Problem Set 1

Chen Li   Introduction to Game Theory

Due: May 26, 2025

1. Iterated Elimination of Strictly Dominated Strategies and Nash Equilibrium (Gibbons[1]). Consider the following normal-form game.

		Player 2		
		L	C	R
	T	(2, 0)	(1, 1)	(4, 2)
Player 1	M	(3, 4)	(1, 2)	(2, 3)
	B	(1, 3)	(0, 2)	(3, 0)

(a) For each player, is there a strictly dominant strategy? Is any strategy strictly dominated by another strategy?

(b) What strategies survive iterated elimination of strictly dominated strategies? (Specify the order of elimination.)

(c) For each player, find her/his best response correspondence.

(d) What are the pure strategy Nash equilibria?

2. Iterated Elimination of Weakly Dominated Strategies. Let s_i and s_i' be two strategies of player i. The strategy s_i is weakly dominated by the strategy s_i' if for any possible strategy profile of the other players, s_{-i}, s_i' does at least as well as s_i, and for some possible strategy profile of the other players, say $s^\sharp_{-i}$, s_i' does strictly better than s_i. That is, the strategy s_i is weakly dominated by the strategy s_i' if for all $s_{-i} \in S_{-i}$,

$$u_i(s_i', s_{-i}) \geq u_i(s_i, s_{-i})$$

[1]Gibbons, Robert. Game Theory for Applied Economists. Princeton University Press, 1992.

and for some $s^\sharp_{-i} \in S_{-i}$,

$$u_i(s'_i, s^\sharp_{-i}) > u_i(s_i, s^\sharp_{-i})$$

Now, consider the following normal-form game.

		Player 2	
		L	R
Player 1	T	(8, 3)	(0, 3)
	M	(4, 2)	(0, 1)
	B	(8, 1)	(-8, 2)

(a) For each player, is any strategy weakly dominated by another strategy?

(b) What strategies survive iterated elimination of weakly dominated strategies if we eliminate all weakly dominated strategies at the same time? Specify the order of elimination.

(c) What strategies survive iterated elimination of weakly dominated strategies if we eliminate only one weakly dominated strategy at each round? If there are two or more weakly dominated strategies for any player, answer the question for all possible orders of elimination. Specify the order of elimination for each case.

3. Second Price Auction (Osborne and Rubinstein[2]). An object is to be assigned to a player in the set $\{1, ..., n\}$ in exchange for a payment. Player i's valuation of the object is v_i, and $v_1 > v_2 > \cdots > v_n > 0$. The mechanism used to assign the object is a (sealed-bid) auction: the players simultaneously submit bids (nonnegative numbers), and the object is given to the player with the lowest index among those who submit the highest bid, in exchange for a payment.

In a *second price* auction the payment that the winner makes is the highest bid among those submitted by the players who do not win (so that if only one player submits the highest bid then the price paid is the *second* highest bid).

Show that in a second price auction the bid v_i of any player i is a *weakly dominant* action: player i's payoff when he bids v_i is at least as high as his payoff when he submits any other bid, regardless of the actions of the other

[2]Osborne, Martin J. and Rubinstein, Ariel. *A Course in Game Theory.* The MIT Press, 1994.

players. Show that nevertheless there are ("inefficient" Nash) equilibria in which the winner is not player 1.

4. Splitting the Dollar (Gibbons). Player 1 and 2 are bargaining over how to split one dollar. Both players simultaneously name shares they would like to have, s_1 and s_2, where $0 \leq s_1, s_2 \leq 1$. If $s_1 + s_2 \leq 1$, then the players receive the shares they named; if $s_1 + s_2 > 1$, then both players receive zero. What are the pure strategy Nash equilibria of this game?

5. Hotelling's Location Game. Two groups of students, Group A and Group B, are seeking locations for their Takoyaki stands at the November Festival. Suppose there is only one road in the university, represented by the interval $[0, 1]$. Each group has to choose a single point within the interval, which represents the location of the stand along the road. Customers are uniformly distributed along the road and are assumed to go to the stand nearest to them. If the two groups choose the same location, they will each attract half the customers. Each group maximizes the share of customers.

(a) Formulate the above story as a normal-form game. Note that this game cannot be represented by a matrix since the strategy set of each player is infinite. So you have to formally write down the set of players, each player's strategy set, and each player's payoff function.

(b) Find the set of pure strategy Nash equilibria.

(c) Group C joins this Takoyaki stand location game. Now, each of Group A, Group B and Group C has to choose a single point within the interval $[0, 1]$. Customers are uniformly distributed along the road and are assumed to go to the stand nearest to them. If two groups choose the same location, they will each attract half the customers. If all groups choose the same location, they will each attract $\frac{1}{3}$ of all the customers. Each group maximizes the share of customers. Find the set of pure strategy Nash equilibria of this three-player game.