# Classification

Fundamentals of Artificial Intelligence

Instructor: Chenhui Chu

Email: chu@i.kyoto-u.ac.jp

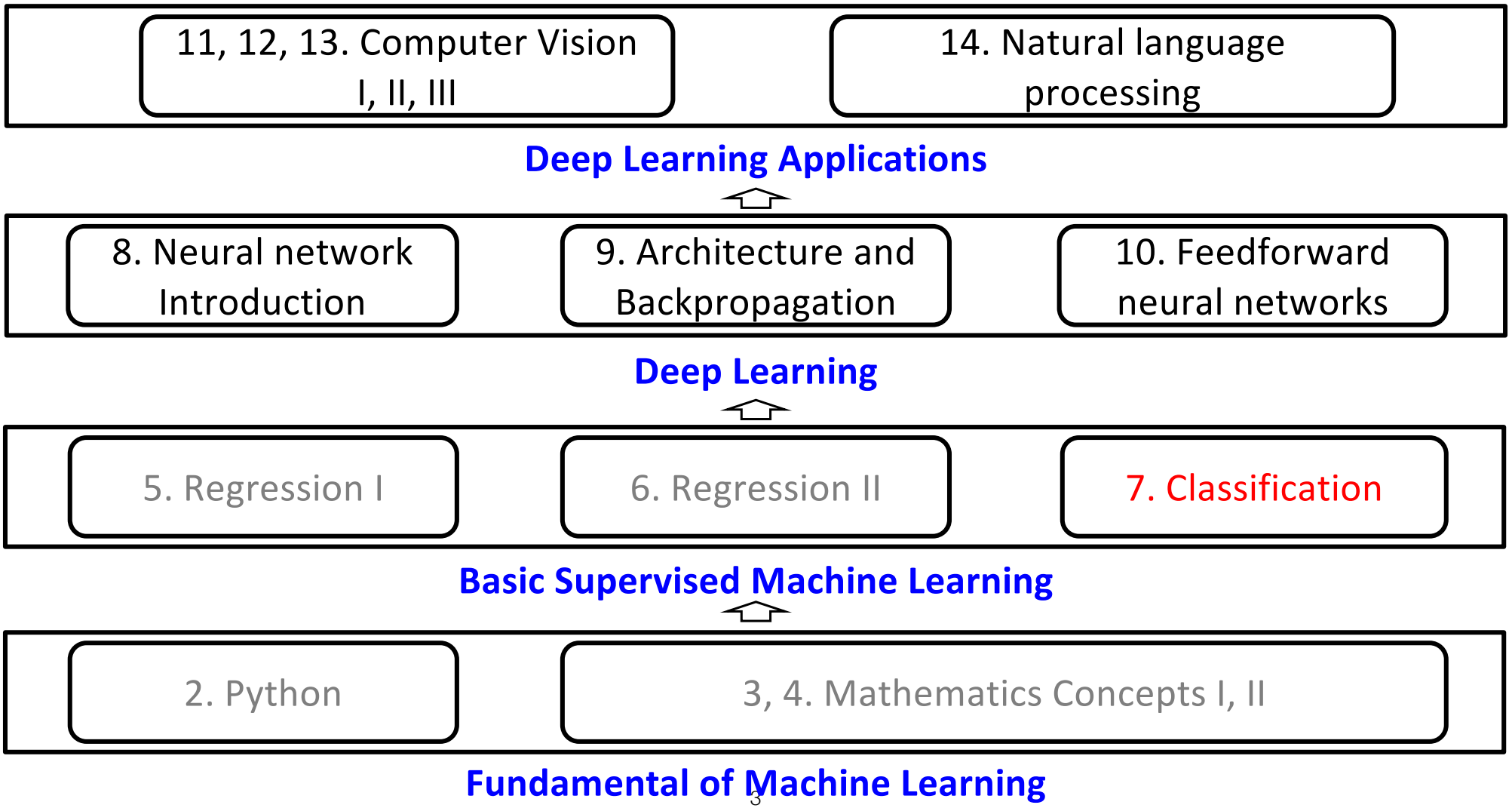
Teaching Assistant: Youyuan Lin

E-mail: youyuan@nlp.ist.i.kyoto-u.ac.jp

# Schedule

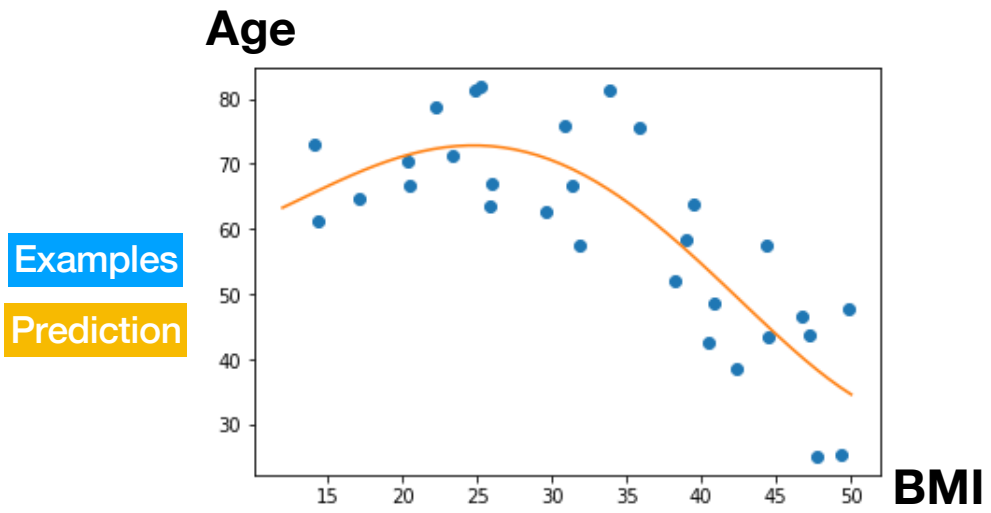
- 1. Overview of AI and this Course (4/14)
- 2. Introduction to Python (4/21)
- 3, 4. Mathematics Concepts I, II (4/28, 5/12)
- 5, 6. Regression I, II (5/19, 5/26)
- 7. Classification (6/2)
- 8. Introduction to Neural Networks (6/9)
- 9. Neural Networks Architecture and Backpropagation (6/16)
- 10. Fully Connected Layers (6/23)
- 11, 12, 13. Computer Vision I, II, III (6/30, 7/7, 7/14)
- 14. Natural Language Processing (7/17)

# Overview of This Course



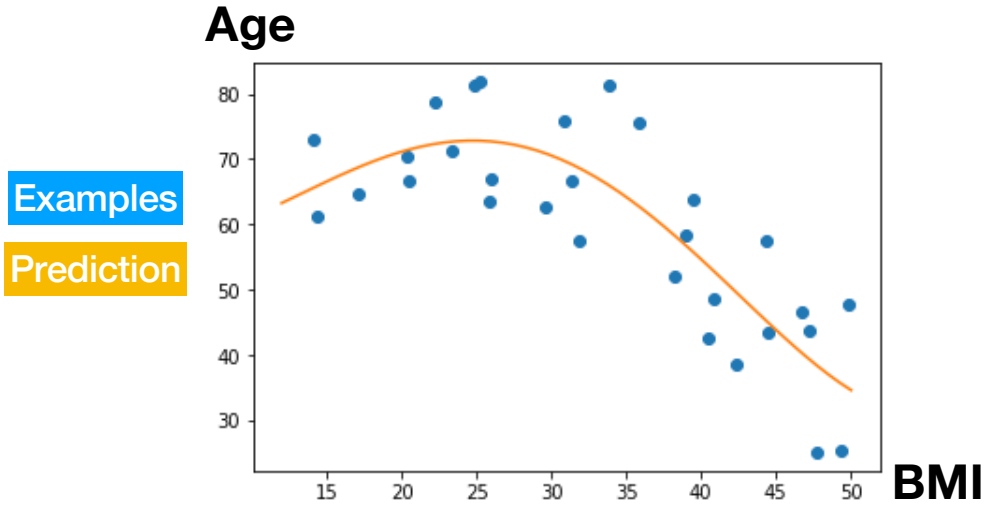
# Regression

- In the last two sessions, we considered the task of ***regression***
- Given examples, predict a number
  - Age of Death
  - Price of a stock
  - Temperature of an object
  - ….



# Regression vs. Classification

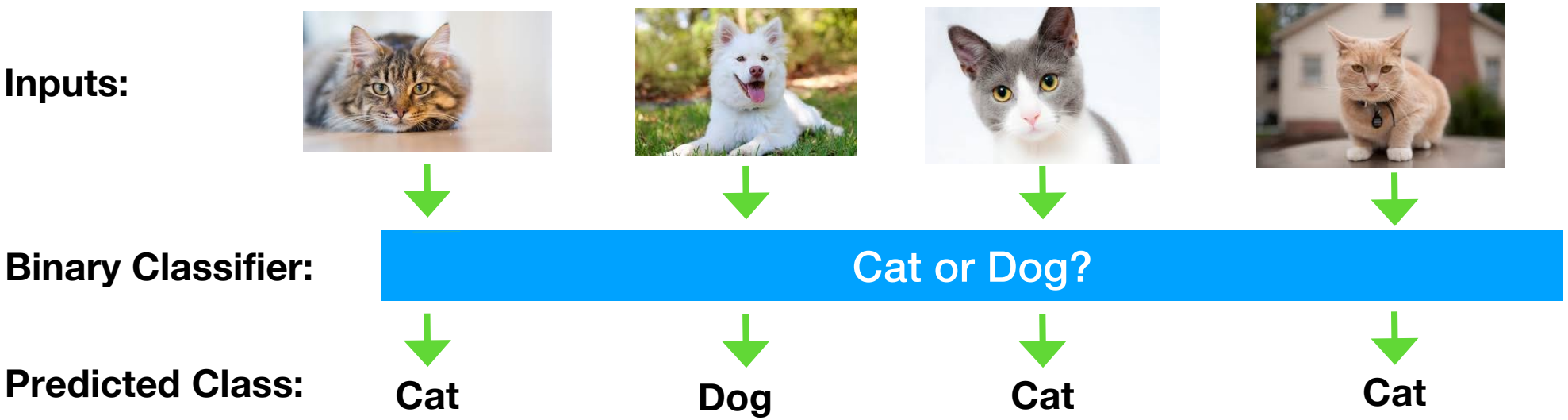
- In the last two sessions, we considered the task of ***regression***
- Given examples, predict a number
  - Age of Death
  - Price of a stock
  - Temperature of an object
  - ….
- In contrast, today, we are going to consider the task of ***classification***
  - Given examples, predict a **class**



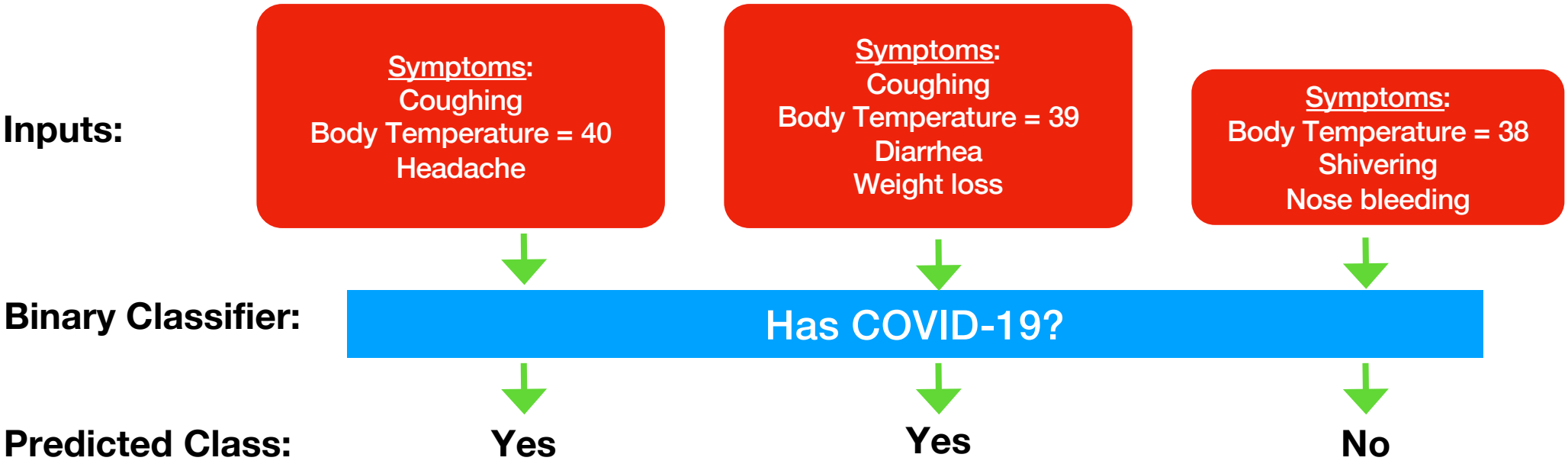
# Binary Classification (1/3)

- The simple type of classification is called ***binary classification***
- **binary** means that we have 2 classes
- A binary classifier can answer questions of the type:
  - Yes or no?
  - Cat or Dog?
  - Red or Blue?
  - Healthy or Unhealthy?

# Binary Classification (2/3)



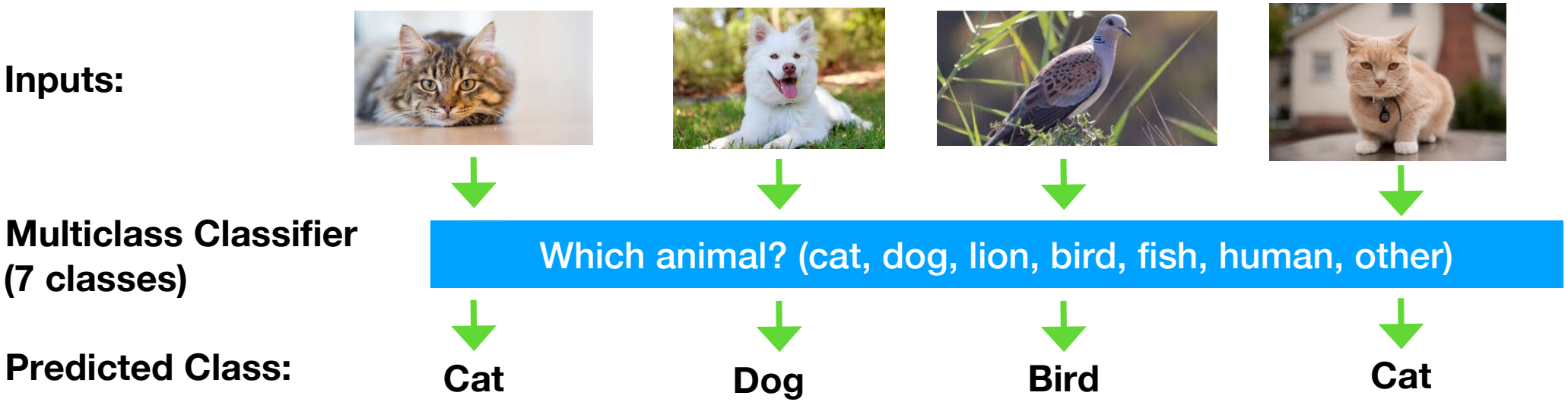
# Binary Classification (3/3)



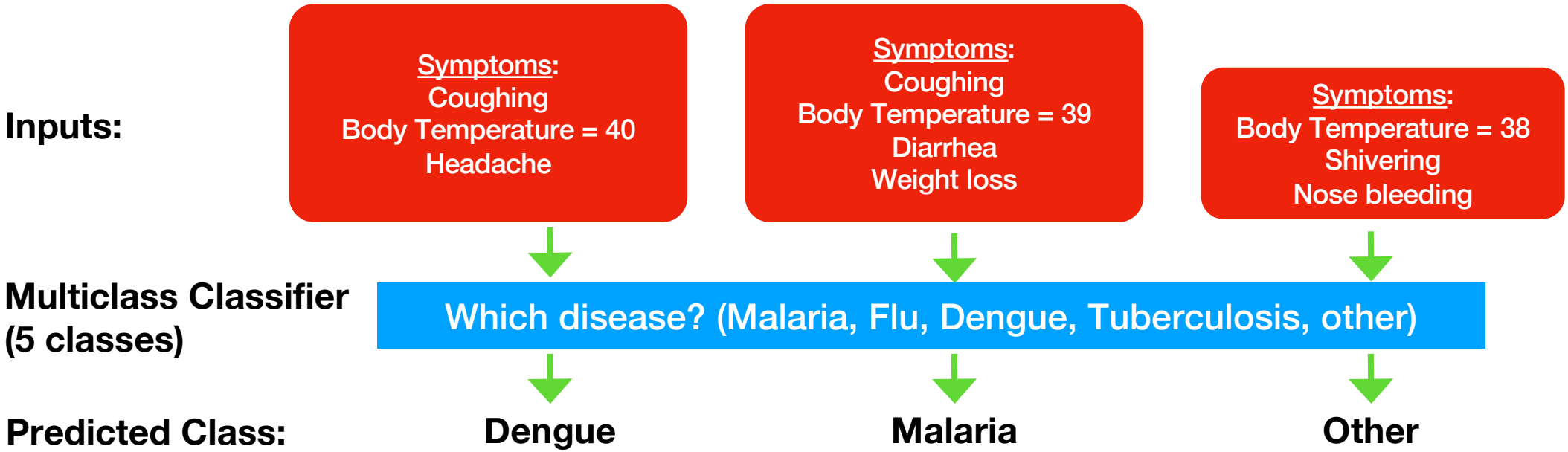
# Multiclass Classification (1/3)

- ***Multiclass classification*** is when we have more than 2 possible answers:
  - What animal is that? (cat, dog, lion, bird, fish, human, other) [7 classes]
  - Which disease is that? (Malaria, Flu, Dengue, Tuberculosis, other) [5 classes]
  - Which color is that? (red, blue, white, yellow, green, black) [6 classes]

# Multiclass Classification (2/3)

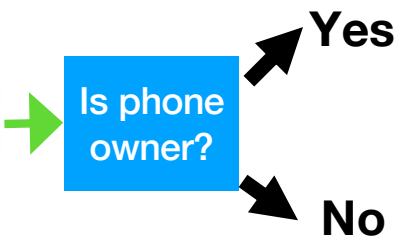


# Multiclass Classification (3/3)

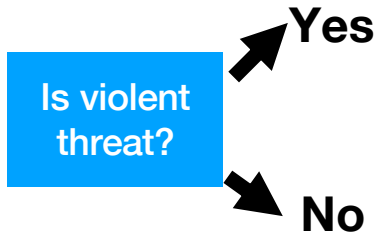


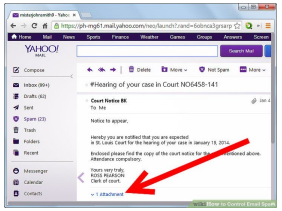
# Classification (1/2)

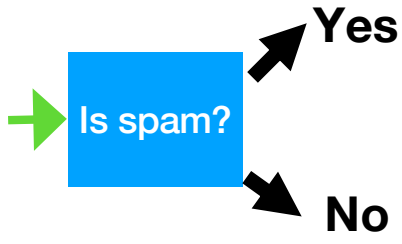
- Many interesting problems in AI can be seen as Classification problems
- Binary Classification:
  - Biometry-based identification (such as Face-id):
  - Spam detection
  - Automatic detection of use of bullying in social media
  - …


**“I am going to punch you till your head explodes”**

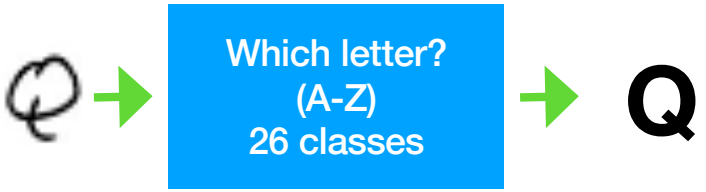


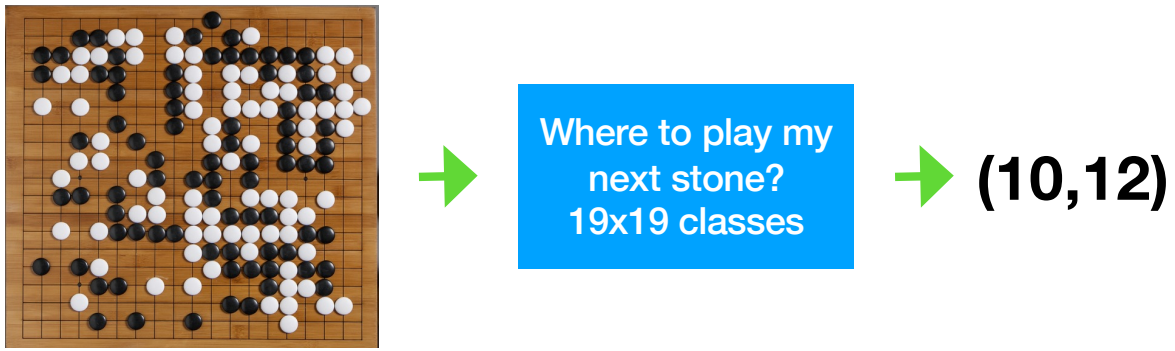





# Classification (2/2)

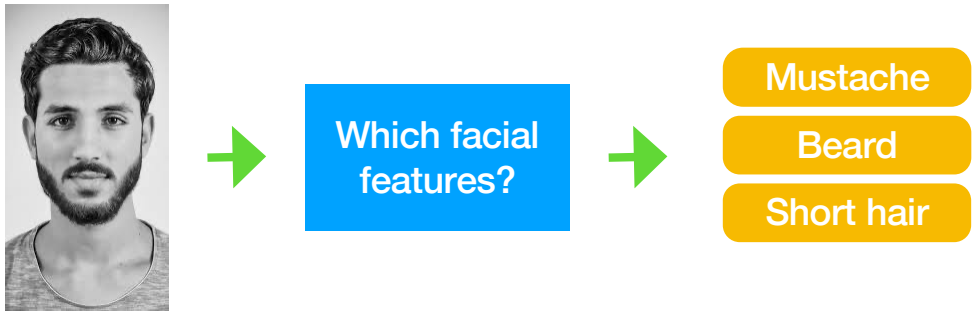
- Some interesting problems that can be seen as **multiclass** classification:
  - Optical Character Recognition (OCR)
  - AI for games
  - Tagging photos with people name
  - …..



# Multiclass vs Multilabel

- In multiclass classification, classes are **exclusives**
  - For each input there is one and only one correct class
- If we want that an input may have more than one class, this is called ***multilabel classification***
  - But we will **not** consider this case for now



# Binary Classification: Predicting Vote (1/4)

- As a practical example of **binary** classification, let us try to predict the **vote** of somebody given some features
  - For now, we only consider income as a feature (consider it expressed in 100,000$/year or 1,000万円/年 to get a concrete idea)
  - We only consider 2 parties for now: Left-wing party and Right-wing party
- We did a survey of 30 persons that accepted to answer us

| | income | vote |
|---|---|---|
| **0** | 39.0 | L |
| **1** | 30.0 | L |
| **2** | 47.0 | L |
| **3** | 69.0 | R |
| **4** | 52.0 | R |
| **5** | 110.0 | R |
| **6** | … | …. |

- Now, given the income of somebody else, we want to predict for which party he is going to vote
- Useful task to:
  - Predict the result of an election (if we know the income of each person in the country, we can predict their vote efficiently)
  - Do efficient marketing on an individual

# Binary Classification: Predicting Vote (2/4)

- Numbers are easier to use than letters in Machine Learning
- As a first step, we assign number 0 to class "*Left-Wing Party*" and number 1 to class "*Right-Wing Party*"
  - Which number you assign to which category will have no influence on prediction

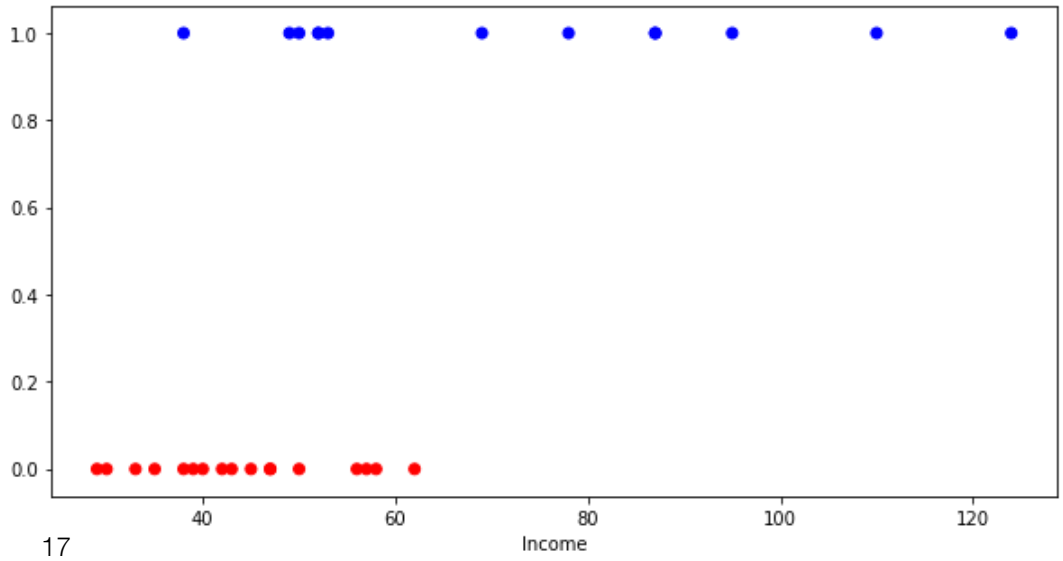
| | income | vote |
|---|---|---|
| **0** | 39.0 | L |
| **1** | 30.0 | L |
| **2** | 47.0 | L |
| **3** | 69.0 | R |
| **4** | 52.0 | R |
| **5** | 110.0 | R |
| **6** | … | …. |

| | income | vote |
|---|---|---|
| **0** | 39.0 | 0 |
| **1** | 30.0 | 0 |
| **2** | 47.0 | 0 |
| **3** | 69.0 | 1 |
| **4** | 52.0 | 1 |
| **5** | 110.0 | 1 |
| **6** | … | …. |

# Binary Classification: Predicting Vote (3/4)

- Now, our 30 persons can be visualized like this:
  - Red: Left-wing party
  - Blue: Right-wing party

| | income | vote |
|---|---|---|
| **0** | 39.0 | 0 |
| **1** | 30.0 | 0 |
| **2** | 47.0 | 0 |
| **3** | 69.0 | 1 |
| **4** | 52.0 | 1 |
| **5** | 110.0 | 1 |
| **6** | … | …. |



# Binary Classification: Predicting Vote (4/4)

- How to predict a class?
- We are going to give a **score** to the possibility that a given voter is of **class 1** (ie. vote for the *right-wing party*)
- Because it is a **binary classification**, we do not need to compute a score for the other class

| | income | vote |
|---|---|---|
| **0** | 39.0 | 0 |
| **1** | 30.0 | 0 |
| **2** | 47.0 | 0 |
| **3** | 69.0 | 1 |
| **4** | 52.0 | 1 |
| **5** | 110.0 | 1 |
| **6** | … | …. |

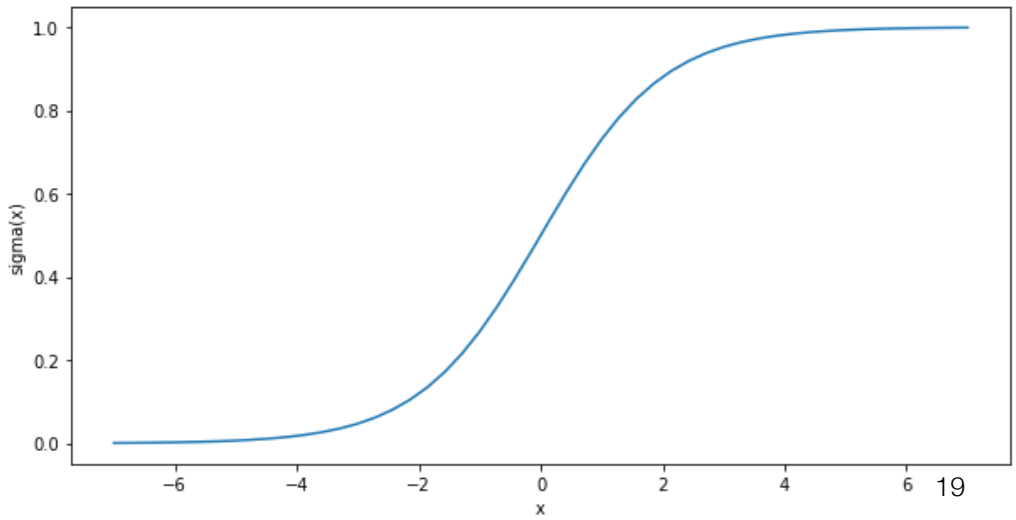
- The score could be any function of the features
- But this time, we will use a linear function again:

$$score(income) = \theta_0 + \theta_1 \times income$$

# The Logistic Function

- We will then apply the ***logistic function*** to the score
- The logistic function (also called **sigmoid/*sigma function***) is defined by:

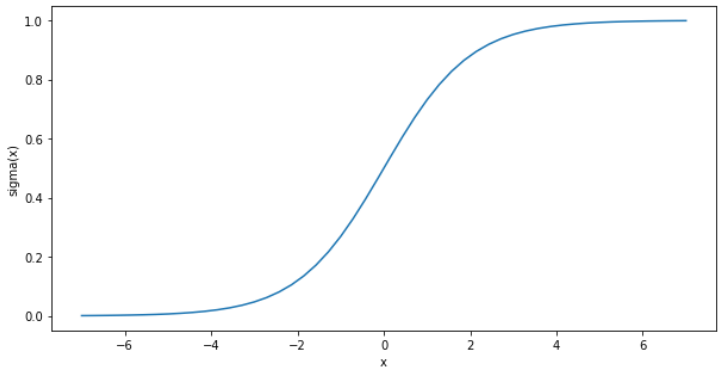
$$\sigma(x) = \frac{1}{1 + exp(-x)}$$




- If *score* is very large, σ(*score*) will be close to 1
- If *score = 0 then* σ(*score*) = 0.5
- If *score* is very negative, σ(*score*) will be close to 0

# Binary Classification: Predicting Vote

| | income | vote |
|---|---|---|
| **0** | 39.0 | 0 |
| **1** | 30.0 | 0 |
| **2** | 47.0 | 0 |
| **3** | 69.0 | 1 |
| **4** | 52.0 | 1 |
| **5** | 110.0 | 1 |
| **6** | … | …. |




- Therefore, our classification model is like this:

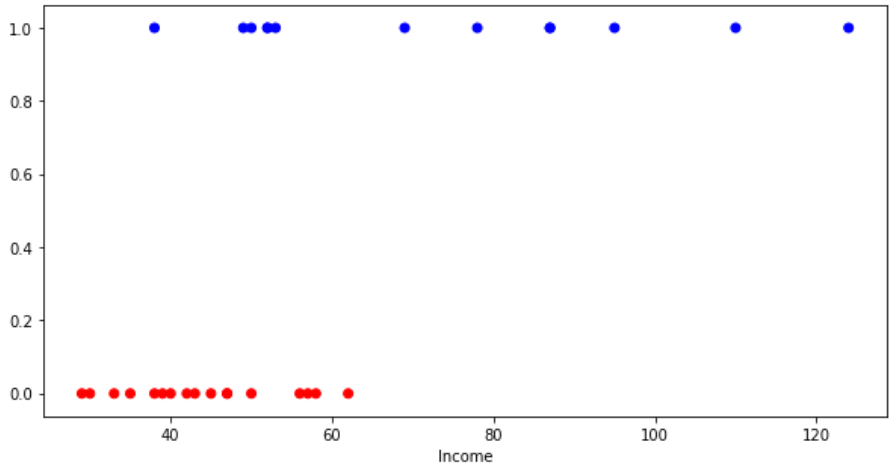
$$score(income) = \theta_0 + \theta_1 \times income$$

$$V_{model} = \sigma(score)$$

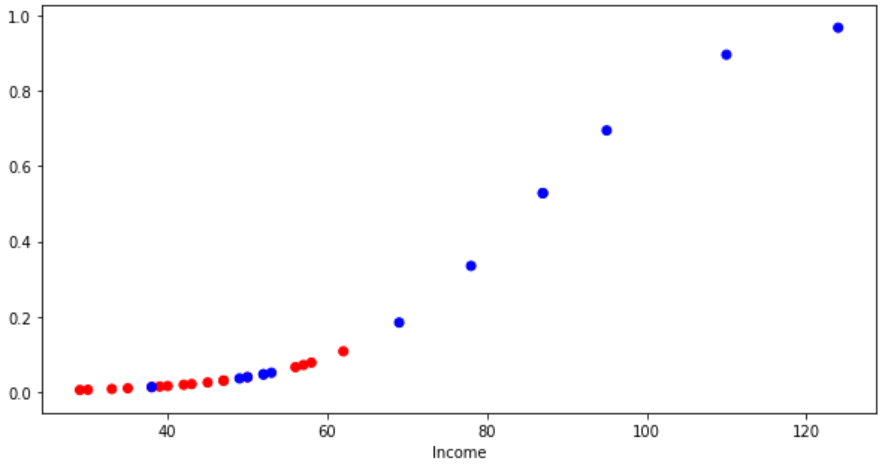
- In practice, the predicted *V* will not be equal to 0 or 1
- The predicted *V* is a value between 0 and 1
- We interpret *V* as the probability that a given voter will vote for the right wing party
- We can also interpret *V* as the confidence of the model that a given voter will vote for the right wing party

# Difference between Prediction and Data (1/2)

- Our examples represent binary values
- Either **0** or **1**

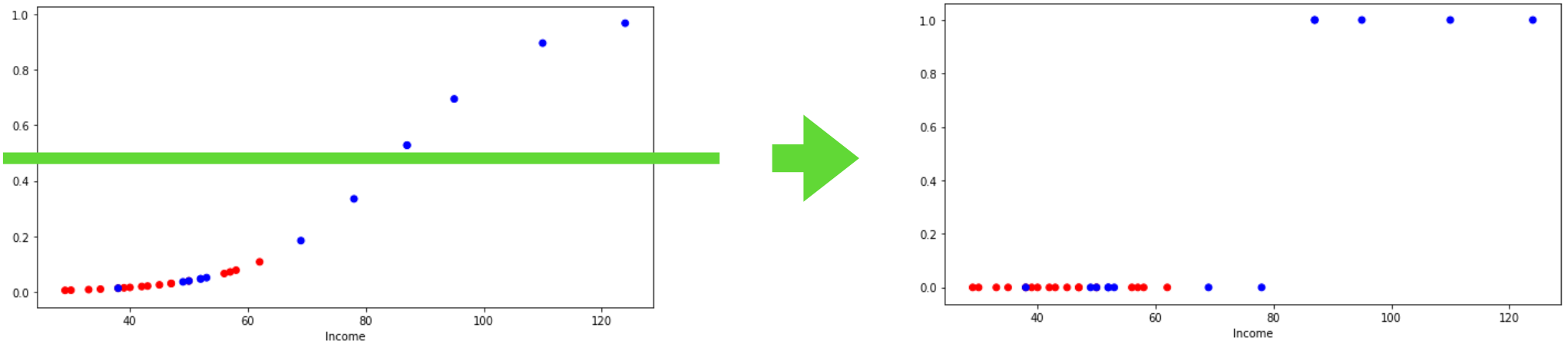



- Our predictions are probabilities between 0 or 1
- We want to predict the data: either 0 or 1



# Difference between Prediction and Data (2/2)

- Our predictions are probabilities between 0 or 1
- We want to predict the data: either 0 or 1
- -> We choose a decision boundary
  - (usually we check if predicted value is higher or lower than 0.5)



# Our Model Predictions

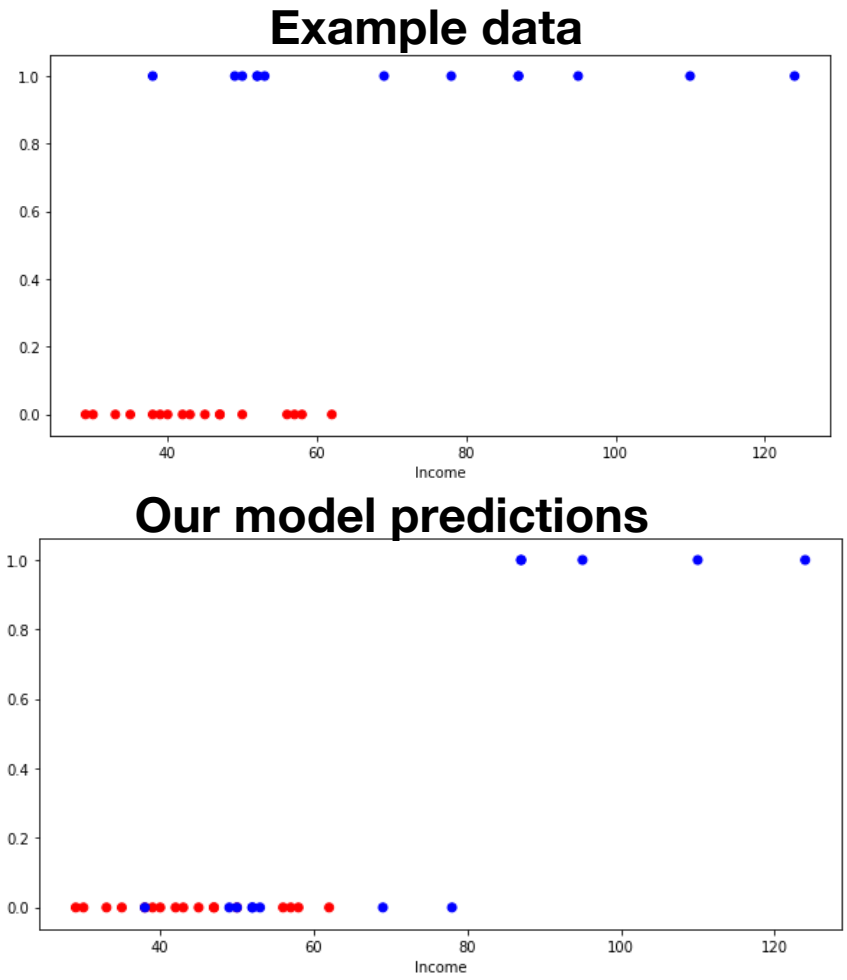
- Finally, we now have predictions from our model

$$score(income) = \theta_0 + \theta_1 \times income$$

$$V_{model} = \sigma(score)$$

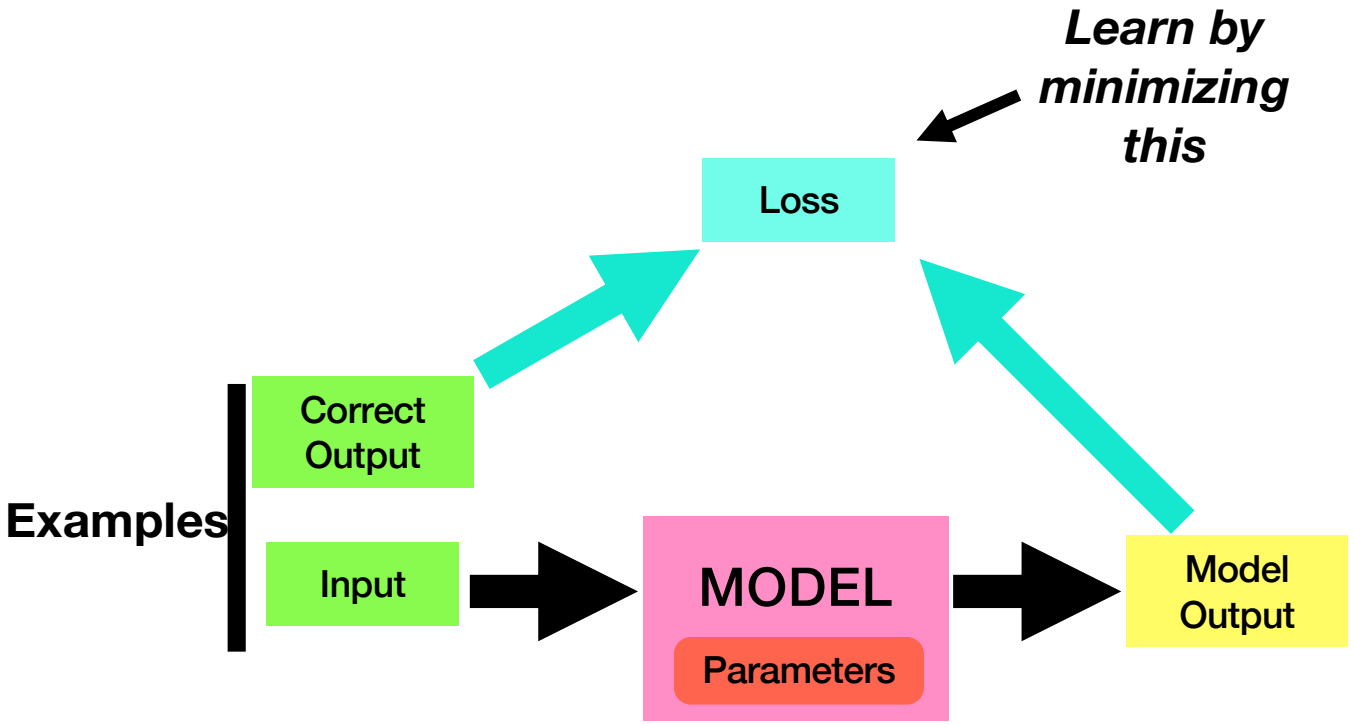
**Class = 1 if V >0.5 else 0**

- Remaining question: how to find the parameters θ ?



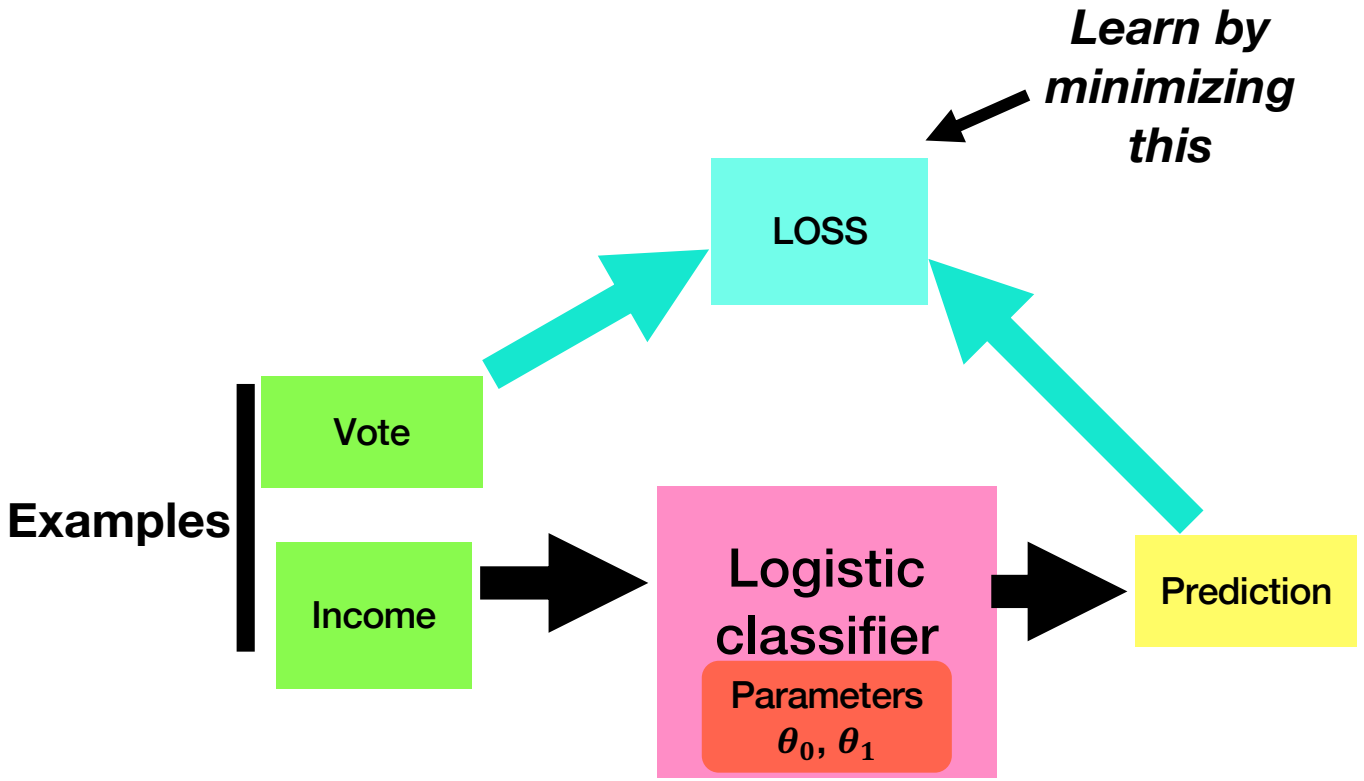
# Supervised Learning (1/3)

- In supervised learning, we usually have:
  - A **MODEL**: a "parameterized" function that takes input and produces output
  - A ***Loss***: A function that computes how different the model output is from the correct output
  - ***Examples*** of input and correct output



# Supervised Learning (2/3)

- In supervised learning, we usually have:
  - A **MODEL**: a "parameterized" function that takes input and produces output
  - A ***Loss***: A function that computes how different the model output is from the correct output
  - ***Examples*** of input and correct output (cigarettes smoked, age of death)

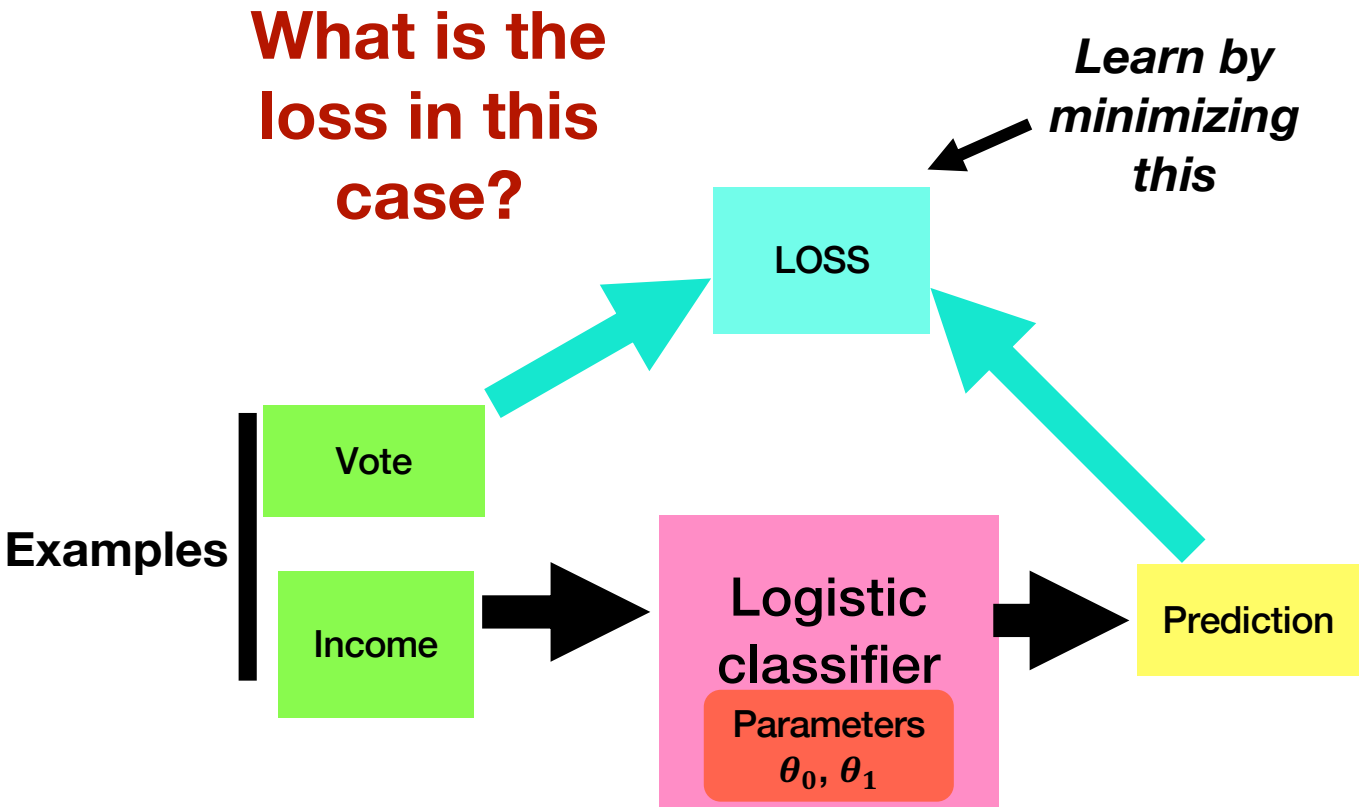



$$score(income) = \theta_0 + \theta_1 \times income$$

$$V_{model} = \sigma(score)$$

# Supervised Learning (3/3)

- In supervised learning, we usually have:
  - A **MODEL**: a "parameterized" function that takes input and produces output
  - A ***Loss***: A function that computes how different the model output is from the correct output
  - ***Examples*** of input and correct output (cigarettes smoked, age of death)




$$score(income) = \theta_0 + \theta_1 \times income$$

$$V_{model} = \sigma(score)$$

# What is the Loss We Should Use? (1/4)

- We need a loss that tells us how bad are the predictions given the examples
- One possibility: we count how many predictions were wrong

**7 right-wing voters were predicted to vote left**

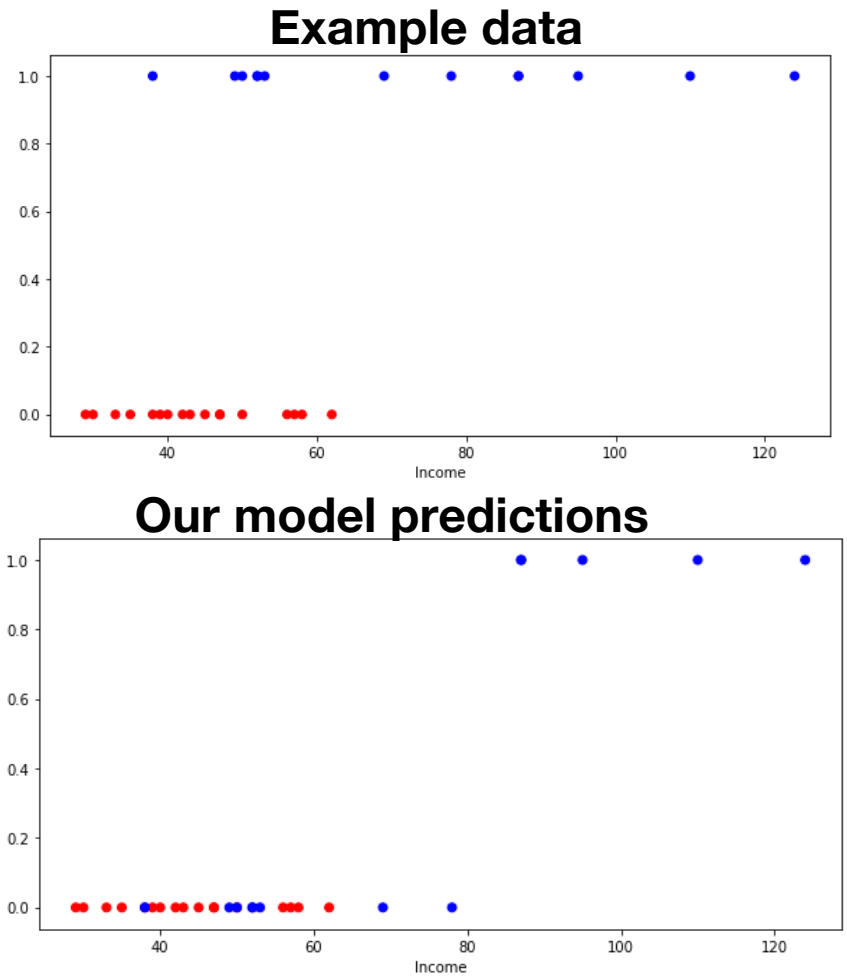
**0 left-wing voters were predicted to vote right**

**23 out of 30 predictions are correct**

**Accuracy: 23/30**

**Loss: 7/30**

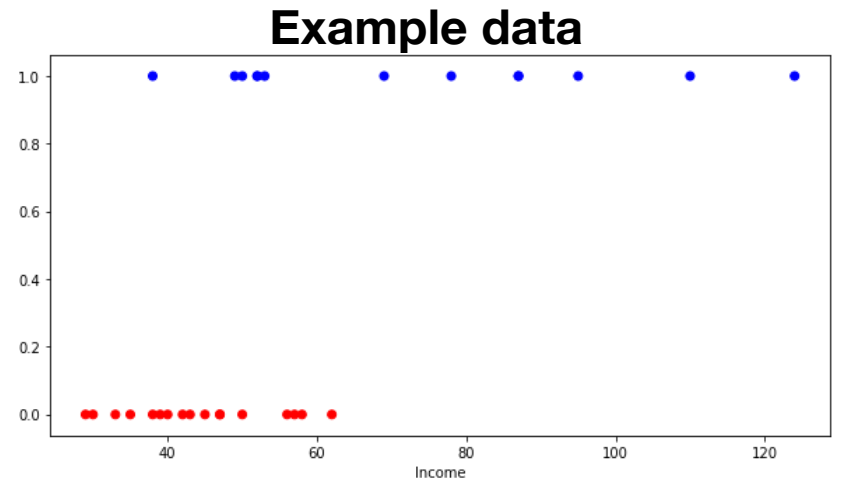
- Intuitively, it seems a good way to evaluate the model
- Unfortunately, it is not a good loss for learning parameters

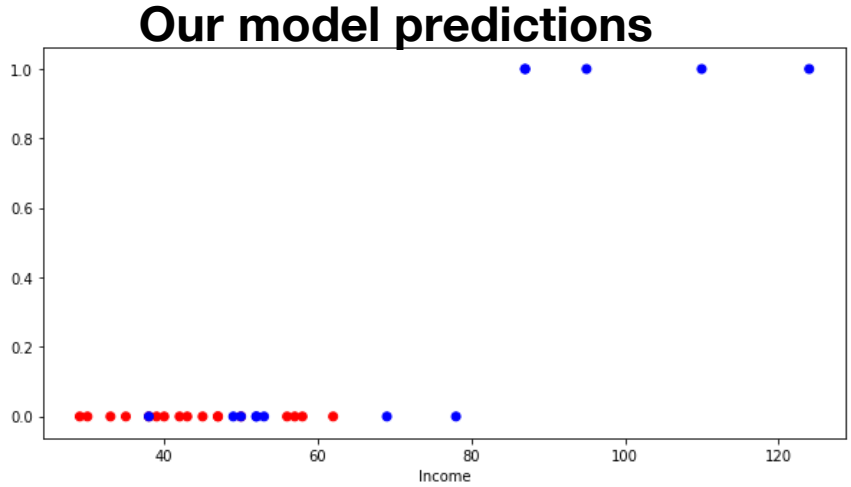


# What is the Loss We Should Use? (2/4)

- We need a loss that tells us how bad are the predictions given the examples
- One possibility: we count how many predictions were wrong

- Intuitively, it seems a good way to evaluate the model
- Unfortunately, it is not a good loss for learning parameters
- Because this loss is not a continuous function of the parameters θ
- We cannot compute a gradient -> we cannot use gradient descent



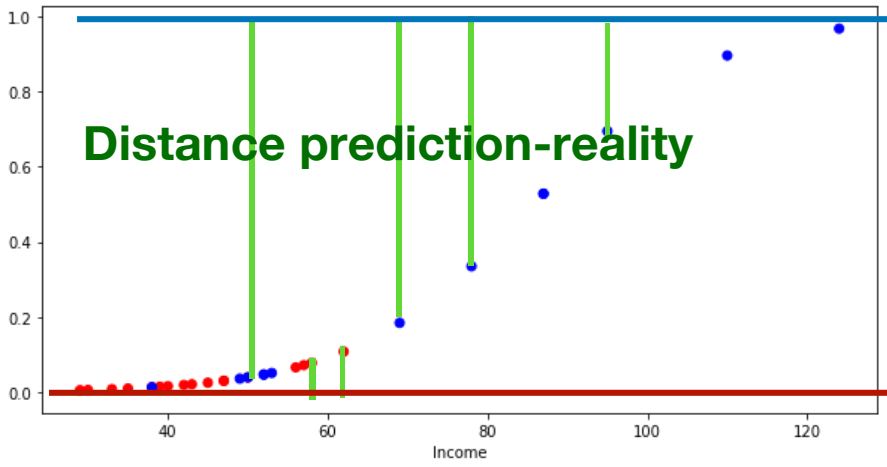
# What is the Loss We Should Use? (3/4)

- How about the ***Mean Squared Error***? If we use the predicted probability instead of the predicted class, then the same situation as with regression: we compute the distance between the prediction and the real value
- In theory it works; **in practice, it does not work**

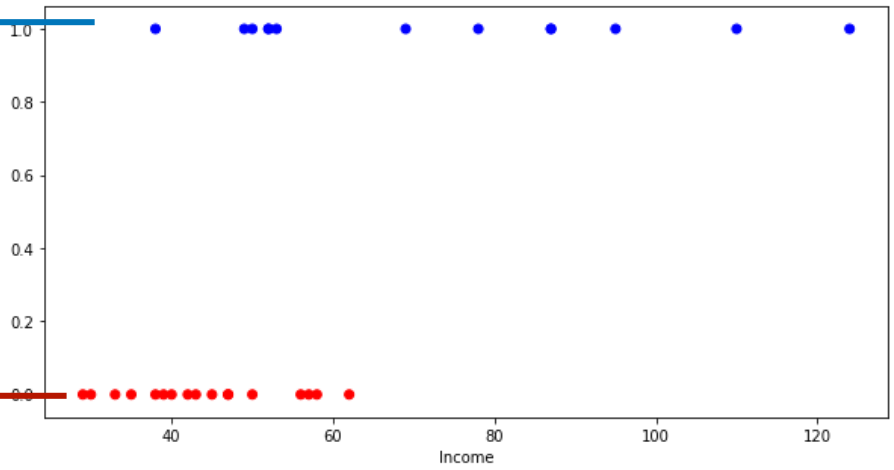
$$V_{model} = \sigma(score)$$

- The Mean Squared Error loss of a logistic classifier is very difficult to minimize in practice (for technical reasons we will not discuss here)

**Model's Prediction**

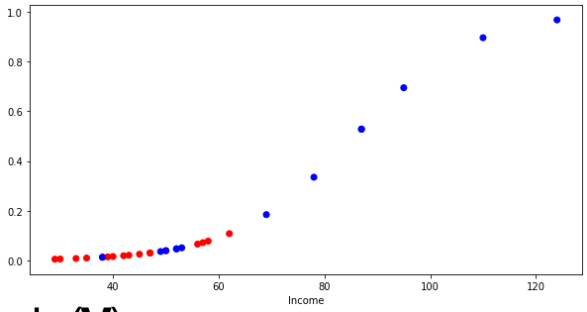



**Example data**



# What is the Loss We Should Use? (4/4)

$$V_{model} = \sigma(score)$$

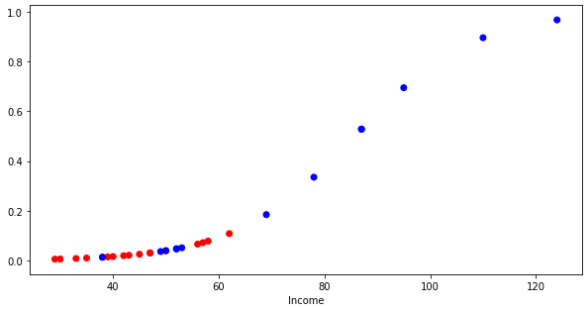



- We will use something called the ***cross-entropy loss***
- It is defined as follow
- For each example:
  - If the correct class is **1**, then the ***cost*** of the prediction **V** is -ln(**V**)
  - If the correct class is **0**, then the ***cost*** of the prediction **V** is -ln(1-**V**)
- Then we average the cost over each example

$$CrossEntropyLoss = \frac{1}{N}\sum_{i=1}^{N} cost_i$$

# The Cross Entropy Loss (1/6)

$$V_{model} = \sigma(score)$$




- For each example:
  - If the correct class is **1**, then the ***cost*** of the prediction **V** is -ln(**V**)
  - If the correct class is **0**, then the ***cost*** of the prediction **V** is -ln(1-**V**)
- Then we average the cost over each example

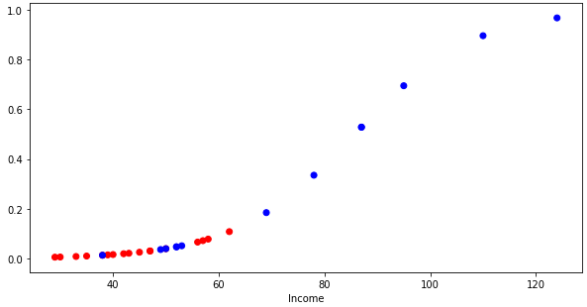
$$CrossEntropyLoss = \frac{1}{N}\sum_{i=1}^{N} cost_i$$

- Why it might work?

# The Cross Entropy Loss (2/6)

- For each example:
  - If the correct class is **1**, then the ***cost*** of the prediction **V** is -ln(**V**)
  - If the correct class is **0**, then the ***cost*** of the prediction **V** is -ln(1-**V**)
- Then we average the cost over each example

$$V_{model} = \sigma(score)$$

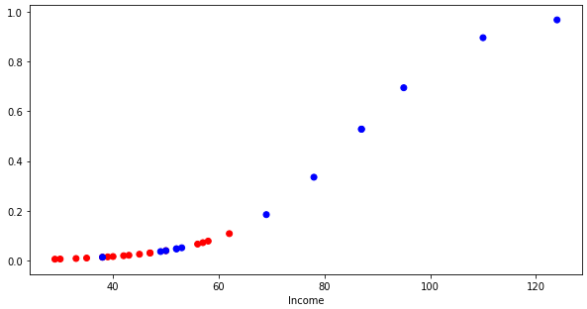



- If, for a given person, V=1: the model says that this person will certainly vote for the right wing party
  - If the given person actually voted for the right-wing party (correct class is 1), then the cost is -ln(1) = 0
  - If the given person actually voted for the left-wing party (correct class is 0), then the cost is -ln(1-1) = ∞

# The Cross Entropy Loss (3/6)

- For each example:
  - If the correct class is **1**, then the ***cost*** of the prediction **V** is -ln(**V**)
  - If the correct class is **0**, then the ***cost*** of the prediction **V** is -ln(1-**V**)
- Then we average the cost over each example

$$V_{model} = \sigma(score)$$

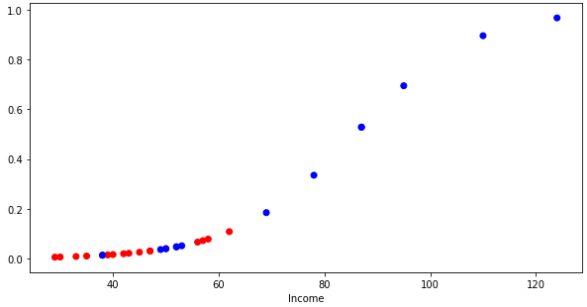



- If, for a given person, V=0: the model says that this person will certainly vote for the left wing party
  - If the given person actually voted for the right-wing party (correct class is 1), then the cost is -ln(0) = ∞
  - If the given person actually voted for the left-wing party (correct class is 0), then the cost is -ln(1-0) = 0

# The Cross Entropy Loss (4/6)

- For each example:
  - If the correct class is **1**, then the ***cost*** of the prediction **V** is -ln(**V**)
  - If the correct class is **0**, then the ***cost*** of the prediction **V** is -ln(1-**V**)
- Then we average the cost over each example

$$V_{model} = \sigma(score)$$




- If, for a given person, V=0.5: the model says that there is an equal probability that the voter will vote for any party
  - If the given person actually voted for the right-wing party (correct class is 1), then the cost is -ln(0.5) = 0.69
  - If the given person actually voted for the left-wing party (correct class is 0), then the cost is -ln(1-0.5) = 0.69

# The Cross Entropy Loss (5/6)

- In short, the cost will be zero only if the model made a **very confident** correct prediction
- The cost will be very large if the model made a **very confident** incorrect prediction
- If the model made prediction with little confidence (V is close to 0.5), the cost will be moderate whatever the correct answer was
- If the model always make correct predictions with high confidence, the average of the cost will be close to zero

$$CrossEntropyLoss = \frac{1}{N}\sum_{i=1}^{N} cost_i$$

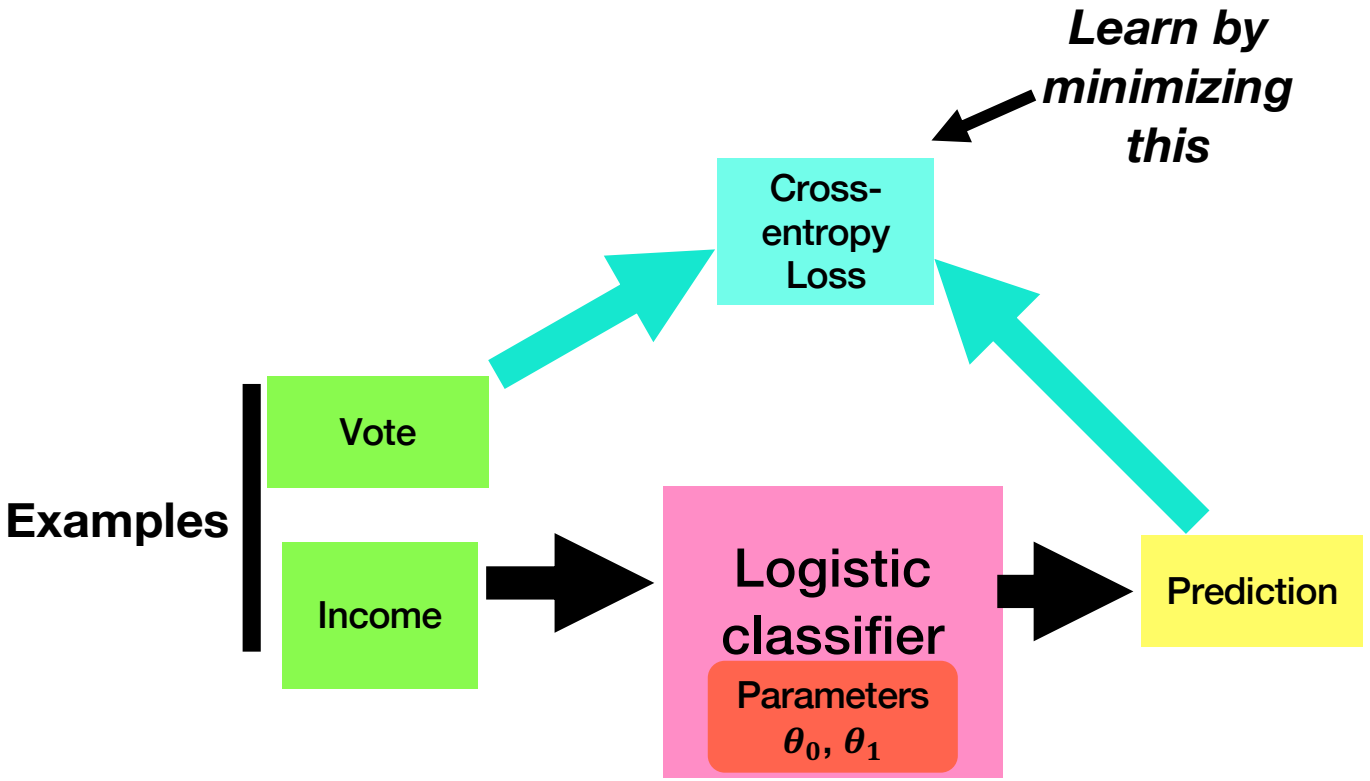
# The Cross Entropy Loss (6/6)

- From this it follows that the Cross Entropy Loss is a good loss for our setting
- If we minimize it, we will find a good model
- It is a continuous (and smooth) function of the parameters
- It is easy to minimize in practice

$$CrossEntropyLoss = \frac{1}{N}\sum_{i=1}^{N} cost_i$$

# Supervised Learning

- In supervised learning, we usually have:
  - A **MODEL**: a "parameterized" function that takes input and produce output
  - A ***Loss***: A function that compute how different the model output is from the correct output
  - ***Examples*** of input and correct output (cigarets smoked, age of death)




$$score(income) = \theta_0 + \theta_1 \times income$$

$$V_{model} = \sigma(score)$$

# Minimizing the Cross-entropy Loss

- As before, to minimize the loss, we need to compute its gradient with respect to the parameters

$$score(income) = \theta_0 + \theta_1 \times income$$

$$V_{model} = \sigma(score)$$

$$CrossEntropyLoss = \frac{1}{N}\sum_{i=1}^{N} cost_i$$

**We can find that:**

$$\frac{\partial}{\partial \theta_0} CrossEntropyLoss = \frac{1}{N}\sum_{i=1}^{N} (V_{model}(income_i) - V_i)$$

$$\frac{\partial}{\partial \theta_1} CrossEntropyLoss = \frac{1}{N}\sum_{i=1}^{N} (V_{model}(income_i) - V_i) \times income_i$$

# Minimizing the Cross-entropy Loss

- As before, to minimize the loss, we need to compute its gradient with respect to the parameters

$$score(income) = \theta_0 + \theta_1 \times income$$

$$V_{model} = \sigma(score)$$

$$CrossEntropyLoss = \frac{1}{N}\sum_{i=1}^{N} cost_i$$

**We can find that:**

$$\frac{\partial}{\partial \theta_0} CrossEntropyLoss = \frac{1}{N}\sum_{i=1}^{N} error_i$$

$$\frac{\partial}{\partial \theta_1} CrossEntropyLoss = \frac{1}{N}\sum_{i=1}^{N} error_i \times income_i$$

**With:**

$$error_i = V_{model}(income_i) - V_i$$

$$grad \cdot CEL = [\frac{\partial}{\partial \theta_0} CEL, \frac{\partial}{\partial \theta_1} CEL] = [\frac{1}{N}\sum_{i=1}^{N} error_i, \frac{1}{N}\sum_{i=1}^{N} error_i \times income_i]$$

**Our data**

| | income | vote |
|---|---|---|
| **0** | 39.0 | 0 |
| **1** | 30.0 | 0 |
| **2** | 47.0 | 0 |
| **3** | 69.0 | 1 |
| **4** | 52.0 | 1 |
| **5** | 110.0 | 1 |
| **6** | … | …. |

# Minimizing the Cross-entropy Loss

- Now that we know the gradient of the loss, we can minimize the loss to find the best parameters $\theta$
- To make things a bit less abstract, let's try to do it in practice on a small example

# Example (1/5)

**Example Data**

| | income | vote |
|---|---|---|
| **1** | 40 | 0 |
| **2** | 70 | 1 |
| **3** | 20 | 0 |

- Suppose these parameters: $\theta_0 = -8$ $\theta_1 = 0.1$

$$score(income) = \theta_0 + \theta_1 \times income$$

$$V_{model} = \sigma(score) \qquad \sigma(x) = \frac{1}{1 + exp(-x)}$$

| | income | vote | Score | Prediction V | Predicted Class | Cost |
|---|---|---|---|---|---|---|
| **1** | 40 | 0 | | | | |
| **2** | 70 | 1 | | | | |
| **3** | 20 | 0 | | | | |

# Example (2/5)

**Example Data**

| | income | vote |
|---|---|---|
| **1** | 40 | 0 |
| **2** | 70 | 1 |
| **3** | 20 | 0 |

- Suppose these parameters: $\theta_0 = -8$ $\theta_1 = 0.1$

$$score(income) = \theta_0 + \theta_1 \times income$$

$$V_{model} = \sigma(score) \qquad \sigma(x) = \frac{1}{1 + exp(-x)}$$

| | income | vote | Score | Prediction V | Predicted Class | Cost |
|---|---|---|---|---|---|---|
| **1** | 40 | 0 | -4 | 0.018 | 0 | |
| **2** | 70 | 1 | -1 | 0.269 | 0 | |
| **3** | 20 | 0 | -6 | 0.002 | 0 | |

# Example (3/5)

**Example Data**

| | income | vote |
|---|---|---|
| **1** | 40 | 0 |
| **2** | 70 | 1 |
| **3** | 20 | 0 |

- Suppose these parameters: $\theta_0 = -8$ $\theta_1 = 0.1$

$$score(income) = \theta_0 + \theta_1 \times income$$

$$V_{model} = \sigma(score) \qquad \sigma(x) = \frac{1}{1 + exp(-x)}$$

| | income | vote | Score | Prediction V | Predicted Class | Cost |
|---|---|---|---|---|---|---|
| **1** | 40 | 0 | -4 | 0.018 | 0 | |
| **2** | 70 | 1 | -1 | 0.269 | 0 | |
| **3** | 20 | 0 | -6 | 0.002 | 0 | |

- if the correct class is 1, then the *cost* of the prediction V is -ln(V)
- If the correct class is 0, then the *cost* of the prediction V is -ln(1-V)

$$CrossEntropyLoss = \frac{1}{N} \sum_{i=1}^{N} cost_i$$

# Example (4/5)

**Example Data**

| | income | vote |
|---|---|---|
| **1** | 40 | 0 |
| **2** | 70 | 1 |
| **3** | 20 | 0 |

- Suppose these parameters: $\theta_0 = -8$ $\theta_1 = 0.1$

$$score(income) = \theta_0 + \theta_1 \times income$$

$$V_{model} = \sigma(score) \qquad \sigma(x) = \frac{1}{1 + exp(-x)}$$

| | income | vote | Score | Prediction V | Predicted Class | Cost |
|---|---|---|---|---|---|---|
| **1** | 40 | 0 | -4 | 0.018 | 0 | 0.018 |
| **2** | 70 | 1 | -1 | 0.269 | 0 | 1.310 |
| **3** | 20 | 0 | -6 | 0.002 | 0 | 0.002 |

- if the correct class is 1, then the *cost* of the prediction V is -ln(V)
- If the correct class is 0, then the *cost* of the prediction V is -ln(1-V)

$$CrossEntropy = \frac{1}{3}(0.018 + 1.31 + 0.002) = 0.44$$

# Example (5/5)

**Example Data**

| | income | vote |
|---|---|---|
| **1** | 40 | 0 |
| **2** | 70 | 1 |
| **3** | 20 | 0 |

$\theta_0 = -8 \qquad \theta_1 = 0.1$

- Now suppose we want to improve our parameters
- Let us do one iteration of gradient descent

| | income | vote | Score | Prediction V | Predicted Class | Cost |
|---|---|---|---|---|---|---|
| **1** | 40 | 0 | -4 | 0.018 | 0 | 0.018 |
| **2** | 70 | 1 | -1 | 0.269 | 0 | 1.310 |
| **3** | 20 | 0 | -6 | 0.002 | 0 | 0.002 |

$$CrossEntropy = \frac{1}{3}(0.018 + 1.31 + 0.002) = 0.44$$

# Computing the Gradient (1/2)

$$\theta_0 = -8 \quad \theta_1 = 0.1$$

| | income | vote | Score | Prediction V | Predicted Class | Cost | Error |
|---|---|---|---|---|---|---|---|
| **1** | 40 | 0 | -4 | 0.018 | 0 | 0.018 | |
| **2** | 70 | 1 | -1 | 0.269 | 0 | 1.31 | |
| **3** | 20 | 0 | -6 | 0.002 | 0 | 0.002 | |

$$error_i = V_{model}(income_i) - V_i$$

$$grad \cdot CEL = [\frac{\partial}{\partial \theta_0} CEL, \frac{\partial}{\partial \theta_1} CEL] = [\frac{1}{N} \sum_{i=1}^{N} error_i, \frac{1}{N} \sum_{i=1}^{N} error_i \times income_i]$$

# Computing the Gradient (2/2)

$$\theta_0 = -8 \quad \theta_1 = 0.1$$

| | income | vote | Score | Prediction V | Predicted Class | Cost | Error |
|---|---|---|---|---|---|---|---|
| **1** | 40 | 0 | -4 | 0.018 | 0 | 0.018 | 0.018 |
| **2** | 70 | 1 | -1 | 0.269 | 0 | 1.310 | -0.731 |
| **3** | 20 | 0 | -6 | 0.002 | 0 | 0.002 | 0.002 |

$$error_i = V_{model}(income_i) - V_i$$

$$grad \cdot CEL = [\frac{1}{N}\sum_{i=1}^{N} error_i, \frac{1}{N}\sum_{i=1}^{N} error_i \times income_i]$$

$$= \frac{1}{3}[0.018 - 0.731 + 0.002, 0.018 \times 40 - 0.731 \times 70 + 0.002 \times 20] = [-0.23, -16.8]$$

# Doing One Iteration of Gradient Descent (1/2)

| | income | vote | Score | Prediction V | Predicted Class | Cost | Error |
|---|---|---|---|---|---|---|---|
| **1** | 40 | 0 | -4 | 0.018 | 0 | 0.018 | 0.018 |
| **2** | 70 | 1 | -1 | 0.269 | 0 | 1.31 | -0.731 |
| **3** | 20 | 0 | -6 | 0.002 | 0 | 0.002 | 0.002 |

$\theta_0 = -8 \quad \theta_1 = 0.1$

$grad \cdot CrossEntropy = [-0.23, -16.8]$

- Let us do one iteration of gradient descent with lr = 0.005
- What are new parameters $\theta$?

# Doing One Iteration of Gradient Descent (2/2)

| | income | vote | Score | Prediction V | Predicted Class | Cost | Error |
|---|---|---|---|---|---|---|---|
| **1** | 40 | 0 | -4 | 0.018 | 0 | 0.018 | 0.018 |
| **2** | 70 | 1 | -1 | 0.269 | 0 | 1.31 | -0.731 |
| **3** | 20 | 0 | -6 | 0.002 | 0 | 0.002 | 0.002 |

$\theta_0 = -8 \quad \theta_1 = 0.1$

$grad \cdot CrossEntropy = [-0.23, -16.8]$

- Let us do one iteration of gradient descent with lr = 0.005
- What are new parameters $\theta$?

$\theta_0 = -8 - 0.005 \times -0.23 = -7.999 \qquad \theta_1 = 0.1 - 0.005 \times -16.8 = 0.184$

# Trying the New Parameters

**Example Data**

| | income | vote |
|---|---|---|
| **1** | 40 | 0 |
| **2** | 70 | 1 |
| **3** | 20 | 0 |

- Let us try these new parameters

$$\theta_0 = -7.999$$

$$\theta_1 = 0.184$$

$$score(income) = \theta_0 + \theta_1 \times income$$

$$V_{model} = \sigma(score) \qquad \sigma(x) = \frac{1}{1 + exp(-x)}$$

| | income | vote | Score | Prediction V | Predicted Class | Cost |
|---|---|---|---|---|---|---|
| **1** | 40 | 0 | | | | |
| **2** | 70 | 1 | | | | |
| **3** | 20 | 0 | | | | |

# Exercise 1: Compute the New Loss by Filling the Table as Before

**Example Data**

| | income | vote |
|---|---|---|
| **1** | 40 | 0 |
| **2** | 70 | 1 |
| **3** | 20 | 0 |

- Let us try these new parameters

$$\theta_0 = -7.999$$

$$\theta_1 = 0.184$$

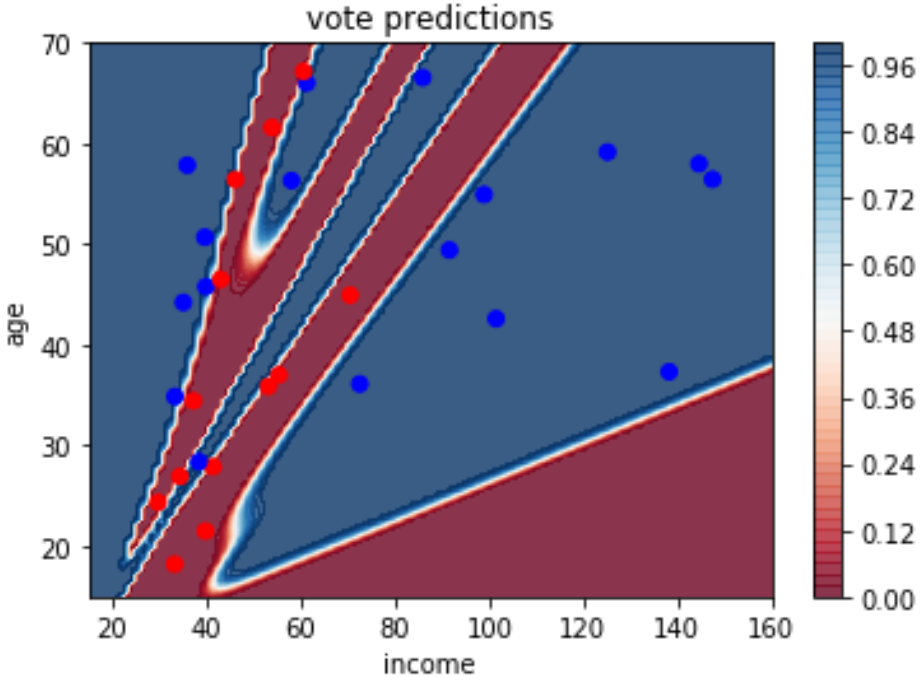
$$score(income) = \theta_0 + \theta_1 \times income$$

$$V_{model} = \sigma(score) \qquad \sigma(x) = \frac{1}{1 + exp(-x)}$$

| | income | vote | Score | Prediction V | Predicted Class | Cost |
|---|---|---|---|---|---|---|
| **1** | 40 | 0 | | | | |
| **2** | 70 | 1 | | | | |
| **3** | 20 | 0 | | | | |

Beyond binary classification with one feature:

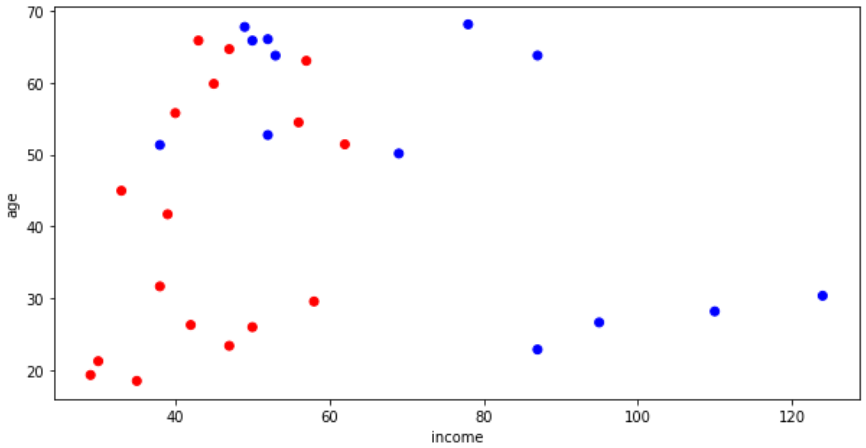
# Many features, complex models



# Classification with More Than One Feature (1/3)

- Let us now suppose that, on top of **income**, we have information about **age**
- How do we change our Logistic model to take into account the new information?

| | income | age | vote |
|---|---|---|---|
| **0** | 39.0 | 42.0 | L |
| **1** | 30.0 | 21.0 | L |
| **2** | 47.0 | 65.0 | L |
| **3** | 69.0 | 50.0 | R |
| **4** | 52.0 | 53.0 | R |
| **5** | 110.0 | 28.0 | R |
| **…** | … | … | … |



# Classification with More Than One Feature (2/3)

- Let us now suppose that, on top of **income**, we have information about **age**
- How do we change our Logistic model to take into account the new information?

| | income | age | vote |
|---|---|---|---|
| **0** | 39.0 | 42.0 | L |
| **1** | 30.0 | 21.0 | L |
| **2** | 47.0 | 65.0 | L |
| **3** | 69.0 | 50.0 | R |
| **4** | 52.0 | 53.0 | R |
| **5** | 110.0 | 28.0 | R |
| **…** | … | … | … |

**EASY! We just add a new parameter to the score function:**

$$score(income) = \theta_0 + \theta_1 \times income$$

$$V_{model} = \sigma(score)$$

$$score(income, age) = \theta_0 + \theta_1 \times income + \theta_2 \times age$$

$$V_{model} = \sigma(score)$$

# Classification with More Than One Feature (3/3)

- Let us now suppose that, on top of **income**, we have information about **age**
- How do we change our Logistic model to take into account the new information?

| | income | age | vote |
|---|---|---|---|
| **0** | 39.0 | 42.0 | L |
| **1** | 30.0 | 21.0 | L |
| **2** | 47.0 | 65.0 | L |
| **3** | 69.0 | 50.0 | R |
| **4** | 52.0 | 53.0 | R |
| **5** | 110.0 | 28.0 | R |
| **…** | … | … | … |

**EASY! We just add a new parameter to the score function:**

$$score(income) = \theta_0 + \theta_1 \times income$$

$$V_{model} = \sigma(score)$$

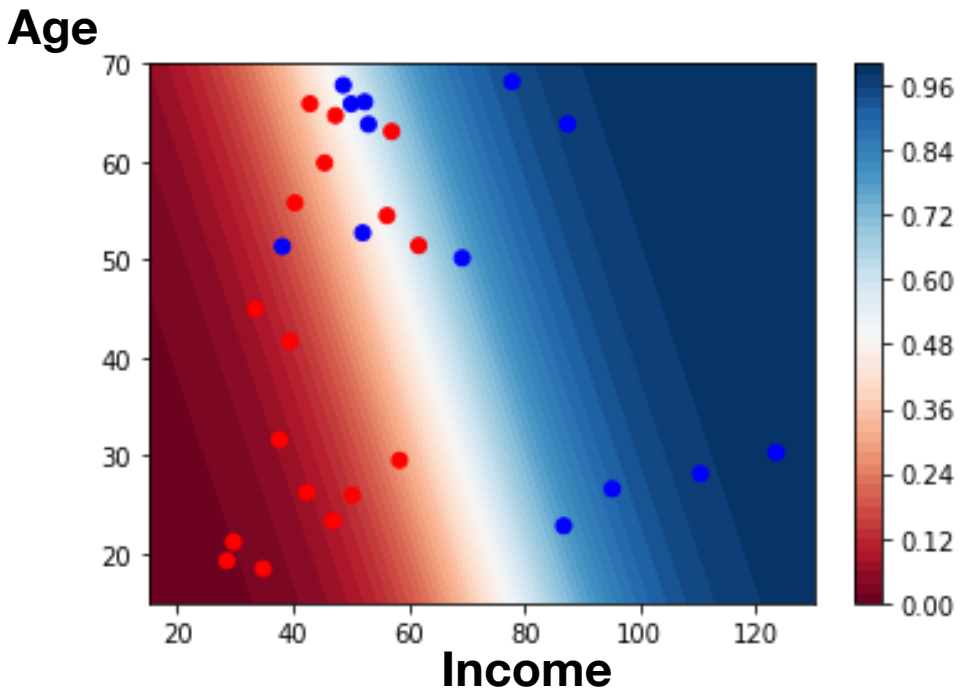
$$score(income, age) = \theta_0 + \theta_1 \times income + \theta_2 \times age$$

$$V_{model} = \sigma(score)$$

**Nothing else to change:**
**Everything we have seen about cost, loss, gradient descent work the same**

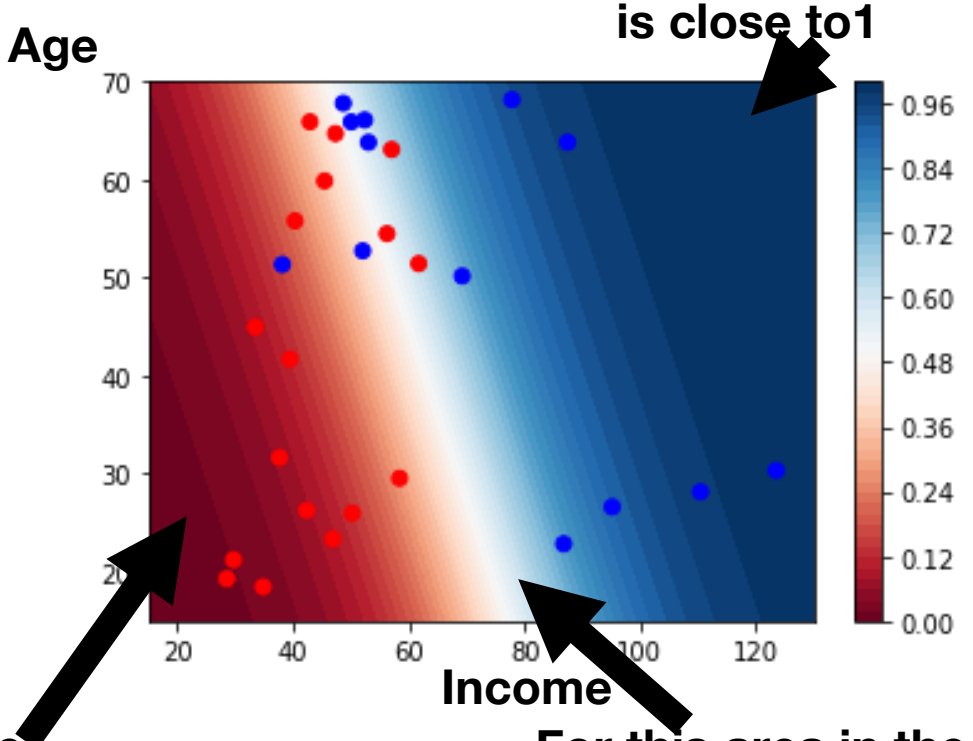
# Visualizing the Decision Boundary (1/2)

- If we represent the value outputted by the classifier for all age and all income, we can visualize the boundary decision



# Visualizing the Decision Boundary (2/2)

- If we represent the value outputted by the classifier for all age and all income, we can visualize the boundary decision



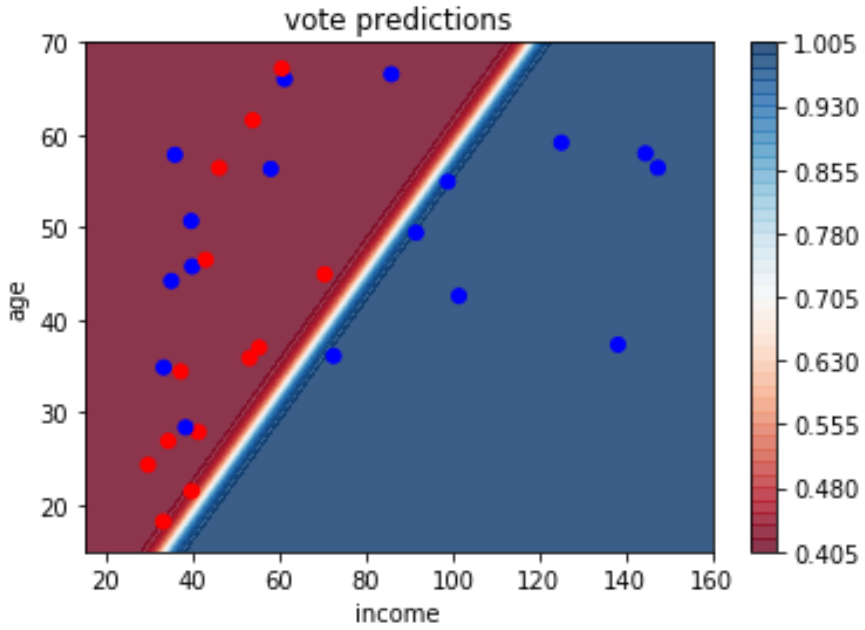
# More Complex Models

- In fact, the score that we compute is like the linear models that we were using for linear regression.
- Therefore, we can apply the same tricks in term of feature expansion to learn a more complex model

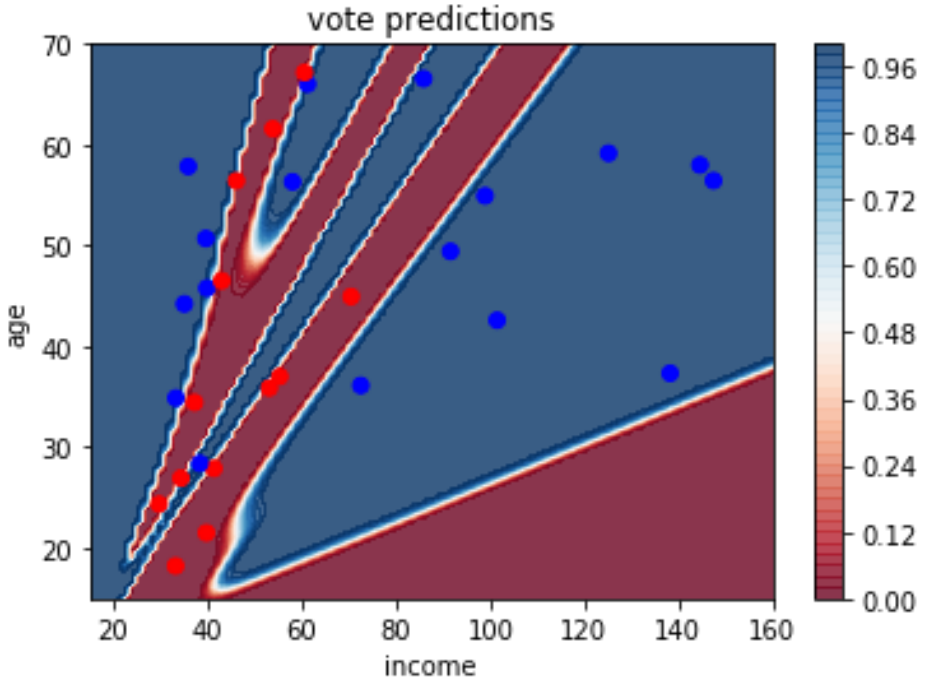
$$score(income, age) = \theta_0 + \theta_1 \times income + \theta_2 \times age + \theta_3 \times income^2 + \theta_4 \times age^2 + \theta_5 \times income \times log(age) + \ldots$$

- But beware overfitting

# Overfitting




$$score(income, age) = \theta_0 + \theta_1 \times income + \theta_2 \times age$$




$$score(income, age) = \theta_0 + \theta_1 \times income + \theta_2 \times age + \theta_3 \times income^2 + \theta_4 \times age^2 + \theta_5 \times income \times log(age) + \ldots$$

$$V_{model} = \sigma(score)$$

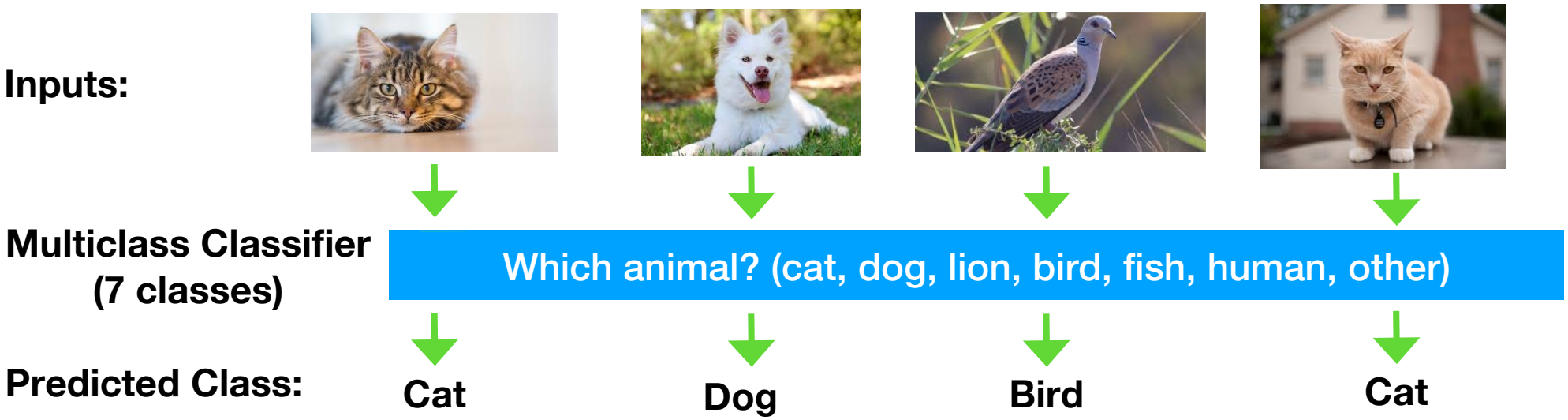
# Multiclass Classification

Which animal? (cat, dog, lion, bird, fish, human, other)

**Cat**

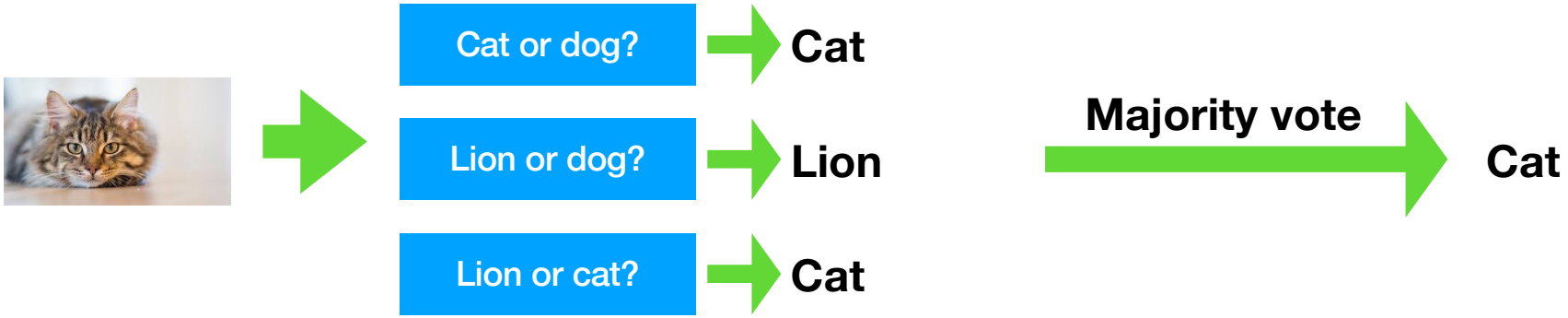
# Multiclass Classification (1/6)

- Remember that multi class classification is when we have more than 2 possible answers



# Multiclass Classification (2/6)

- How do we do multi class classification?
- Mainly two options:
  - 1- Reduce to binary classification
    - e.g. One vs One approach: train a binary classifier for each pair of category



# Multiclass Classification (3/6)

- How do we do multi class classification?
- Mainly two options:
  - 1- Reduce to binary classification
    - e.g. One vs One approach: train a binary classifier for each pair of category
  - 2- Use multinomial Logistic Regression (a.k.a **Softmax** classification)

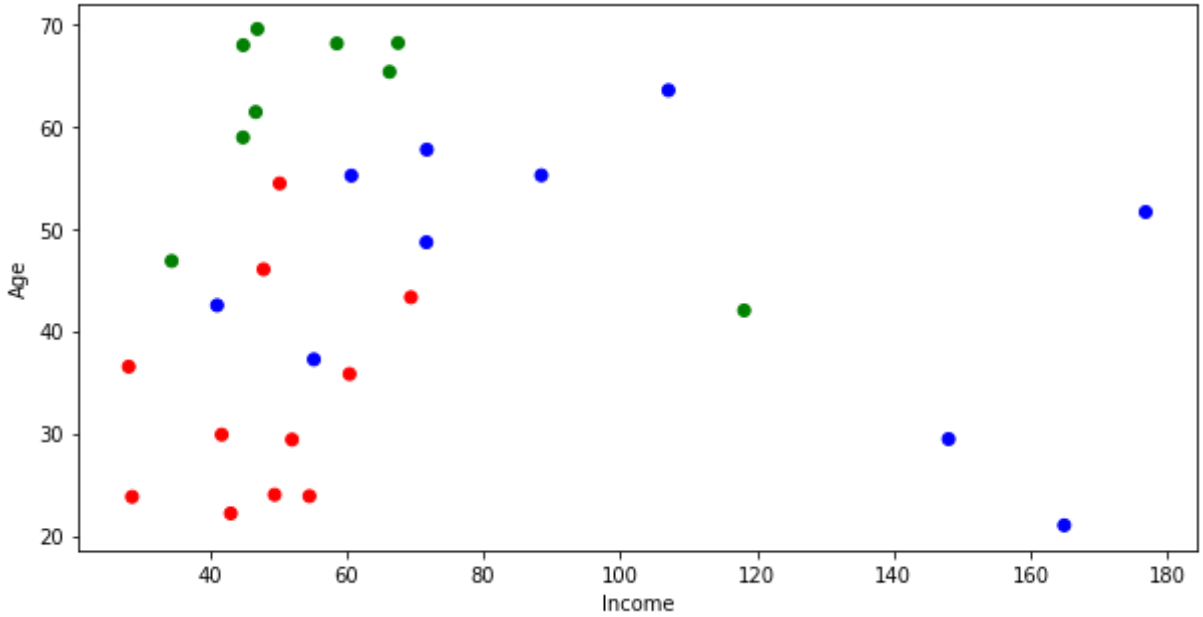
# Multiclass Classification (4/6)

- How do we do multi class classification?
- Mainly two options:
  - 1- Reduce to binary classification
    - e.g. One vs One approach: train a binary classifier for each pair of category
  - 2- Use multinomial Logistic Regression (a.k.a **Softmax** classification)
    - Similar to Binary Logistic Classification
    - But we are going to compute a ***score*** function for each of the classes

# Multiclass Classification (5/6)

- Let us now suppose that we have 3 parties: Left-Wing, Right-Wing and “Green” party
- How are we going to predict the votes with Softmax classification?

| | income | age | vote |
|---|---|---|---|
| **0** | 48.0 | 46.0 | L |
| **1** | 72.0 | 58.0 | R |
| **2** | 72.0 | 49.0 | R |
| **3** | 29.0 | 24.0 | L |
| **4** | 47.0 | 70.0 | G |
| **5** | 177.0 | 52.0 | R |
| **6** | … | … | … |



# Multiclass Classification (6/6)

- Let us now suppose that we have 3 parties: Left-Wing, Right-Wing and “Green”
- How are we going to predict the votes with Softmax classification?

| | income | age | vote |
|---|---|---|---|
| **0** | 48.0 | 46.0 | L |
| **1** | 72.0 | 58.0 | R |
| **2** | 72.0 | 49.0 | R |
| **3** | 29.0 | 24.0 | L |
| **4** | 47.0 | 70.0 | G |
| **5** | 177.0 | 52.0 | R |
| **6** | … | … | … |

$$score_L(income, age) = \theta_0^L + \theta_1^L \times income + \theta_2^L \times age$$

$$score_R(income, age) = \theta_0^R + \theta_1^R \times income + \theta_2^R \times age$$

$$score_G(income, age) = \theta_0^G + \theta_1^G \times income + \theta_2^G \times age$$

$$V_{model} = softmax(score_L, score_R, score_G)$$

**We now have 3 x 3 = 9 parameters**

# The Softmax Function (1/3)

- The ***softmax*** function (sometimes called *multinomial logistic function*) is used for multiclass classification
- The input of a softmax function is a vector of N dimensions
- The output is a N-dimension vector that can represent probabilities for each class
- It is computed by taking the exponential of the components of a vector, then dividing by the sum of the exponentiated components

$$softmax(\vec{x})_i = \frac{exp(x_i)}{\sum_j exp(x_j)}$$

# The Softmax Function (2/3)

- Example: computing $softmax([1, -1, 0, 2])$

$[1, -1, 0, 2]$ **1- Exponential** $[e^1, e^{-1}, e^0, e^2] \approx [2.71, 0.36, 1, 7.39]$

**2- Sum:** $2.71 + 0.36 + 1 + 7.39 = 11.46$

**3- Divide by sum** $\left[\frac{2.71}{11.46}, \frac{0.36}{11.46}, \frac{1}{11.46}, \frac{7.39}{11.46}\right] = [0.23, 0.03, 0.09, 0.65]$

**Softmax**

# The Softmax Function (3/3)

- Example: computing $softmax([1, -1, 0, 2])$

$[1, -1, 0, 2]$ **1- Exponential** $[e^1, e^{-1}, e^0, e^2] \approx [2.71, 0.36, 1, 7.39]$

**This step makes all values > 0**

**2- Sum:** $2.71 + 0.36 + 1 + 7.39 = 11.46$

**3- Divide by sum**

$$\left[\frac{2.71}{11.46}, \frac{0.36}{11.46}, \frac{1}{11.46}, \frac{7.39}{11.46}\right] = [0.23, 0.03, 0.09, 0.65]$$

**This step makes all values sum to 1**

**Softmax**

**Positive values summing to 1: Can be seen as probabilities**

# Multiclass Classification

- Let us now suppose that we have 3 parties: Left-Wing, Right-Wing and “Green”
- How are we going to predict the votes?

| | income | age | vote |
|---|---|---|---|
| **0** | 48.0 | 46.0 | L |
| **1** | 72.0 | 58.0 | R |
| **2** | 72.0 | 49.0 | R |
| **3** | 29.0 | 24.0 | L |
| **4** | 47.0 | 70.0 | G |
| **5** | 177.0 | 52.0 | R |
| **6** | … | … | … |

$$score_L(income, age) = \theta_0^L + \theta_1^L \times income + \theta_2^L \times age$$

$$score_R(income, age) = \theta_0^R + \theta_1^R \times income + \theta_2^R \times age$$

$$score_G(income, age) = \theta_0^G + \theta_1^G \times income + \theta_2^G \times age$$

$$V_{model} = softmax(score_L, score_R, score_G)$$

**This is now a vector of dimension 3 that contains probabilities of voting for each party**

# Exercise 2: Compute the Softmax

$\theta_0^L = -3 \quad \theta_1^L = 0.1 \quad \theta_2^L = 0.2$

$\theta_0^R = -5 \quad \theta_1^R = 0.3 \quad \theta_2^R = 0.4$

$\theta_0^G = 0 \quad \theta_1^G = 0.1 \quad \theta_2^G = 0.1$

$$score_L(income, age) = \theta_0^L + \theta_1^L \times income + \theta_2^L \times age$$

$$score_R(income, age) = \theta_0^R + \theta_1^R \times income + \theta_2^R \times age$$

$$score_G(income, age) = \theta_0^G + \theta_1^G \times income + \theta_2^G \times age$$

$$V_{model} = softmax(score_L, score_R, score_G)$$

**Somebody is 40 year old, with income of 50**
**Which party is predicted by this model?**

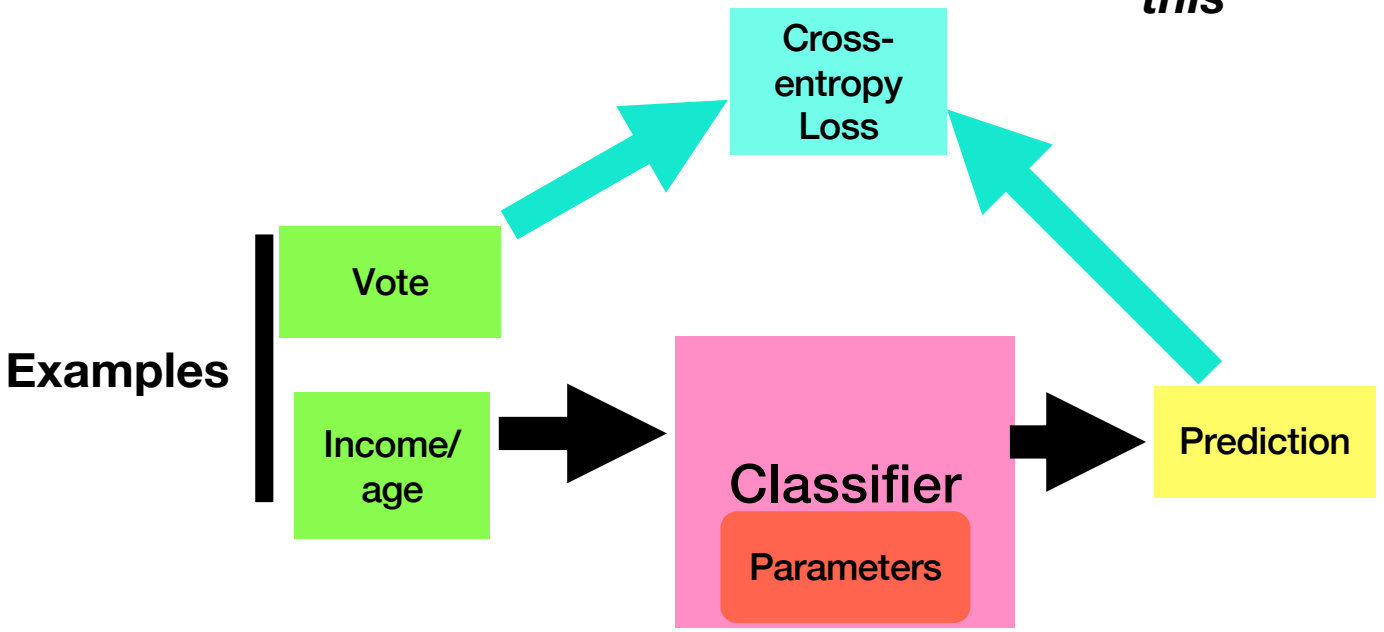
# Multiclass Classification

- Finding the optimal parameters is once again very similar to the case of binary classification

- We will use a cross-entropy loss again

# Supervised Learning

- In supervised learning, we usually have:
  - A **MODEL**: a "parameterized" function that takes input and produce output
  - A ***Loss***: A function that compute how different the model output is from the correct output
  - ***Examples*** of input and correct output (cigarets smoked, age of death)




$$score_L(income, age) = \theta_0^L + \theta_1^L \times income + \theta_2^L \times age$$

$$score_R(income, age) = \theta_0^R + \theta_1^R \times income + \theta_2^R \times age$$

$$score_G(income, age) = \theta_0^G + \theta_1^G \times income + \theta_2^G \times age$$

$$V_{model} = softmax(score_L, score_R, score_G)$$

# Multiclass Classification

- Finding the optimal parameters is once again very similar to the case of binary classification
- We will use a cross-entropy loss again (adapted for multi-class case)

- For each example:
  - If the correct class is i, then the *cost* of the prediction -ln(V[i])
- Then we average the cost over each example

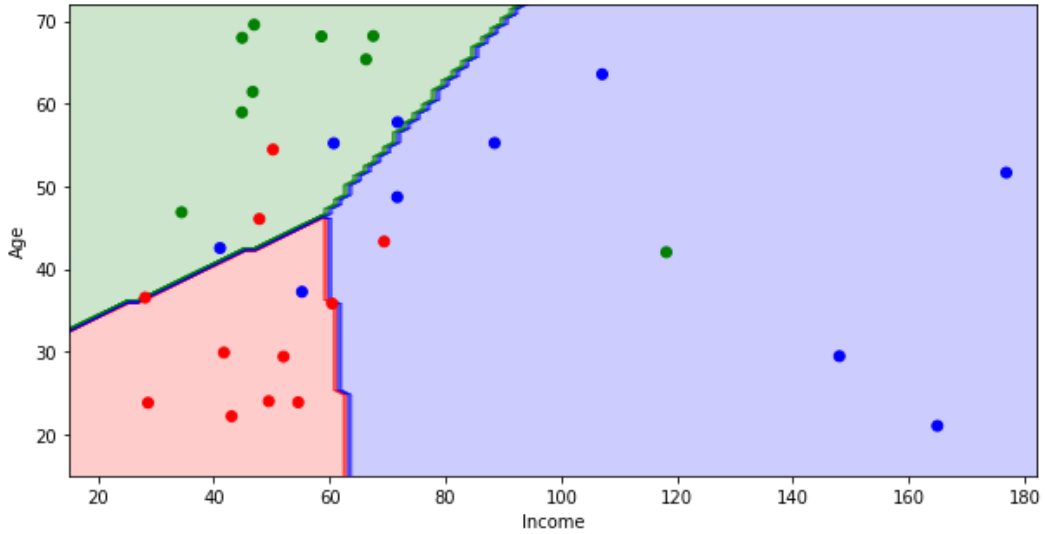
# Visualization of Boundary Decisions

After learning good parameters, we can visualize the model that was learned.

The graph is colored according to the party that get the maximum score for a given age/income pair

We see the areas are separated  by straight lines: this is because our score functions are linear functions of age and income

| | income | age | vote |
|---|---|---|---|
| **0** | 48.0 | 46.0 | L |
| **1** | 72.0 | 58.0 | R |
| **2** | 72.0 | 49.0 | R |
| **3** | 29.0 | 24.0 | L |
| **4** | 47.0 | 70.0 | G |
| **5** | 177.0 | 52.0 | R |
| **6** | … | … | … |



# Summary

- We have seen the two most important type of tasks in supervised learning:
  - **Regression** (predict a value) : e.g. age of death
  - **Classification** (predict a category): e.g. Political party
- Using “simple” models:
  - **Linear Regression** for regression
  - **Logistic Regression** for classification
    - (Note: Logistic Regression should be called “Logistic Classification”, but it is a historical term)

# Next Time

- Next time, we will start discussing the fancy things:
  - **Neural Networks** and Deep Learning
- These will let us solve the same tasks of **classification** and **regression**, but with much more powerful models

# Report

- Submit the report of exercises 1 and 2 **in pdf** via PandA
  - Submission due: **next lecture**
  - Name the pdf file as **student id_name**.