
Index

- absorbing state 11, 111
- absorption probability 12, 112
- action 198
- adapted 129
- alleles 175
- aperiodicity 40
- average number of customers 181

- backward equation 62, 96
- Bayesian statistics 211
- biological models 6, 9, 16, 82, 170
- birth process 81
 - infinitesimal definition 85
 - jump chain/holding time definition 85
 - transition probability definition 85
- birth-and-death chain 16
- boundary 138
- boundary theory for Markov chains 147
- branching process 171
 - with immigration 179
- Brownian motion 159
 - as limit of random walks 164
 - existence 161
 - in \mathbb{R}^d 165
 - scaling invariance 165
 - starting from x 165
 - transition density 166
- busy period 181

- capacity 151
- central limit theorem 160
- charge 151
- closed class 11, 111
- closed migration process 184
- communicating class 11, 111
- conditional expectation 129
- conductivity 151
- continuous-time Markov chains 87
 - construction 89
 - infinitesimal definition 94
 - jump chain/holding time definition 94, 97
 - transition probability definition 94, 97
- continuous-time random process 67
- convergence to equilibrium 41, 121, 168
- countable set 217
- coupling method 41
- current 151

- detailed balance 48, 124
- discounted value function 202
- distribution 1
- \mathbb{E} , expectation 222
- $E(\lambda)$, exponential distribution of
 - parameter λ 70
- e^Q , exponential of Q 62
- effective conductivity 156
- electrical network 151
- energy 154
- epidemic model 173
- equilibrium distribution 33, 117
- ergodic theorem 53, 126, 168
- Erlang's formula 183
- excursions 24
- expectation 222
- expected hitting time 12, 113
- expected return time 37, 118
- explosion
 - for birth processes 83
 - for continuous-time chains 90
- explosion time 69
- explosive Q -matrix 91
- exponential distribution 70
- \mathcal{F}_n , filtration 129
- fair price 135
- filtration 129
- finite-dimensional distributions 67
- first passage decomposition 28
- first passage time 19, 115
- flow 151
- forward equation 62, 100
 - for birth processes 84
 - for Poisson process 78
- Fubini's theorem 223
 - discrete case 219
- full conditional distributions 212
- fundamental solution 145
- γ_i^j , expected time in i between visits to j 35
- Galton–Watson process 171
- gambling 131
- Gaussian distribution 160
- generator 166
- generator matrix 94, 97
- Gibbs sampler 210
- gravity 134, 169
- Green matrix 144, 145
- harmonic function 146
- Hastings algorithm 210
- hitting probability 12
- hitting time 12, 111
- holding times 69
- I , state-space 2
- infective 173
- integrable 129, 222
- integral form of the backward
 - equation 98
- integral form of the forward equation 101
- inter-arrival times 180
- invariant distribution 33, 117
 - computation of 40
- irreducibility 11, 111
- Ising model 214
- jump chain 69
- jump matrix 87
- jump times 69
- last exit time 20
- long-run proportion of time 53, 126
- m_i , expected return time 37, 118
- μ_i^j , expected time in i between visits to j 118
- Markov(λ, P) 2
- Markov(λ, Q) 94, 97
- Markov chain
 - continuous-time 88
 - discrete-time 2
- Markov chain Monte Carlo 206, 208
- Markov decision process 197
 - expected total cost 198
 - expected total discounted cost 202
 - long-run average costs 204
- Markov property 3
 - for birth processes 84
 - for continuous-time chains 93
 - for Poisson process 75
- martingale 129, 141, 176, 204

- associated to a Markov chain 132
 - associated to Brownian motion 169
- matrix exponentials 105
- maximum likelihood estimate 56
- measure 1
- memoryless property 70
- Metropolis algorithm 210
- minimal non-negative solution 13
- minimal process 69
- monotone convergence 223
 - discrete case 219
- Moran model 177
- mutation 6, 176
- non-minimal chains 103
- null recurrence 37, 118
- $o(t), O(t)$, order notation 63
- Ohm's law 151
- open migration process 185
- optional stopping theorem 130
- \mathbb{P} , probability 222
- P , transition matrix 2
- \widehat{P} , transition matrix of reversed chain 47
- $P(t)$, transition semigroup 96
- $\widehat{P}(t)$, semigroup of reversed chain 124
- $p_{ij}^{(n)}$, n -step transition probability 5
- Π , jump matrix 87
- π -system 228
- Poisson process 74
 - infinitesimal definition 76
 - jump chain/holding time definition 76
 - transition probability definition 76
- policy 198
- policy improvement 201
- population genetics 175
- population growth 171
- positive recurrence 37, 118
- potential 138
 - associated to a Markov chain 138
 - associated to Brownian motion 169
 - gravitational 134
 - in electrical networks 151
 - with discounted costs 142
- potential theory 134
- probability generating function 171
- probability measure 222
- probability space 222
- Q -matrix 60
- \widehat{Q} , generator matrix of reversed chain 124
- q_i , rate of leaving i 61
- q_{ij} , rate of going from i to j 61
- queue 179
 - M/G/1 187
 - M/G/ ∞ 191
 - M/M/1 180
 - M/M/s 182
- queueing network 183–185
- queues in series 183
- random chessboard knight 50
- random walk
 - on \mathbb{Z}^d 29
 - on a graph 49
- recurrence 24, 114, 167
- recurrence relations 57
- reflected random walks 195
- reservoir model 194, 195
- resolvent 146
- resource management 192
- restocking a warehouse 192
- return probability 25
- reversibility 48, 125
- right-continuous process 67
- ruin
 - gambler 15
 - insurance company 196
- selective advantage 176
- semigroup 96
- semigroup property 62
- service times 180
- shopping centre 185
- simple birth process 82
- simulation 206
- skeleton 122
- state-space 1

- stationary distribution 33, 117
- stationary increments 76
- stationary policy 198
- statistics 55, 211, 215
- stochastic matrix 2
- stopping time 19
- strong law of large numbers 52
- strong Markov property 19, 93, 227
- success-run chain 38
- susceptible 173

- telephone exchange 183
- texture parameter 216
- time reversal 47, 123
- transience 24, 114, 167
- transition matrix 2
 - irreducible 11
 - maximum likelihood estimate 56
- transition semigroup 165
- truncated Poisson distribution 183

- unit mass 3

- $V_i(n)$, number of visits to i before n 53
- valency 50
- value function 198

- weak convergence 164
- Wiener process 159
- Wiener's theorem 161
- Wright–Fisher model 175

- ζ , explosion time 69