(3):

(a): Let X_n = number of type - α individuals at generation n.
if $X_n = i$:

$P($ type α individual is chosed to reproduce $) = \frac{i}{N}$

$\rightarrow P($ type A $\sim$ $) = \frac{N-i}{N}$

$\Rightarrow P(X_n = X_n + 1)$: $p_{i,i+1} = \underbrace{\frac{i}{N}}_{\boxed{\text{chose } \alpha}} \cdot \underbrace{\frac{N-i}{N}}_{\boxed{\text{reproduce by } \alpha \text{, chose A}}}$

$\Rightarrow P(X_n = X_n - 1)$: $p_{i,i-1} = \frac{N-i}{N} \cdot \frac{i}{N}$ (opposite)

$\Rightarrow P(X_n = X_n)$: $p_{i,i} = \underbrace{\frac{i^2}{N^2}}_{\boxed{\text{double choose } \alpha}} + \underbrace{\frac{(N-i)^2}{N^2}}_{\boxed{\text{double choose A}}}$

boundary state:

$p_{0,0} = 1 = p_{N,N}$ (absorbing state)

(b): Let $u(i) = P(\underbrace{\text{absorbed at } N}_{(\alpha)}, \underbrace{\text{before hitting } 0}_{(A)})$

$u(i) = p_{i,i-1} u(i-1) + p_{i,i} u(i) + p_{i,i+1} u(i+1)$

$\rightarrow p_{i,i+1} u(i+1) + (p_{i,i} - 1) u(i) + p_{i,i-1} u(i-1)$

boundary: $u(0) = 0$, $u(N) = 1$

(c): recurrence to eliminate $u(i+1)$, get:

$p_{i,i+1} \cdot \alpha(i) = p_{i,i-1} \cdot \alpha(i-1)$

and: $p_{i,i+1} = p_{i,i-1} = \frac{i(N-i)}{N^2}$ $\rightarrow$: $\alpha(i) = \alpha(i-1) \Rightarrow \alpha(i)$ is a constant.

$\rightarrow u(i) = u(0) + \sum_{k=0}^{i-1} \alpha = i \cdot \alpha$ $\rightarrow$ By condition $u(N) = 1$: $\longrightarrow$

$\alpha = \frac{1}{N} \Rightarrow u(i) = \frac{1}{N}$ for every step is ± 1

(1): $h_0^{\{0\}} = 0$ and $h_N^{\{0\}} = \infty$, we can get easily

For the other state $1 \le i \le N-1$:

$$\underline{h_i^{\{0\}}} = 1 + \frac{1}{2}\underline{h_{i+1}^{\{0\}}} + \frac{1}{2}\underline{h_{i-1}^{\{0\}}}$$

we set a quadratic form:

$$\underline{A + Bi + Ci^2} = 1 + \frac{1}{2}(h_{i+1}^{\{0\}} + h_{i-1}^{\{0\}})$$

$$= 1 + \frac{1}{2}(2A + B(i+1+i-1) + C(i^2+2i+1+i^2-2i+1))$$

$$= 1 + \frac{1}{2}(2A + 2Bi + 2Ci^2 + 2C)$$

$$= 1 + \underline{A + Bi + Ci^2} + C$$

$$\Rightarrow 0 = 1 + C \longrightarrow C = -1$$

$$h_i^{\{0\}} = A + Bi - i^2$$

when $i = 0$:

$$0 = A \longrightarrow A = 0$$

$\longrightarrow h_i^{\{0\}} = i(B - i) \longrightarrow$ if $B = N$, expected hitting time to either.

But we only conside one edge, so $h_i^{\{0\}} = i(2N - i)$ $(0 \le i \le \infty)$

(2): let $p_{i,i-1} = q_i$

$\Rightarrow p_{i,i+1} = (\frac{i+1}{i})^\beta q_i \longrightarrow q_i + (\frac{i+1}{i})^\beta q_i = 1 \longrightarrow q_i = \frac{1}{1 + (\frac{i+1}{i})^\beta}$

cosider i:

when $i \to \infty$: $\frac{i+1}{i} \simeq 1 + \frac{1}{i} \Rightarrow (\frac{i+1}{i})^\beta \simeq 1 + \frac{\beta}{i}$

$q_i \longrightarrow \frac{1}{1+1+\frac{\beta}{i}} = \frac{1}{2+\frac{\beta}{i}} \simeq \frac{1}{2} - \frac{\beta}{4i} \longrightarrow \begin{cases} \beta > 0, \text{ ~~close~~ far away} \\ \beta < 0, \text{ ~~more~~ closer} \end{cases}$

$\sum_{i=1}^{\infty} (\frac{i+1}{i})^\beta = \lim_{n \to \infty} (n+1)^\beta \longrightarrow \beta$ needs: $\beta < 0$ to closed touch to 0