministic (stochastic) environments?

Variants of A* algorithm

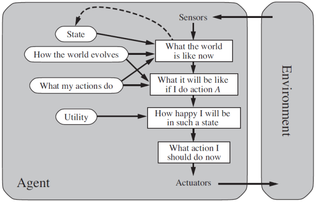


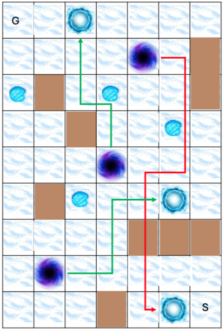
- MEU principle:

$$EU(a|\mathbf{e}) = \sum_{s'} P(Result(a) = s'|a, \mathbf{e})U(s')$$

- But we can just consider $U(s) = g(n) + h(n)$ (A*) or simply $h(n)$ (best-first).
- We can still apply an A* like algorithm on a set of belief states rather than working with single states.
- One may aggregate the heuristics over belief states — expected values!.
- We also need to update the heuristics for each node $h(n)$ during exploration.

Utility-based Agents



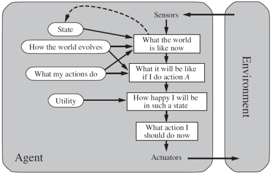


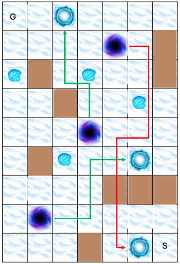
- Conflicting goals — Avoid falling into a hole, and minimize the time spent in the maze.
- MEU principle: (Almost!) the *theory of everything* in AI.

$$EU(a|\mathbf{e}) = \sum_{s'} P(Result(a) = s'|a, \mathbf{e})U(s')$$

- All we then need to do is to recursively compute the $U(s)$ for each state.

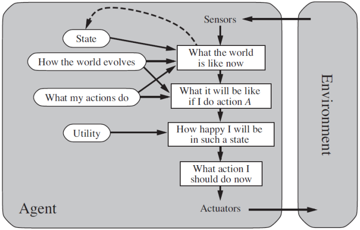
Utility-based Agents

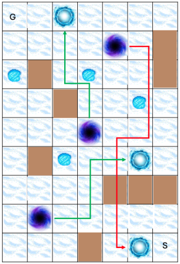




- If you actually compute recursively the utility of each state, it's more like a "**plan-ahead**" without actually exploring the environment.
- For this plan-ahead strategy, an agent needs to **know the probabilities of the stochastic states**. Without any information (e.g., the portals, you have to assume a uniform distribution).
- Works well for static environments.
- However, this plan-ahead strategy is slow as actual exploration is delayed.
- One may also work with **estimated utilities** rather than true ones.

Utility-based Agents in Dynamic Environments





- Consider a more difficult environment.
  - Parts of the frozen surface melting — new holes appear.
- The agent's beliefs about the environment should change.
- $EU(\alpha|\mathbf{e}, e_j) = \max_a \sum_{s'} P(Result(a) = s'|a, \mathbf{e}, e_j)U(s')$
- Useful when there's a performance cost involved in procuring the information.

That's it for the AI course

- You should now be able to apply some of the knowledge gained from the course in **intelligent agent design**.
- Let me know if you're interested for a PhD.
  - Good scope to increase your technical knowledge before joining the industry and start your first job much higher up the company ladder.
  - Or you could just continue to remain in the academia and teach new students.
  - My research spans: **Agentic Information Retrieval** and **Agentic Generative (Multi-modal) Models**.