ILAS Seminar in Probability: Assignment 7

1. Compute the extinction probability q of a branching process with offspring distribution given by
$$\mathbf{P}(N = k) = (1 - p)^k p, \qquad k = 0, 1, 2, \ldots,$$
where $p \in (0, 1)$ is a constant. Sketch a plot to show how q depends on p.

2. Consider a branching process with offspring distribution given $\mathbf{P}(N = k) = p_k$ for $k = 0, 1, 2, 3$, where
$$p_0 = a, \qquad p_1 = 1 - a - b - c, \qquad p_2 = b, \qquad p_3 = c,$$
where $a, b, c > 0$ are such that $a + b + c < 1$.

   (a) Give an inequality in terms of a, b, c that holds if and only if the branching process survives with strictly positive probability.

   (b) Compute the extinction probability of the branching process.

   (c) Compute the extinction probability of a branching process with $\mathrm{Bin}(3, \frac{1}{2})$ offspring distribution.