Introduction to Neural Networks

Fundamentals of Artificial Intelligence

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Schedule

- 1. Overview of AI and this Course (4/14)
- 2. Introduction to Python (4/21)
- 3, 4. Mathematics Concepts I, II (4/28, 5/12)
- 5, 6. Regression I, II (5/19, 5/26)
- 7. Classification (6/2)
- 8. Introduction to Neural Networks (6/9)
- 9. Neural Networks Architecture and Backpropagation (6/16)
- 10. Fully Connected Layers (6/23)
- 11, 12, 13. Computer Vision I, II, III (6/30, 7/7, 7/14)
- 14. Natural Language Processing (7/17)

Overview of This Course

11, 12, 13. Computer Vision I, II, III

14. Natural language processing

Deep Learning Applications

8. Neural network Introduction

9. Architecture and Backpropagation

10. Feedforward neural networks

Deep Learning

5. Regression I

6. Regression II

7. Classification

Basic Supervised Machine Learning

2. Python

3, 4. Mathematics Concepts I, II

Fundamental of Machine Learning

Introduction to Neural Networks

- Finally, the fancy part!

- **Neural Networks** are currently the most efficient and most used models for Machine Learning (and for AI in general, actually)

- Today, we discuss neurons, both the **biological neurons** and the **artificial neurons**

The Real (Biological) Neurons

- We are going to spend a good amount of time discussing **biological neurons**

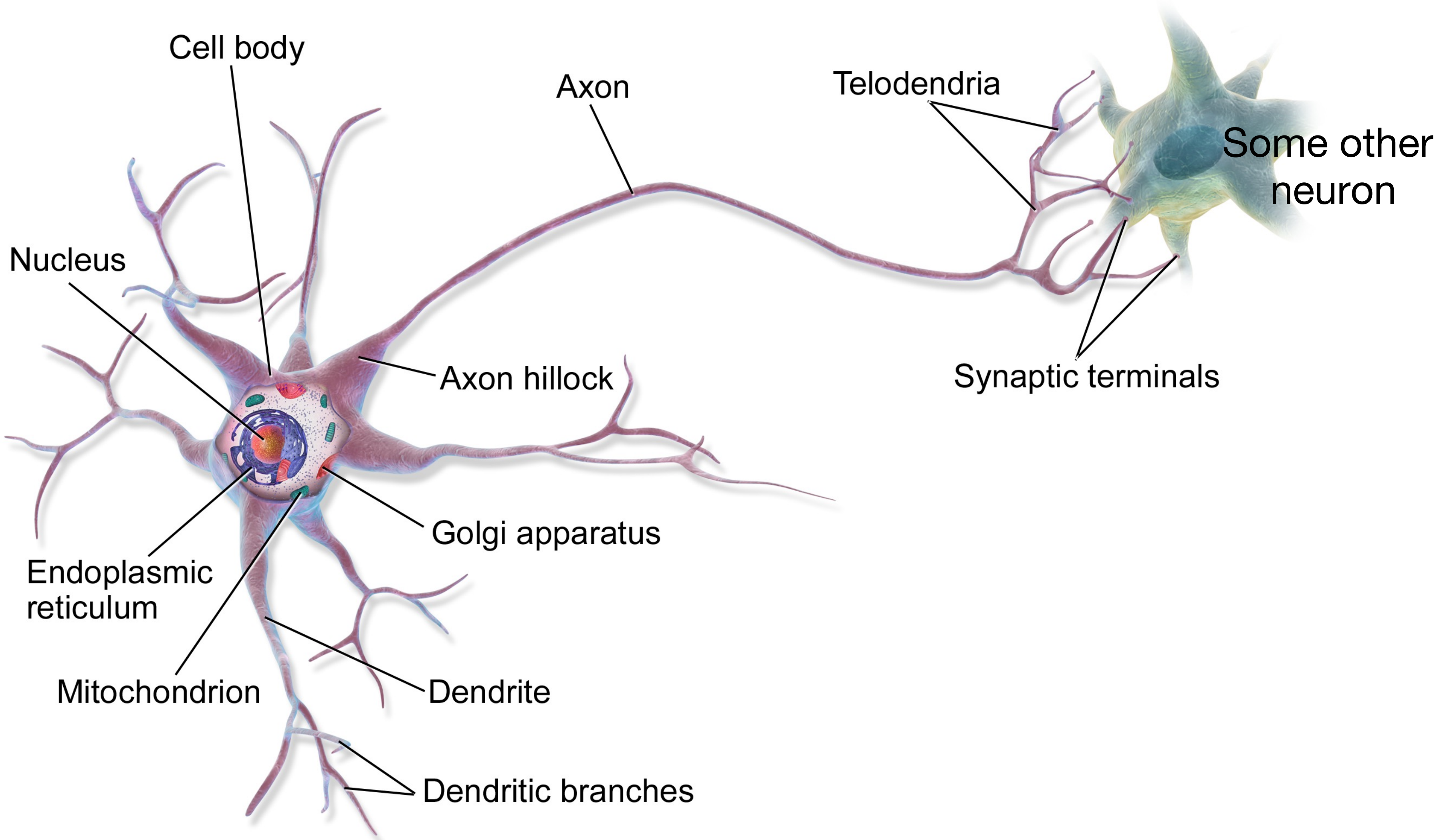
- But keep in mind that the goal of AI is not to simulate precisely a human brain

- The goal is to have software that do what we want (image recognition, etc.)

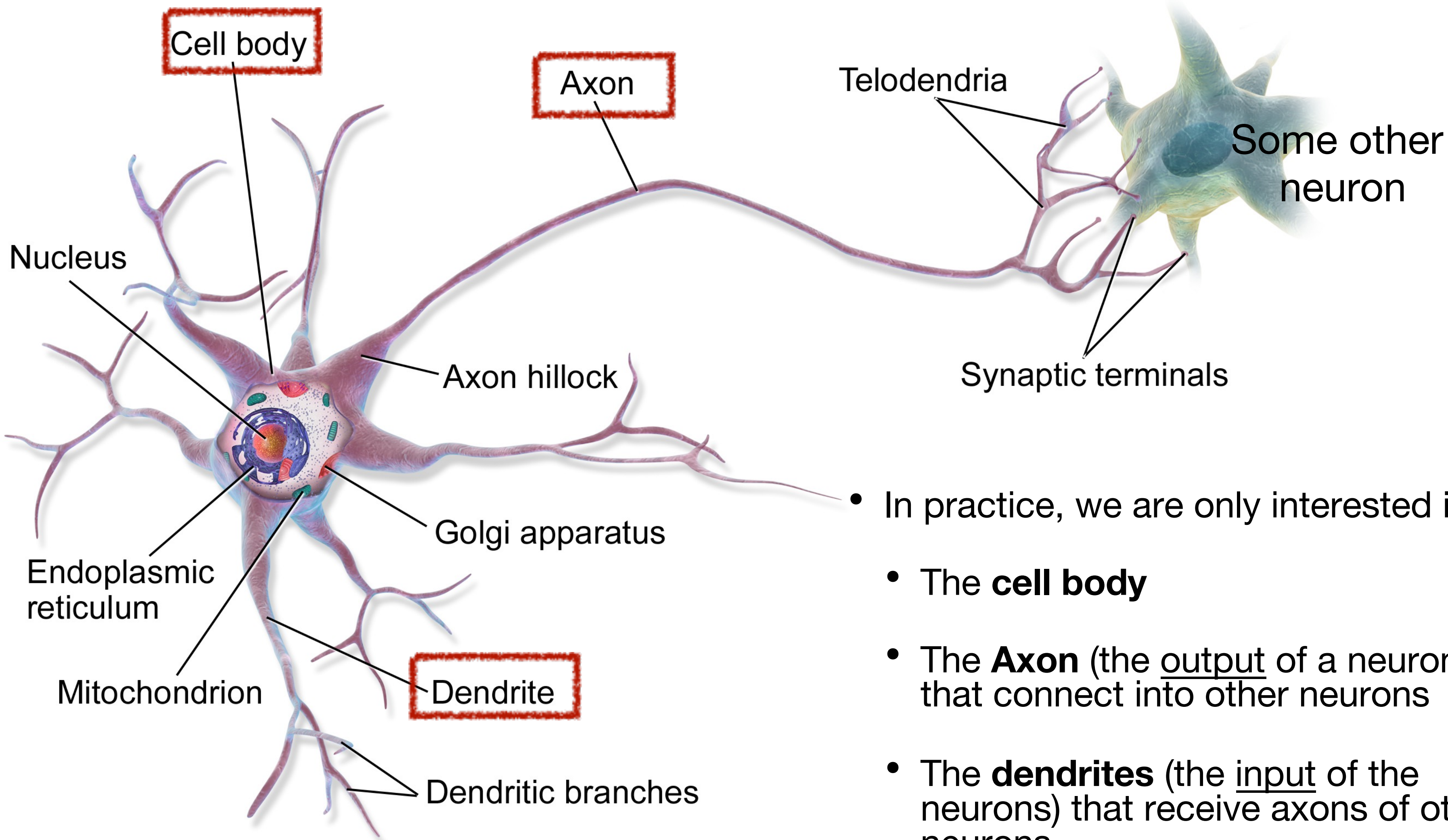
- It is actually perfectly possible to explain **Artificial Neural Networks** without mentioning how they relate to the brain

- But I think it is very interesting to see the parallel

Anatomy of a Neuron (1/2)



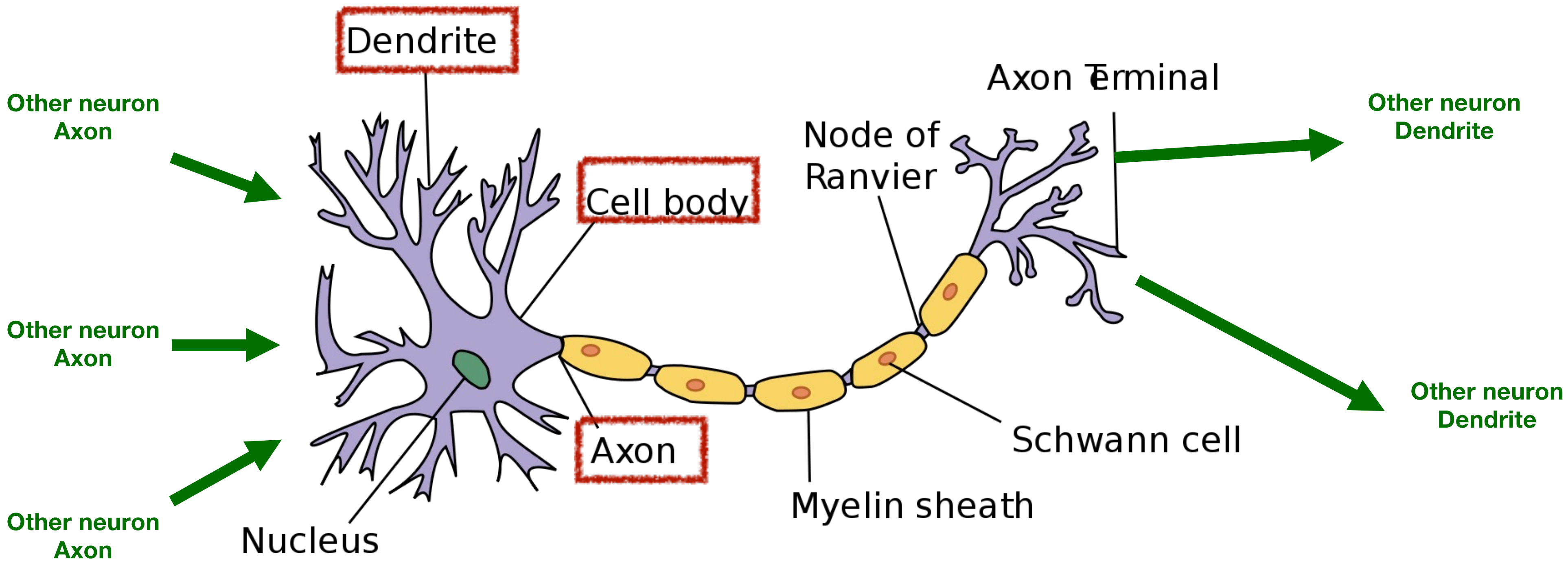
Anatomy of a Neuron (2/2)



- In practice, we are only interested in:
  - The **cell body**
  - The **Axon** (the output of a neuron) that connect into other neurons
  - The **dendrites** (the input of the neurons) that receive axons of other neurons

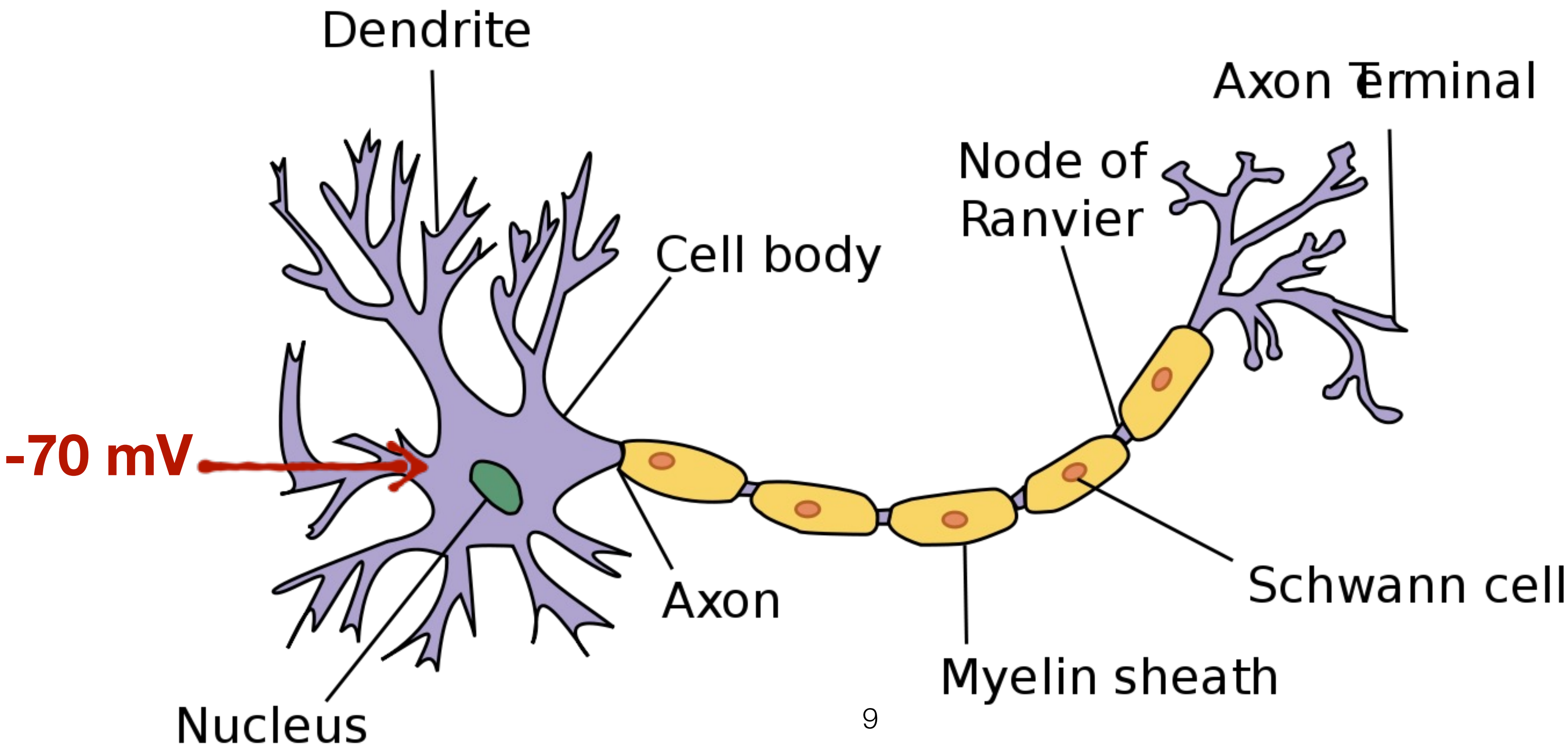
What a Neuron Does (1/4)

- The neuron receives input from other neurons from its **dendrites**
- If the sum of the input is above a certain **threshold**, it sends a signal (called *Action Potential*) on its **axon** to other neurons



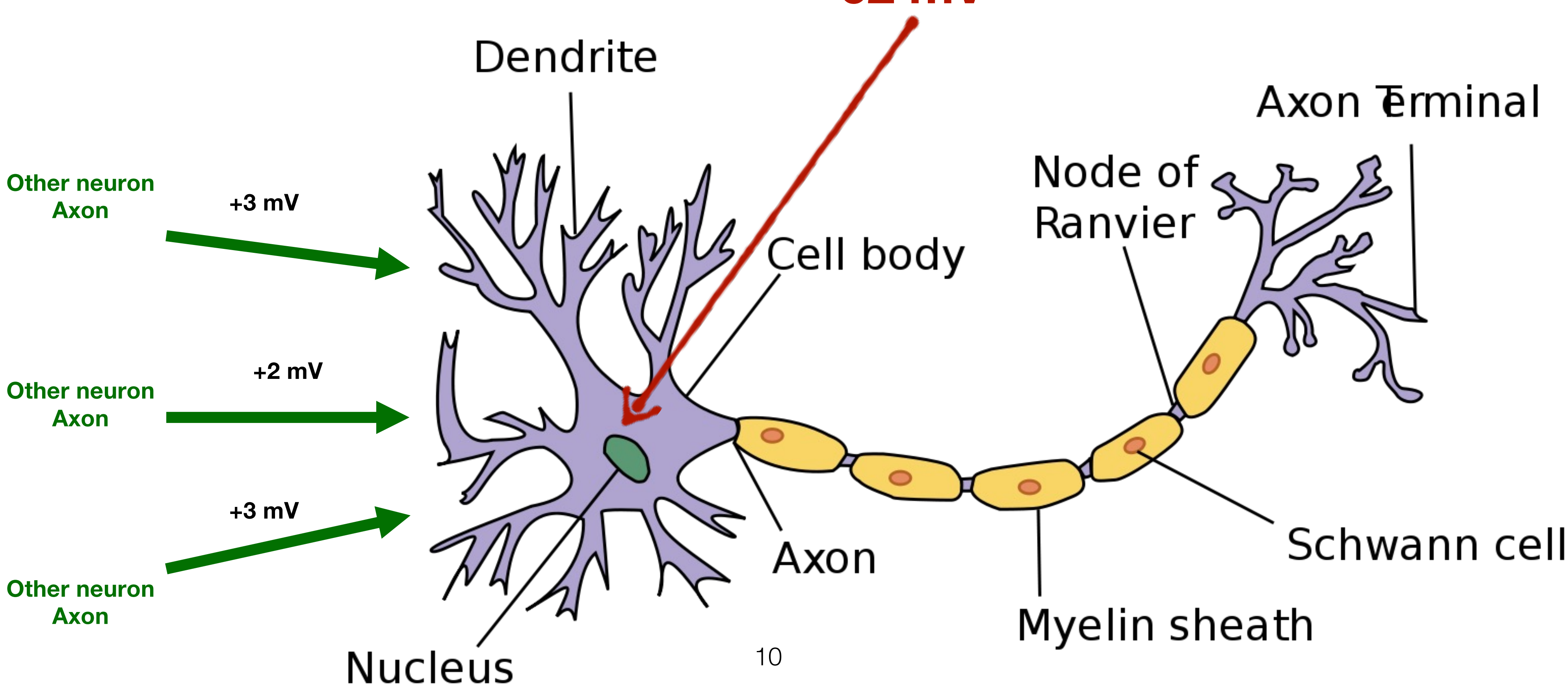
What a Neuron Does (2/4)

- At rest, the inside of a neuron has an electric potential of -70mv
- It is the “**Resting Potential**”
- Chemically created by Potassium and Sodium ions



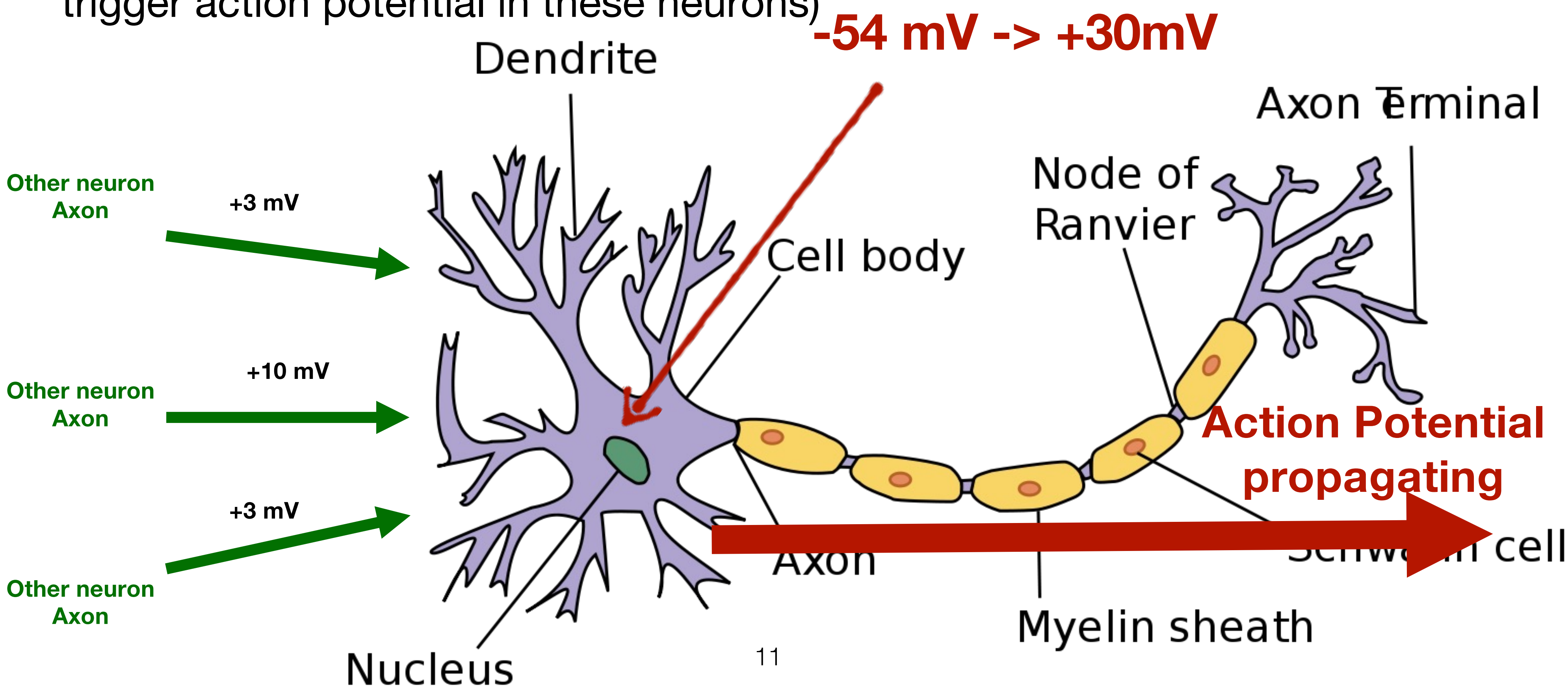
What a Neuron Does (3/4)

- The neuron might <u>receive</u> electric potential from <u>other neurons</u> on its **dendrite**
- The <u>potential</u> inside the cell body <u>can then increase</u>
- It is then called “**Graded potential**”

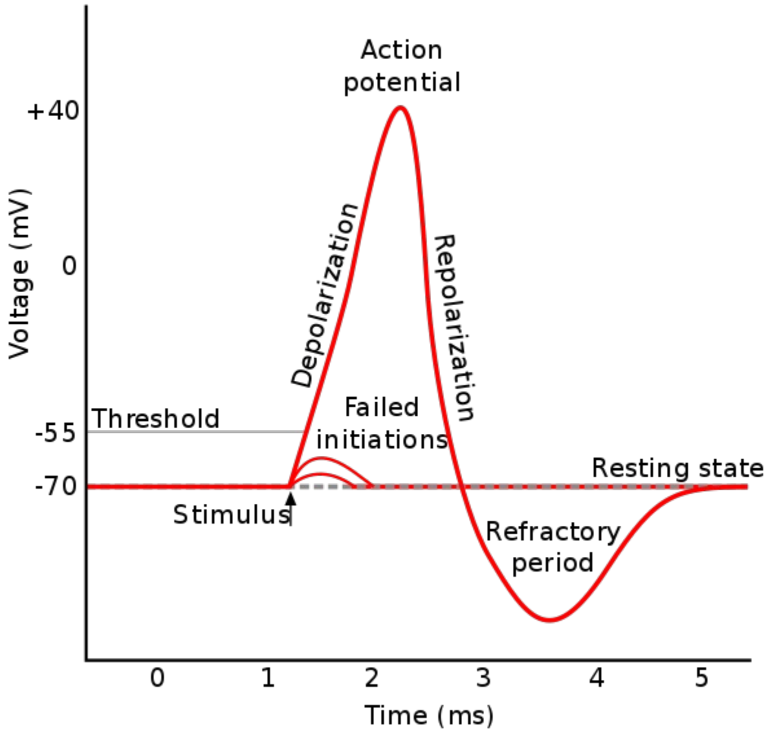


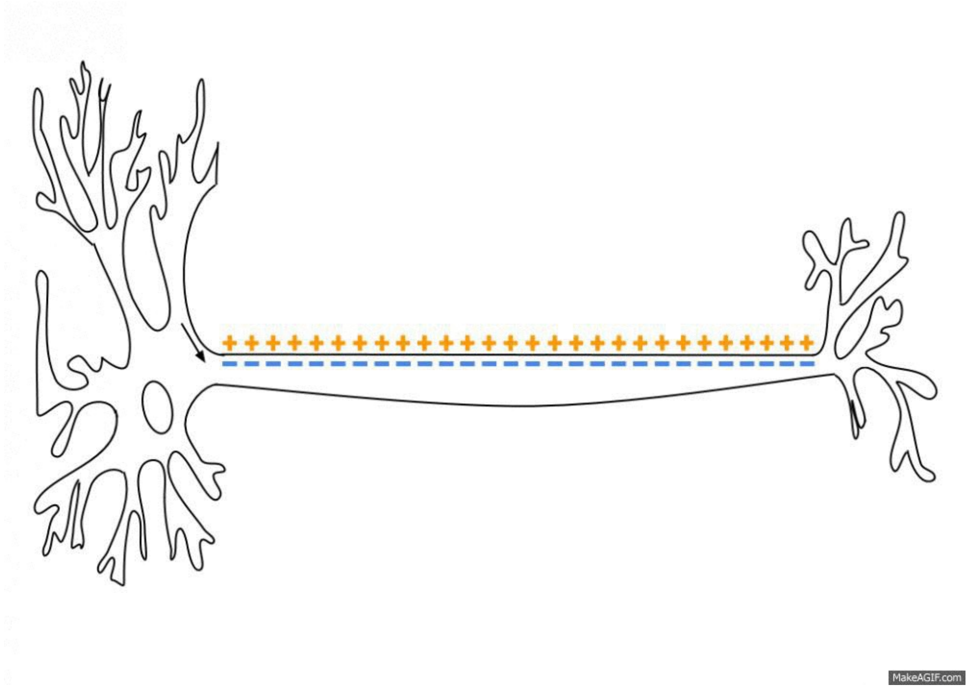
What a Neuron Does (4/4)

- If the potential inside a neuron goes above -55mV something happens
- A chemical chain reaction will suddenly increase the potential to +30mV (called the **Action Potential**)
- This potential will propagate on the **axon** to other neurons (and may then trigger action potential in these neurons)



The Action Potential

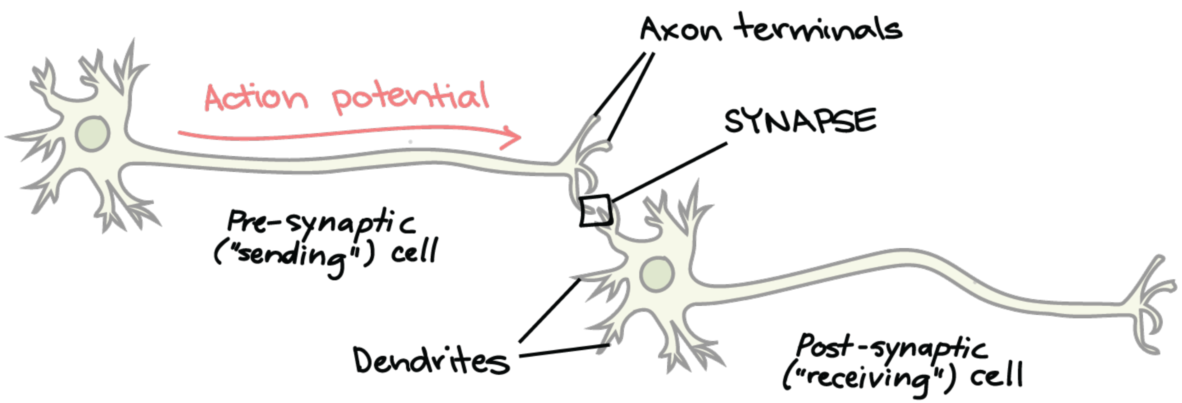




From Wikipedia, by users Chris 73 and Laurentaylorj

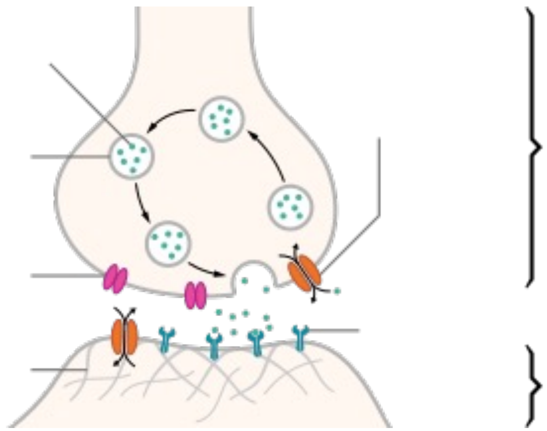
Synaptic Connections (1/2)

- The axon of a neuron is connected to the dendrites of other neurons through a “connector” called a **synapse**
- The actual potential passed to the dendrite will depend on the synapse
- There can be **excitatory synapses** or **inhibitory synapses**



From Khan Academy

Synapse



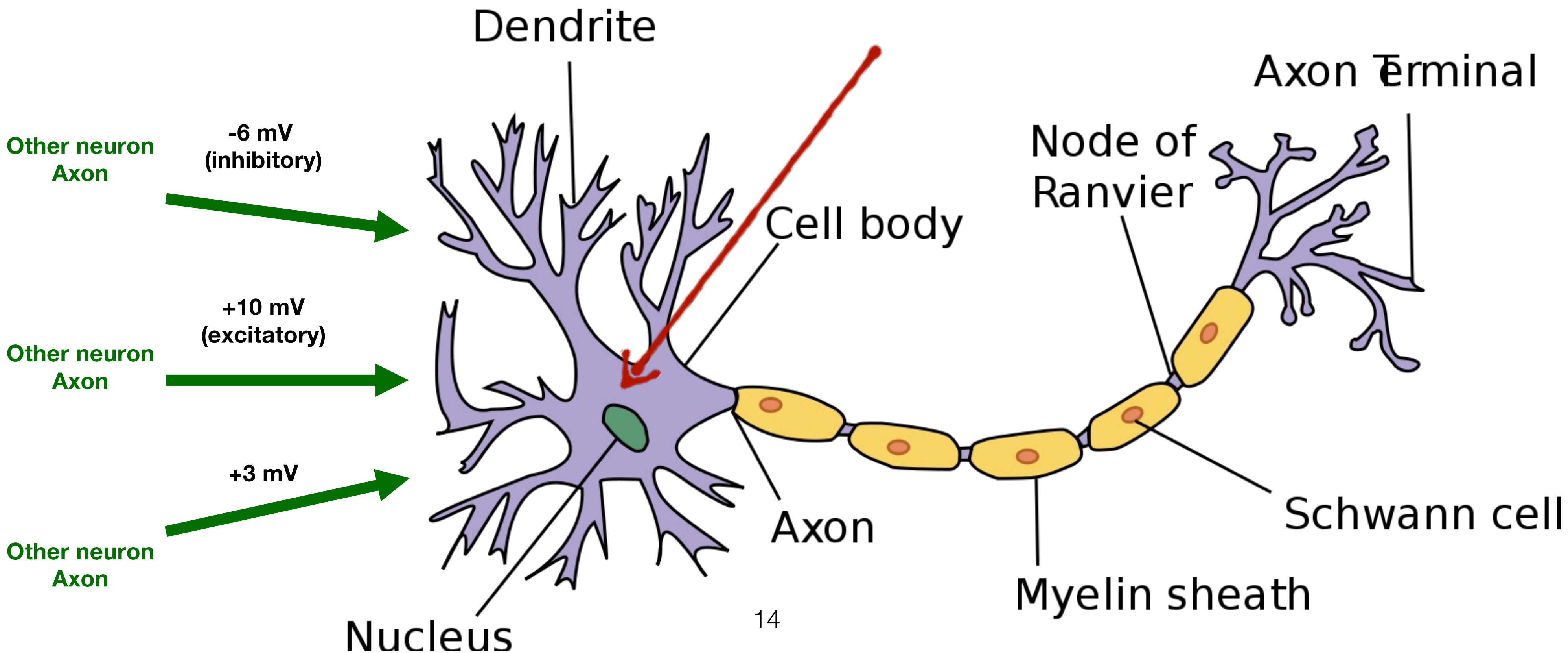
FromWikipedia

Synaptic Connections (2/2)

- The type of synaptic connection influence the effect of the action potential of previous neurons

(70mV = Resting Potential)

-70mV - 6mV + 10mV + 3mV = -63 mV

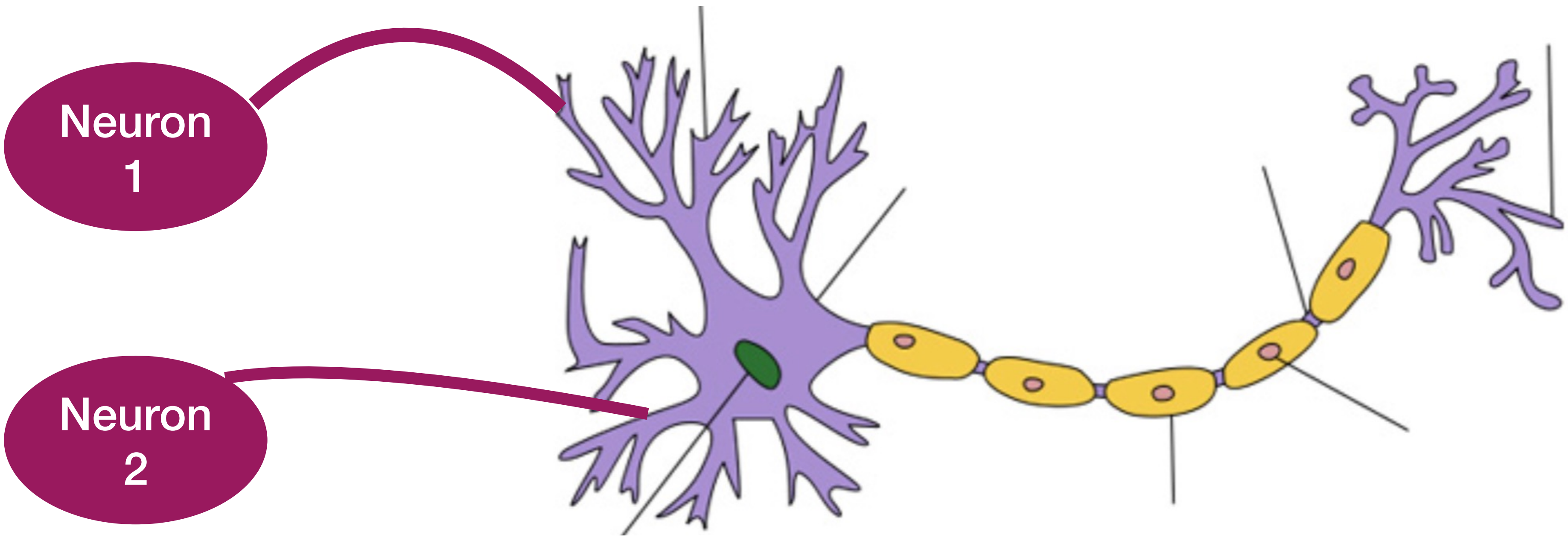


The Biological Process in a Neuron

- For a biological view about how all of this is actually happening in the neurons, let us watch a 14 minutes video by Paul Andersen:
  - https://www.youtube.com/watch?v=HYLyhXRp298
- Lot of Biology and Chemistry in this video; but **do not worry if you do not understand everything**.

From Biology to Math and Machine Learning

- Now, let us try to describe “mathematically” what a neuron is doing
- Let us consider a neuron with only 2 dendrites connected to other neurons (2 inputs)



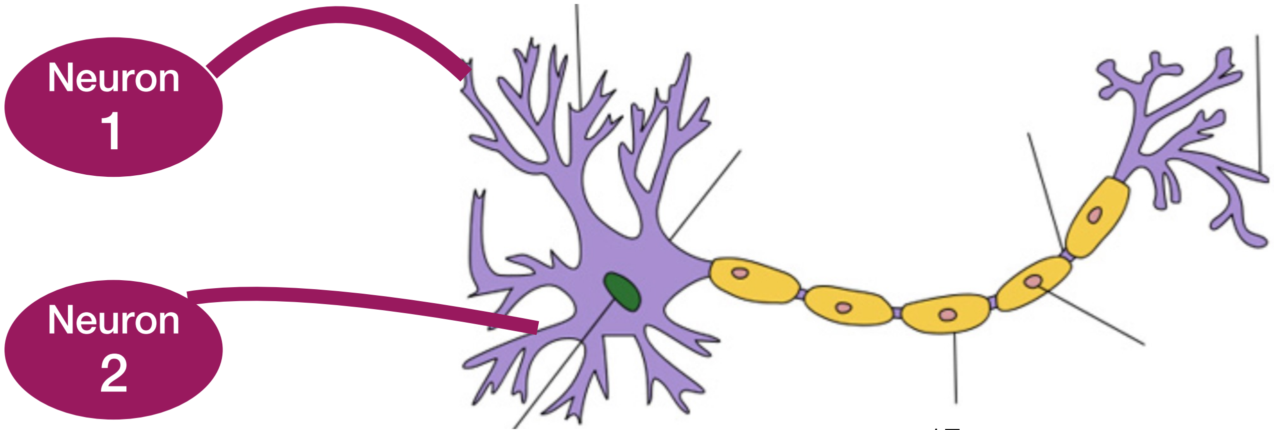
Equation of a Neuron with 2 Dendrites (1/8)

- Now, let us try to describe "mathematically" what this neuron is doing:

  1. Some input potentials arrives on its **dendrites**

  2. The potential are accumulated in the **body** (*graded potential*)

  3. If the *graded potential* is <u>larger than a given threshold</u>**,** the neuron fires an <u>*action potential*</u> on the **axon**



Equation of a Neuron with 2 Dendrites (2/8)

- Now, let us try to describe “mathematically” what this neuron is doing:

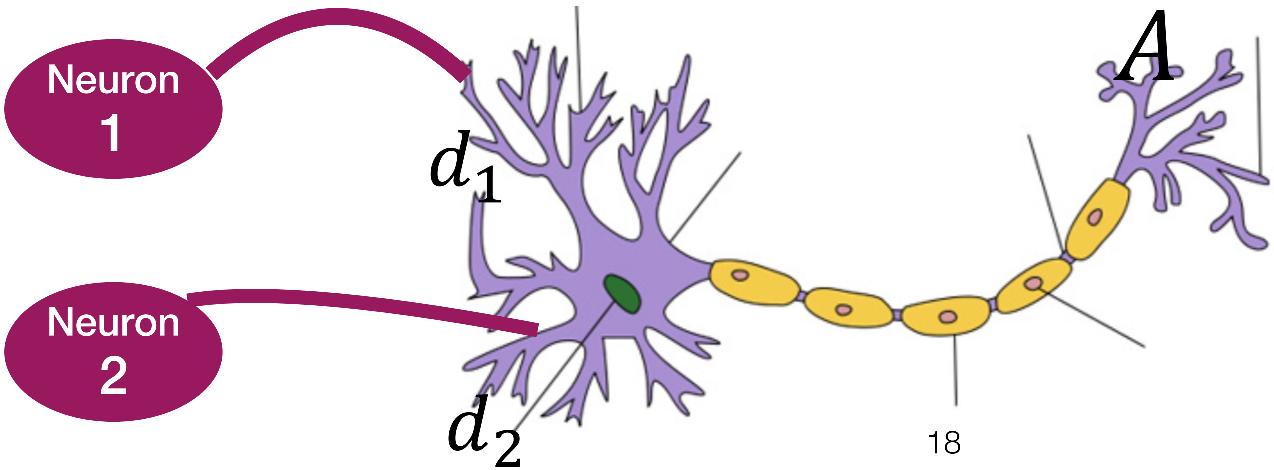
  1. Some input potentials arrives on its **dendrites**

$$\boxed{d_1 \quad d_2}$$

  2. The potentials are accumulated in the **body** (*graded potential*)

$$\boxed{gradedP = -70mV + d_1 + d_2}$$

  3. If the *graded potential* is <u>larger than a given threshold</u>, the neuron fires an <u>*action potential*</u> on the **axon**



$A = -70mV$ If $gradedP <= -55mV$

$A = +30mV$ If $gradedP > -55mV$

Equation of a Neuron with 2 Dendrites (3/8)

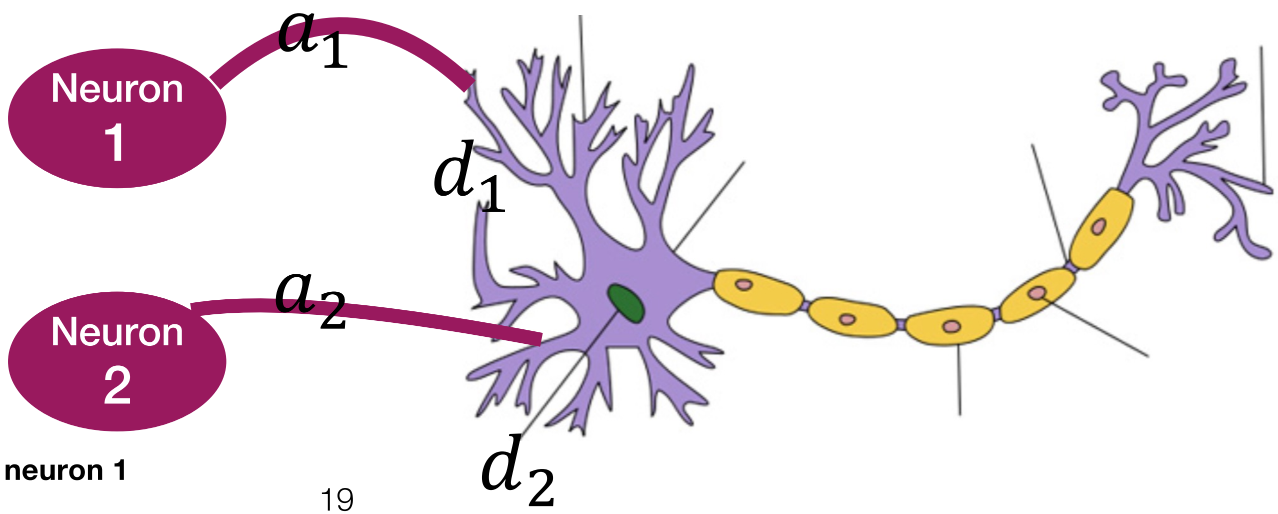
- The <u>dendrite potential</u> will <u>depend</u> on the **type of the synaptic connection.** Let us suppose the synaptic connection can be represented by a <u>parameter θ</u>

$$\boxed{\begin{aligned} d_1 &= \theta_1 \times a_1 \\ d_2 &= \theta_2 \times a_2 \end{aligned}}$$

If <u>strong excitatory connection</u> between the axon of Neuron1 and the dendrite: <u>θ is large</u>

If <u>weak excitatory connection</u> between the axon of Neuron1 and the dendrite: <u>θ is small</u>

If <u>inhibiting connection</u>: <u>θ is negative</u>



a_1 is action potential on axon of neuron 1

Equation of a Neuron with 2 Dendrites (4/8)

- Now, let us try to describe “mathematically” what this neuron is doing:

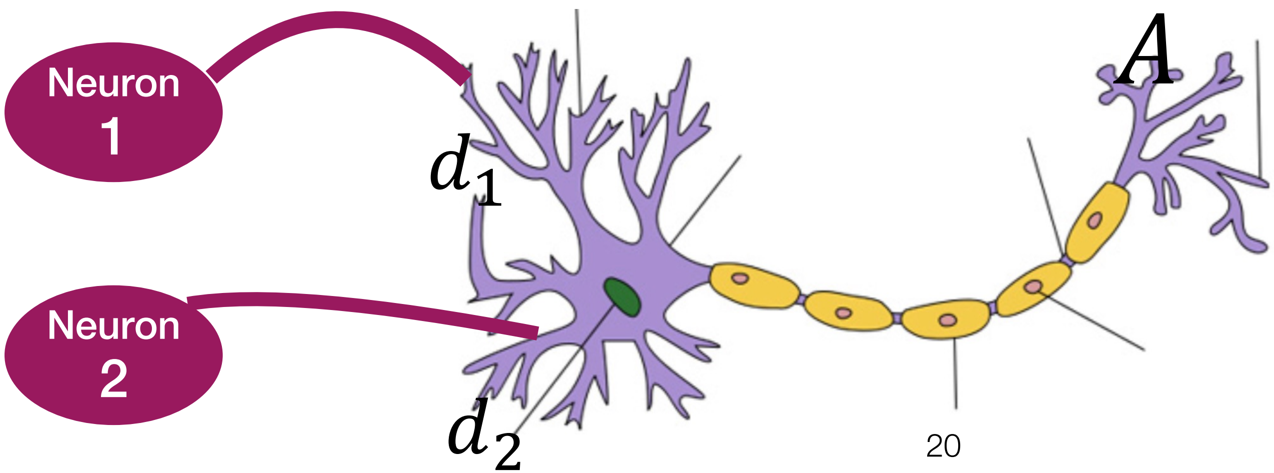
1. Some input potentials arrives on its **dendrites**

$$\boxed{d_1 = \theta_1 \times a_1 \quad d_2 = \theta_2 \times a_2}$$

2. The potentials are accumulated in the **body** (*graded potential*)

$$\boxed{gradedP = -70mV + \theta_1 \times a_1 + \theta_2 \times a_2}$$

3. If the *graded potential* is larger than a given threshold, the neuron fires an *action potential* on the **axon**



$$A = -70mV$$

If $gradedP <= -55mV$

$$A = +30mV$$

If $gradedP > -55mV$

Equation of a Neuron with 2 Dendrites (5/8)

- Let us combine the two equations:

$$\boxed{gradedP = -70mV + \theta_1 \times a_1 + \theta_2 \times a_2}$$

$A = -70mV$

If $gradedP \leq -55mV$

$A = +30mV$

If $gradedP > -55mV$

$A = -70mV$

If $-70mV + \theta_1 \times a_1 + \theta_2 \times a_2 \leq -55mV$

$A = +30mV$

If $-70mV + \theta_1 \times a_1 + \theta_2 \times a_2 > -55mV$

$A = -70mV$

If $\theta_1 \times a_1 + \theta_2 \times a_2 - 15mV \leq 0$

$A = +30mV$

If $\theta_1 \times a_1 + \theta_2 \times a_2 - 15mV > 0$

Equation of a Neuron with 2 Dendrites (6/8)

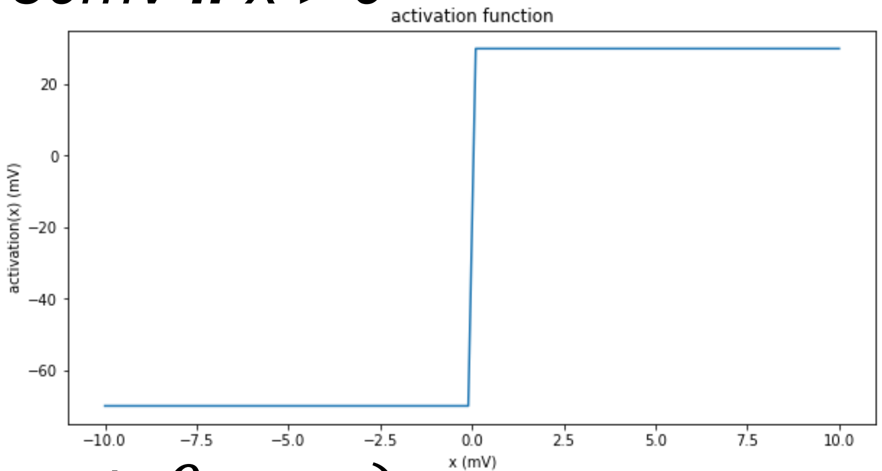
$A = -70mV$

If $\theta_1 \times a_1 + \theta_2 \times a_2 - 15mV \leq 0$

$A = +30mV$

If $\theta_1 \times a_1 + \theta_2 \times a_2 - 15mV > 0$

- We can define an **activation** function:
  - activation(*x*) = - *70mV* **if** *x <= 0 // 30mV* ***if*** *x > 0*



Then we can write:

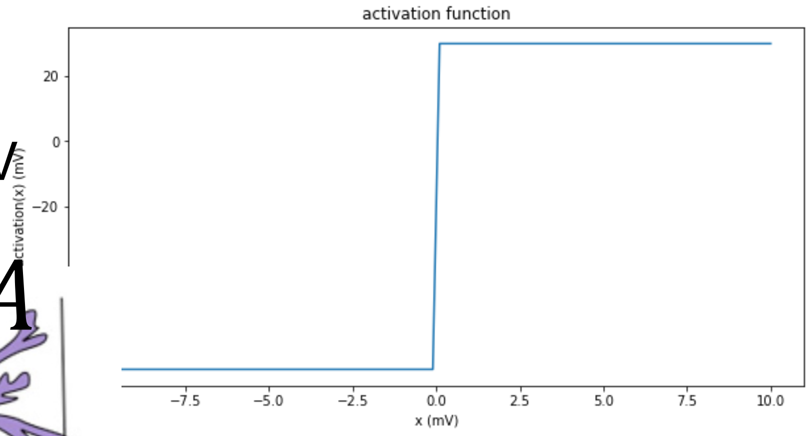
$$A = activation(-15mV + \theta_1 \times a_1 + \theta_2 \times a_2)$$

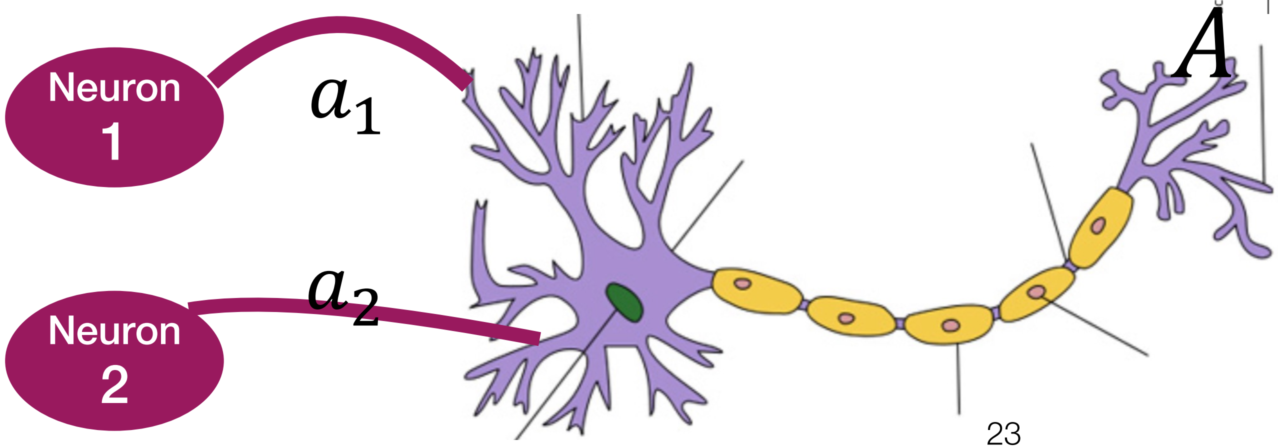
Equation of a Neuron with 2 Dendrites (7/8)

- Our final equation:

$$\boxed{A = activation(\theta_1 \times a_1 + \theta_2 \times a_2)}$$

  - A: Action Potential of our Neuron
  - a_1: action potential from Neuron1
  - θ_1 : represent the strength of synaptic connection between Neuron1 and our neuron
  - *activation(x)* = -70mV **if** x < 15mV **else** +30mV

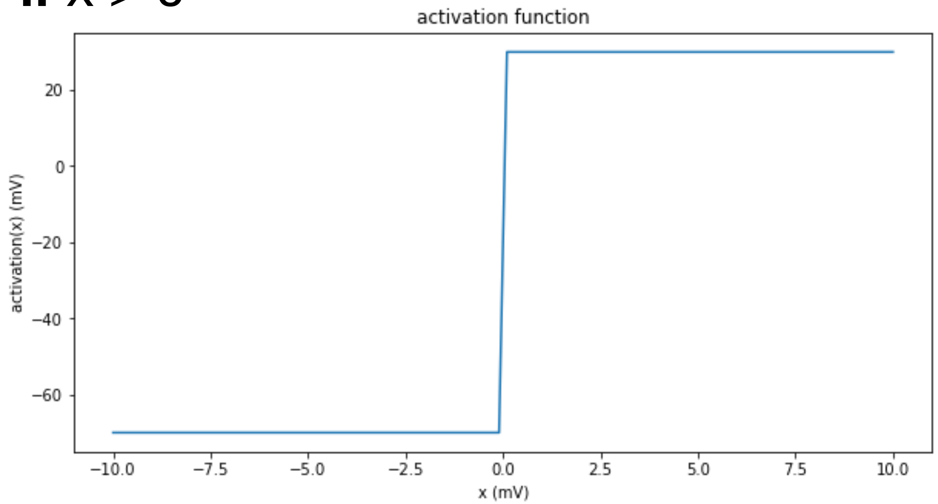




Equation of a Neuron with 2 Dendrites (8/8)

- Our final equation: $\boxed{A = activation(-15mV + \theta_1 \times a_1 + \theta_2 \times a_2)}$
  - *A*: Action Potential of <u>our Neuron</u>
  - a_1: action potential <u>from Neuron1</u>
  - θ_1 : represent the <u>strength of synaptic connection</u> between Neuron1 and our neuron
  - *activation(x)* = -70mV **if** x < 0 **//** +30mV **if** x > 0

This equation reminds me something



From Biology to Machine Learning (1/6)

$$\boxed{A = activation(-15mV + \theta_1 \times a_1 + \theta_2 \times a_2)}$$

activation(x) = -70mV if x < 0 // +30mV if x > 0

- -70mV, +30mV and -15mV are <u>values that come from the chemical process </u>in a Neuron
- But from the general point of view of how a neuron is working, <u>their precise value is not important</u>
- <u>What matter is the "**All or None**" behavior</u>: the neurons is activated, or it is not
- Then, let us say <u>our action potential is one</u>, and our<u> resting potential is zero</u>
  - -70mV —> 0 and +30 mV —> 1

From Biology to Machine Learning (2/6)

$$\boxed{A = activation(-15mV + \theta_1 \times a_1 + \theta_2 \times a_2)}$$

activation(x) = *0* if x < 0 // *1* if x > 0

- Then, let us say <u>our action potential is one</u>, and our <u>resting potential is zero</u>
  - -70mV —> 0 and +30 mV —> 1
- Similarly, let us replace the threshold value -15mV by a parameter θ_0

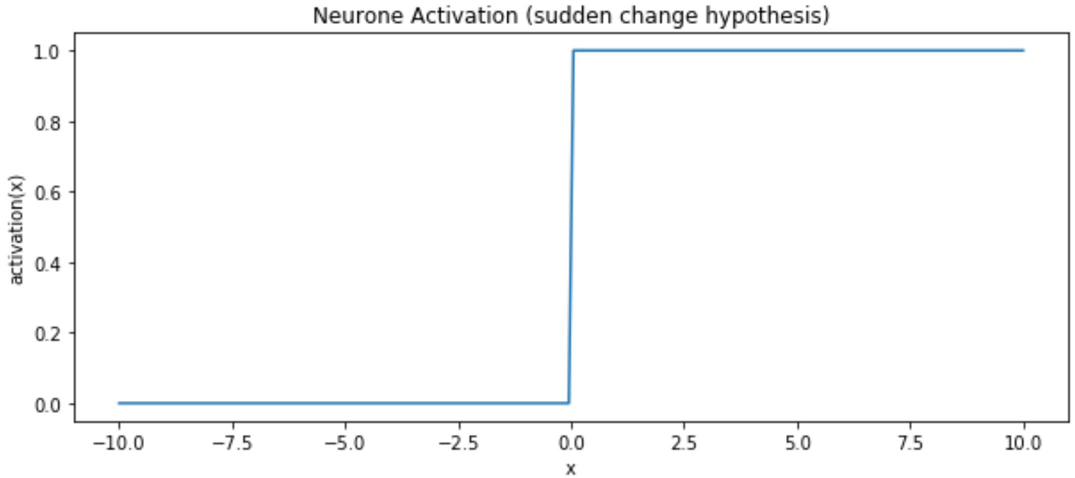
$$\boxed{A = activation(\theta_0 + \theta_1 \times a_1 + \theta_2 \times a_2)}$$

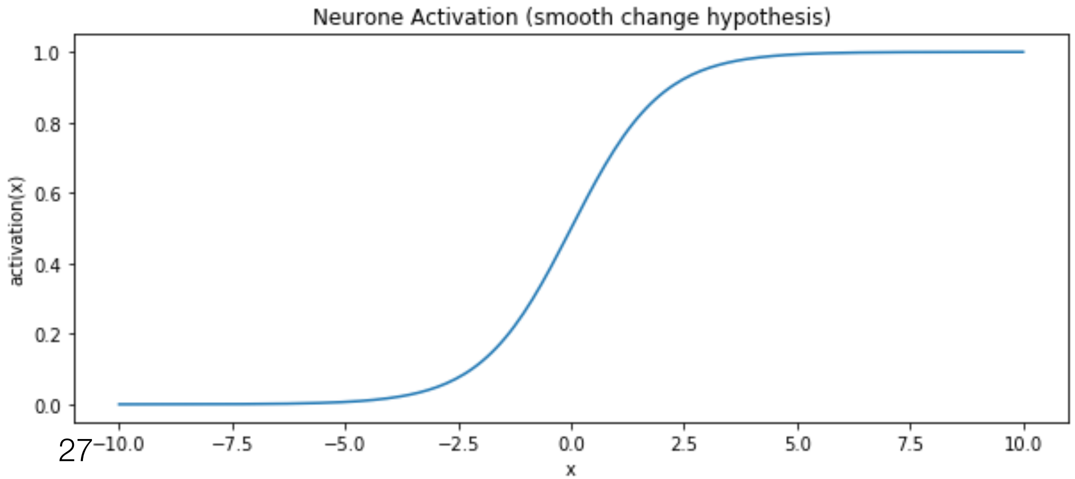
activation(x) = *0* if x < 0 // *1* if x > 0

From Biology to Machine Learning (3/6)

- Finally, let us consider once more the activation function
- Maybe in practice, the change on the output is not so sudden
- We could consider the activation to be smoother

activation(x) = *0* if x < 0 // *1* if x > 0

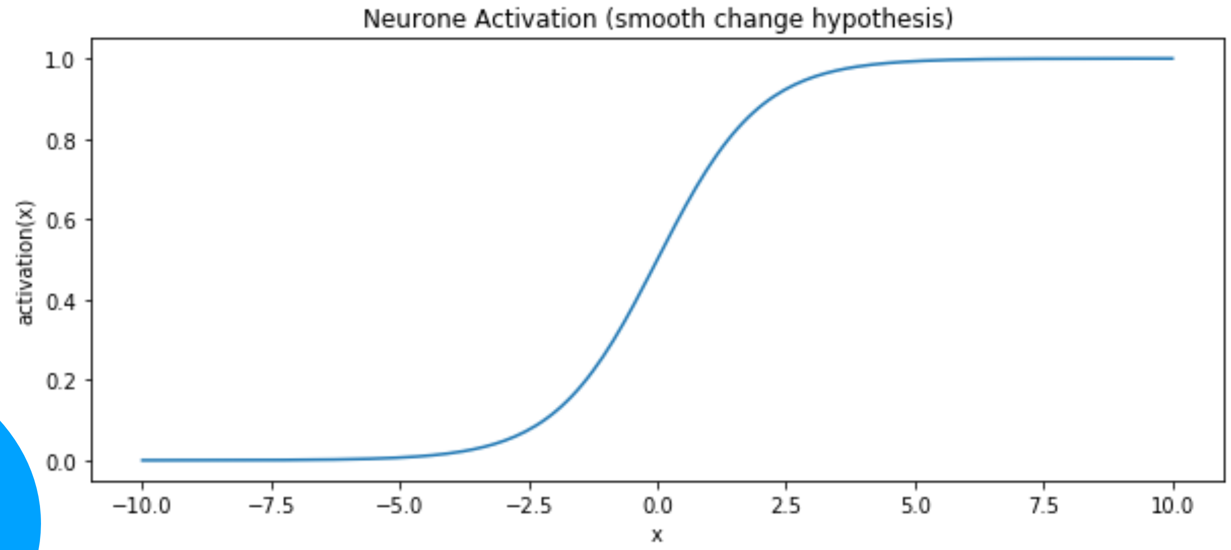




From Biology to Machine Learning (4/6)

- So, finally, we have this:

$$\boxed{A = activation(\theta_0 + \theta_1 \times a_1 + \theta_2 \times a_2)}$$

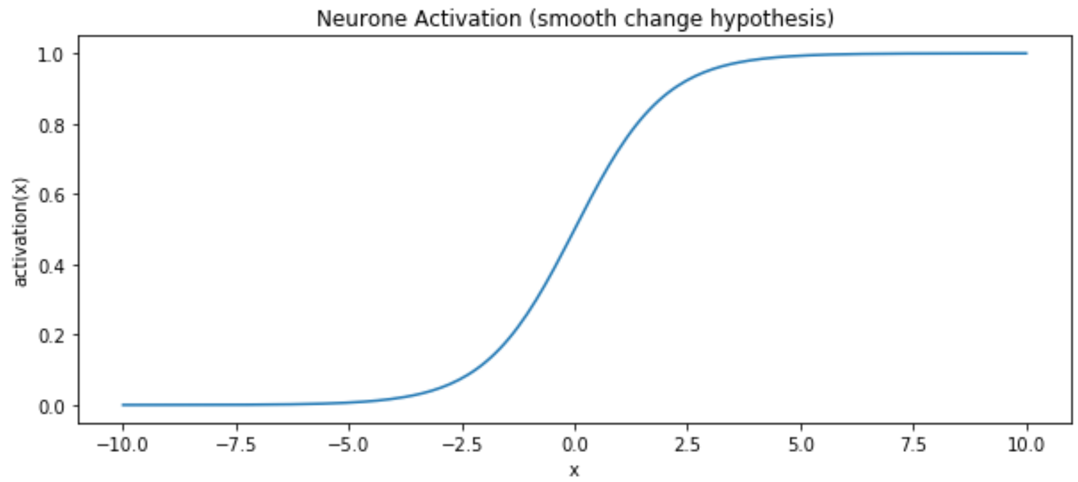


From Biology to Machine Learning (5/6)

Neuron Equation:

$$A = activation(\theta_0 + \theta_1 \times a_1 + \theta_2 \times a_2)$$

$$\sigma(x) = \frac{1}{1 + exp(-x)}$$



Logistic Classifier Equation:

$$score(income, age) = \theta_0 + \theta_1 \times income + \theta_2 \times age$$

$$V_{model} = \sigma(score)$$

From Biology to Machine Learning (6/6)

$$A = activation(\theta_0 + \theta_1 \times a_1 + \theta_2 \times a_2)$$

- Under a <u>few simplifying assumption</u>, the **logistic classifier** is a <u>good approximation</u> of the processing done by a **neuron**
- Therefore, we can also call a logistic classifier an **artificial Neuron**
- **Or** we can say that a **Neuron** is a **biological logistic classifier**

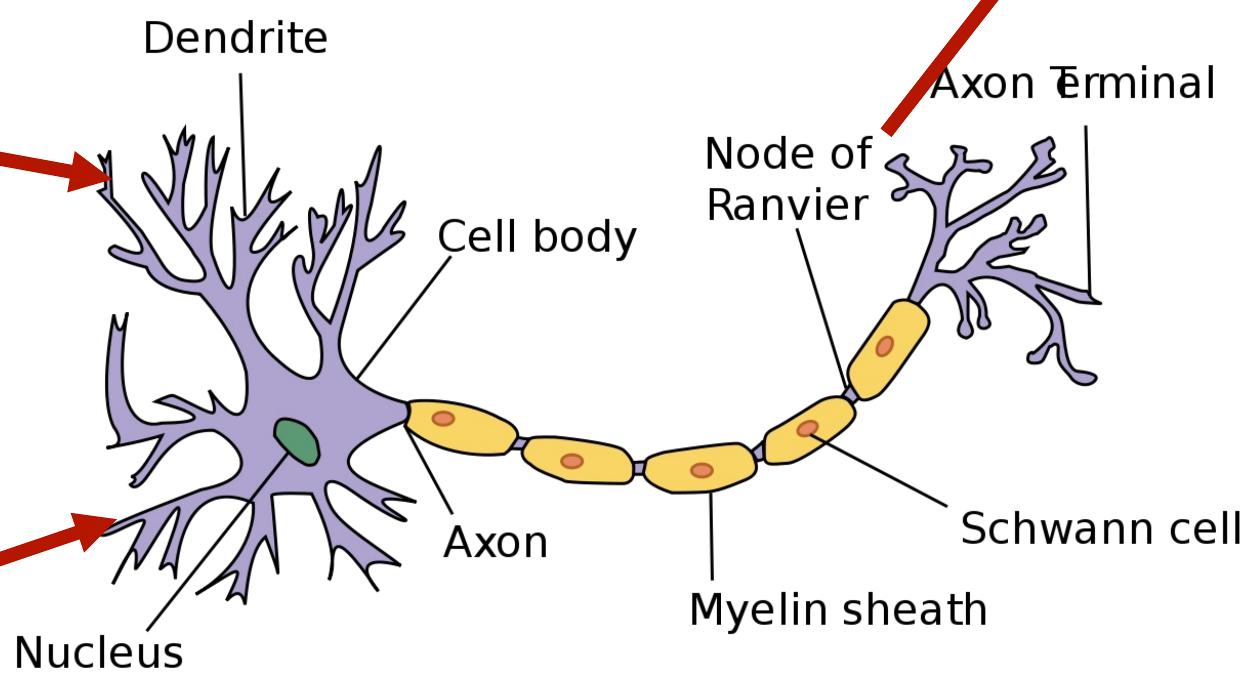
A Neuron as a Binary Classifier

-70mV or 30mV impulse
-70mV: Left-Wing
30mV: Right Wing

VOTE

INCOME

eg. 12 mV impulse



AGE

eg. 3.2 mV impulse

Remember this?

Example

Example Data

	income	vote
1	40	0
2	70	1
3	20	0

- Suppose these parameters:

$$\theta_0 = -8$$

$$\theta_1 = 0.1$$

$$score(income) = \theta_0 + \theta_1 \times income$$

$$V_{model} = \sigma(score) \qquad \sigma(x) = \frac{1}{1 + exp(-x)}$$

	income	vote	Score	Prediction V	Predicted Class	Cost
1	40	0	-4	0.018	0	
2	70	1	-1	0.269	0	
3	20	0	-6	0.002	0	

You were actually doing what a neuron does!

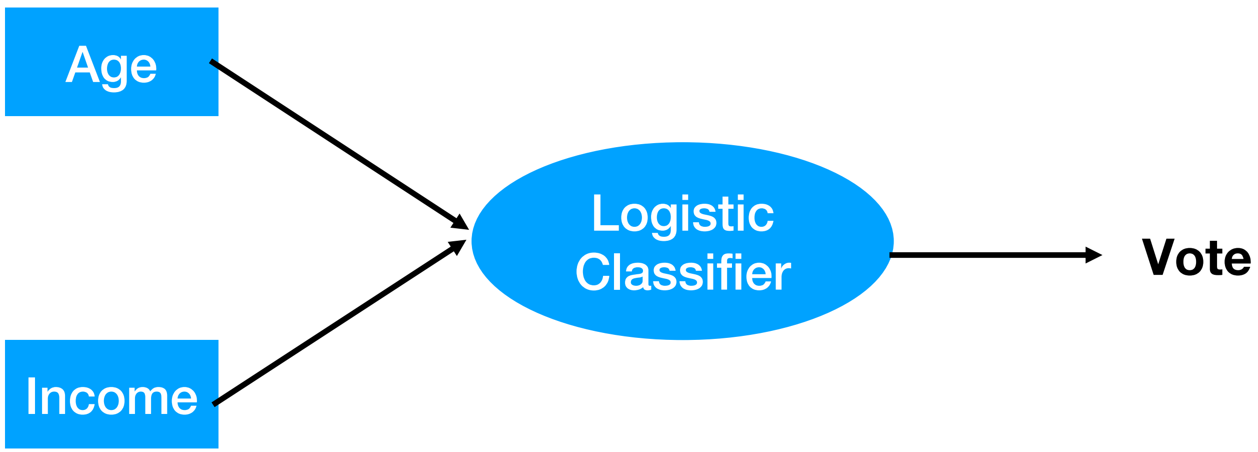
Neural Networks

- Neurons in the brain are very interconnected
- In fact, an average neuron has about 7,000 dendrite connections

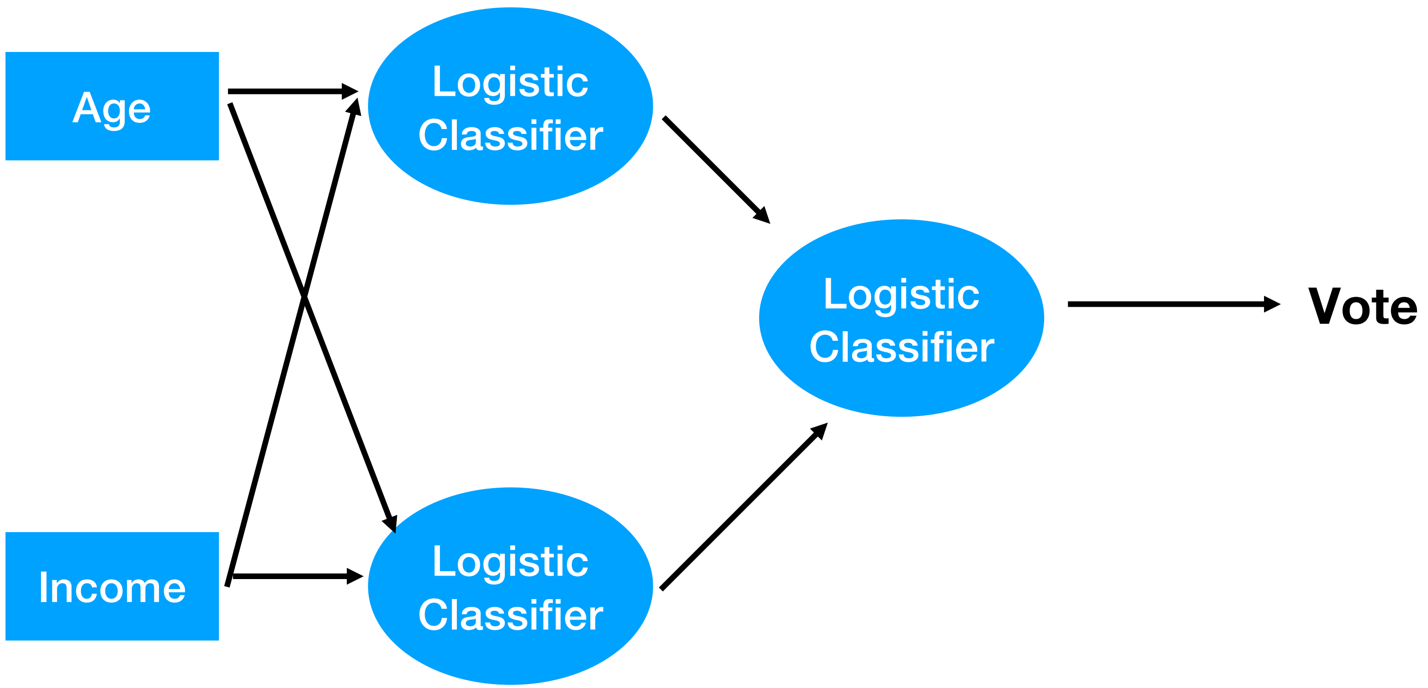
$$A = activation(\theta_0 + \theta_1 \times a_1 + \theta_2 \times a_2 + \ldots + \theta_{7000} \times a_{7000})$$

- How about interconnecting logistic classifiers? Will it work?

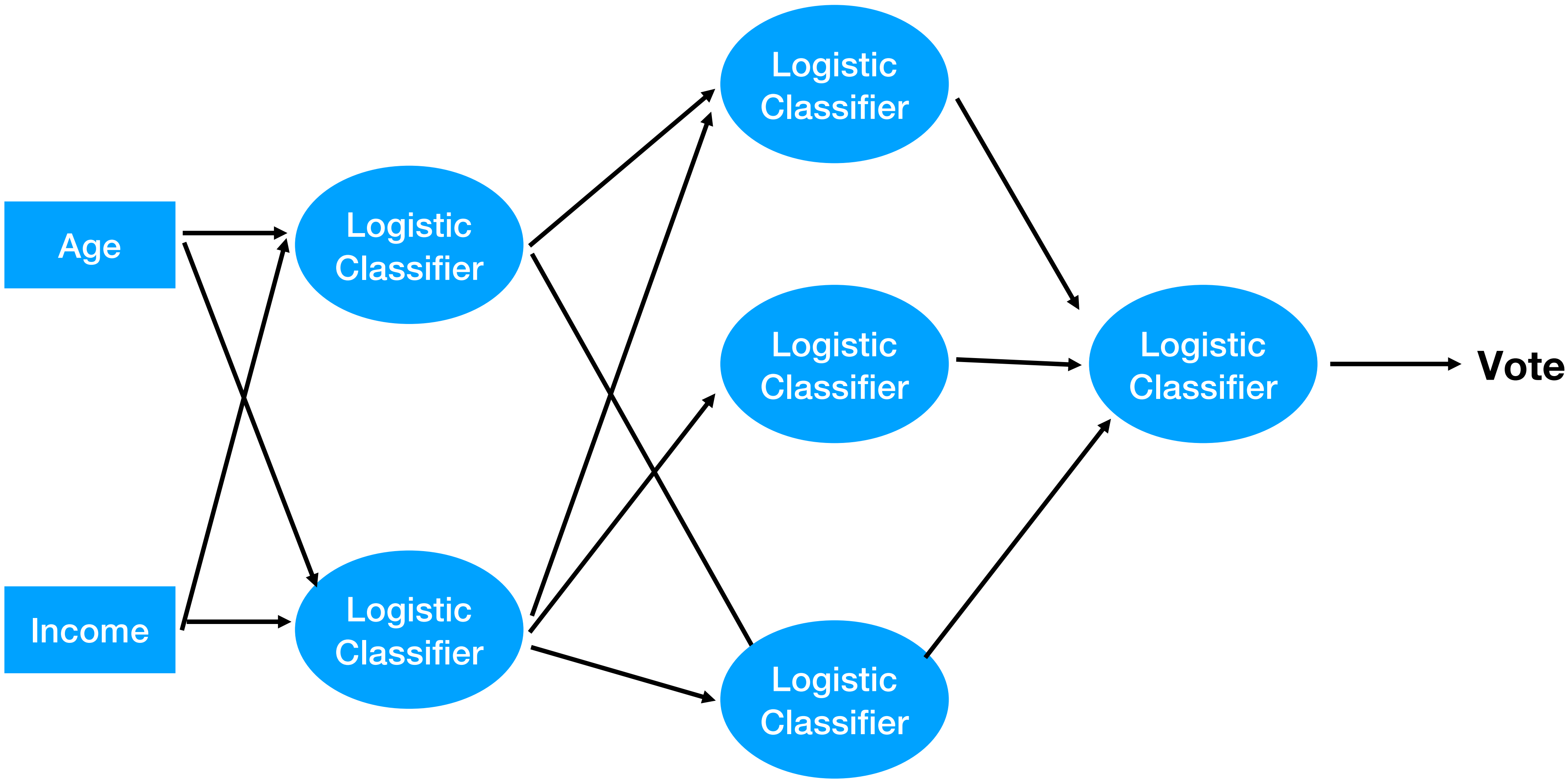
One Neuron



Three Neurons

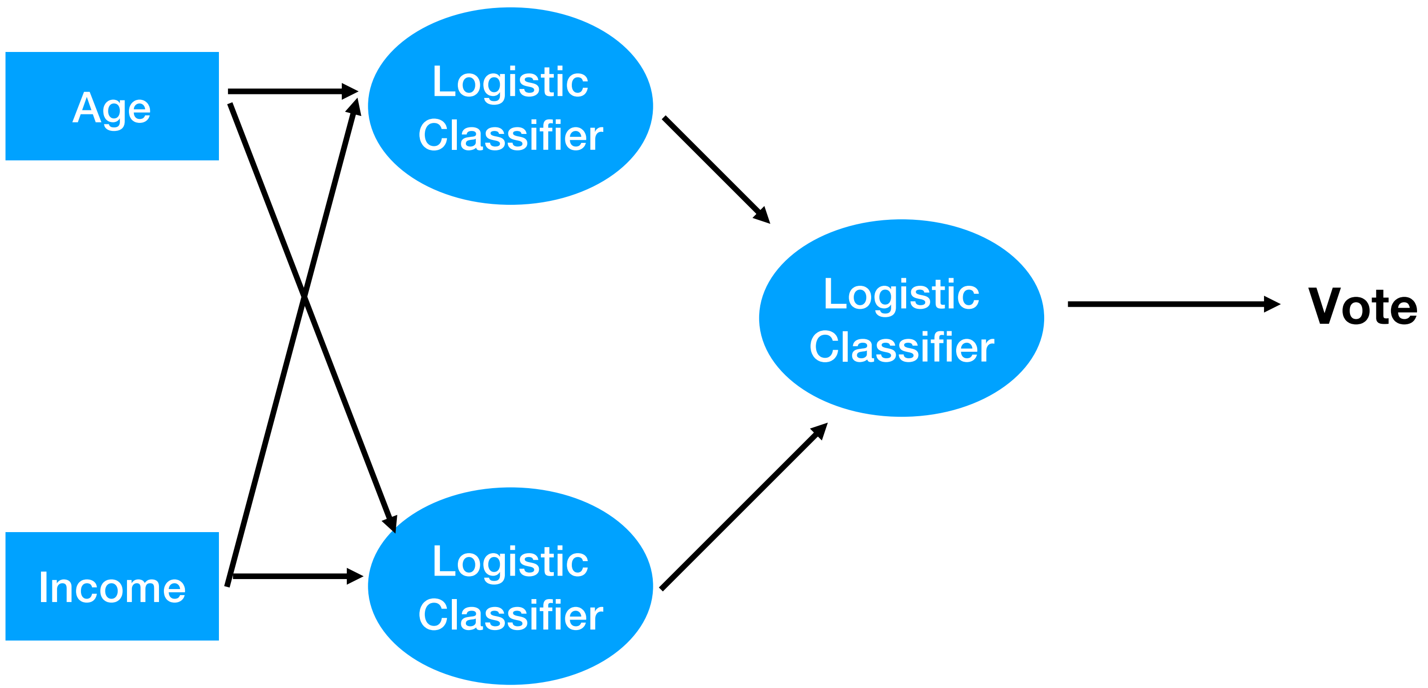


More Neurons



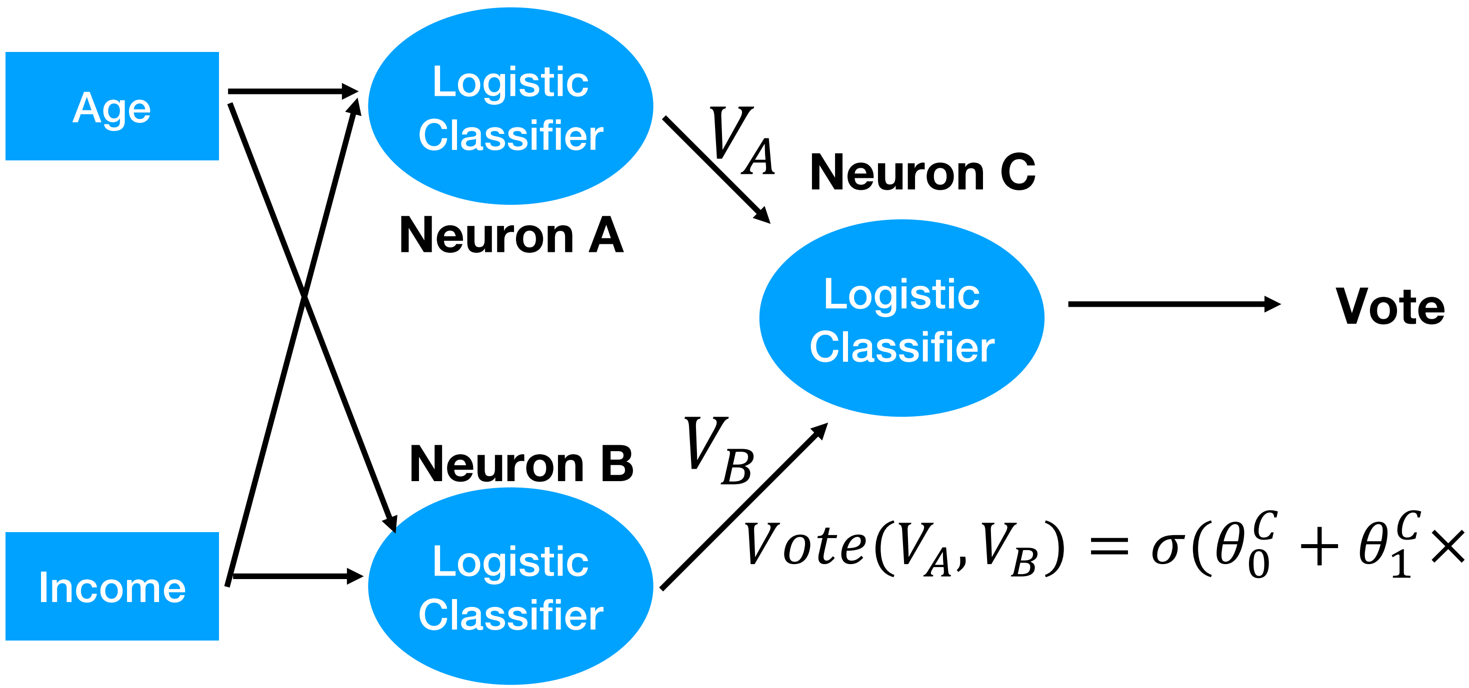
Three Neurons (1/3)

What do we compute in this case?



Three Neurons (2/3)

$$V_A(income, age) = \sigma(\theta_0^A + \theta_1^A \times income + \theta_2^A \times age)$$



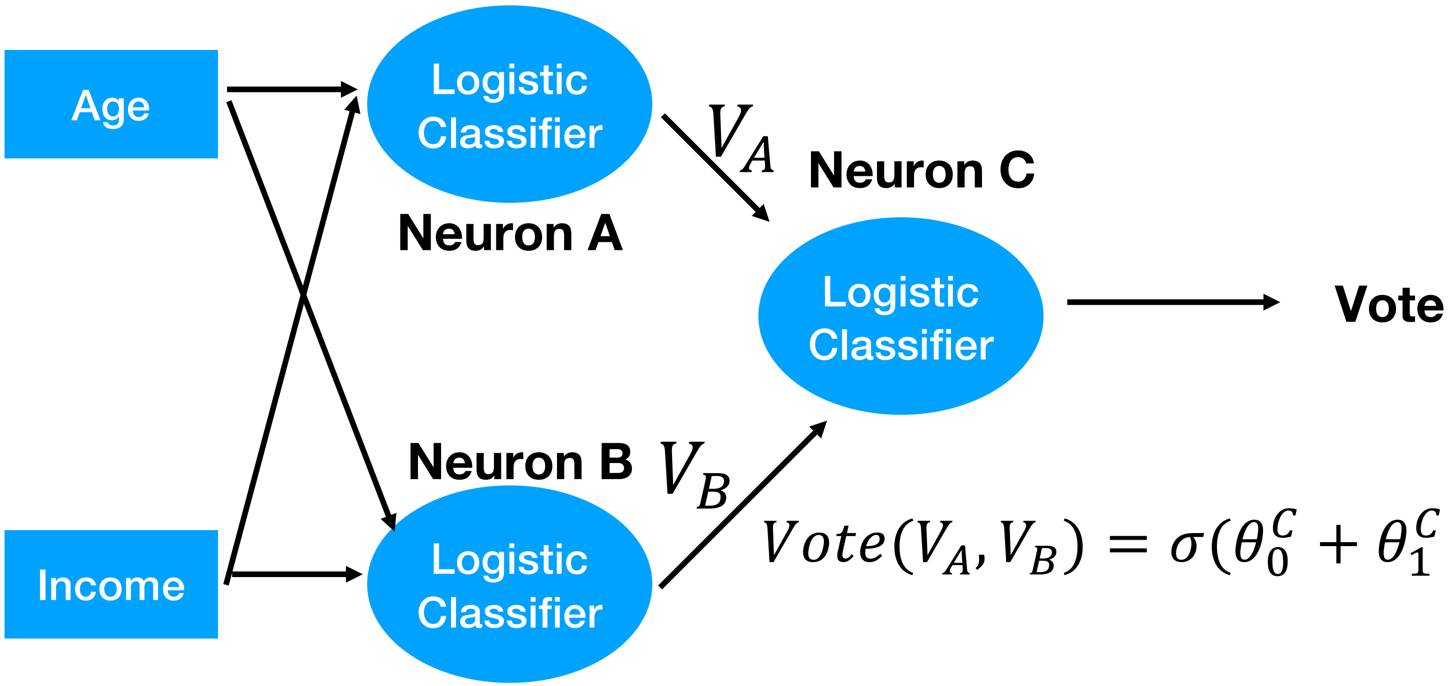
$$Vote(V_A, V_B) = \sigma(\theta_0^C + \theta_1^C \times V_A + \theta_2^C \times V_B)$$

$$V_B(income, age) = \sigma(\theta_0^B + \theta_1^B \times income + \theta_2^B \times age)$$

Three Neurons (3/3)

We now have a model with 9 parameters

$$V_A(income, age) = \sigma(\theta_0^A + \theta_1^A \times income + \theta_2^A \times age)$$



$$Vote(V_A, V_B) = \sigma(\theta_0^C + \theta_1^C \times V_A + \theta_2^C \times V_B)$$

$$V_B(income, age) = \sigma(\theta_0^B + \theta_1^B \times income + \theta_2^B \times age)$$

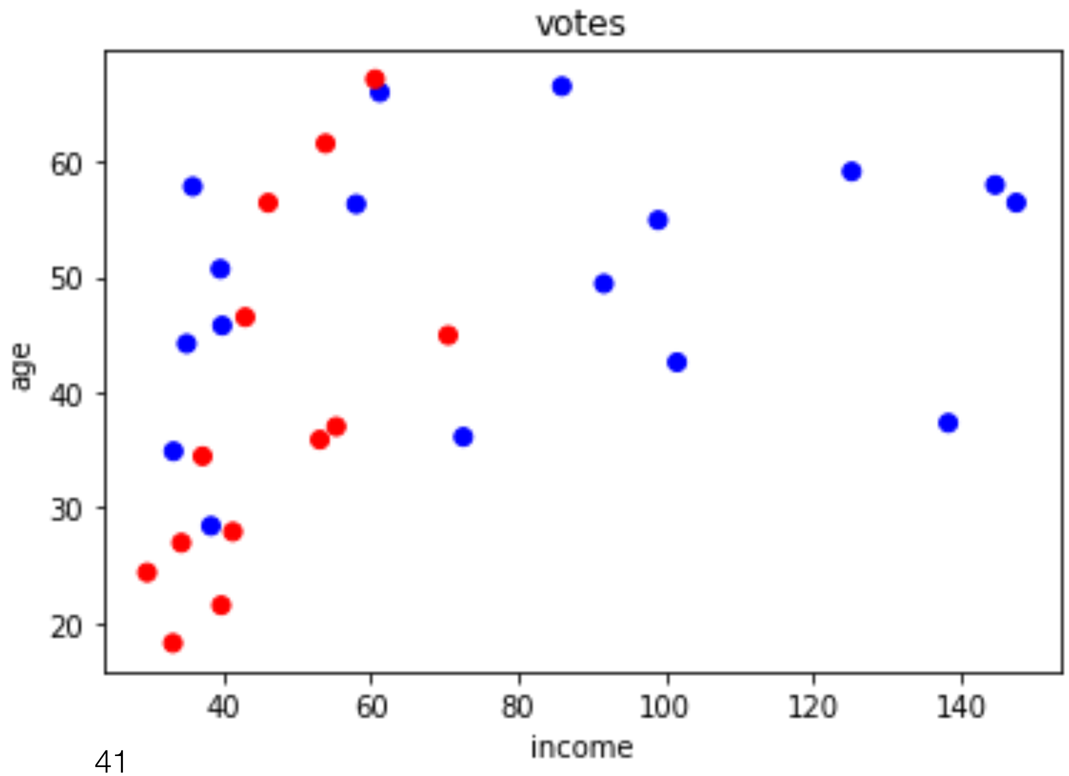
Does it Work?

- Can we really get something better by <u>connecting artificial neurons</u> like this?
- Yes!
- The “power” of a network increase with the number of neurons
- If we connect many simple logistic classifiers, we get a more powerful binary classifier

Back to Our Example

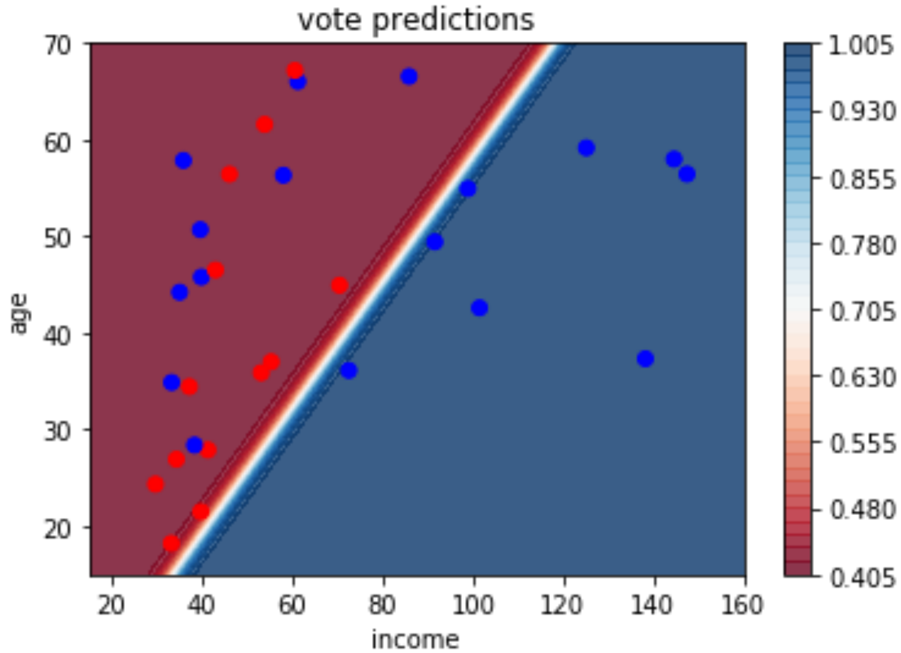
- Let us look again at our voting data
- Let us try to make different neural networks learn to predict vote knowing the income and age of somebody

	income	age	vote
0	39.0	42.0	L
1	30.0	21.0	L
2	47.0	65.0	L
3	69.0	50.0	R
4	52.0	53.0	R
5	110.0	28.0	R
...	...	...	...



Single Neuron

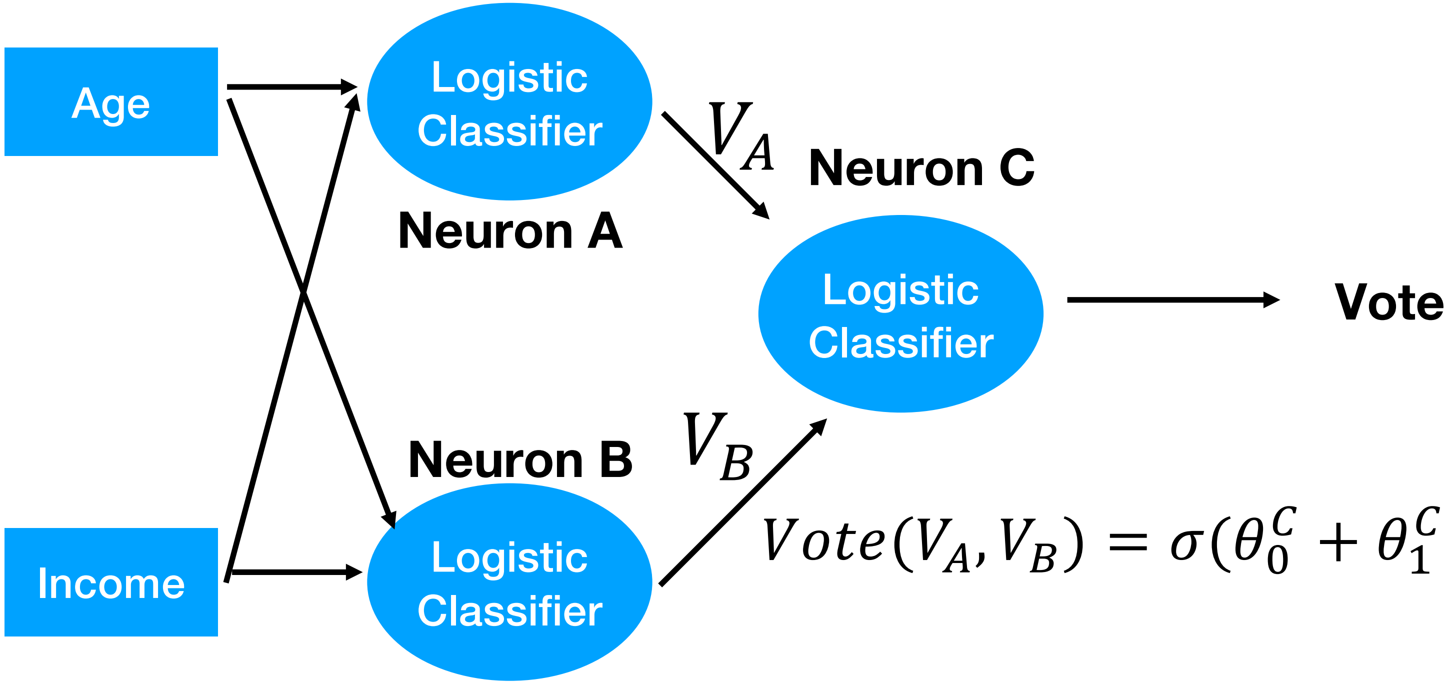
- A **Single Neuron** is just a simple **logistic classifier**
    - Like we saw last lecture
- Class boundary is a straight line
- Only 2 zones



- Darker red area means the classifier is certain people in this age/income zone will vote for the Left-Wing party
- Darker blue area means the classifier is certain people in this age/income zone will vote for the Right-Wing party
- White-ish area means the classifier is uncertain

Three Neurons

$$V_A(income, age) = \sigma(\theta_0^A + \theta_1^A \times income + \theta_2^A \times age)$$

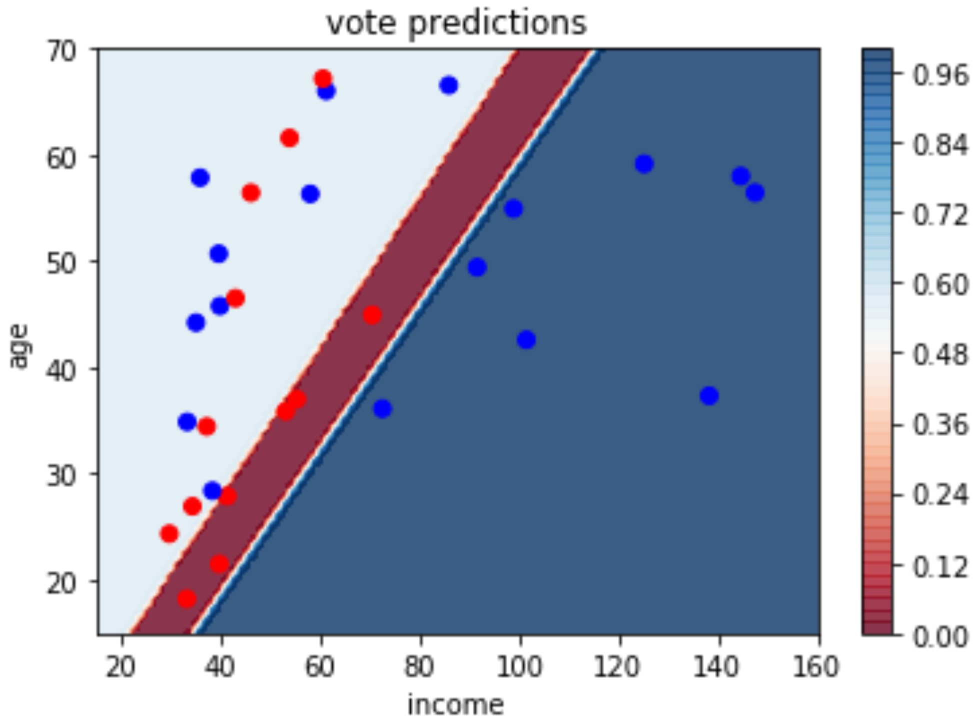


$$Vote(V_A, V_B) = \sigma(\theta_0^C + \theta_1^C \times V_A + \theta_2^C \times V_B)$$

$$V_B(income, age) = \sigma(\theta_0^B + \theta_1^B \times income + \theta_2^B \times age)$$

Classification Boundary for Three Neurons

- With three neurons: the classifier can define a third zone of uncertainty



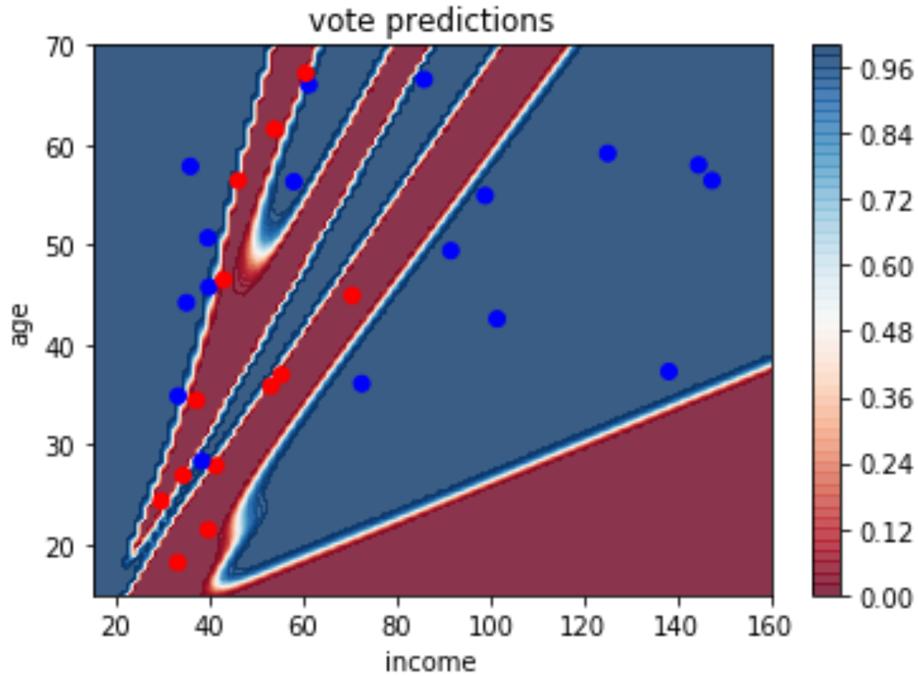
- Darker red area means the classifier is certain people in this age/income zone will vote for the Left-Wing party

- Darker blue area means the classifier is certain people in this age/income zone will vote for the Right-Wing party

- White-ish area means the classifier is uncertain

Classification Boundary for ~100 Neurons

- The Neural Network can now perfectly predict each example
- There seems to be some overfitting



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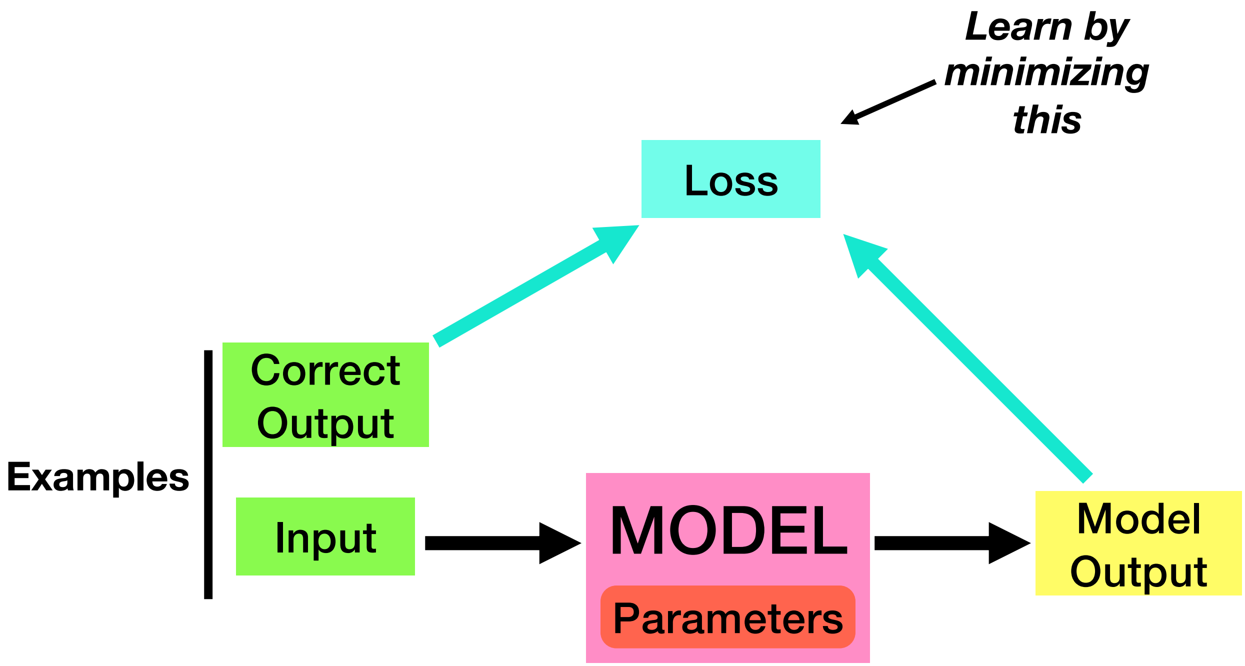
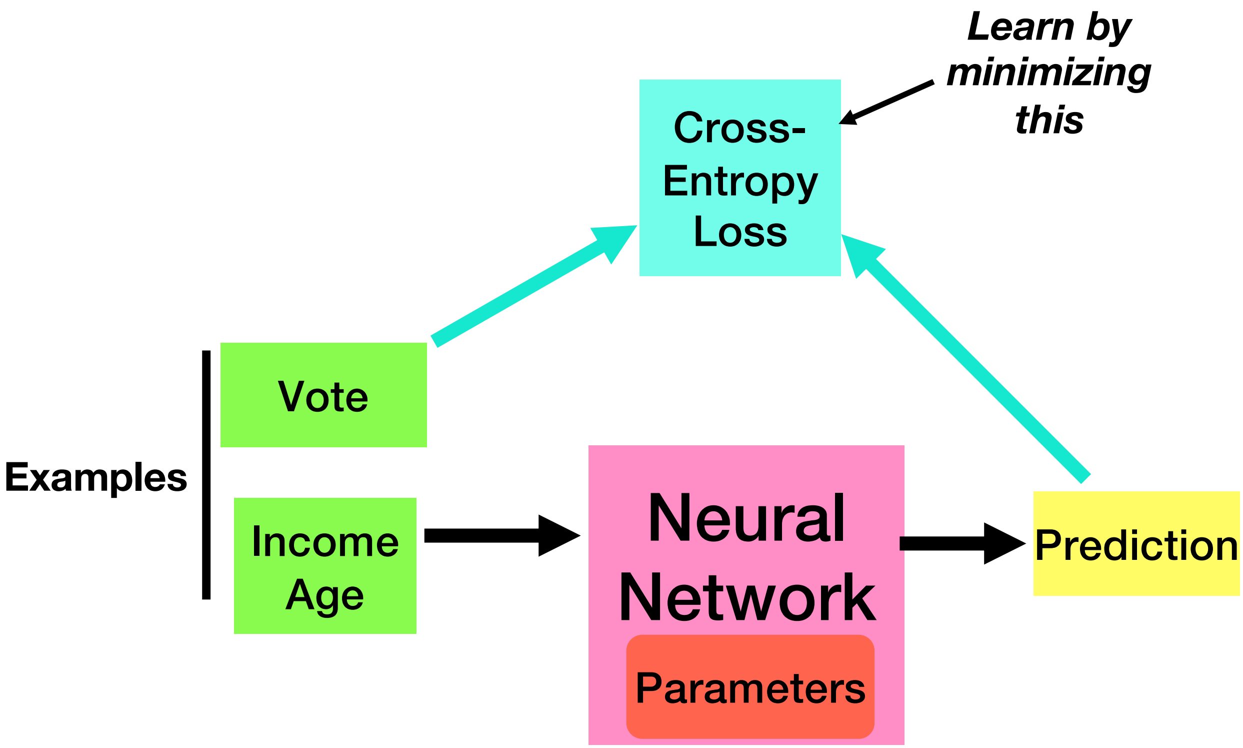
Supervised Learning

- In supervised learning, we usually have:
  - A **MODEL**: a "parameterized" function that takes input and produces output
  - A ***Loss***: A function that computes how different the model output is from the correct output
  - ***Examples*** of input and correct output



Supervised Learning for Neural Network

- In supervised learning, we usually have:
  - A **MODEL**: a "parameterized" function that takes input and produces output
  - A ***Loss***: A function that computes how different the model output is from the correct output
  - ***Examples*** of input and correct output (cigarettes smoked, age of death)



Making the Connections

- What we have seen so far:

Increasing complexity

Linear Regression

$$f(x, y) = \theta_0 + \theta_1 \times x + \theta_2 \times y$$

Logistic Classifier

$$score(x, y) = \theta_0 + \theta_1 \times x + \theta_2 \times y$$

$$prediction = \sigma(score)$$

Neural Network: Combination of logistic classifier

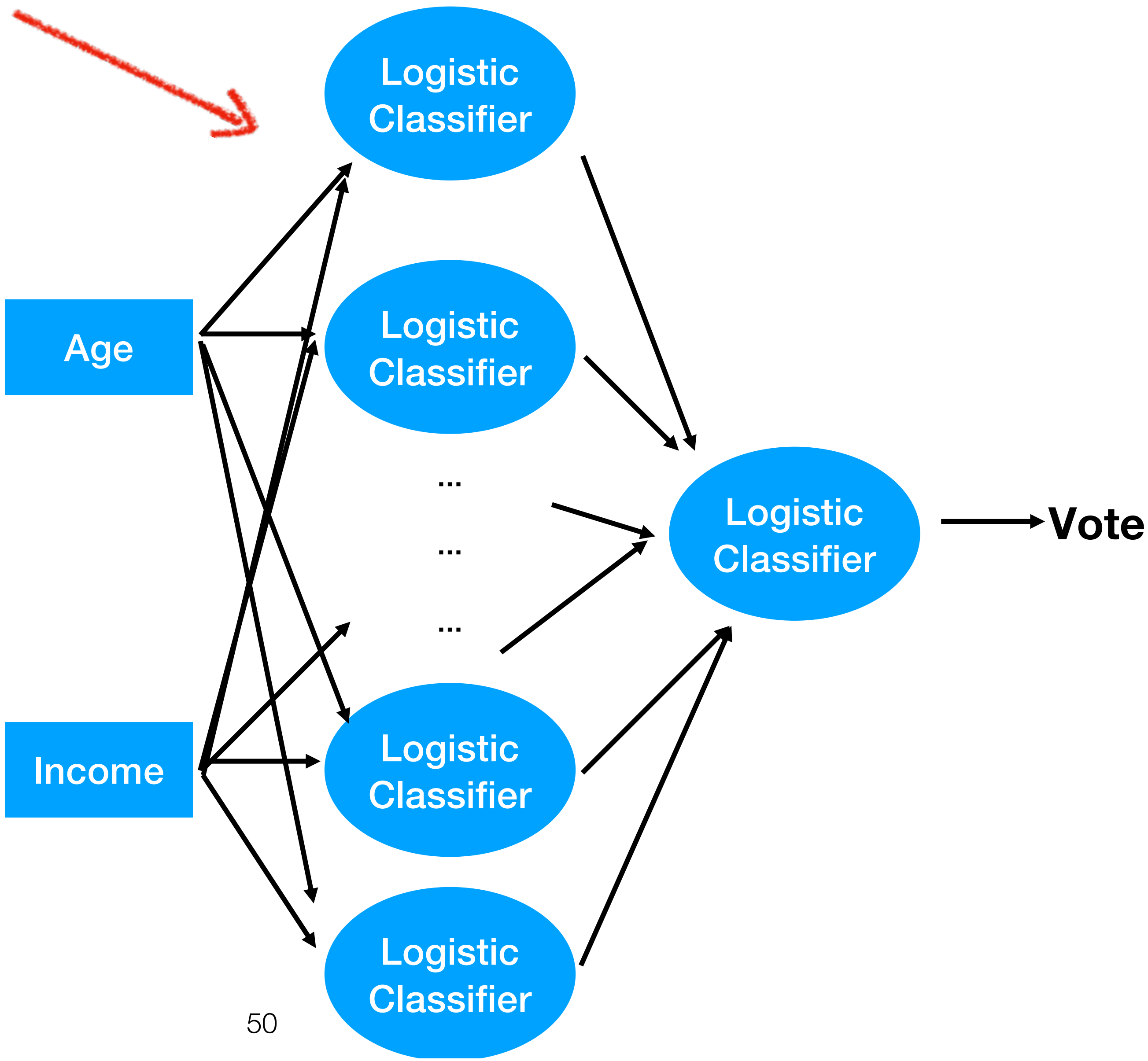
All trained by gradient descent on the loss

Does it Work?
A Theoretical Answer

- Indeed, we do get better classifiers by connecting simple logistic classifiers
- <u>Universal approximation theorem:</u>
    - **Any** function can be **approximated** by a neural network with **2 layers of neurons**

Universal Approximation Theorem

Stack enough neurons here, and you can learn from any data, no matter how complex it is

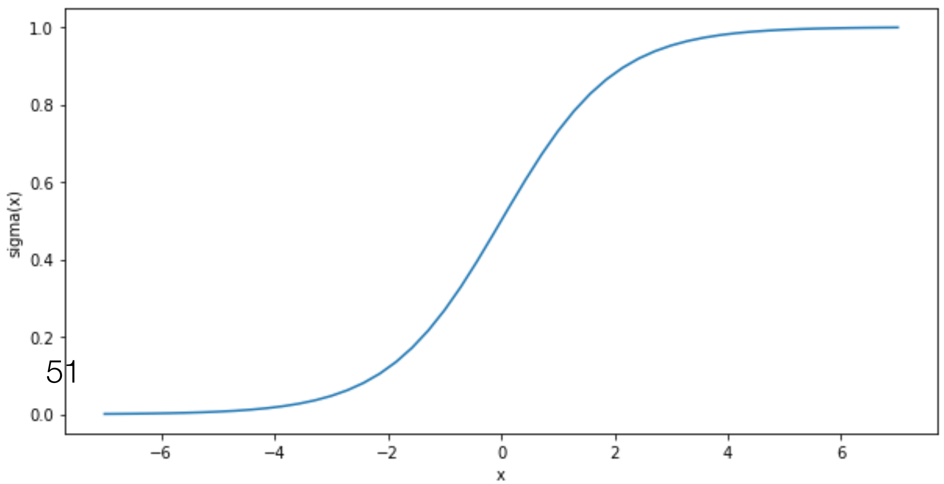


But note that this 2-layers architecture is usually not the most efficient

On “Activation” Functions

- So far, we have applied the “sigmoid function” (a.k.a “logistic function”) to the output of the Neuron
  - Historically, the first to be used
  - It behaves similarly to Biological Neurons, as we have seen
  - But, not the most efficient in practice

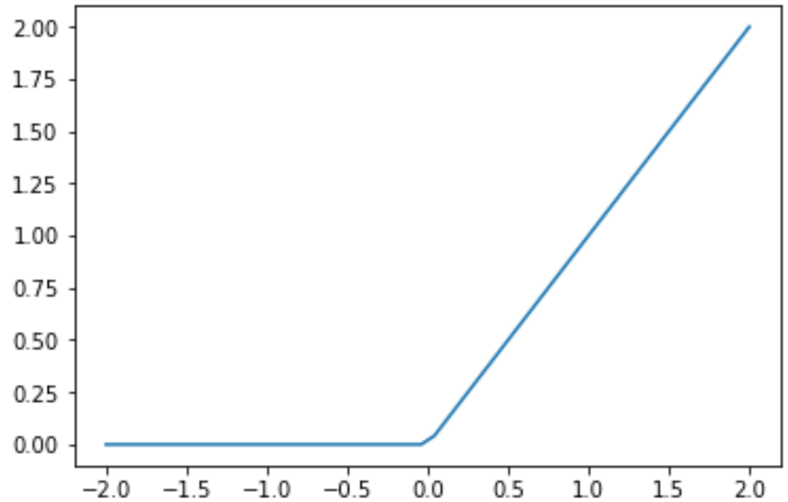
$$\sigma(x) = \frac{1}{1 + exp(-x)}$$



Rectified Linear Unit

- There are many possible functions to choose from
- One that is simple and works very well: “Rectified Linear Unit”
- Very fast to compute
- Very efficient
- Less similar to biological Neurons

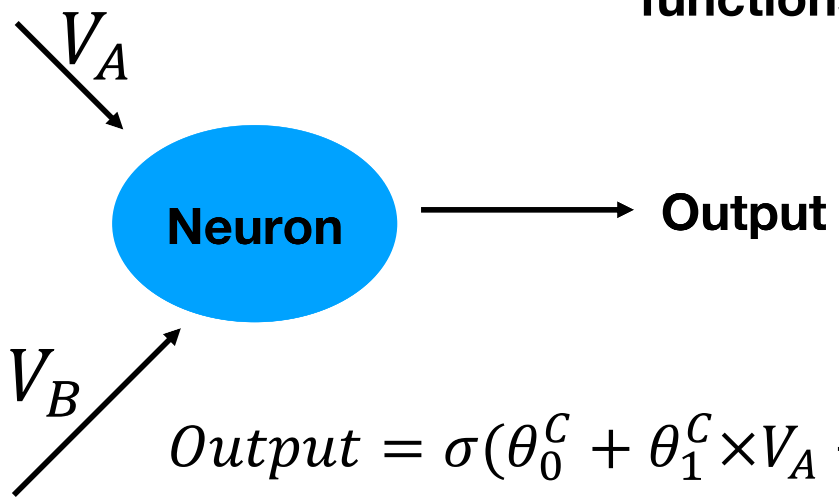
$$ReLU(x) = max(x, 0)$$



Different Activations

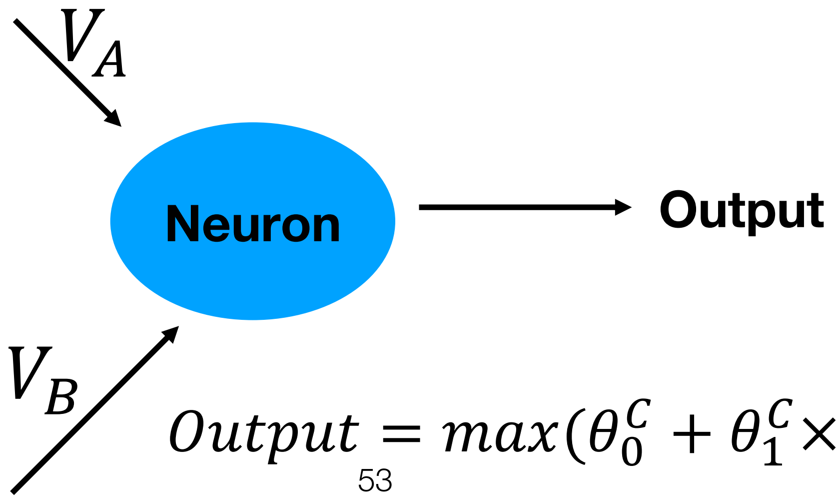
(Finding the best activation functions is an active area of research)

Neuron with sigmoid activation



$$Output = \sigma(\theta_0^C + \theta_1^C \times V_A + \theta_2^C \times V_B)$$

Neuron with ReLU activation



$$Output = max(\theta_0^C + \theta_1^C \times V_A + \theta_2^C \times V_B, 0)$$

Funny Calculations: What is the Power of the Brain? (1/5)

- We mentioned before that on average, a **human neuron** has 7000 dendrite connections:

$$A = activation(\theta_0 + \theta_1 \times a_1 + \theta_2 \times a_2 + \ldots + \theta_{7000} \times a_{7000})$$

- Therefore, to “simulate” one neuron computation, we need about 7000 multiplications and 7000 additions

- It seems a neuron cannot activate more than 200 times per seconds

- To simulate a neuron in real time, we therefore need to compute about (7000 + 7000) x 200 = 2 800 000 operations per second

Funny Calculations: What is the Power of the Brain? (2/5)

- To simulate a neuron in real time, we therefore need to compute about (7000 + 7000) x 200 = 2 800 000 operations per second
- In computer technology, we use the term FLOPS (floating point operation per seconds)
- A current computer with an intel processor should have a power of about 200 Gflops
- Therefore, it can simulate, in real time, about 200x10^9 / 2 800 000 = 71 000 neurons
- How many neurons in the brain?

Funny Calculations: What is the Power of the Brain? (3/5)

- A intel CPU can simulate, in real time, about 71 000 neurons
- How many neurons in the brain?
  - About 100 billions !
    - (actually more like 80 billions)

Funny Calculations: What is the Power of the Brain? (4/5)

- What is the number of FLOPS for the brain?
  - 1e11 x 2 800 000 = 2.8e17 FLOPS
  - = 280 000 TeraFLOPS = 280 PetaFLOPS

Most Powerful Computer in the World?

- As of 2021, one of the most powerful (known) computer in the world is the Fukaku, located in Riken/Fujitsu
  - Made of almost 158,976 CPU nodes
  - Computation power: about 488 Petaflops
- -> Enough to simulate one human brain!

Funny Calculations: What is the Power of the Brain? (5/5)

- Previous calculations should not be considered **too** seriously
  - We did a lot of biological and mathematical approximations about how the brain works
- Still, it gives us <u>some idea</u> about <u>how powerful</u> the human brain is and <u>why AI is difficult</u>:
  - Even a <u>supercomputer filling several rooms</u> cannot match the computation power of a <u>human brain that is 1000s of times smaller</u>
- At the same time, computers are **beginning** to be competitive with the brain
  - Might be an explanation of why AI is now starting to become more successful

On Biology and Machine Learning

- Today, we spent a good time discussing biological neurons and the brain
- But do not think Neural Networks and AI are about simulating a human brain!
    - In practice, we do not mind doing things that would not happen with a biological neurons if it suits us
- In practice, we use Artificial Neural Networks because they are efficient for the tasks we want to do
    - And they also have some mathematic justification
- You could say it is “almost” a coincidence that the tools we use are similar to the way the brain work
- Also note that gradient descent is the way Artificial Neural Networks learn, but not the way the brain learn

Next Time

- Start discussing Neural Network architectures
  - Feed-Forward Neural Networks
  - Convolutional Neural Networks
  - Recurrent Neural Networks
- Mathematical aspect: BackPropagation, Matrix multiplication

Report

- Write a report discussing the relationship between linear regression, classification, and neural networks **in pdf** and submit via PandA
  - Submission due: **next lecture**
  - Name the pdf file as **student id_name**.