Neural Networks with Fully Connected Layers in Practice

Fundamentals of Artificial Intelligence

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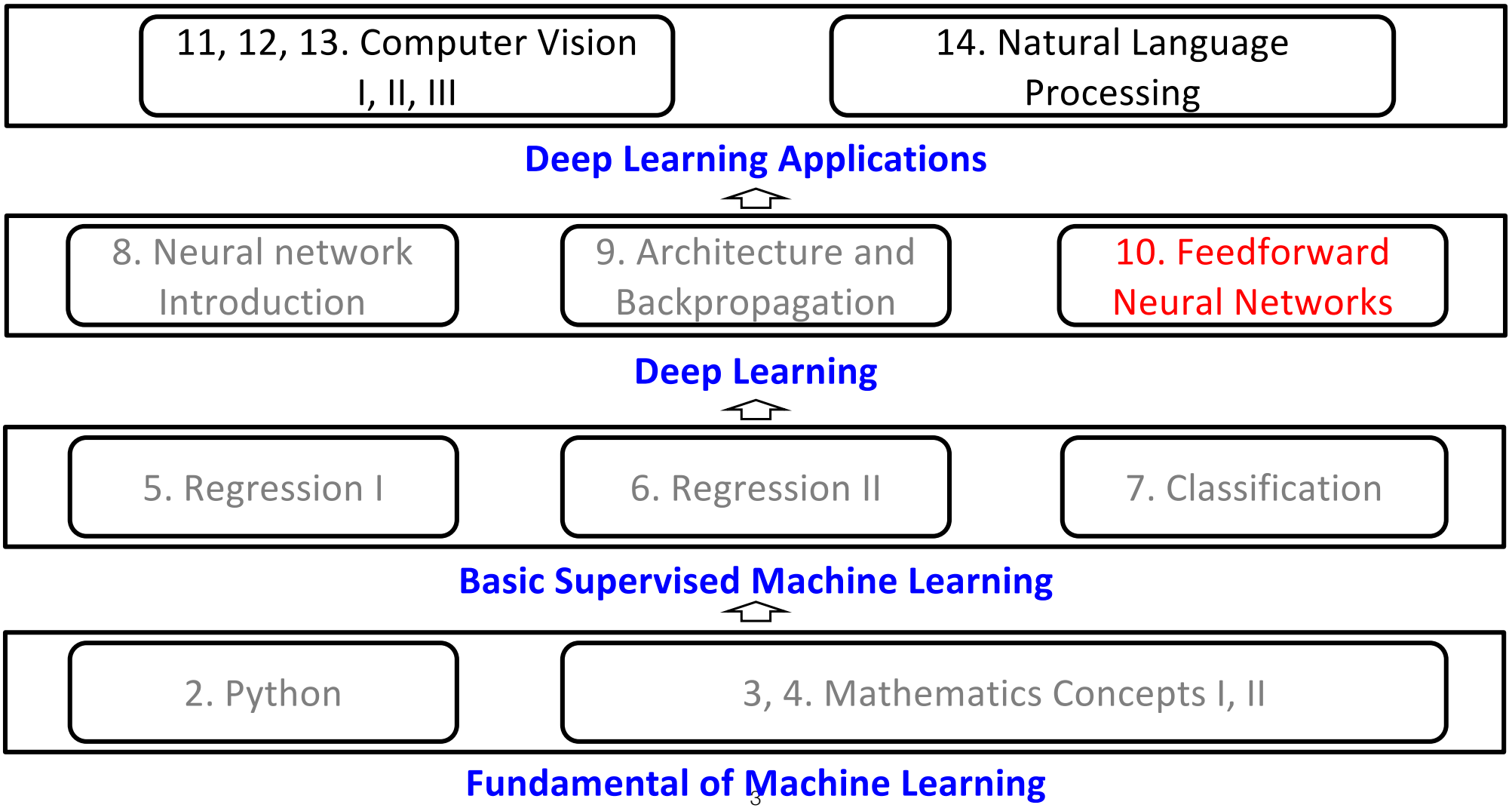
Schedule

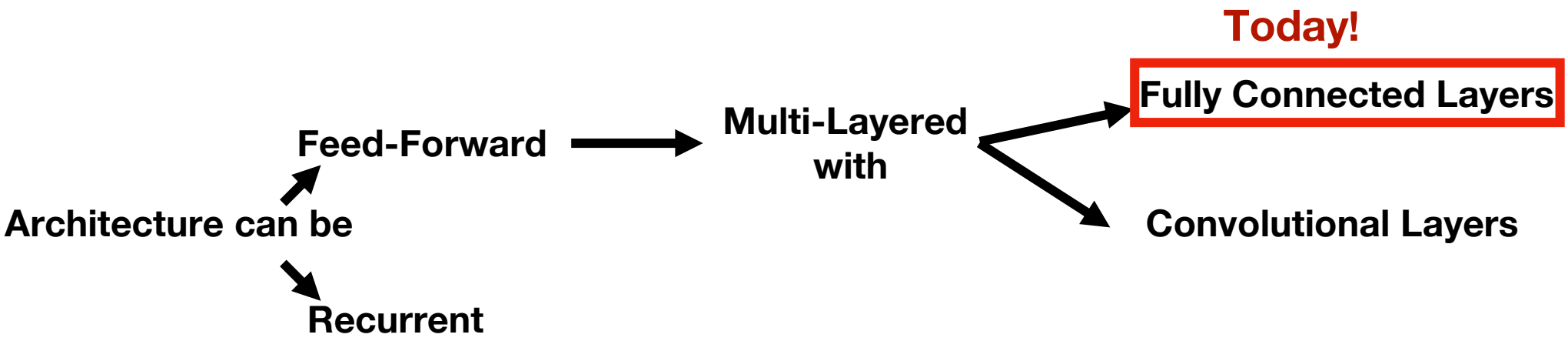
- 1. Overview of AI and this Course (4/14)
- 2. Introduction to Python (4/21)
- 3, 4. Mathematics Concepts I, II (4/28, 5/12)
- 5, 6. Regression I, II (5/19, 5/26)
- 7. Classification (6/2)
- 8. Introduction to Neural Networks (6/9)
- 9. Neural Networks Architecture and Backpropagation (6/16)
- 10. Fully Connected Layers (6/23)
- 11, 12, 13. Computer Vision I, II, III (6/30, 7/7, 7/14)
- 14. Natural Language Processing (7/17)

Overview of This Course



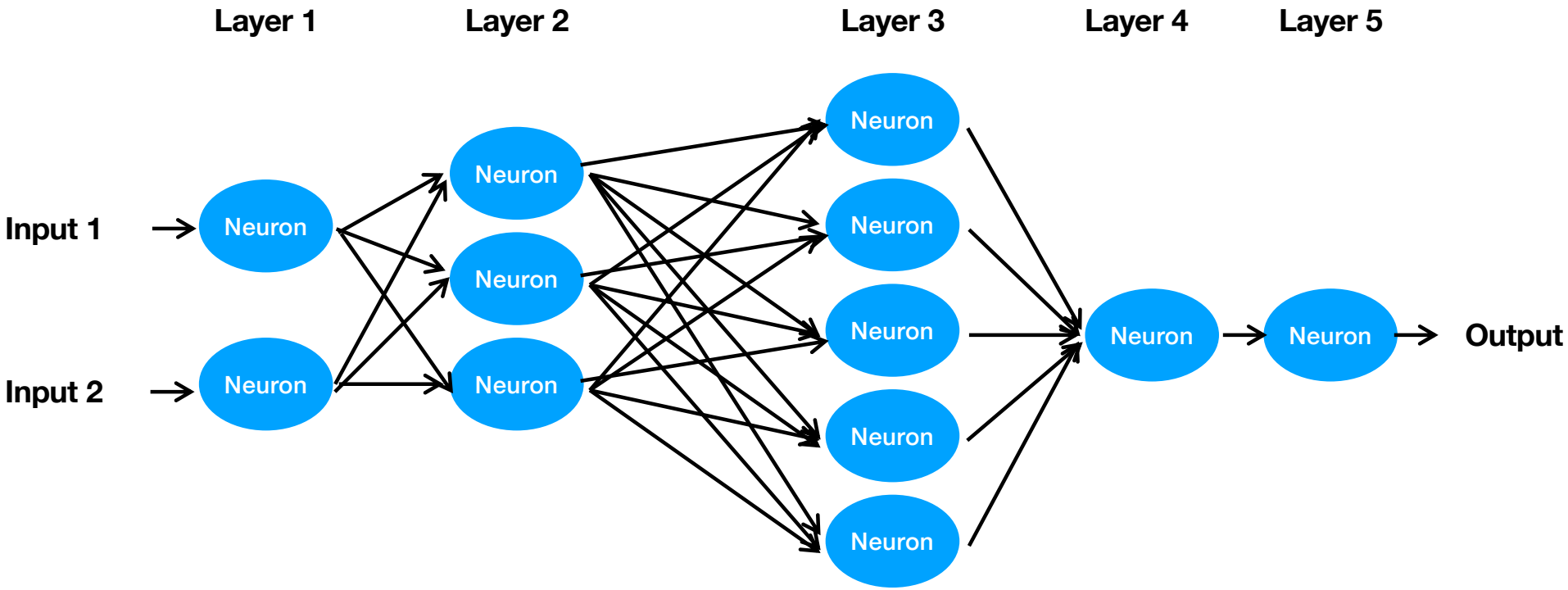
Neural Network Architectures

- We are still here:



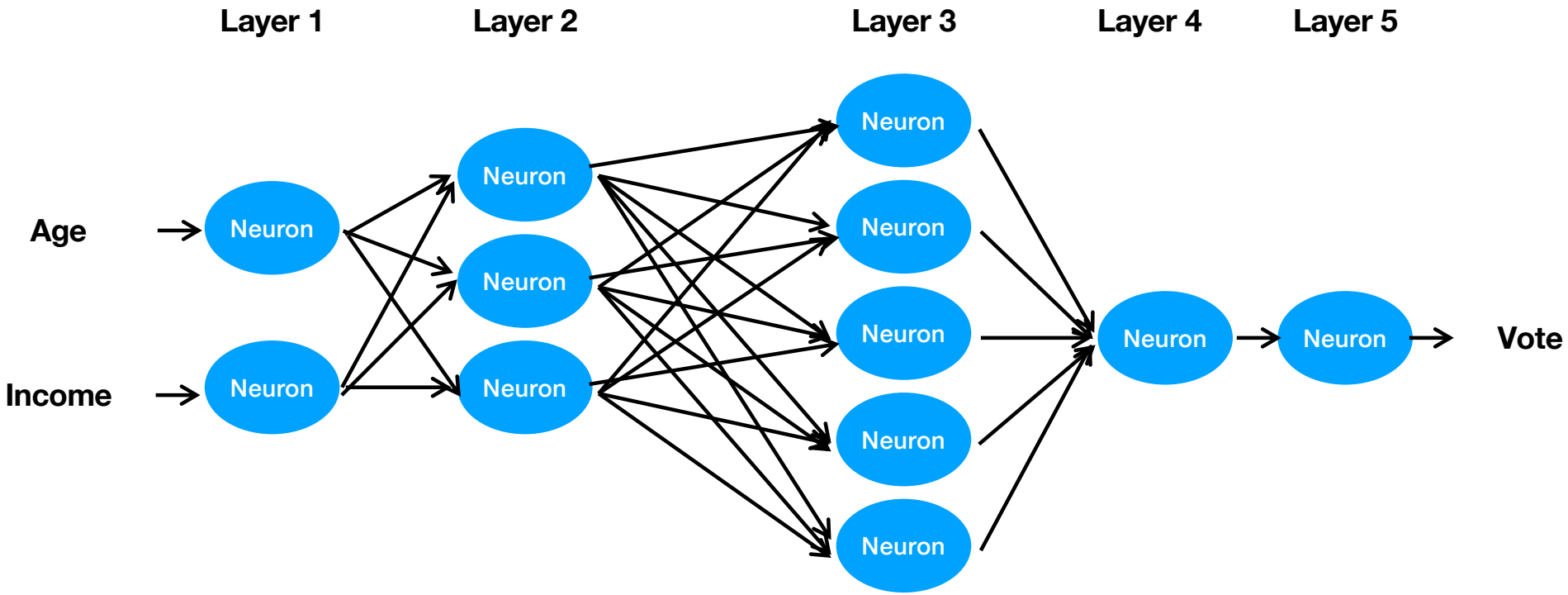
Feed-Forward Networks with Fully Connected Layers (1/2)

- Therefore, we are going to consider this type of Neural Network:



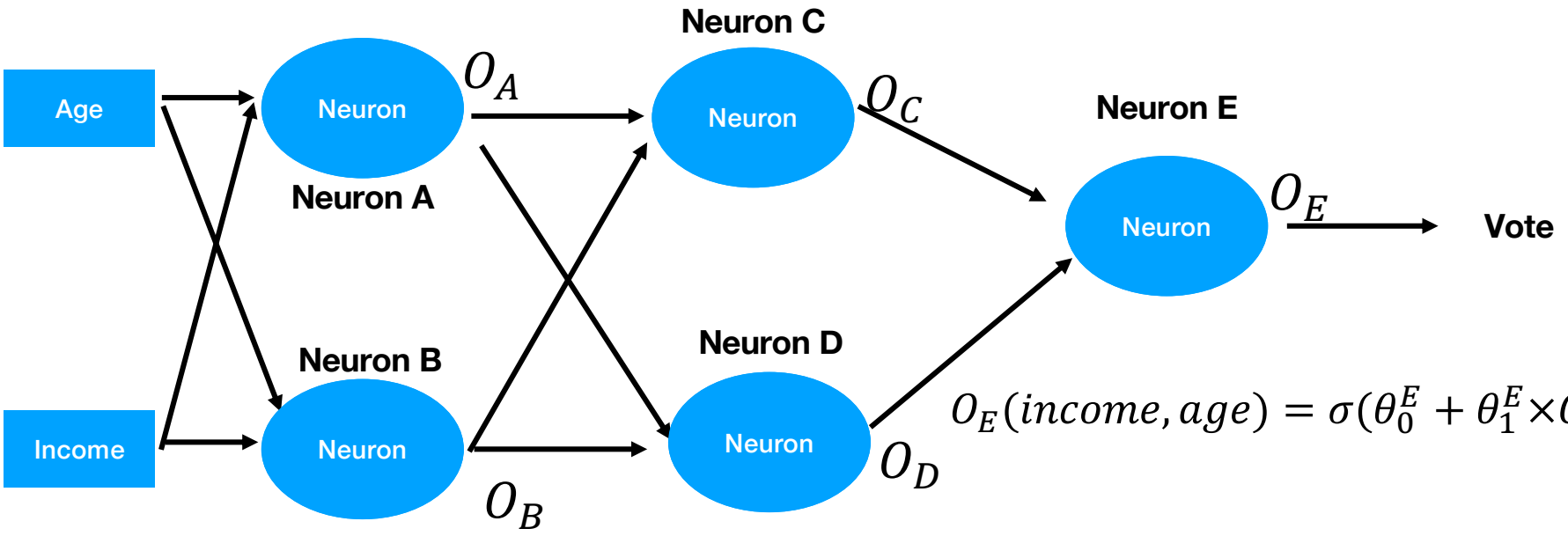
Feed-Forward Networks with Fully Connected Layers (2/2)

- Therefore, we are going to consider this type of Neural Network:



Keeping in Mind What This Type of Graph Mean (1/3)

$O_A(income, age) = \sigma(\theta_0^A + \theta_1^A \times income + \theta_2^A \times age)$ $O_C(income, age) = \sigma(\theta_0^C + \theta_1^C \times O_A + \theta_2^C \times O_B)$

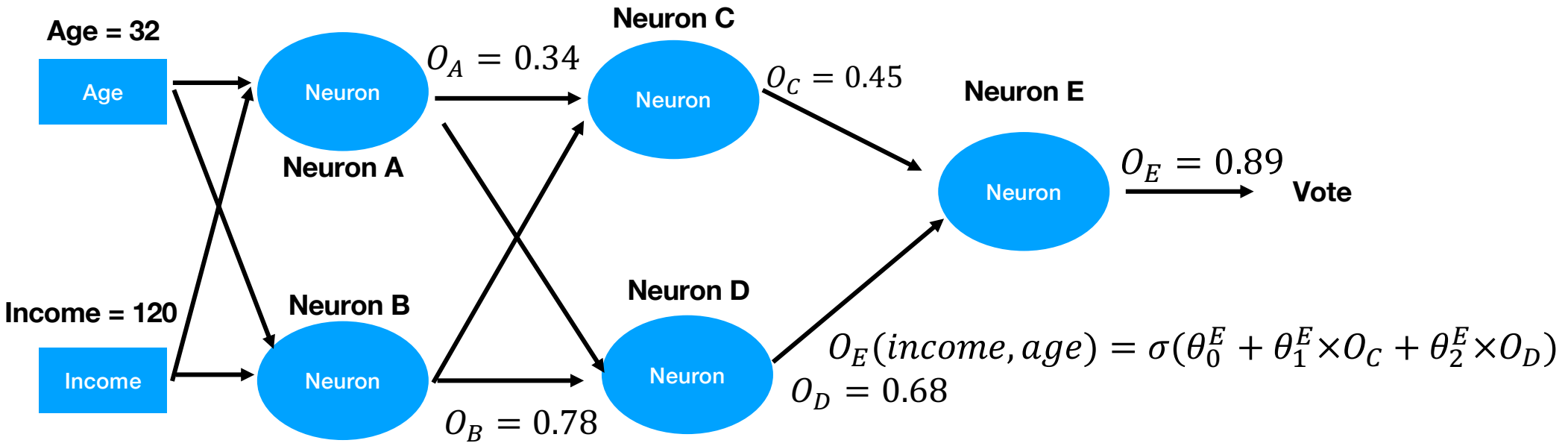


$O_E(income, age) = \sigma(\theta_0^E + \theta_1^E \times O_C + \theta_2^E \times O_D)$

$O_B(income, age) = \sigma(\theta_0^B + \theta_1^B \times income + \theta_2^B \times age)$ $O_D(income, age) = \sigma(\theta_0^D + \theta_1^D \times O_A + \theta_2^D \times O_B)$

Keeping in Mind What This Type of Graph Mean (2/3)

$O_A(income, age) = \sigma(\theta_0^A + \theta_1^A \times income + \theta_2^A \times age)$ $O_C(income, age) = \sigma(\theta_0^C + \theta_1^C \times O_A + \theta_2^C \times O_B)$



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Keeping in Mind What This Type of Graph Mean (3/3)

—> Each Neural Network architecture defines a function of the input with parameters θ

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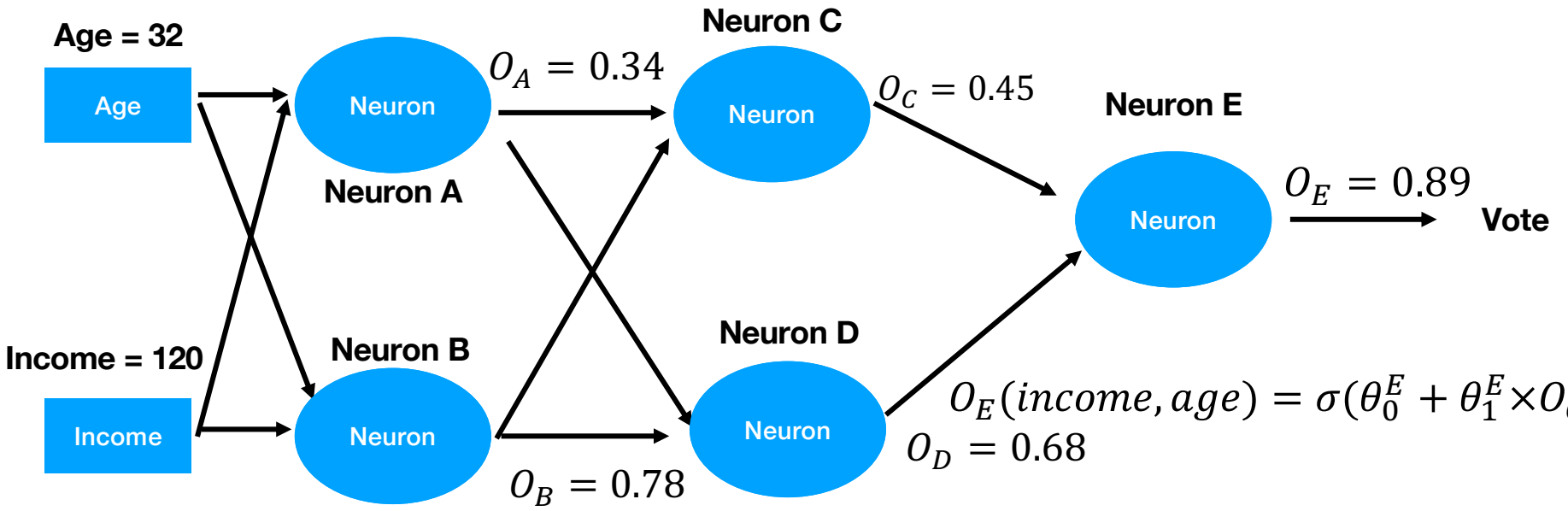

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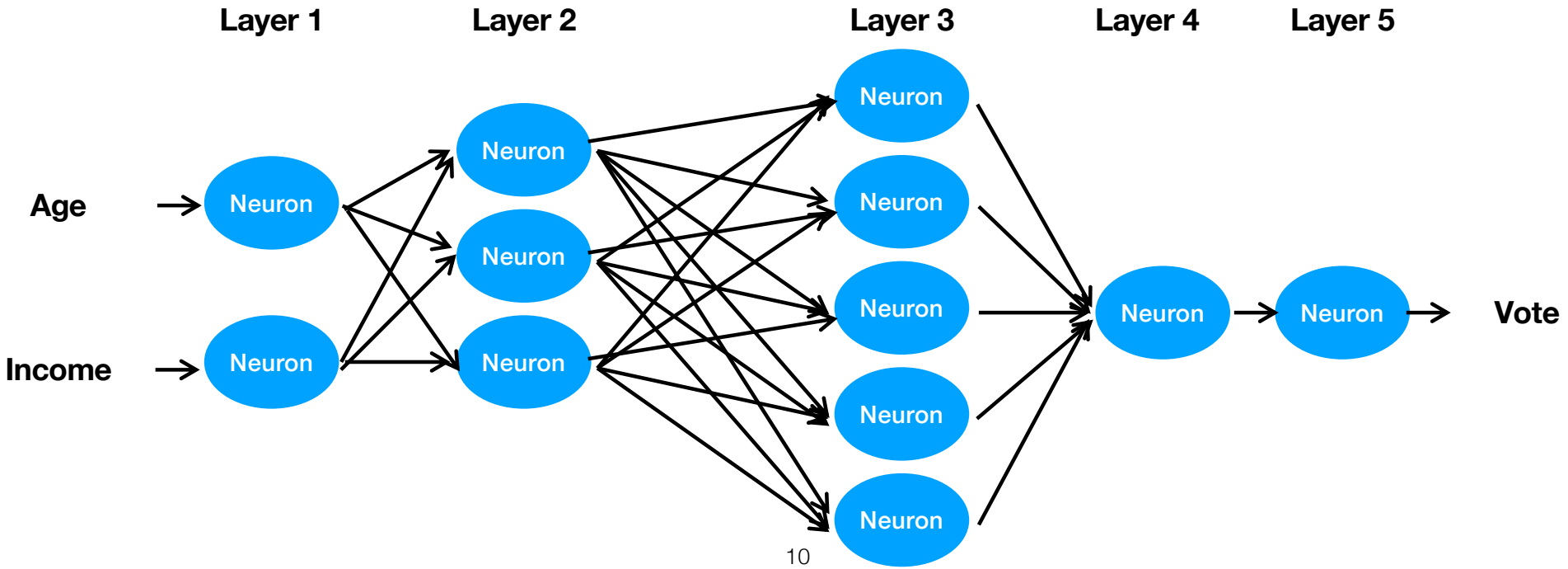
Feed-Forward Networks with Fully Connected Layers

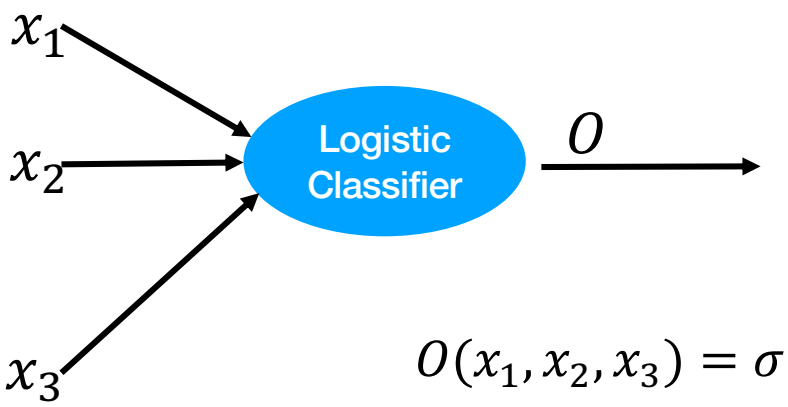
—> Each Neural Network architecture defines a function of the input with parameters θ

- Therefore, this is just a visual way of defining a complicated parameterized function of ***Vote*** given ***Age*** and ***Income***:



Parameters (1/2)

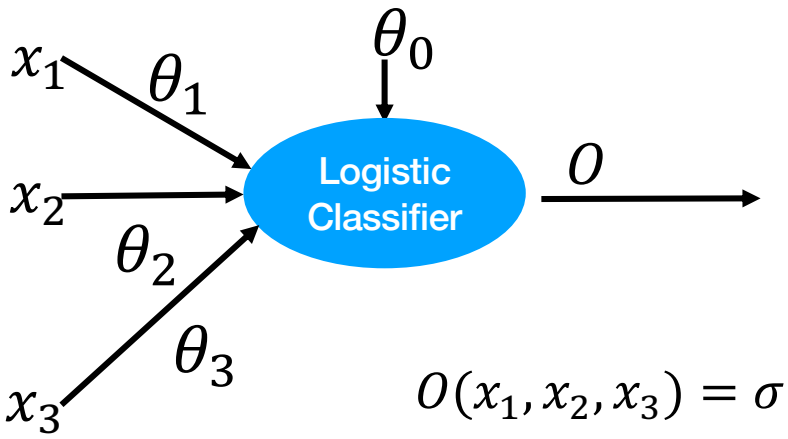
- If a neuron has N inputs, it has N+1 parameters



$$O(x_1, x_2, x_3) = \sigma(\theta_0 + \theta_1 \times x_1 + \theta_2 \times x_2 + \theta_3 \times x_3)$$

Parameters (2/2)

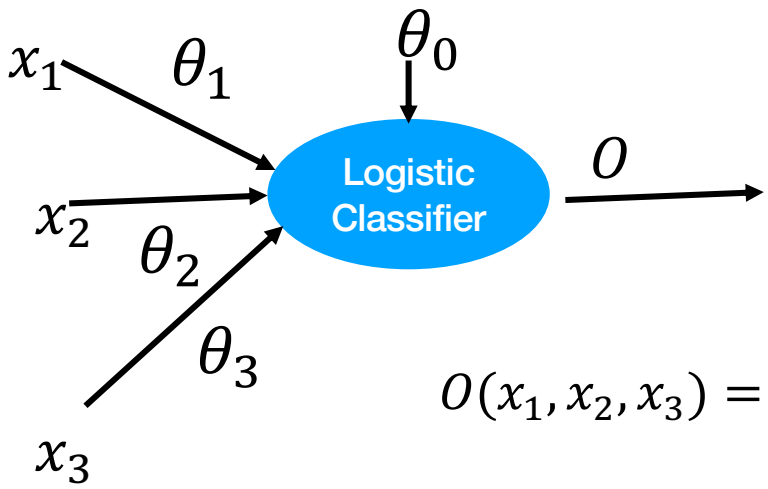
- If a neuron has N inputs, it has N+1 parameters
  - Visually, we can associate a parameter to each input, and show θ_0 separately



$$O(x_1, x_2, x_3) = \sigma(\theta_0 + \theta_1 \times x_1 + \theta_2 \times x_2 + \theta_3 \times x_3)$$

Parameters: Terminology (1/2)

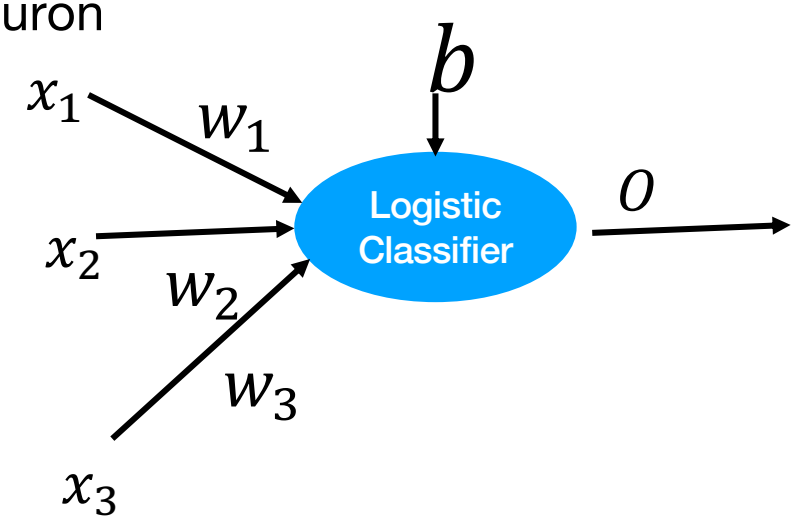
- θ_1, θ_2, θ_3 are often called the ***weights*** of the neuron
- θ_0 is often called the ***bias*** of the neuron



$$O(x_1, x_2, x_3) = \sigma(\theta_0 + \theta_1 \times x_1 + \theta_2 \times x_2 + \theta_3 \times x_3)$$

Parameters: Terminology (2/2)

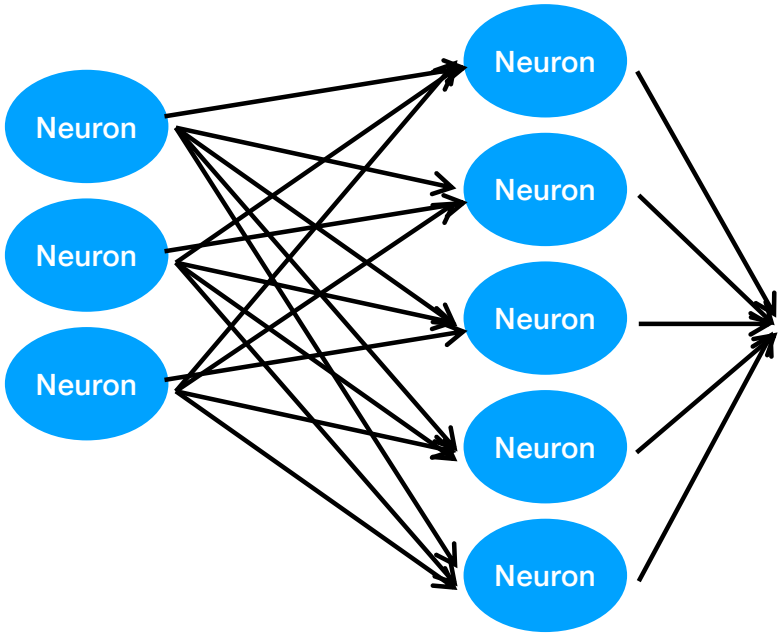
- θ_1, θ_2, θ_3 are often called the ***weights*** of the neuron
  - They are therefore often also noted w_1, w_2, w_3
- θ_0 is often called the ***bias*** of the neuron
  - It is often noted ***b***



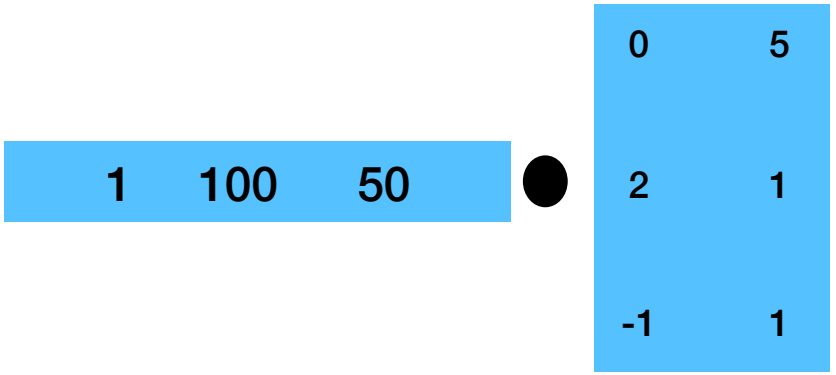
$$O(x_1, x_2, x_3) = \sigma(b + w_1 \times x_1 + w_2 \times x_2 + w_3 \times x_3)$$

“Connectionist” View vs. “Mathematical” View (1/2)

- “Connections” viewpoint
  - *Fully Connected Layers*

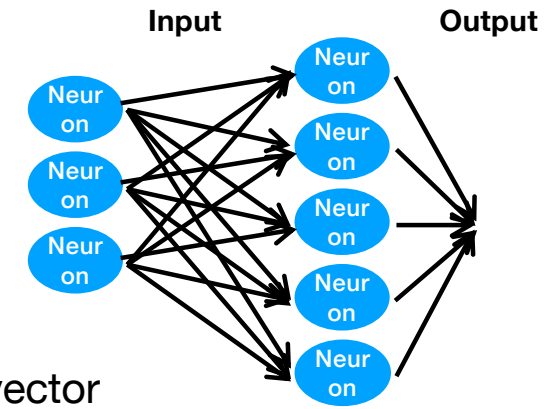


- Math operations viewpoint
  - *Matrix multiplications*



“Connectionist” View vs. “Mathematical” View (2/2)

- In practice, computing the output of a fully connected layer is equivalent to computing:
  - a Matrix multiplication
  - followed by an activation
- Equivalences:
  - Input of the layer (= output of the previous layer) <-> row vector
  - Output of the layer (= set of output of each neuron in a layer) <-> row vector
  - Weights of a neuron <-> column vector
  - Set weights of all neurons in a fully connected layer <-> matrix
  - Batched input of a layer (= several inputs at once) <-> matrix



Linear Algebra

- In order to discuss the way Neural Networks are actually implemented, we need to discuss some mathematical concepts
- We discuss:
  - Vector scalar products (a.k.a inner product)
  - Matrices
  - Matrix-vector multiplication
  - Matrix-Matrix multiplication
- Warning: it is a bit ambitious to explain that in 30 minutes. Hopefully you understand most of it.

Row Vectors and Column Vectors

- We have seen that ***vectors*** are just a “list” of numbers
- We are actually going to distinguish 2 “types” of vectors: ***row vectors*** and ***column vectors***

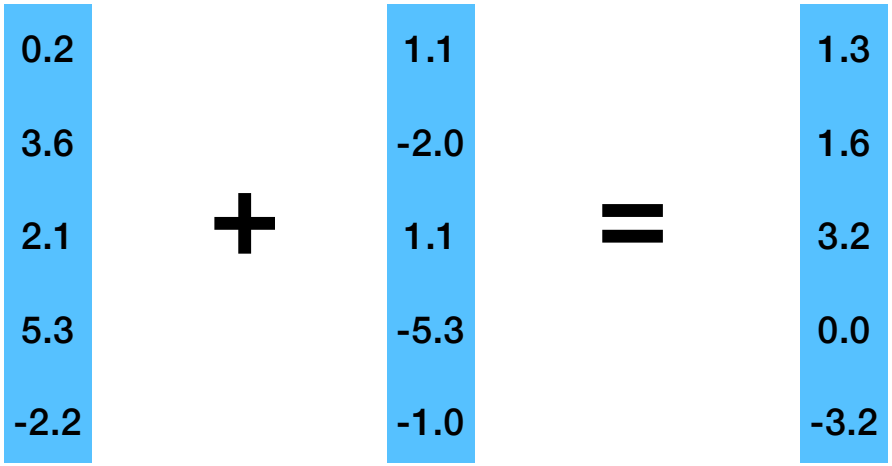
(5-dimensional) Column vector:

(5-dimensional) Row vector:

Column Vectors: Add

- Column vectors of the same dimension can be added

(5-dimensional) Column vector + **(5-dimensional) Column vector** = **(5-dimensional) Column vector**



Column Vectors: Multiply

- Column vectors can be multiplied by a number

(5-dimensional) Column vector **(5-dimensional) Column vector**

$$2.0 \times \begin{bmatrix} 1.1 \\ -2.0 \\ 1.1 \\ -5.3 \\ -1.0 \end{bmatrix} = \begin{bmatrix} 2.2 \\ -4.0 \\ 2.2 \\ -10.6 \\ -2.0 \end{bmatrix}$$

Row Vectors

- Row vectors have the same operations as Column vectors
- They can be added:

(5-dimensional) Row vector

0.2	3.6	2.1	5.3	-2.2

+

(5-dimensional) Row vector

1.1	-2.0	1.1	-5.3	-1.0

=

(5-dimensional) Row vector

1.3	1.6	3.2	0.0	-3.2

- They can be multiplied by a number:

2.0 X

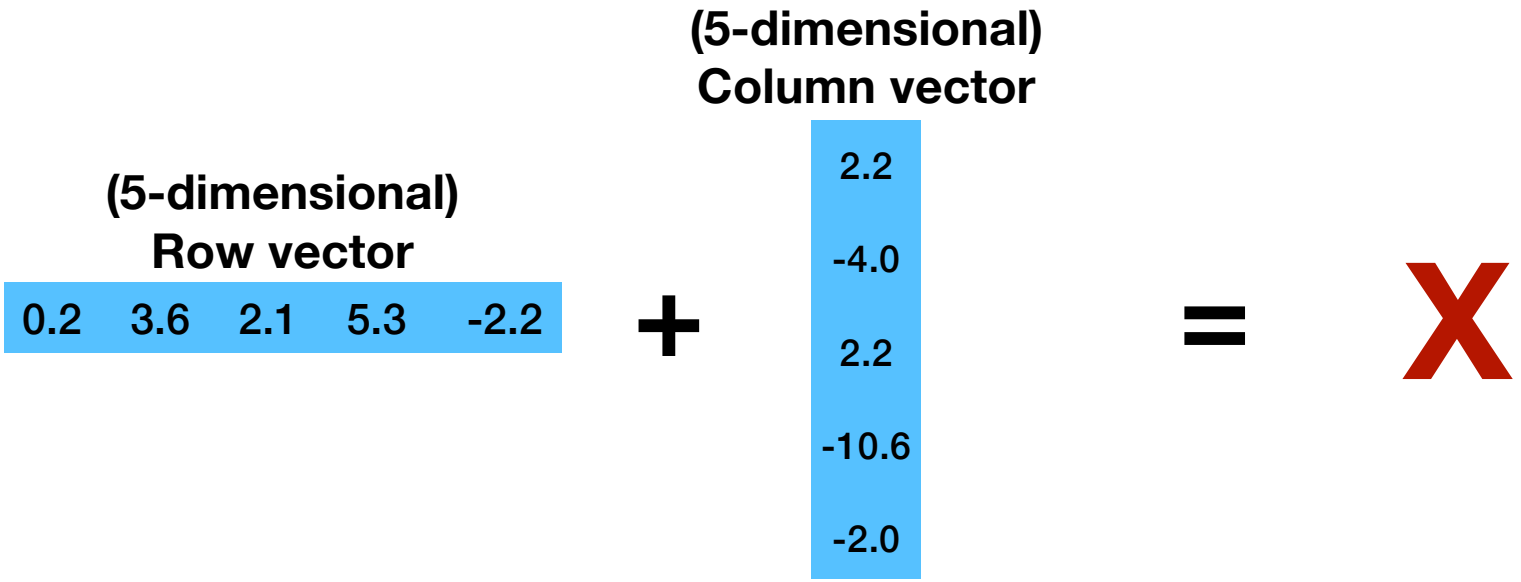
0.2	3.6	2.1	5.3	-2.2

=

0.4	7.2	4.2	10.6	-4.4

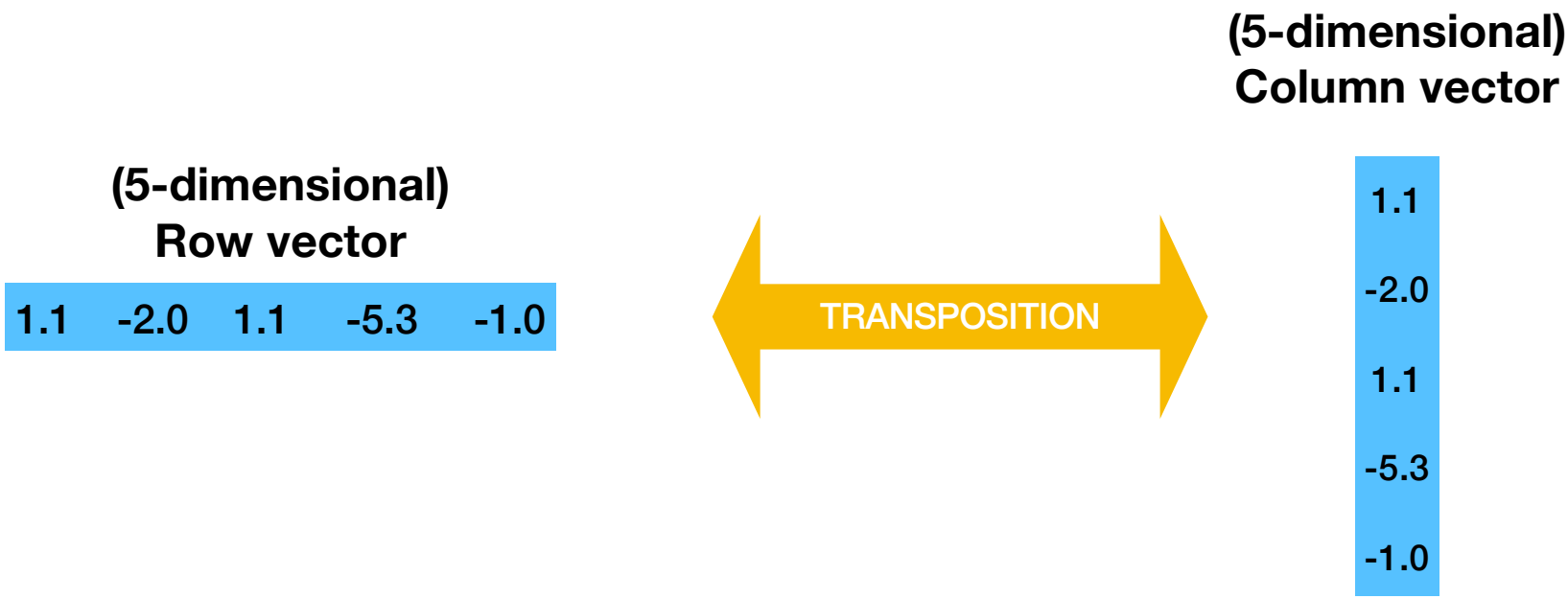
Row Vectors and Column Vectors

- Row vectors and column vectors cannot be added together



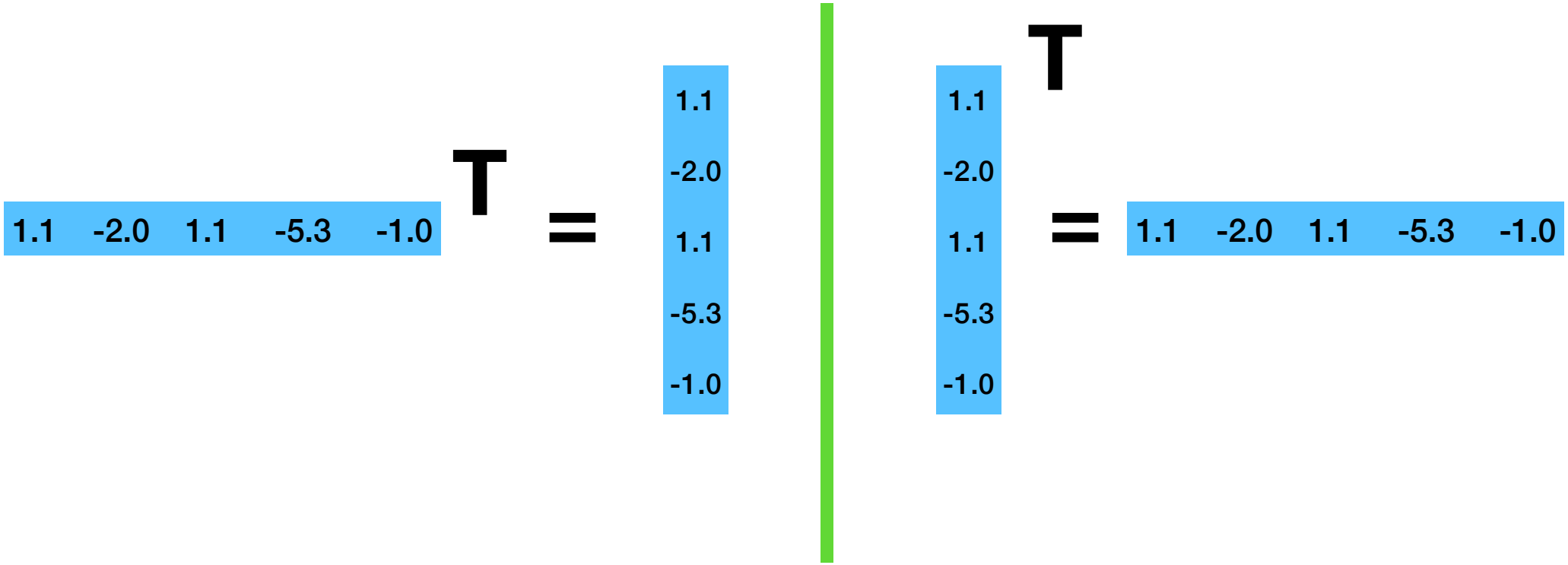
Transposition (1/2)

- However, *row vectors* can be transformed in *column vectors* by an operation called ***transposition*** (and vice-versa)



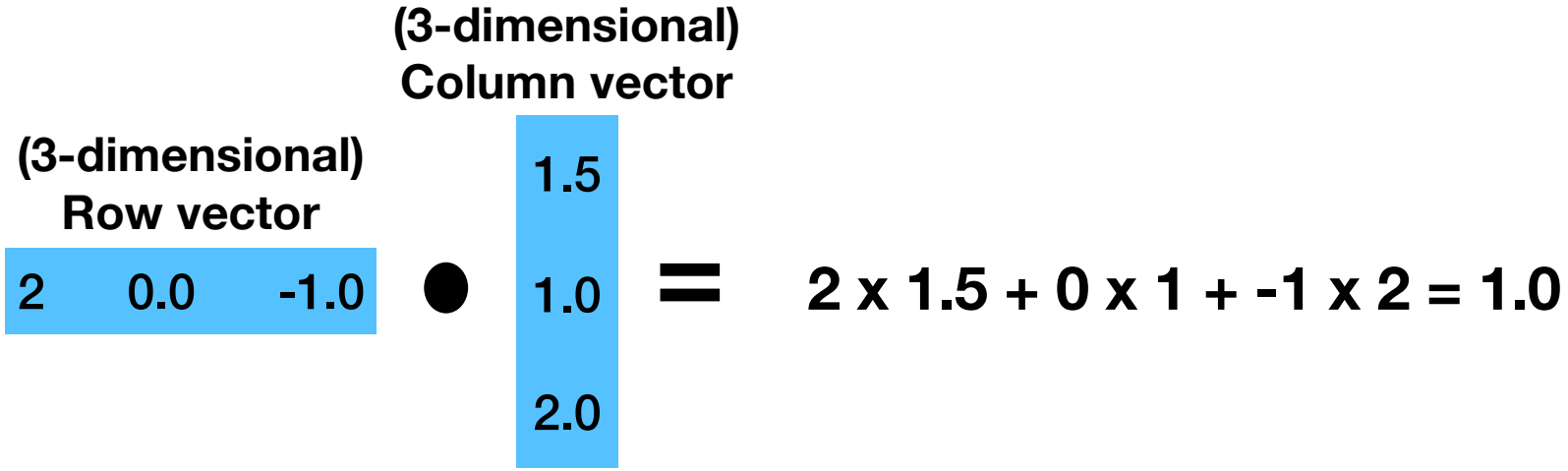
Transposition (2/2)

- Transposition is usually noted with a ***T*** in exponent:



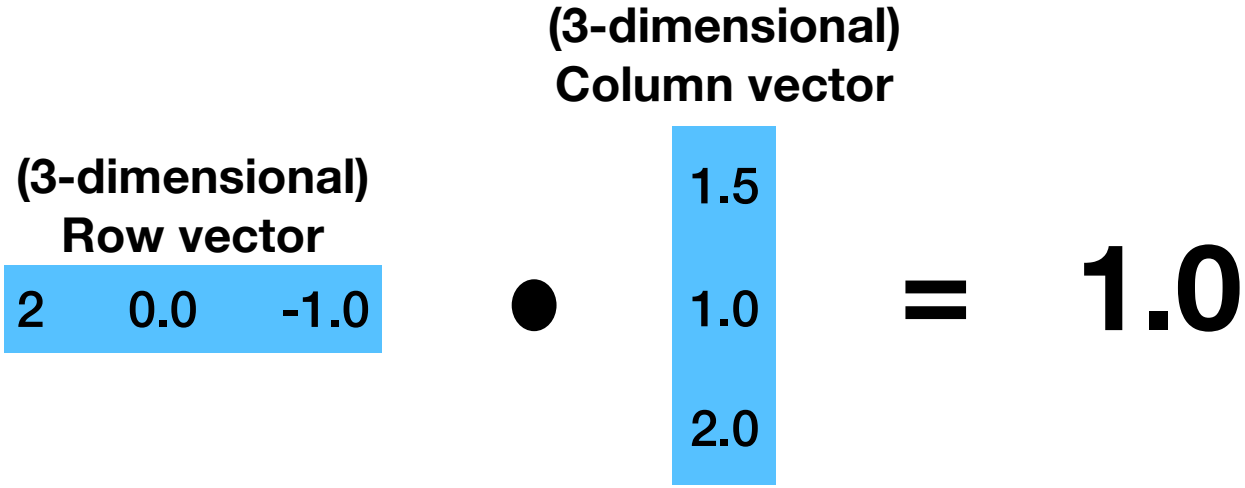
Inner product (1/4)

- We can compute the ***inner product*** of a *row vector* and a *column vector* to obtain a single number
- It consists in taking the product of the number dimension by dimension and then taking the sum

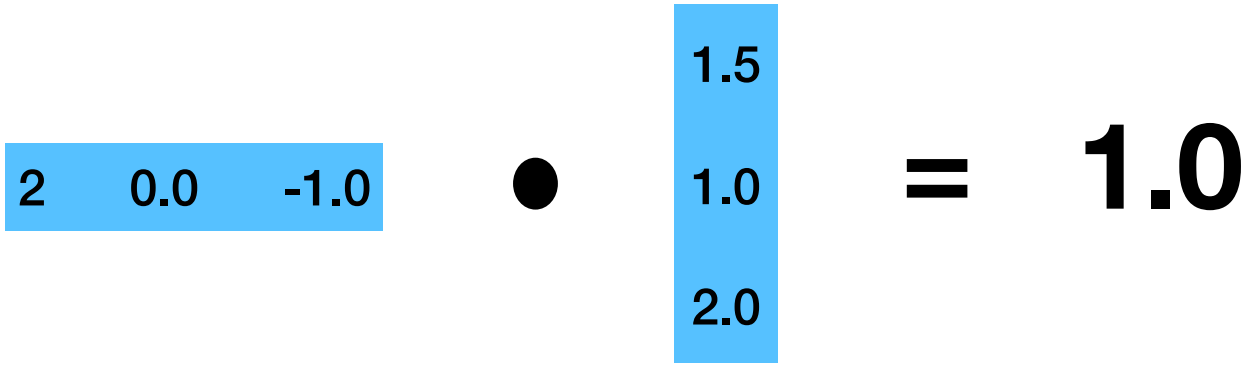


Inner product (2/4)

- The vectors should have the **same dimension**
- You should have the *row vector* on the left, and the *column vector* on the right
  - (if it is the opposite, the operation is called *outer product* and gives a different result)

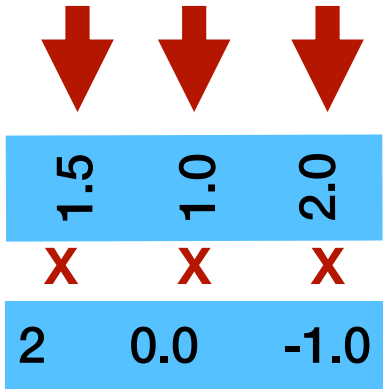


Inner Product (3/4)



You can try to visualize this as the column vector lying down on the row vector to produce the number

Inner Product (4/4)



$\bullet$ = **1.0**

3.0 + 0.0 + -2.0 → **1.0**

You can try to visualize this as the column vector lying down on the row vector to produce the number

Why Inner Product is Important? (1/2)

- It allows us to **express linear functions efficiently**
- And remember that linear functions are one of the **fundamental components** of Machine Learning

Linear regression

$$f(x, y) = \theta_0 + \theta_1 \times x + \theta_2 \times y$$

Logistic Classifier

$$score(x, y) = \theta_0 + \theta_1 \times x + \theta_2 \times y$$

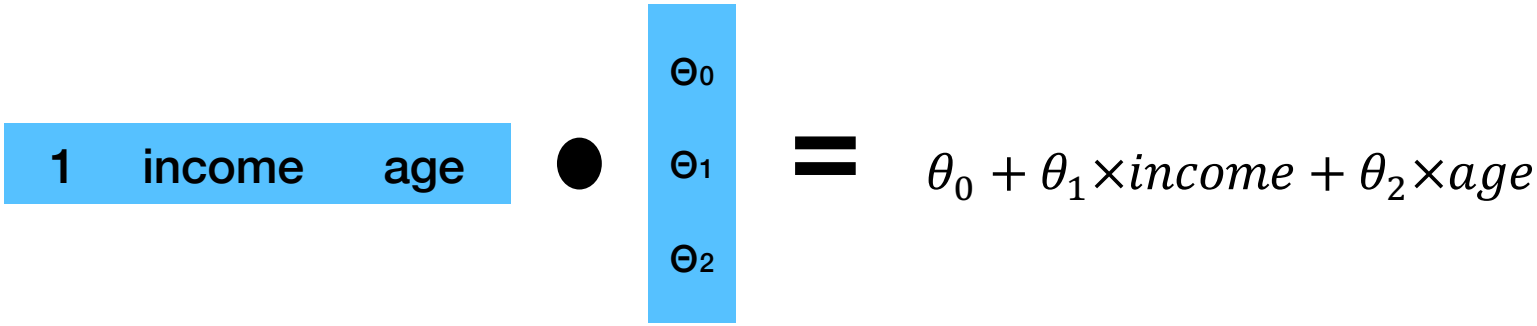
$$prediction = \sigma(score)$$

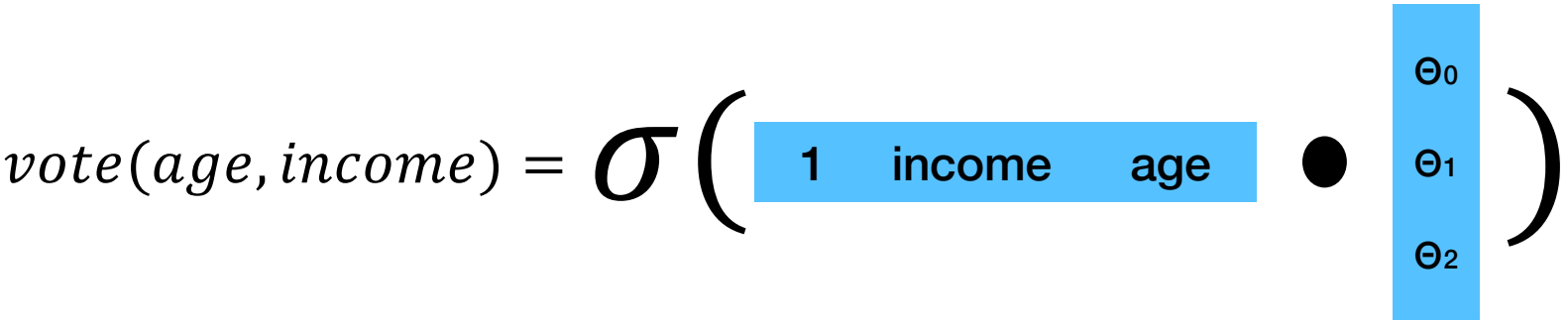
Neuron (same formula as Logistic Classifier)

$$output(x, y) = \sigma(\theta_0 + \theta_1 \times x + \theta_2 \times y)$$

Why Inner Product is Important? (2/2)

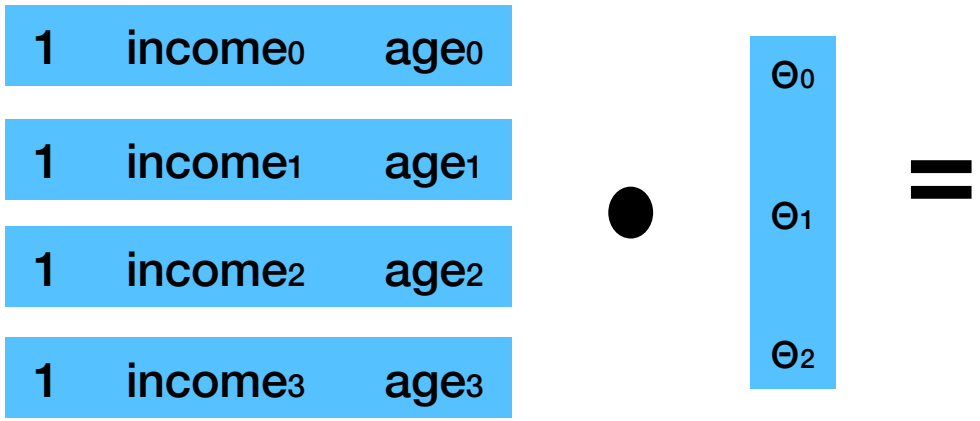
$$vote(age, income) = \sigma(\theta_0 + \theta_1 \times age + \theta_2 \times income)$$





Representing Many Inner Product at Once

$$vote(age, income) = \sigma(\theta_0 + \theta_1 \times age + \theta_2 \times income)$$



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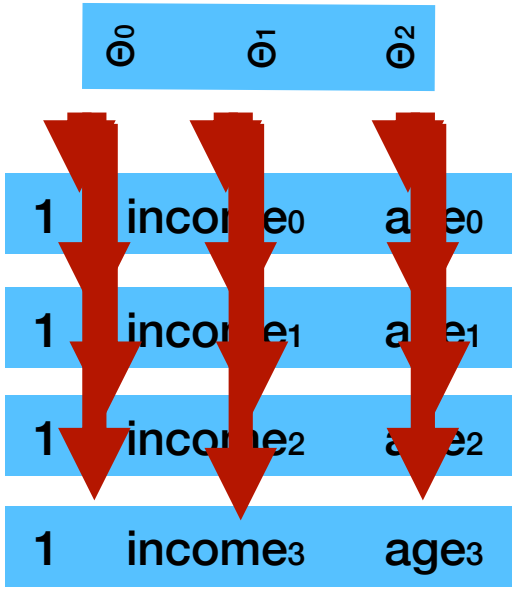


$$\bullet \quad =$$

$$\theta_0 + \theta_1 \times income_0 + \theta_2 \times age_0$$

$$\theta_0 + \theta_1 \times income_1 + \theta_2 \times age_1$$

$$\theta_0 + \theta_1 \times income_2 + \theta_2 \times age_2$$

$$\theta_0 + \theta_1 \times income_3 + \theta_2 \times age_3$$

Representing Many Inner Product at Once

$$vote(age, income) = \sigma(\theta_0 + \theta_1 \times age + \theta_2 \times income)$$

3-dimensional Row vectors (4 times)

Batch of 4 inputs

1	$income_0$	age_0
1	$income_1$	age_1
1	$income_2$	age_2
1	$income_3$	age_3

$\bullet$

3-dimensional Column vector

Θ_0

Θ_1

Θ_2

Parameters of one neuron

$=$

4-dimensional Column vector

$$\theta_0 + \theta_1 \times income_0 + \theta_2 \times age_0$$

$$\theta_0 + \theta_1 \times income_1 + \theta_2 \times age_1$$

$$\theta_0 + \theta_1 \times income_2 + \theta_2 \times age_2$$

$$\theta_0 + \theta_1 \times income_3 + \theta_2 \times age_3$$

This represents the computation of the score output of a single neuron for 4 inputs at the same time!

Representing Many Inner Product at Once

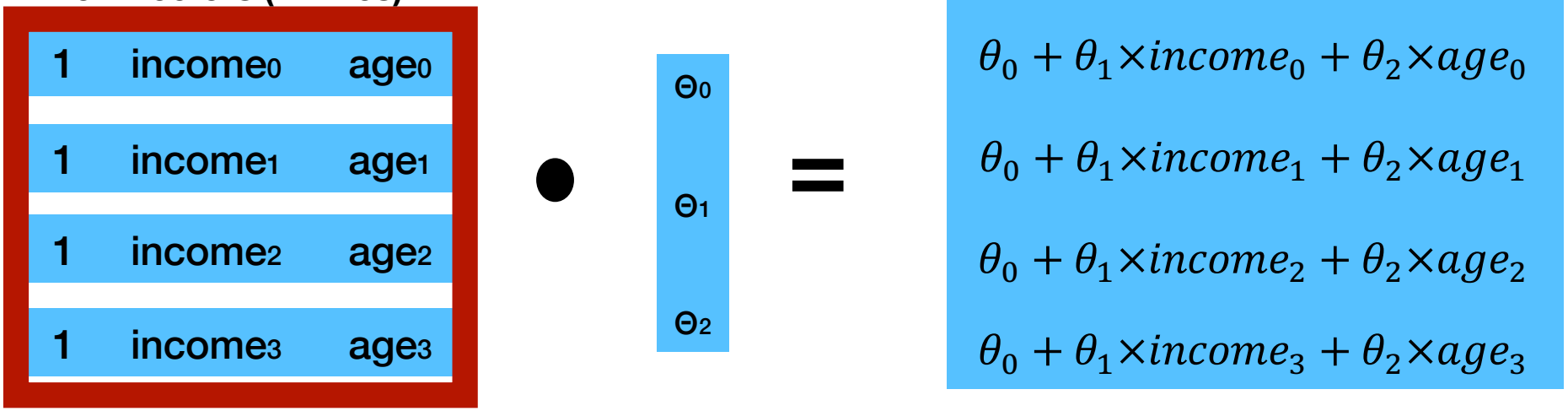
$$vote(age, income) = \sigma(\theta_0 + \theta_1 \times age + \theta_2 \times income)$$

3-dimensional Row vectors (4 times)

3-dimensional Column vector

4-dimensional Column vector

Batch of 4 inputs



This is called a 4x3 matrix

Representing Many Inner Product at Once

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3-dimensional Row vectors (4 times)

1	$income_0$	age_0
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1	$income_2$	age_2
1	$income_3$	age_3

This is called a 4x3 matrix

$\bullet$

3-dimensional Column vector

Θ_0

Θ_1

Θ_2

$=$

4-dimensional Column vector

$$\theta_0 + \theta_1 \times income_0 + \theta_2 \times age_0$$

$$\theta_0 + \theta_1 \times income_1 + \theta_2 \times age_1$$

$$\theta_0 + \theta_1 \times income_2 + \theta_2 \times age_2$$

$$\theta_0 + \theta_1 \times income_3 + \theta_2 \times age_3$$

Representing Many Inner Product at Once

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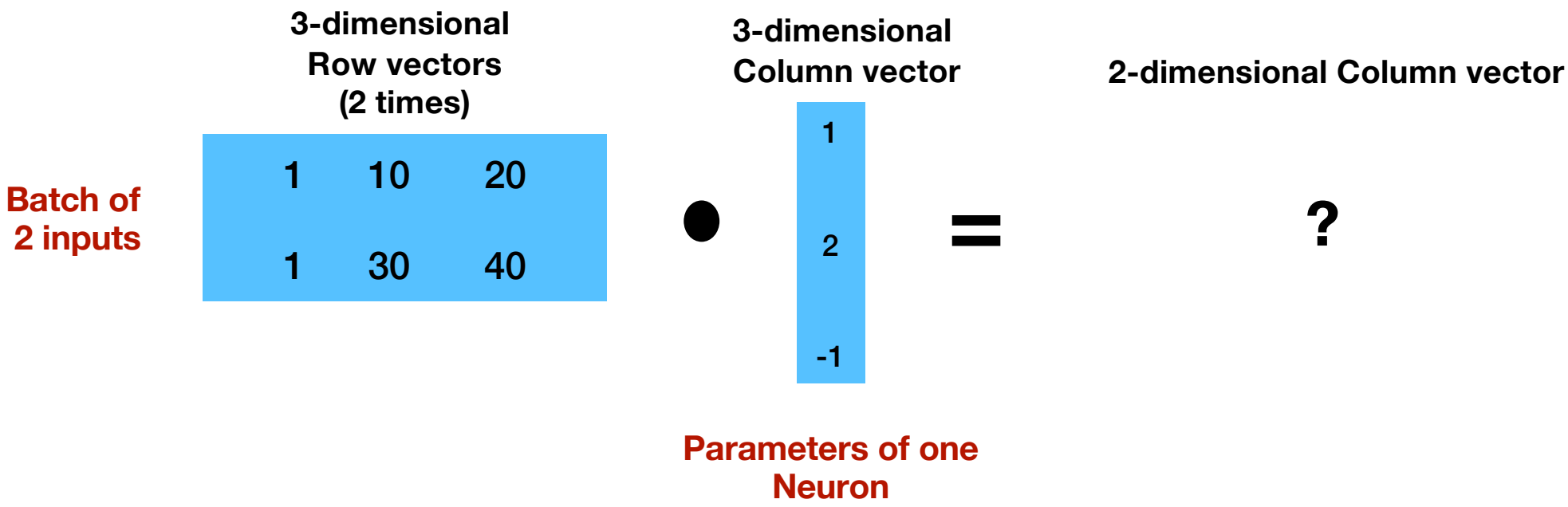

Representing Many Inner Product at Once

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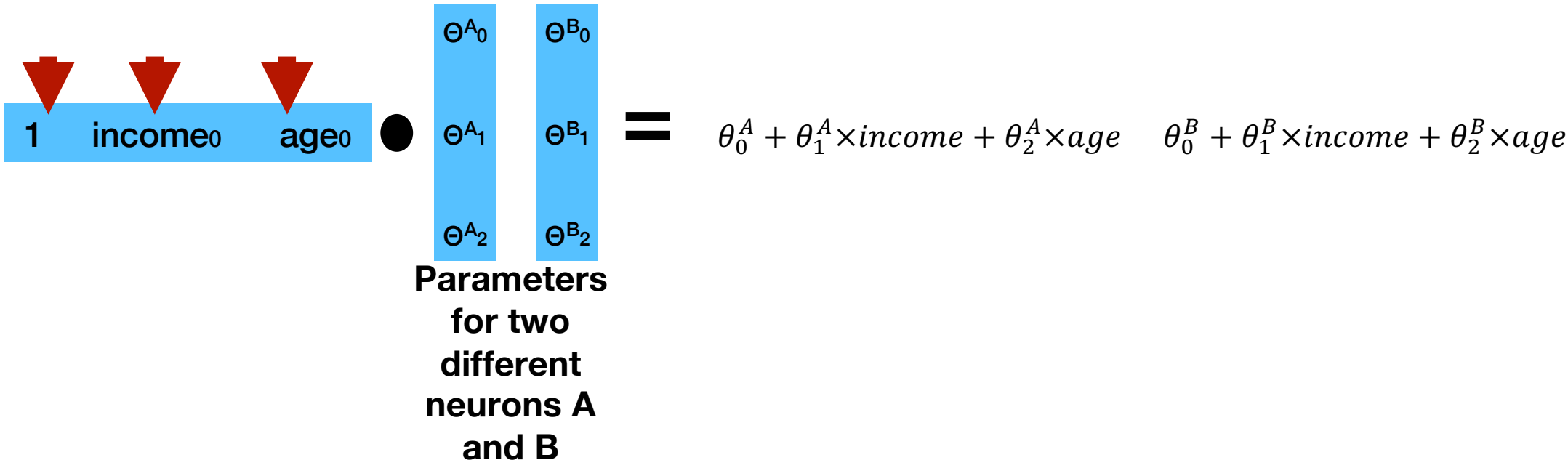

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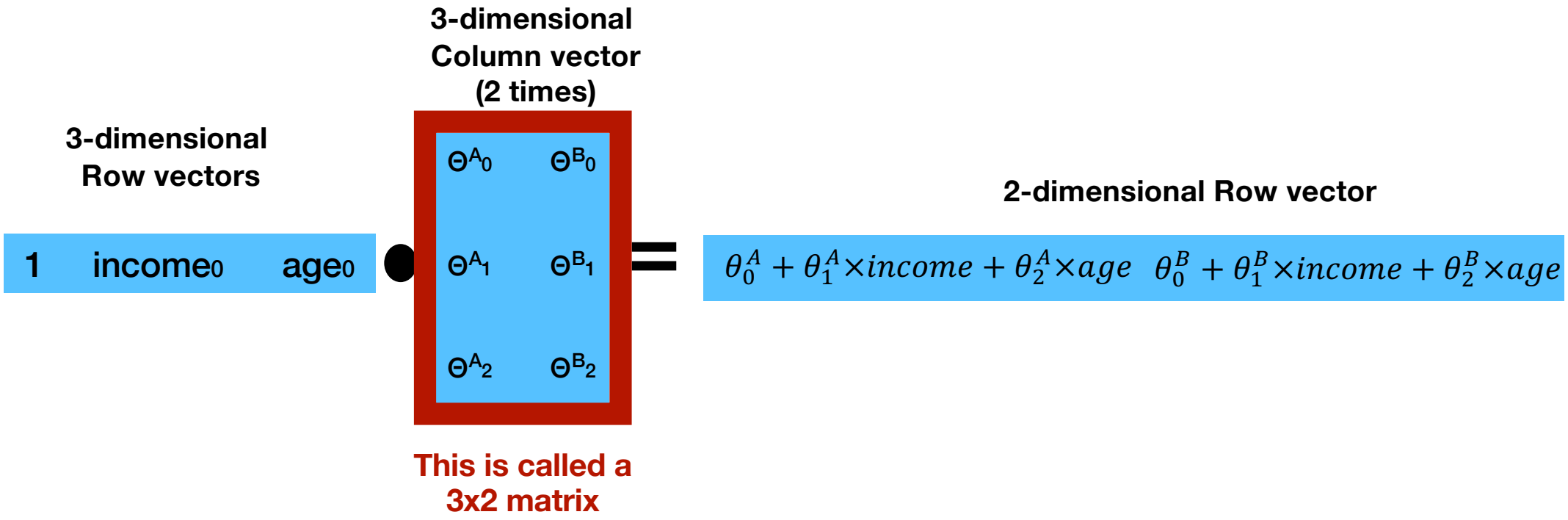

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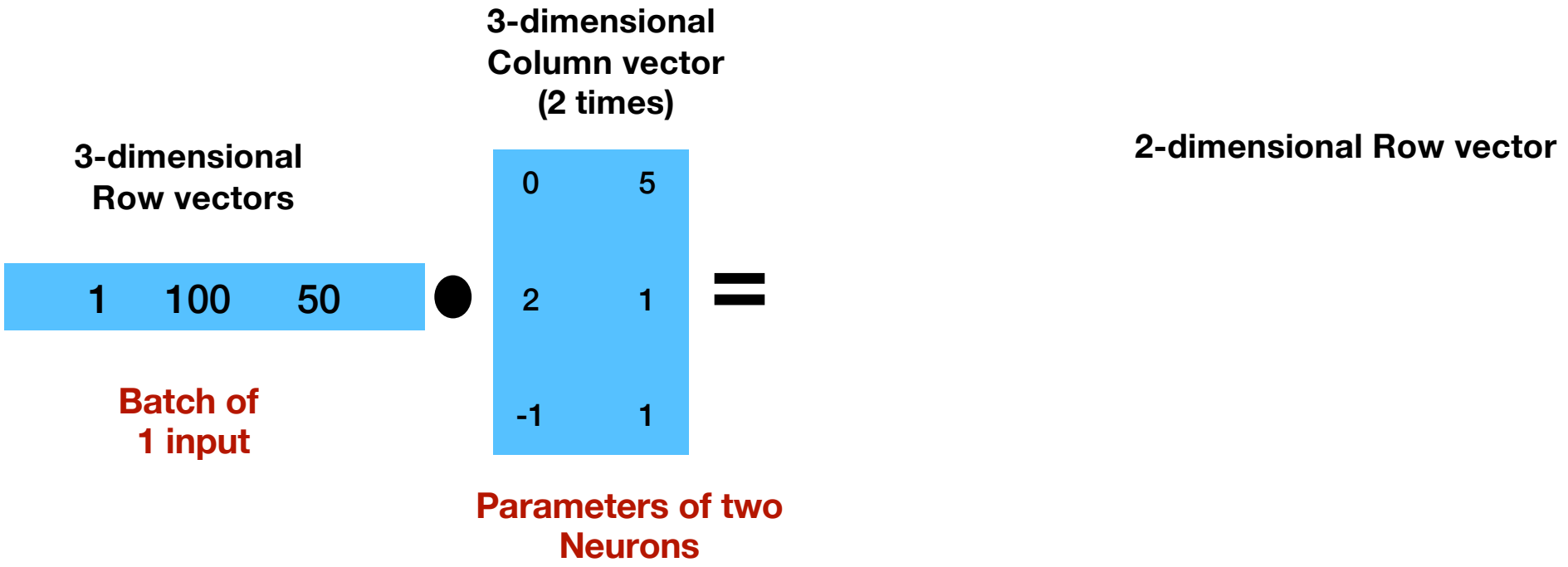
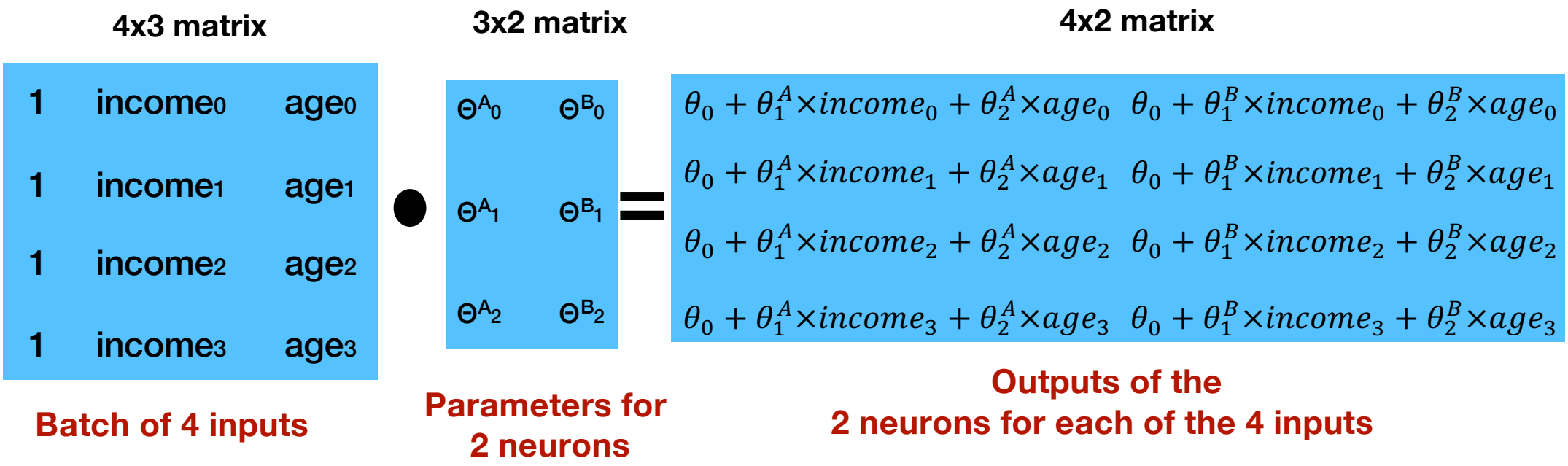
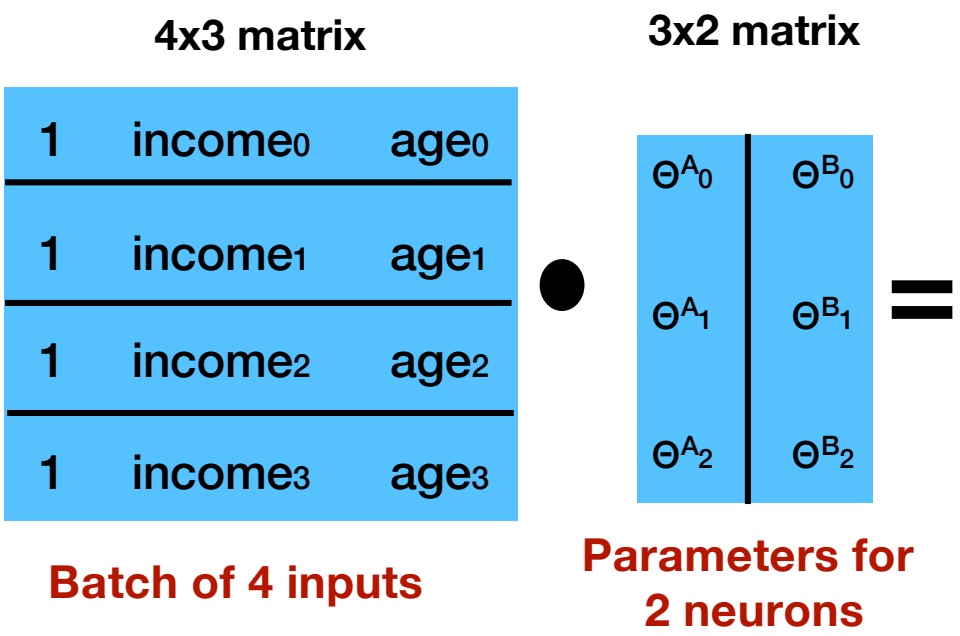


Representing Many Inner Product at Once

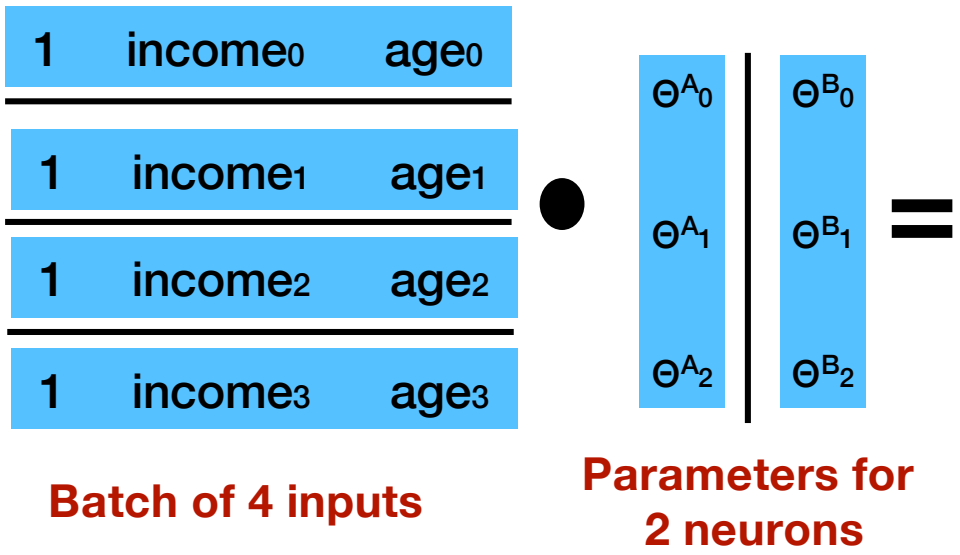
- If we combine the 2 aspects of having several inputs and several neurons, we have what is called a ***matrix multiplication***



Matrix Multiplication (1/4)



Matrix Multiplication (2/4)

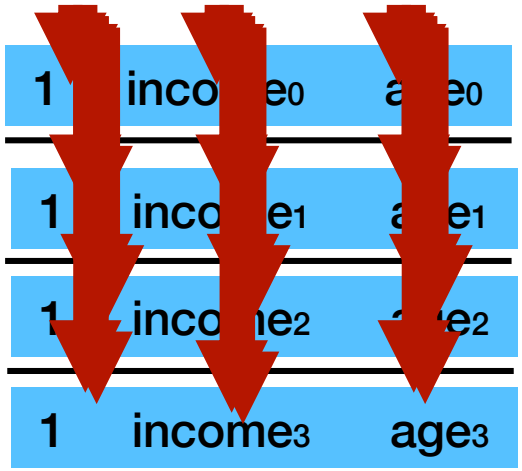


Batch of 4 inputs

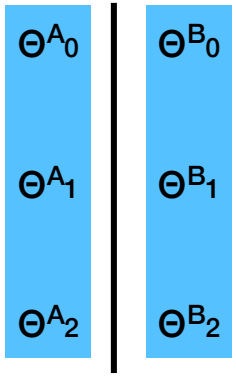
Parameters for 2 neurons

4x2 matrix

Matrix Multiplication (3/4)



Batch of 4 inputs



Parameters for 2 neurons

4x2 matrix

$$\begin{matrix} \theta_0^A + \theta_1^A \times income_0 + \theta_2^A \times age_0 & \theta_0^B + \theta_1^B \times income_0 + \theta_2^B \times age_0 \\ \theta_0^A + \theta_1^A \times income_1 + \theta_2^A \times age_1 & \theta_0^B + \theta_1^B \times income_1 + \theta_2^B \times age_1 \\ \theta_0^A + \theta_1^A \times income_2 + \theta_2^A \times age_2 & \theta_0^B + \theta_1^B \times income_2 + \theta_2^B \times age_2 \\ \theta_0^A + \theta_1^A \times income_3 + \theta_3^A \times age_3 & \theta_0^B + \theta_1^B \times income_3 + \theta_3^B \times age_3 \end{matrix}$$

Matrix Multiplication (4/4)

- We represent many inputs as a matrix
- We represent many neurons as a matrix
- Matrix multiplication compute the output score of all the neurons for all the inputs

4x3 matrix

1	$income_0$	age_0
1	$income_1$	age_1
1	$income_2$	age_2
1	$income_3$	age_3

Batch of 4 inputs

$\bullet$

3x2 matrix

$\Theta^A{}_0$	$\Theta^B{}_0$
$\Theta^A{}_1$	$\Theta^B{}_1$
$\Theta^A{}_2$	$\Theta^B{}_2$

Parameters for 2 neurons

$=$

4x2 matrix

$\theta_0^A + \theta_1^A \times income_0 + \theta_2^A \times age_0$	$\theta_0^B + \theta_1^B \times income_0 + \theta_2^B \times age_0$
$\theta_0^A + \theta_1^A \times income_1 + \theta_2^A \times age_1$	$\theta_0^B + \theta_1^B \times income_1 + \theta_2^B \times age_1$
$\theta_0^A + \theta_1^A \times income_2 + \theta_2^A \times age_2$	$\theta_0^B + \theta_1^B \times income_2 + \theta_2^B \times age_2$
$\theta_0^A + \theta_1^A \times income_3 + \theta_2^A \times age_3$	$\theta_0^B + \theta_1^B \times income_3 + \theta_2^B \times age_3$

Outputs of the 2 neurons for each of the 4 inputs

Exercise 1

2x3 matrix

1	100	50
1	10	20

Batch of 2 inputs

•

3x2 matrix

1	0
2	1
1	-1

Parameters for
2 neurons

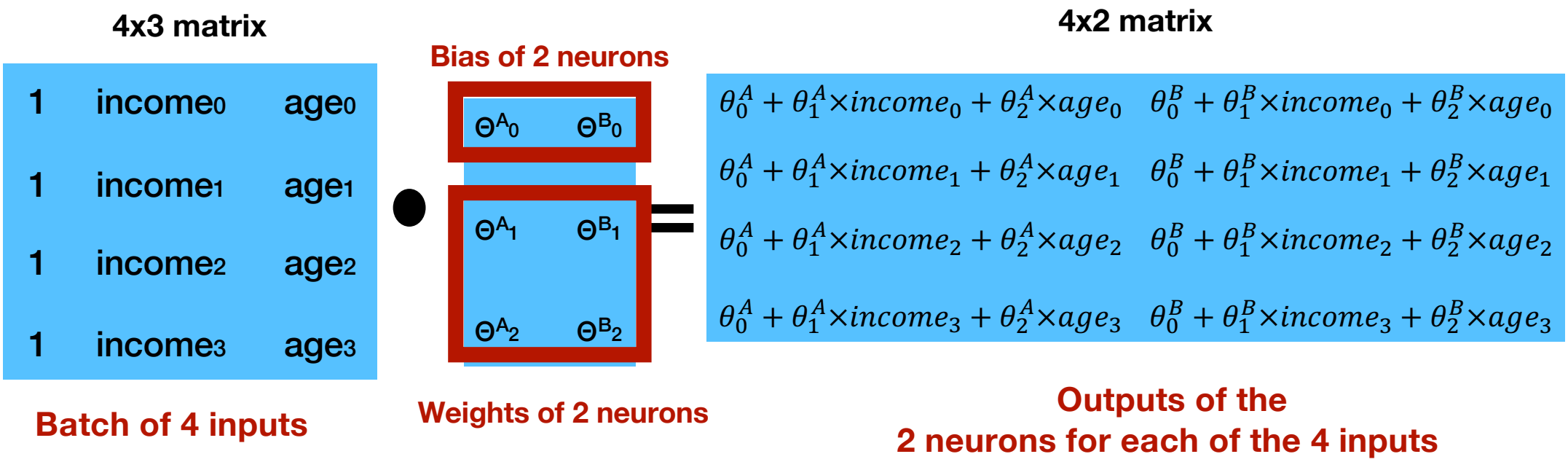
=

2x2 matrix

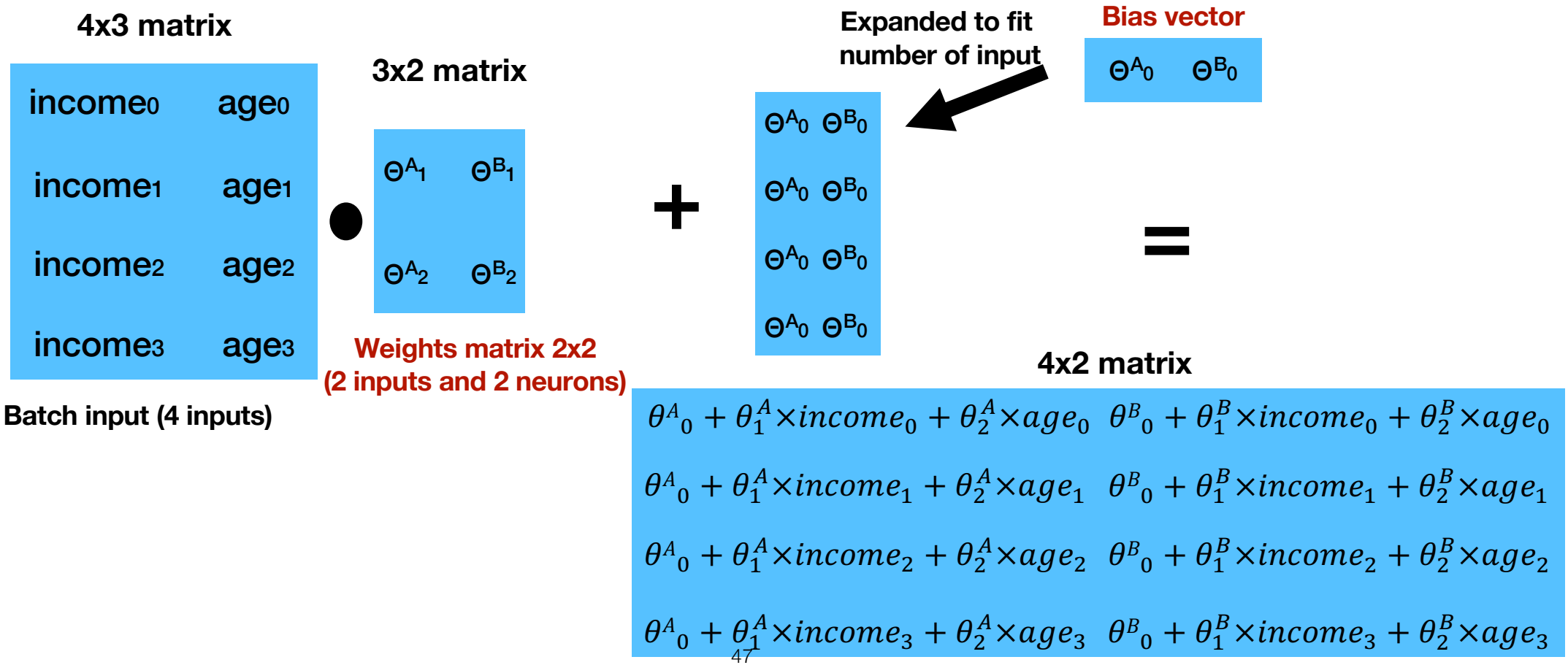
Outputs of the
2 neurons for each of the 2 inputs

Computing the Output of Many Neurons for Many Inputs (1/2)

- In practice, we separate the ***weights*** from the ***bias***



Computing the Output of Many Neurons for Many Inputs (2/2)



Exercise 2

Bias for 2 neurons

1	0

2x2 matrix

100	50
10	20

Batch of 2 inputs

•

2x2 matrix

2	1
1	-1

Weights for 2 neurons

=

2x2 matrix

Score outputs of the 2 neurons for each of the 2 inputs

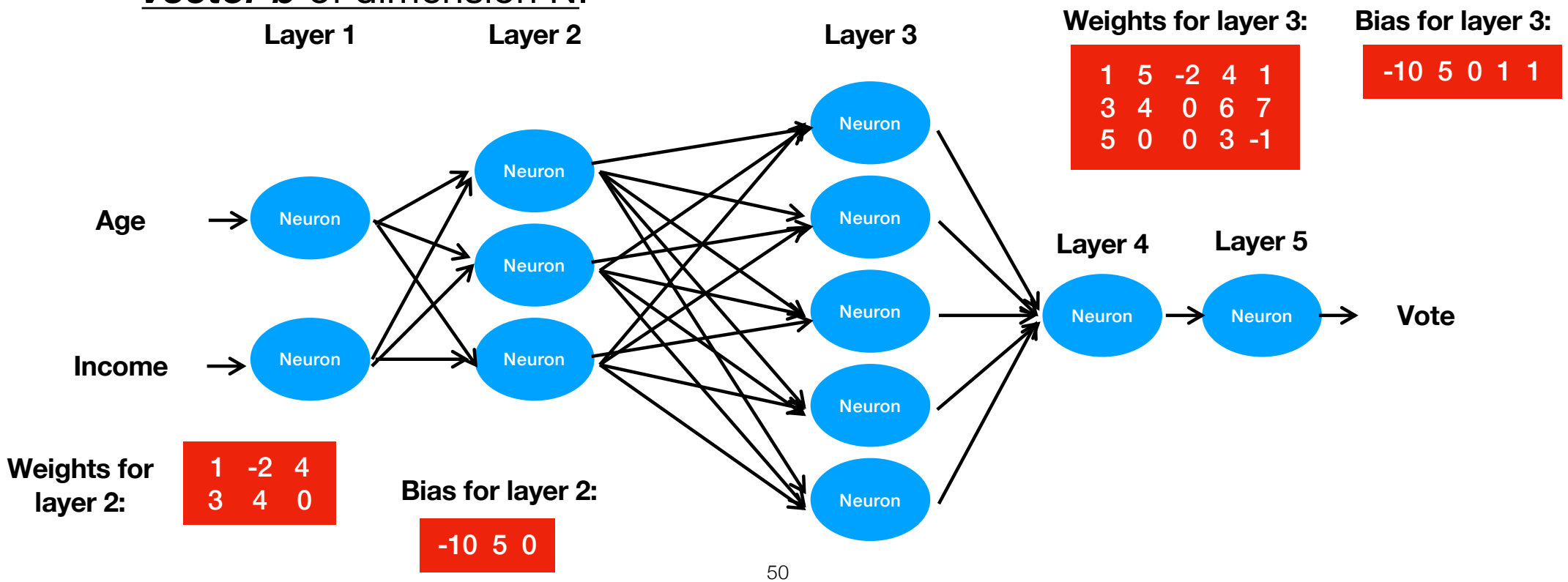
Matrix Multiplication for FNN

- Important: using these matrix representations only work if all the neurons have the same input and are fully connected

- Then, in practice, a **Fully Connected layer of N neurons with K input** can be represented by a ***matrix of weights W*** of shape KxN, and a ***bias vector b*** of dimension N.

Feed-Forward Networks with Fully Connected Layers

- Then, in practice, a **Fully Connected layer of N neurons with K input** can be represented by a ***matrix of weights W*** of shape KxN, and a ***bias vector b*** of dimension N.



Visualization for FNN

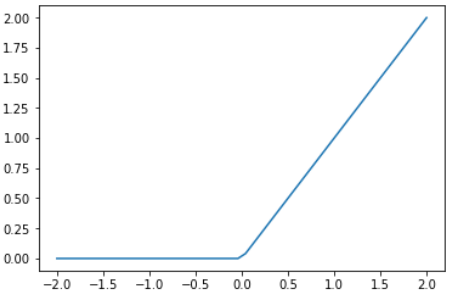
- An interactive Neural Network with Fully Connected Layers
  - From some nice people at Google
  - https://playground.tensorflow.org

Activations

- Some possible activation functions:

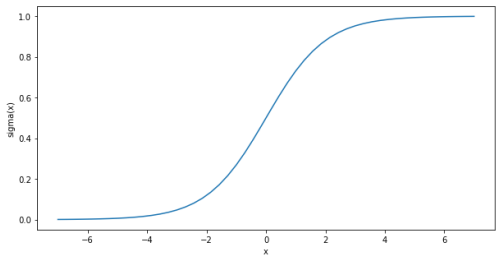
Rectified Linear Unit

$$ReLU(x) = max(x, 0)$$



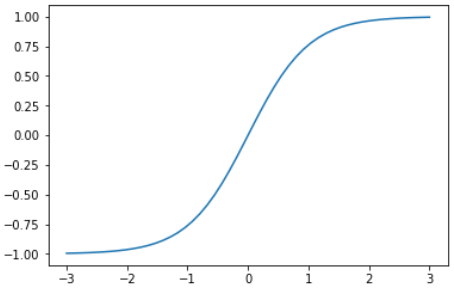
Sigmoid (a.k.a logistic function)

$$\sigma(x) = \frac{1}{1 + exp(-x)}$$



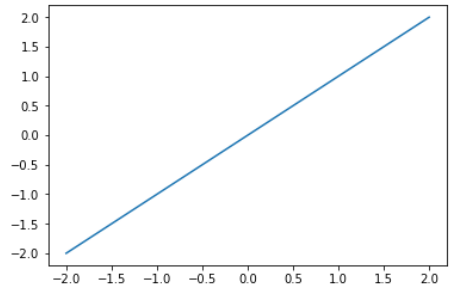
Hyperbolic Tangent (= a linear transformation of the sigmoid)

$$tanh(x) = 2\sigma(2x) - 1$$



Linear function (useless as activation function)

$$f(x) = x$$



L1 and L2 Regularization

- We have discussed that L2 Regularization can be used for reducing overfitting
  - -> *Change Regularization and Regularization Rate to observe the effect of Regularization*

L1 Regularization

- We note $|\Theta|^1$ the sum of the absolute value of all parameters Θ_k (this is called the “L1 Norm”)
- Then we add this quantity to the loss we want to minimize:

$$Loss = MeanSquaredError = \frac{1}{N} \cdot \sum_i (model(cig_i, bmi_i, ismale_i) - age_i)^2$$

$$Loss = \frac{1}{N} \cdot \sum_i (model(cig_i, bmi_i, ismale_i) - age_i)^2 + \lambda|\vec{\theta}|$$

- Then we apply Gradient Descent to this new loss

L2 Regularization

- We note $|\Theta|^2$ the sum of the square of all parameters Θ_k (this is called the "L2 Norm")
- Then we add this quantity to the loss we want to minimize:

$$Loss = MeanSquaredError = \frac{1}{N} \cdot \sum_i (model(cig_i, bmi_i, ismale_i) - age_i)^2$$

$$Loss = \frac{1}{N} \cdot \sum_i (model(cig_i, bmi_i, ismale_i) - age_i)^2 + \lambda |\vec{\theta}|^2$$

- Then we apply Gradient Descent to this new loss

Random Restarts

- Because the weights of a neural network are initialized randomly, the result of a training will change every time we reinitialize the network
- Therefore, to obtain best performances, it is common to train a networks several times with different random initializations, and keep the best result
  - -> *Press the reload button to restart a training with new random weight initialization*

Regularization Methods for Neural Network

- When we train a network with many neurons, the danger of overfitting is large
- There are a few technics that are very efficient at preventing this:
  - Early Stopping
  - Dropout
  - Weight Decay
  - Stochastic Gradient Descent

Early Stopping

- We keep a validation set separate from the training data
- We fix a ***patience*** number (typically patience = 10 or 20)
- During training, if we see no improvement on a validation set after ***patience*** measures, we stop the training
  - *Check the evolution of validation loss to detect when training should be stopped*

Dropout

- During training, we add random noise to disturb the network
- In practice, we randomly “cut” a certain proportion of connections (typically 10% to 50%)
  - *Check the effect of adding noise*

Stochastic Gradient Descent

- Instead of computing the gradient of the loss for all the training data, we compute it for a subsampled part of the training data
- This is actually done anyway to get faster training
- But it is also beneficial to prevent overfitting even if you could afford to compute the gradient for all the examples at once
    - -> *change batch size to observe what happens when we compute gradient on more or less examples*

Input Features

- We see that even for a simple model (like linear regression), adding new functions of the input can increase the capacity of the model
  - -> *Add input features and see the effect when there are few neurons*

Report

- Submit the report of exercises 1 and 2 **in pdf** via PandA
  - Submission due: **next lecture**
  - Name the pdf file as **student id_name**.

Google Colab Notebook

- Let us train FNN with Google Colab notebook

https://shorturl.at/RfCjU