

LOGIC I

PROPOSITIONAL LOGIC

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LOGIC I

**TRUTH FUNCTIONS -
PROPOSITIONS**

TRUTH TABLE - PROPOSITIONS

If and only if a patient does not have MELD score below 13 and they receive dialysis more than twice per week, then they are eligible for a liver transplant.

1. Transform to symbolic proposition $(\sim M \cdot D) \equiv T$
2. Determine number of columns = proposition + operator
3. Determine number of rows = $L = 2^n$

Number of different simple propositions	Number of lines in truth table	Operator	Name	Logical function	Used to translate
1	2	\sim	tilde	negation	not, it is not the case that
2	4	\cdot	dot	conjunction	and, also, moreover
3	8	\vee	wedge	disjunction	or, unless
4	$L = 2^n$	\supset	horseshoe	implication	if ... then ..., only if
5	32	\equiv	triple bar	equivalence	if and only if
6	64				

TRUTH TABLE - PROPOSITIONS

Proposition

M	D	T
T	T	T
T	T	F
T	F	T
T	F	F
F	T	T
F	T	F
F	F	T
F	F	F

Operation

$\sim M$	$\sim M \cdot D$	$(\sim M \cdot D) \equiv T$
F	F	F
F	F	T
F	F	F
F	F	T
T	T	T
T	T	F
T	F	F
T	F	T

If and only if a patient does not have MELD score below 13 and they receive dialysis more than twice per week, then they are eligible for a liver transplant.

$$(\sim M \cdot D) \equiv T$$

TRUTH TABLE - PROPOSITIONS

Alternative Method

$$(\neg M \cdot D) \equiv T$$

T	F	F	T	F	T
T	F	F	T	T	F
T	F	F	F	F	T
T	F	F	F	T	F
F	T	T	T	T	T
F	T	T	T	F	F
F	T	F	F	F	T
F	T	F	F	T	F

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TRUTH TABLE - PROPOSITIONS

Alternative Method

$$(\neg M \cdot D) \equiv T$$

T	F	F	T	F	T
T	F	F	T	T	F
T	F	F	F	F	T
T	F	F	F	T	F
F	T	T	T	T	T
F	T	T	T	F	F
F	T	F	F	F	T
F	T	F	F	T	F

Classifying Statements

Column under main operator

all true

all false

at least one true, at least one false

Statement classification

tautologous (logically true)

self-contradictory (logically false)

contingent

TRUTH TABLE - PROPOSITIONS

Logically True

$[(G \supset H) \cdot G] \supset H$	T T T T T T F F F T F T T F F F T F F F	T T T T F F T T T F F F

Logically False

$(G \vee H) \equiv (\sim G \cdot \sim H)$	T T T T T F F T T F F F	F T F F T F T F T F T F F F T T F T T F

Classifying Statements

Column under main operator	Statement classification
all true	tautologous (logically true)
all false	self-contradictory (logically false)
at least one true, at least one false	contingent

When a statement is either **Logically True** or **Logically False**, the Truth Value only depends on its form. The content of the statement has no impact on the truth. As a result, these statements provide no information about the topic.

Look for these statements from political leaders

TRUTH TABLE - PROPOSITIONS

Logically True

$[(G \supset H) \cdot G] \supset H$			
T	T	T	T
T	F	F	T
F	T	T	T
F	T	F	F

Logically False

$(G \vee H) \equiv (\sim G \cdot \sim H)$			
T	T	T	F
T	T	F	F
F	T	T	F
F	F	F	T

Classifying Statements

Column under main operator	Statement classification
all true	tautologous (logically true)
all false	self-contradictory (logically false)
at least one true, at least one false	contingent

Tautology: Saying the same thing twice = 冗長, 繰り返し

Self-Contradictory: two statements that oppose each other = 矛盾

If our country is great, then we will be great.

If the social democrats believe in gay marriage, then they are against families

TRUTH TABLE - PROPOSITIONS

CLASSIFYING STATEMENTS

Main operator Column:

ALL TRUE = Logically True/Tautologous

ALL FALSE = Logically False/Self-contradictory

1+ TRUE & 1+ FALSE = Contingent

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Classify this truth table:

(A	v	~	B)	⇒	B
T	T	F	T	T	
T	T	T	F	F	
F	F	F	T	T	
F	T	T	F	F	

Logically True
(Tautologous)

Logically False
(self-contradictory)

Contingent

Falsrue

Classify this proposition truth table:

[$G \supset (N \supset \sim G)$]	\bullet	$[(N \equiv G) \bullet (N \vee G)]$]
T	F	T	F	F	T
T	T	F	T	F	F
F	T	T	T	F	T
F	T	F	T	T	F

TRUTH TABLE - PROPOSITIONS

Comparing Statements

We can also compare the truth value of two separate propositions

Columns under main operators	Relation
same truth value on each line	logically equivalent
opposite truth value on each line	contradictory
there is at least one line on which the truth values are both true	consistent
there is no line on which the truth values are both true	inconsistent

TRUTH TABLE - PROPOSITIONS

Comparing Statements

$$(K \supset L)$$

T	T	T
T	F	F
F	T	T
F	T	F

$$(\sim L \supset \sim K)$$

F	T	T	F	T
T	F	F	F	T
F	T	T	T	F
T	F	T	T	F

$$\equiv$$

Logically equivalent

- When the main operators are exactly the same, the two statements are equal.
- They are saying the same thing.
- If we make it conditional (\equiv) the truth value would be Logically True
- Redundant - 冗長

TRUTH TABLE - PROPOSITIONS

Comparing Statements

$$(K \supset L)$$

T	T	T
T	F	F
F	T	T
F	F	F

$$\equiv$$

F
F
F
F

$$(K \cdot \sim L)$$

T	F	T
T	T	F
F	F	T
F	F	F

Contradictory

- Exact opposites are contradictions
- They cancel each other out
- If we make it biconditional (\equiv) the truth value would be Logically False - 矛盾

TRUTH TABLE - PROPOSITIONS

Comparing Statements

$K \vee L$		$K \cdot L$	
K	L	K	L
T	T	T	T
T	F	F	F
F	T	T	F
F	F	F	F



Consistent

- At least one line is true for both
- It is possible for both statements to be true = consistent

TRUTH TABLE - PROPOSITIONS

Comparing Statements

	$K \equiv L$
T	T
T	F
F	T
F	F

	$K \cdot \sim L$
T	F
T	T
F	F
F	T

Inconsistent

- No line where true for both
- They are also not Logically Equivalent or Contradictory

TRUTH TABLE - PROPOSITIONS

CLASSIFYING STATEMENTS

Main Operator Column:

ALL TRUE = Logically True/Tautologous

ALL FALSE = Logically False/Self-contradictory

1+ TRUE & 1+ FALSE = Contingent

COMPARING STATEMENTS (2+ Statements)

Main Operator Column of 2 statements:

SAME VALUES = Logically Equivalent

OPPOSITE VALUES = Contradictory

1+ TRUE FOR BOTH = Consistent

0 TRUE FOR BOTH = Inconsistent

STRUCTURALLY
POSSIBLE TO
MAKE SENSE

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Compare these two propositions:

$$\sim D \vee B$$

F	T	T	T
F	T	F	F
T	F	T	T
T	F	T	F

$$\sim (D \bullet \sim B)$$

T	T	F	F	T
F	T	T	T	F
T	F	F	F	T
T	F	F	T	F

Inconsistent

Consistent

Logically
Equivalent

Contradictory

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Compare these two propositions:

$(E \supset C) \supset L$		
T	T	T
T	T	F
T	F	F
T	F	F
F	T	T
F	T	F
F	F	T
F	F	F

$E \supset (C \supset L)$		
T	T	T
T	F	F
T	T	F
T	F	T
F	T	T
F	F	F
F	T	F
F	F	F

Inconsistent

Consistent

Logically equivalent

Contradictory

LOGIC I

**TRUTH FUNCTIONS -
ARGUMENTS**

TRUTH TABLE - ARGUMENTS

Using truth tables to test the validity of arguments

1. Symbolize the arguments using letters and operators for propositions.
2. Write out the symbolized argument, placing a single slash between the premises and a double slash between the last premise and the conclusion.
3. Draw a truth table for the symbolized argument for each proposition broken into parts, outlining the columns representing the premises and conclusion.
4. Look for a line in which all of the premises are true, and the conclusion is false. If such a line exists, the argument is invalid; if not, it is valid.

TRUTH TABLE - ARGUMENTS

If juvenile killers are as responsible for their crimes as adults, then execution is a justifiable punishment. Juvenile killers are not as responsible for their crimes as adults. Therefore, execution is not a justifiable punishment.

1. Symbolize the arguments using letters and operators for propositions.

$$J \supset E$$

$$\frac{\sim J}{\sim E}$$



TRUTH TABLE- ARGUMENTS

If juvenile killers are as responsible for their crimes as adults, then execution is a justifiable punishment. Juvenile killers are not as responsible for their crimes as adults. Therefore, execution is not a justifiable punishment.

2. Write out the symbolized argument, placing a single slash between the premises and a double slash between the last premise and the conclusion.

$$J \supset E / \sim J // \sim E$$

TRUTH TABLE- ARGUMENTS

If juvenile killers are as responsible for their crimes as adults, then execution is a justifiable punishment. Juvenile killers are not as responsible for their crimes as adults. Therefore, execution is not a justifiable punishment.

3. Draw a truth table for the symbolized argument for each proposition broken into parts, outlining the columns representing the premises and conclusion.

$$J \supset E / \sim J // \sim E$$

T	T	T	T
T	F	T	F
F	T	F	T
F	F	F	F

TRUTH TABLE- ARGUMENTS

If juvenile killers are as responsible for their crimes as adults, then execution is a justifiable punishment. Juvenile killers are not as responsible for their crimes as adults. Therefore, execution is not a justifiable punishment.

3. Draw a truth table for the symbolized argument for each proposition broken into parts, outlining the columns representing the premises and conclusion.

$$J \supset E / \sim J // \sim E$$

T	T	T	T
T	F	T	F
F	T	F	T
F	F	F	F

Notice: 2 Letters for 2 propositions = 4 rows
Make columns identical for the same letter

TRUTH TABLE- ARGUMENTS

If juvenile killers are as responsible for their crimes as adults, then execution is a justifiable punishment. Juvenile killers are not as responsible for their crimes as adults. Therefore, execution is not a justifiable punishment.

3. Draw a truth table for the symbolized argument for each proposition broken into parts, outlining the columns representing the premises and conclusion.

J \supset E		/	$\sim J$ // $\sim E$	
T	T	T	F	T
T	F	F	F	T
F	T	T	T	F
F	T	F	T	T

TRUTH TABLE- ARGUMENTS

If juvenile killers are as responsible for their crimes as adults, then execution is a justifiable punishment. Juvenile killers are not as responsible for their crimes as adults. Therefore, execution is not a justifiable punishment.

4. Look for a line in which all of the premises are true, and the conclusion is false. If such a line exists, the argument is invalid; if not, it is valid.

$$\begin{array}{c} J \supset E \quad / \quad \sim J \quad // \quad \sim E \\ \begin{array}{ccccc} T & \boxed{T} & T & F & T \\ T & \boxed{F} & F & F & T \\ F & T & T & T & F \\ F & T & F & F & F \end{array} \end{array}$$

T	T	T	F	T
T	F	F	F	T
F	T	T	T	F
F	T	F	F	F

TRUTH TABLE - ARGUMENTS

If juvenile killers are as responsible for their crimes as adults, then execution is a justifiable punishment. Juvenile killers are not as responsible for their crimes as adults. Therefore, execution is not a justifiable punishment.

4. Look for a line in which all of the premises are true, and the conclusion is false. If such a line exists, the argument is invalid; if not, it is valid.

J	⊃	E	/	~J	//	~E
T	T	T		F		
T	F	F		T		
F	(T)	T		(T)		
F	T	F		T		

INVALID

TRUTH TABLE - ARGUMENTS

If juvenile killers are as responsible for their crimes as adults, then execution is a justifiable punishment. Juvenile killers are not as responsible for their crimes as adults. Therefore, execution is not a justifiable punishment.

4. Look for a line in which *all of the premises are true*, and the *conclusion is false*. If such a line exists, the argument is invalid; if not, it is valid.

J	\equiv	E	/	$\sim J$	//	$\sim E$
T	T	T		F		
T	F	F		T		
F	F	T		T		
F	T	F		F		

BONUS QUESTION:
How can we change the argument to make it valid?

WORK IT OUT!

If microchips are made from diamond wafers, then computers will generate less heat. Computers will not generate less heat and microchips will be made from diamond wafers. Therefore, synthetic diamonds will be used for jewelry. (M, C, S)