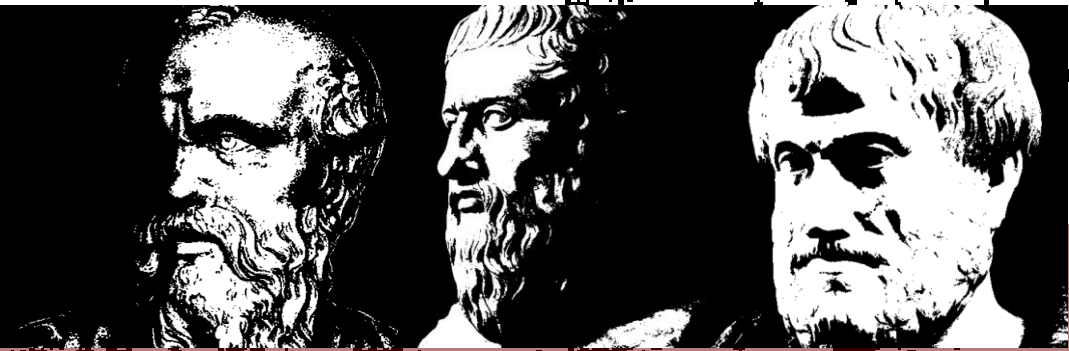# LOGIC I

FORMAL LOGIC: CATEGORICAL SYLLOGISMS

Dr. Ethan SAHKER, PhD

Categorical Propositions

OVERVIEW

Proposition	Letter name	Quantity	Quality	Terms distributed
All S are P.	**A**	universal	affirmative	S
No S are P.	**E**	universal	negative	S and P
Some S are P.	**I**	particular	affirmative	none
Some S are not P.	**O**	particular	negative	P

Categorical Propositions

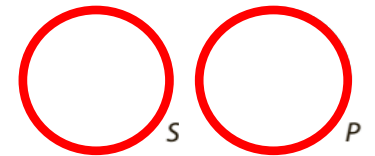| QUALITY QUANTITY	AFFIRMATIVE	NEGATIVE
UNIVERSAL	ALL students are smart	NO students are smart
PARTICULAR	SOME students are smart	SOME students are NOT smart

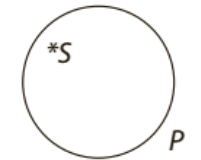# Categorical Propositions

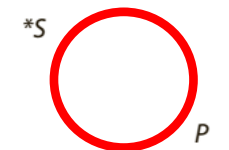**DISTRIBUTION**

A: All S are P = S Distributed
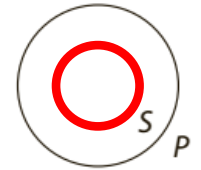
E: No S are P = S & P Distributed

I: Some S are P = Undistributed

O: Some S are not P = P Distributed

Categorical Propositions

The Traditional Square of Opposition

A: All students are smart.   **E:** No students are smart.

I: Some students are smart.   **O:** Some students are not smart.

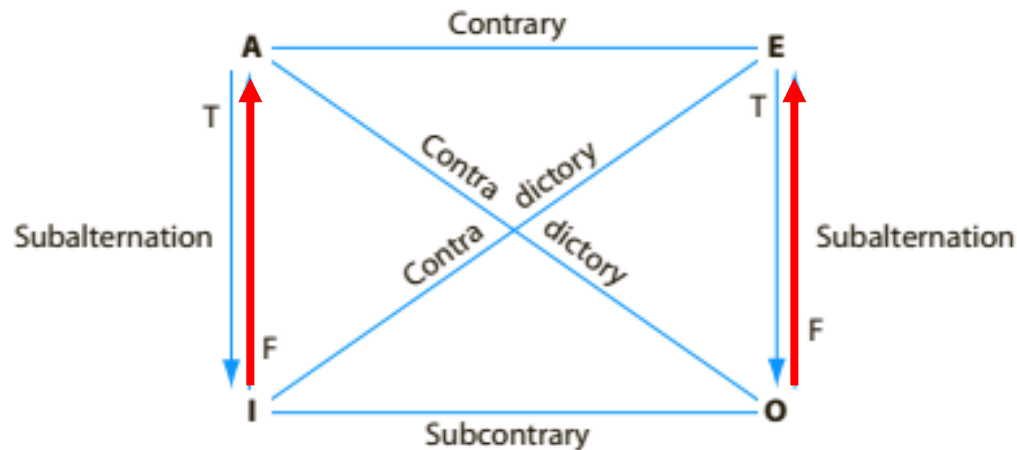**Contradictory**: if one is False the other is True

Contrary: Both can be False, both cannot be True

Subcontrary: both can be True, both cannot be False

Subalternation:
   Superaltern (A, E): If super is True, sub is True
   Subaltern (I, O): if sub is False, super is False

Categorical Propositions

SET THEORY

I the set of As are labeled as s(A) and the set of all Bs are labeled as s(B) then:

LOGIC	LANGUAGE	OPERATORS
All S are P	s(A) is a subset of s(B)	s(A) ⊆ s(B)
No S are P	The intersection of s(A) and s(B) is empty	s(A) ∩ s(B) = Ø
Some S are P	The intersection of s(A) ands(B) is not empty	s(A) ∩ s(B) ≠ Ø
Some S are not P	s(A) is not a subset of s(B)	s(A) ⊄ s(B)

Categorical Syllogisms

- A syllogism is a deductive argument having <u>two premises</u> and <u>one conclusion</u>.
- A categorical syllogism is a syllogism having:
  - <u>3</u> categorical propositions,
  - <u>3</u> different terms
  - Each term appears <u>twice</u> in <u>different propositions</u>.

EXAMPLE:

All soldiers are nationalists.

No traitors are nationalists.

Therefore, no traitors are soldiers.

Categorical Syllogisms

- A syllogism is a deductive argument having <u>two premises</u> and <u>one conclusion</u>.
- A categorical syllogism is a syllogism having:
  ✓ - <u>3</u> categorical propositions,
  - <u>3</u> different terms
  - Each term appears <u>twice</u> in <u>different propositions</u>.

EXAMPLE:

A Proposition ⟶ All soldiers are nationalists.

E Proposition ⟶ No traitors are nationalists.

E Proposition ⟶ Therefore, no traitors are soldiers.

Categorical Syllogisms

- A syllogism is a deductive argument having <u>two premises</u> and <u>one conclusion</u>.
- A categorical syllogism is a syllogism having:
  - ✓ - <u>3</u> categorical propositions,
  - ✓ - <u>3</u> different terms
  - - Each term appears <u>twice</u> in <u>different propositions</u>.

Major Term - Soldiers   **A** Proposition ⟶

Middle Term - Nationalists   **E** Proposition ⟶

Minor Term - Traitors   **E** Proposition ⟶

EXAMPLE:

All <u>soldiers</u> are <u>nationalists</u>.

No <u>traitors</u> are nationalists.

Therefore, no traitors are soldiers.

Categorical Syllogisms

- A syllogism is a deductive argument having <u>two premises</u> and <u>one conclusion</u>.
- A categorical syllogism is a syllogism having:
  - ✓ - <u>3</u> categorical propositions,
  - ✓ - <u>3</u> different terms
  - ✓ - Each term appears <u>twice</u> in <u>different propositions</u>.

1st and 3rd	Major Term - Soldiers	**A** Proposition ⟶	
1st and 2nd	Middle Term - Nationalists	**E** Proposition ⟶	
2nd and 3rd	Minor Term - Traitors	**E** Proposition ⟶	

EXAMPLE:

All soldiers are nationalists.

No traitors are nationalists.

Therefore, no traitors are soldiers.

Categorical Syllogisms

Major Term – Predicate of the conclusion (Soldiers)

Middle Term – Appears once in each premise (Nationalists)

Minor Term – Subject of the conclusion (Traitors)

Major Premise – Contains Major Term (Soldiers)

Minor Premise – Contains Minor Term (Traitors)

Major Premise

Minor Premise

EXAMPLE:

    P   M
All soldiers are nationalists.
    S   M
No traitors are nationalists.

Therefore, no traitors are soldiers.
        S   P

Categorical Syllogisms

Standard Form Categorical Syllogism

1. All three statements are categorical propositions
2. Each term occurs 2 times and each time is identical.
3. Each term is used with the same meaning throughout the argument.
4. The major premise is listed first, the minor premise second, and the conclusion last.

EXAMPLE:

All watercolors are paintings.   ←——— **A** Proposition

Some watercolors are masterpieces.   ←——— **I** Proposition

Thus, some paintings are masterpieces.   ←——— **I** Proposition

Categorical Syllogisms

Standard Form Categorical Syllogism

1. All three statements are categorical propositions
2. Each term occurs 2 times and each time is identical.
3. Each term is used with the same meaning throughout the argument.
4. The major premise is listed first, the minor premise second, and the conclusion last.

EXAMPLE:

All watercolors are paintings. ⟵ **A** Proposition   Major Term - Masterpieces

Some watercolors are masterpieces. ⟵ **I** Proposition   Middle Term - Watercolors

Thus, some paintings are masterpieces. ⟵ **I** Proposition   Minor Term - Paintings

Categorical Syllogisms

Standard Form Categorical Syllogism

1. All three statements are categorical propositions
2. Each term occurs 2 times and each time is identical.
3. Each term is used with the same meaning throughout the argument.
4. The major premise is listed first, the minor premise second, and the conclusion last.

EXAMPLE:

Some watercolors are masterpieces. ← **A** Proposition   Major Term - Masterpieces

All watercolors are paintings. ← **I** Proposition   Middle Term - Watercolors

Thus, some paintings are masterpieces. ← **I** Proposition   Minor Term - Paintings

Categorical Syllogisms

MOOD – Proposition Determined

1. Mood is determined by the combination of propositional statements.
2. If the major premise is an A proposition, the minor premise an O proposition, and the conclusion an E proposition, the **mood** is **AOE**.

Proposition	Letter name
All S are P.	A
No S are P.	E
Some S are P.	I
Some S are not P.	O

EXAMPLE:

Some animals are dangerous.

All dogs are animals.

Therefore, some dogs are dangerous.

Mood = IAI

Categorical Syllogisms

FIGURE – Term Determined

1. Figure is determined by the location of the two occurrences of the middle
2. If **minor** term = S
3. AND **major** term = P
4. AND **middle** term = M
5. THEN there are 4 possible figures:

Figure 1

M	P
S	M
S	P

Figure 2

P	M
S	M
S	P

Figure 3

M	P
M	S
S	P

Figure 4

P	M
M	S
S	P

EXAMPLE:

Some animals are dangerous.

All dogs are animals.

Therefore, some dogs are dangerous.

Categorical Syllogisms

What is the Figure and mood of the Example?
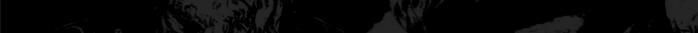
1. minor term = S
2. major term = P
3. middle term = M

Proposition	Letter name
All S are P.	A
No S are P.	E
Some S are P.	I
Some S are not P.	O

Mood = IAI

Figure = 1

Figure 1
M   P
S   M
―――――――
S   P

Figure 2
P   M
S   M
―――――――
S   P

Figure 3
M   P
M   S
―――――――
S   P

Figure 4
P   M
M   S
―――――――
S   P

EXAMPLE:

I — Some animals are dangerous.
               M   P

A — All dogs are animals.
        S   M

I — Therefore, some dogs are dangerous.
                S   P

Categorical Syllogisms

VALID CATEGORICAL SYLLOGISMS

Figure 1: AAA, EAE, AII, EIO

Figure 2: EAE, AEE, EIO, AOO

Figure 3: IAI, AII, OAO, EIO

Figure 4: AEE, IAI, EIO

Proposition	Letter name
All S are P.	A
No S are P.	E
Some S are P.	I
Some S are not P.	O

Figure 1

M	P
S	M
S	P

Figure 2

P	M
S	M
S	P

Figure 3

M	P
M	S
S	P

Figure 4

P	M
M	S
S	P

Categorical *Syllogisms*

VALID CATEGORICAL SYLLOGISMS

Figure 1: AAA, EAE, AII, EIO

Figure 2: EAE, AEE, EIO, AOO

Figure 3: IAI, AII, OAO, EIO

Figure 4: AEE, IAI, EIO

Proposition	Letter name
All S are P.	A
No S are P.	E
Some S are P.	I
Some S are not P.	O

Figure 1

M   P
S   M
―――――
S   P

Figure 2

P   M
S   M
―――――
S   P

Figure 3

M   P
M   S
―――――
S   P

Figure 4

P   M
M   S
―――――
S   P

EXAMPLE:

Some cakes are delicious.

All cheesecakes are cakes.

Some cheesecakes are delicious.

Categorical Syllogisms

VENN DIAGRAMS WITH SYLLOGISMS

Syllogisms have 3 terms

Venn diagrams must contain 3 categories

This is the foundation of multiple correlation analysis in statistics

No P are M

All S are M   **EAE-2**
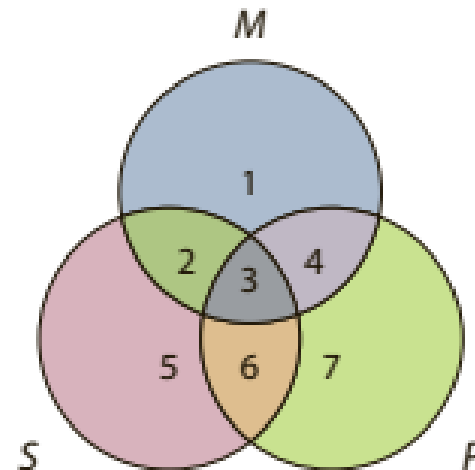
No S are P

Categorical Syllogisms

VENN DIAGRAMS WITH SYLLOGISMS

Focus on the P and M circles and the section eliminated

No P are M

All S are M
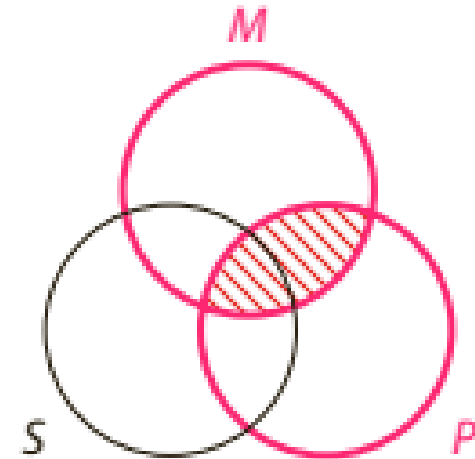
No S are P

EAE-2

Categorical Syllogisms

VENN DIAGRAMS WITH SYLLOGISMS

Next, Focus on S and M

The conclusion claims the S and P overlap should be shaded

No P are M

All S are M

No S are P

All S are M.

EAE-2
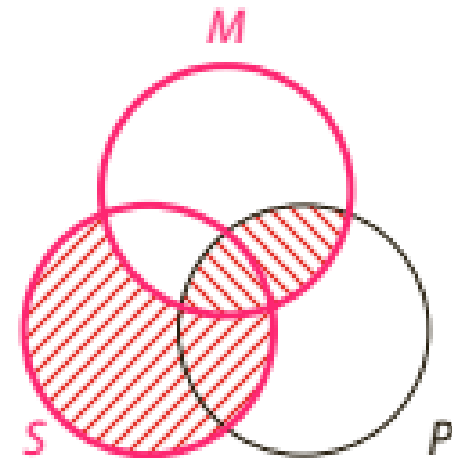
Categorical *Syllogisms*

VENN DIAGRAMS WITH SYLLOGISMS

Next, Focus on S and M

The conclusion claims the S and P overlap should be shaded

The diagram shows this area is indeed shaded to mean non-existence

Thus, the syllogism is valid, based on form.

No P are M

All S are M

No S are P

EAE-2
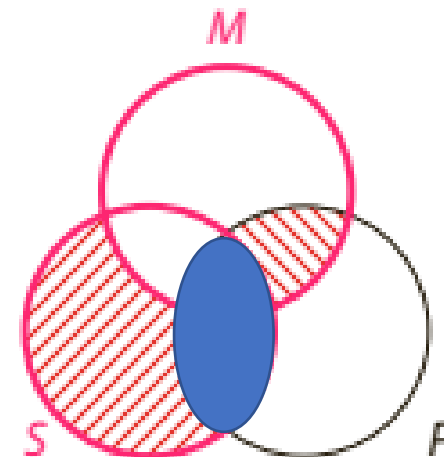
Categorical Syllogisms

VENN DIAGRAMS WITH SYLLOGISMS

Other examples:

Some P are M

All M are S   IAI-4

Some S are P

All M are P

All S are M   AAA-1

All S are P

- **Partial**
  - The relationship between Y and X_1 controlling for effects of X_2 on **both** Y and X_1

- **Part (semi-partial)**
  - The relationship between Y and X_1 removing effects of X_2 from X_1 **but not Y**

$r^2_{Y1.2} = (2)/(2 \text{ and } 3)$

$r^2_{Y1.2(s)} = (2)/(1,2,3 \text{ and } 4)$
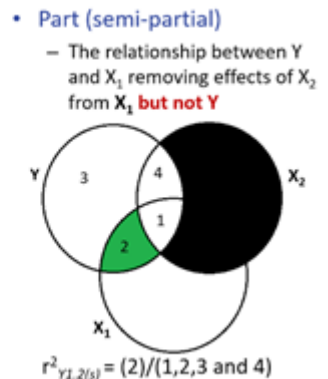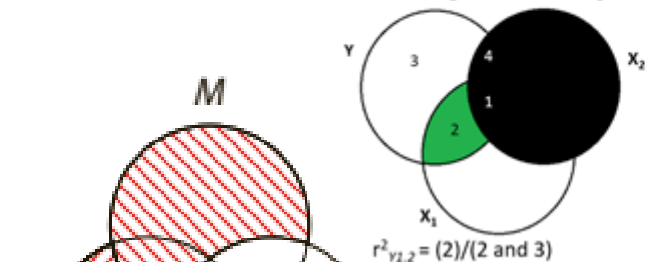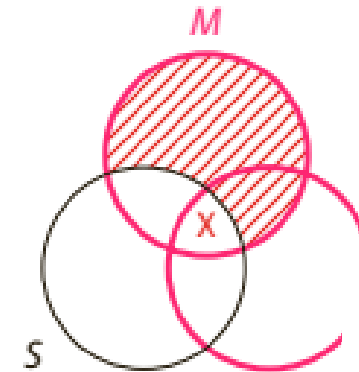
Categorical Syllogisms

LOGIC IN STATISTICS

- **Partial**
  - The relationship between Y and X_1 controlling for effects of X_2 on **both** Y and X_1
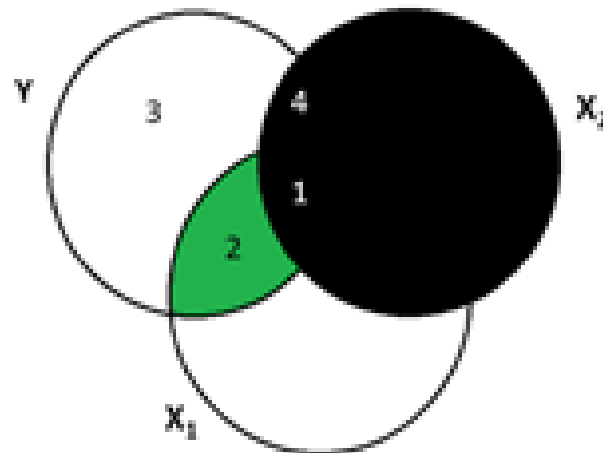
$r^2_{Y1.2} = (2)/(2 \text{ and } 3)$

- **Part (semi-partial)**
  - The relationship between Y and X_1 removing effects of X_2 from X_1 but not Y
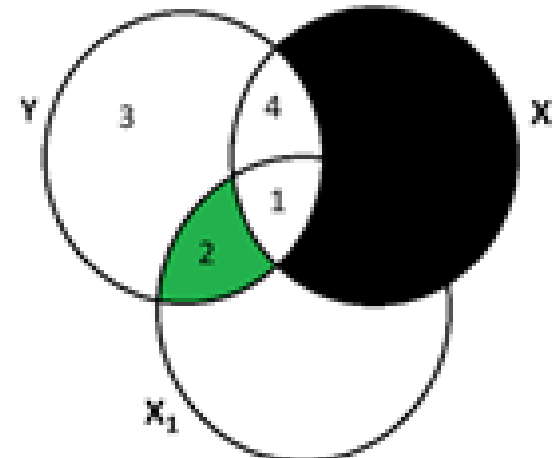
$r^2_{Y1.2(s)} = (2)/(1,2,3 \text{ and } 4)$

Categorical Syllogisms

RULES AND FORMAL FALLACIES

Rule 1: The Middle Term Must Be Distributed at Least Once.

Fallacy: Undistributed middle – The conclusion doesn't follow the premises.

Hey man! I'm a Salmon Shark
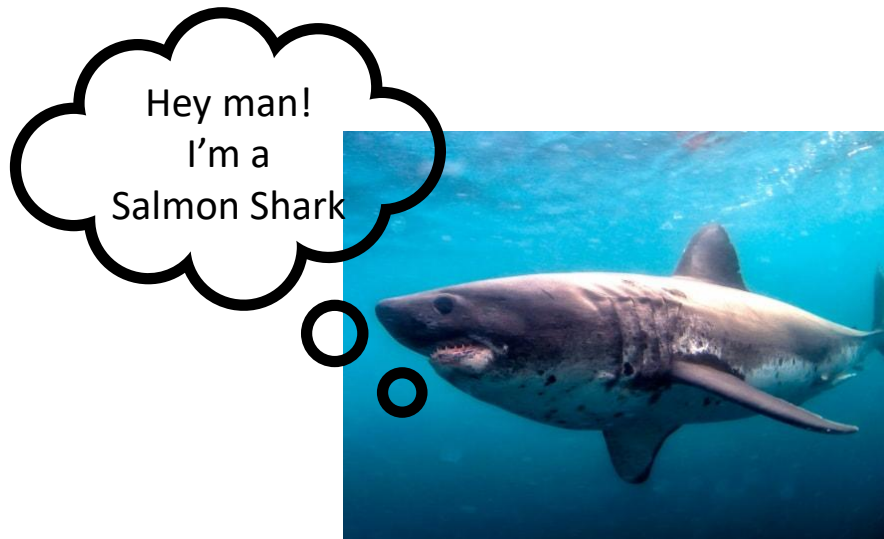
Example:

All sharks are <u>fish</u>.

<u>Some fish are salmon</u>.

All salmon are sharks.

Distribution:

- If every P or every S is something, it is distributed
- A = S Distributed
- E = S & P Distributed
- I = Undistributed
- O = P Distributed

Categorical *Syllogisms*

UNDISTRIBUTED MIDDLE

Figure 1: AAA, EAE, AII, EIO

Figure 2: EAE, AEE, EIO, AOO

Figure 3: IAI, AII, OAO, EIO

Figure 4: AEE, IAI, EIO

Proposition	Letter name
All S are P.	**A**
No S are P.	**E**
Some S are P.	**I**
Some S are not P.	**O**

Figure 1
Ⓜ   P
S   Ⓜ
―――――――
S   P

Figure 2
P   Ⓜ
S   Ⓜ
―――――――
S   P

Figure 3
Ⓜ   P
Ⓜ   S
―――――――
S   P

Figure 4
P   Ⓜ
Ⓜ   S
―――――――
S   P

EXAMPLE: **IAI-1**

Some cakes are delicious.

All cheesecakes are cakes.

Some cheesecakes are delicious.

Categorical *Syllogisms*

RULES AND FORMAL FALLACIES

Rule 2: If a Term Is Distributed in the Conclusion, Then It Must Be Distributed in a Premise.

Fallacies: Illicit major; illicit minor – Conclusion doesn't follow the premises

Examples:

Some horses are animals.

<u>Some dogs </u>are animals.

Some horses are <u>not dogs</u>.

Distribution:
- If every P or every S is something, it is distributed
- A = S Distributed
- E = S & P Distributed
- I = Undistributed
- O = P Distributed

Categorical *Syllogisms*

ILLICIT MAJOR; ILLICIT MINOR

Figure 1: AAA, EAE, AII, EIO

Figure 2: EAE, AEE, EIO, AOO

Figure 3: IAI, AII, OAO, EIO

Figure 4: AEE, IAI, EIO

Proposition	Letter name
All S are P.	A
No S are P.	E
Some S are P.	I
Some S are not P.	O

Figure 1

M	P
S	M
S	P

Figure 2

P	M
S	M
S	P

Figure 3

M	P
M	S
S	P

Figure 4

P	M
M	S
S	P

Example: **IIO-2**

Some horses are animals.

Some dogs are animals.

Some horses are not dogs.

Categorical Syllogisms

RULES AND FORMAL FALLACIES

Rule 3: Two Negative Premises Are Not Allowed.

Fallacies: Exclusive premises – nonexistence cannot serve as evidence

Examples:

No fish are mammals.

Some dogs are not fish.

Some dogs are not mammals.

Quality:
- all S are P = affirming S = Positive
- no S are P = denies S = Negative

Categorical Syllogisms

EXPCLUSIVE PREMISES

Figure 1: AAA, EAE, AII, EIO

Figure 2: EAE, AEE, EIO, AOO

Figure 3: IAI, AII, OAO, EIO

Figure 4: AEE, IAI, EIO

Proposition	Letter name
All *S* are *P*.	**A**
No *S* are *P*.	**E**
Some *S* are *P*.	**I**
Some *S* are not *P*.	**O**

Figure I

Ⓜ   P
S   Ⓜ
―――――
S   P

Figure 2

P   Ⓜ
S   Ⓜ
―――――
S   P

Figure 3

Ⓜ   P
Ⓜ   S
―――――
S   P

Figure 4

P   Ⓜ
Ⓜ   S
―――――
S   P

Examples: **EOO-1**

<u>No fish</u> are mammals.

Some dogs are <u>not fish</u>.

Some dogs are not mammals.

Categorical *Syllogisms*

RULES AND FORMAL FALLACIES

Rule 4: One Negative Premise Requires a Negative Conclusion

Fallacies: Drawing an affirmative from a negative

You can't affirm a relationship that the premises deny

Examples:

All crows are birds.

<u>No wolves </u>are crows.

All birds *are* <u>wolves</u>.

Categorical *Syllogisms*

AFFIRMATIVE FROM NEGATIVE

Figure 1: AAA, EAE, AII, EIO

Figure 2: EAE, AEE, EIO, AOO

Figure 3: IAI, AII, OAO, EIO

Figure 4: AEE, IAI, EIO

Proposition	Letter name
All *S* are *P*.	**A**
No *S* are *P*.	**E**
Some *S* are *P*.	**I**
Some *S* are not *P*.	**O**

Figure 1
M — P
S — M
—
S — P

Figure 2
P — M
S — M
—
S — P

Figure 3
M — P
M — S
—
S — P

Figure 4
P — M
M — S
—
S — P

Examples: **AEA-1**

All crows are birds.

No wolves are crows.

All birds *are* wolves.

Categorical Syllogisms

RULES AND FORMAL FALLACIES

Rule 5: If Both Premises Are Universal, the Conclusion Cannot Be Particular.

Fallacies: Existential Fallacy

Assumes existence of a specific without discussion of specific evidence

Examples:

All mammals are animals.

All tigers are mammals.

Some tigers are animals.

Quantity:
- all versus some of the group members
- no S are P = every member included = Universal
- some S are P = some members included = Particular

Categorical *Syllogisms*

EXISTENTIAL

Figure 1: AAA, EAE, AII, EIO

Figure 2: EAE, AEE, EIO, AOO

Figure 3: IAI, AII, OAO, EIO

Figure 4: AEE, IAI, EIO

Proposition	Letter name
All *S* are *P*.	**A**
No *S* are *P*.	**E**
Some *S* are *P*.	**I**
Some *S* are not *P*.	**O**

Figure 1
Ⓜ   P
S   Ⓜ
―――――
S   P

Figure 2
P   Ⓜ
S   Ⓜ
―――――
S   P

Figure 3
Ⓜ   P
Ⓜ   S
―――――
S   P

Figure 4
P   Ⓜ
Ⓜ   S
―――――
S   P

Examples: **AAI-1**

All mammals are animals.
All tigers are mammals.
Some tigers are animals.

Categorical Syllogisms

PRACTICE

Create an <u>IAI-4 standard syllogism</u> where the <u>middle term</u> is the word <u>"cool."</u>

Proposition	Letter name
All S are P.	A
No S are P.	E
Some S are P.	I
Some S are not P.	O

Figure 1
Ⓜ   P
S   Ⓜ
‾‾‾‾‾‾‾‾‾‾‾
S   P

Figure 2
P   Ⓜ
S   Ⓜ
‾‾‾‾‾‾‾‾‾‾‾
S   P

Figure 3
Ⓜ   P
Ⓜ   S
‾‾‾‾‾‾‾‾‾‾‾
S   P

Figure 4
P   Ⓜ
Ⓜ   S
‾‾‾‾‾‾‾‾‾‾‾
S   P

Standard Form Categorical Syllogism
1. All three statements are categorical propositions
2. Each term occurs 2 times and each time is identical.
3. Each term is used with the same meaning throughout the argument.
4. The major premise is listed first, the minor premise second, and the conclusion last.

Categorical Syllogisms

PRACTICE

Create an OAE-3 standard syllogism where the major term is the word "hotdogs."

Proposition	Letter name
All S are P.	A
No S are P.	E
Some S are P.	I
Some S are not P.	O

Figure 1

M	P
S	M
S	P

Figure 2

P	M
S	M
S	P

Figure 3

M	P
M	S
S	P

Figure 4

P	M
M	S
S	P

Standard Form Categorical Syllogism
1. All three statements are categorical propositions
2. Each term occurs 2 times and each time is identical.
3. Each term is used with the same meaning throughout the argument.
4. The major premise is listed first, the minor premise second, and the conclusion last.