Problem Set 2

Chen Li Introduction to Game Theory

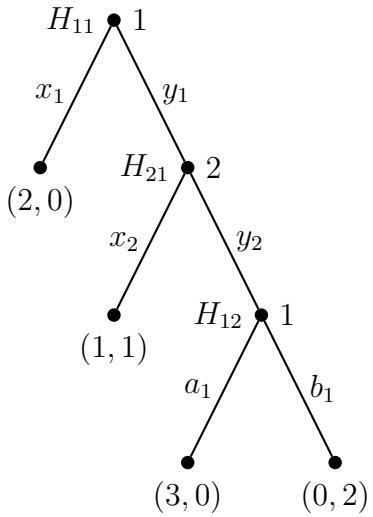
Due: June 23, 2025

1. Mixed Strategy Nash Equilibrium (Gibbons). Find all the mixed strategy Nash equilibria of the following normal-form game.

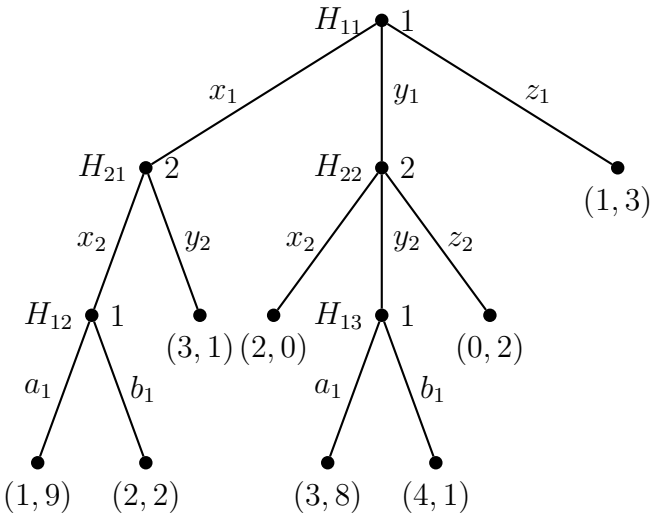
		Player 2	
		x_2	y_2
Player 1	x_1	(2, 1)	(0, 2)
	y_1	(1, 2)	(3, 0)

2. Backward Induction Solutions. Find the backward induction solutions for the following games. In the game trees, the number beside a decision node represents the corresponding player, and H_{im} stands for player i's m-th information set. The way to read payoffs is standard: the number left is player 1's payoff, and the number right is player 2's payoff. For this question, only write down the final answers. Remember that a solution should be a strategy profile of all the players, and a strategy in an extensive form game is a *complete contingent plan*.

(a)



(b)



3. Normal-Form Representation of Extensive Form Games.

(a) Consider game (a) in Question 2. What is the normal-form representation of this game? Find all the Nash equilibria of the normal-form game.

(b) Consider game (b) in Question 2. What is the normal-form representation of this game? Find all the Nash equilibria of the normal-form game.

4. Assassination Game. In this game, n cabinet ministers decide sequentially whether to participate in a coup to assassinate the king. To be specific, there is a ranking from Minister 1 to Minister n, and only Minister 1 can decide whether to join the coup or not. If Minister 1 successfully assassinated the king, then he becomes the new king, and now Minister 2 can decide whether to join the coup to assassinate (former) Minister 1 or not. In general, only Minister $k+1$ can decide whether to join the coup to assassinate the new king before him, that is, former Minister $k(k = 1, \cdots, n-1)$ or not. Assume that a minister can only be assassinated after he becomes the new king. Also assume that when a minister decides to join the coup, then he can successfully assassinate the king with probability 1. The preference of each minister is described as follows: Each prefers to being the king himself and not being assassinated, to remaining a minister (at least he is alive), to being the king himself and being assassinated. Answer the following questions.

(a) Let $n = 2$. Draw the game tree for this case and find the backward induction solution.

(b) Let $n = 3$. Draw the game tree for this case and find the backward induction solution.

(c) For general positive integer n, find the backward induction solution of this game.

5. Modified Entry Game. Consider the following variation of the entry game discussed in class. The players are still Incumbent and Entrant. At the beginning of the game, the Entrant chooses whether to enter the market or not. Now suppose that the Entrant has two ways to enter, that is, the actions available for the Entrant are: Out, In_1, and In_2. If the Entrant chooses Out, the game terminates, and the payoffs for the Incumbent and the Entrant are 3 and 0, respectively. If the Entrant chooses In_1 or In_2, then the Incumbent chooses whether to $Conform$ or to $Fight$. However, in this case, the Incumbent cannot tell whether the Entrant's entry is due to action In_1 or In_2. Payoffs in this case are as follows:

- If the Entrant chooses In_1 and the Incumbent chooses $Conform$, the Entrant's payoff is 1 and the Incumbent's payoff is 2;
- If the Entrant chooses In_1 and the Incumbent chooses $Fight$, the Entrant's payoff is -1 and the Incumbent's payoff is 0;
- If the Entrant chooses In_2 and the Incumbent chooses $Conform$, the Entrant's payoff is 1 and the Incumbent's payoff is 2;
- If the Entrant chooses In_2 and the Incumbent chooses $Fight$, the Entrant's payoff is -1 and the Incumbent's payoff is 0.

(a) Draw the game tree of this game.

(b) How many subgames does this game have? Find all the subgame perfect equilibria.