Problem Set 3

Chen Li Introduction to Game Theory

Due: July 21, 2025

1. Subgame Perfect Equilibrium.

(a) Find all the subgame perfect equilibria of the following 2-stage game:

- Stage 1: The following normal-form game is played.

		Player 2	
		x_2	y_2
Player 1	x_1	(1, 1)	(0, 1)
	y_1	(1, 0)	(0, 0)

- Stage 2: After both players observe the outcome of the play of Stage 1, the following normal-form game is played.

		Player 2	
		z_2	w_2
Player 1	z_1	(1, -1)	(0, 0)
	w_1	(0, 0)	(2, 2)

- Each player's payoff is the (undiscounted) sum of Stage 1 and Stage 2 payoffs.

(b) (Optional) Find all the subgame perfect equilibria when the Stage 2 game of (a) is replaced by the following normal-form game:

		Player 2	
		z_2	w_2
Player 1	z_1	(1, 1)	(0, 0)
	w_1	(0, 0)	(2, 2)

2. Finitely Repeated Game (Gibbons). The accompanying simultaneous-move game is played twice, with the outcome of the first stage observed before the second stage begins. There is no discounting. The variable x is greater than 4, so that (4, 4) is not an equilibrium payoff in the one-shot game. For what values of x is the following strategy (played by both players) a subgame perfect equilibrium?

Strategy: Play Q_i in the first stage. If the first-stage outcome is (Q_1, Q_2), play P_i in the second stage. If the first-stage outcome is (y, Q_2) where $y \neq Q_1$, play R_i in the second stage. If the first-stage outcome is (Q_1, z) where $z \neq Q_2$, play S_i in the second stage. If the first-stage outcome is (y, z) where $y \neq Q_1$ and $z \neq Q_2$, play P_i in the second stage.

		Player 2			
		P_2	Q_2	R_2	S_2
Player 1	P_1	(2, 2)	(x, 0)	(-1, 0)	(0, 0)
	Q_1	(0, x)	(4, 4)	(-1, 0)	(0, 0)
	R_1	(0, 0)	(0, 0)	(0, 2)	(0, 0)
	S_1	(0, -1)	(0, -1)	(-1, -1)	(2, 0)

3. Repeated Game. Consider the following game:

		Player 2	
		C	D
Player 1	C	(0, 0)	(-5, 2)
	D	(2, -5)	(-3, -3)

(a) Find the subgame perfect equilibrium when the above game is repeated twice.

For the following questions, consider infinitely repetition of the above game.

(b) Suppose both players play the "Tit-for-Tat" strategy. That is, both players play C at the first stage and, at stage $t (t \geq 2)$, both play what the other player has played at stage $t-1$. Hence, if this strategy profile forms a subgame perfect equilibrium, we get the ideal outcome (C, C) for every stage. Can this Tit-for-Tat strategy profile form a subgame perfect equilibrium? If yes, find the range of discount factor δ supporting this strategy profile as a subgame perfect equilibrium. If not, explain the reason in detail.

(c) Suppose player 1 plays the "Defect Forever" strategy and player 2 plays the "Grim Trigger" strategy. Can this strategy profile form a subgame perfect equilibrium? If yes, find the range of discount factor δ supporting this strategy profile as a subgame perfect equilibrium. If not, explain the reason in detail.