
Contents

Preface	ix
Introduction	xiii
1. Discrete-time Markov chains	1
1.1 Definition and basic properties	1
1.2 Class structure	10
1.3 Hitting times and absorption probabilities	12
1.4 Strong Markov property	19
1.5 Recurrence and transience	24
1.6 Recurrence and transience of random walks	29
1.7 Invariant distributions	33
1.8 Convergence to equilibrium	40
1.9 Time reversal	47
1.10 Ergodic theorem	52
1.11 Appendix: recurrence relations	57
1.12 Appendix: asymptotics for $n!$	58
2. Continuous-time Markov chains I	60
2.1 Q -matrices and their exponentials	60
2.2 Continuous-time random processes	67
2.3 Some properties of the exponential distribution	70

Contents

2.4 Poisson processes	73
2.5 Birth processes	81
2.6 Jump chain and holding times	87
2.7 Explosion	90
2.8 Forward and backward equations	93
2.9 Non-minimal chains	103
2.10 Appendix: matrix exponentials	105
3. Continuous-time Markov chains II	108
3.1 Basic properties	108
3.2 Class structure	111
3.3 Hitting times and absorption probabilities	112
3.4 Recurrence and transience	114
3.5 Invariant distributions	117
3.6 Convergence to equilibrium	121
3.7 Time reversal	123
3.8 Ergodic theorem	125
4. Further theory	128
4.1 Martingales	128
4.2 Potential theory	134
4.3 Electrical networks	151
4.4 Brownian motion	159
5. Applications	170
5.1 Markov chains in biology	170
5.2 Queues and queueing networks	179
5.3 Markov chains in resource management	192
5.4 Markov decision processes	197
5.5 Markov chain Monte Carlo	206
6. Appendix: probability and measure	217
6.1 Countable sets and countable sums	217
6.2 Basic facts of measure theory	220
6.3 Probability spaces and expectation	222
6.4 Monotone convergence and Fubini's theorem	223
6.5 Stopping times and the strong Markov property	224
6.6 Uniqueness of probabilities and independence of σ -algebras	228
Further reading	232
Index	234