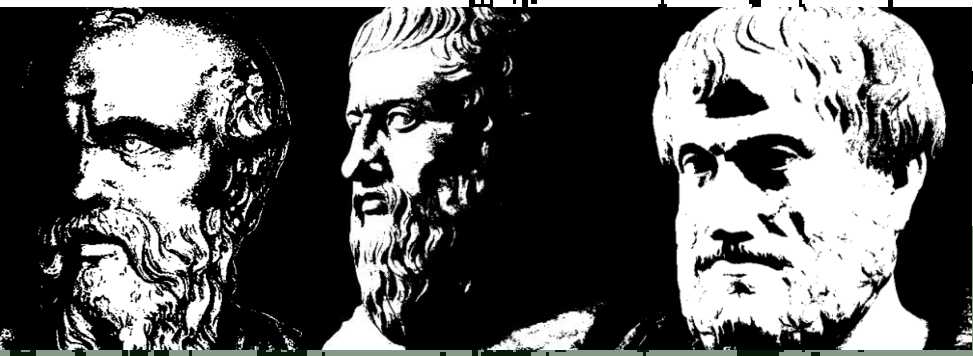
LOGIC I



PROPOSITIONAL LOGIC

Dr. Ethan SAHKER, PhD

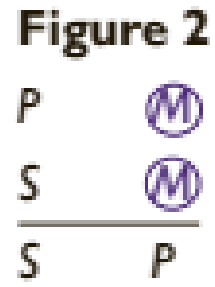
Propositional Logic

11. (2 points)
Select the Major Term in the following Categorical Syllogism:
All neutron stars are things that produce intense gravity.
All neutron stars are extremely dense objects.
Therefore, all extremely dense objects are things that produce intense gravity.

a. Neutron Stars
b. Extremely dense objects
c. Therefore
*d. Things that produce intense gravity

12. (5 points)
Create a categorical syllogism in standard form with EIO-2 mood.
Use real categories.
No mathematicians are clever.
Some economists are clever.
Therefore, some economists are not mathematicians.

Proposition	Letter name
All *S* are *P*.	**A**
No *S* are *P*.	**E**
Some *S* are *P*.	**I**
Some *S* are not *P*.	**O**



Standard Form Categorical Syllogism

1. 3 categorical propositions
2. Each term occurs 2 times identically
3. Term use same meaning
4. The major premise is listed first

LOGIC I

TRUTH FUNCTIONS

Propositional Logic

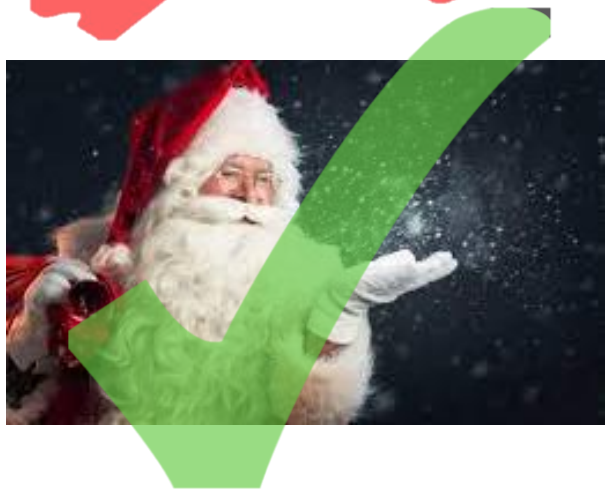
Remember the Operators

Operator	Name	Logical function	Used to translate
~	tilde	negation	not, it is not the case that
•	dot	conjunction	and, also, moreover
V	wedge	disjunction	or, unless
⊃	horseshoe	implication	if . . . then . . . , only if
≡	triple bar	equivalence	if and only if

Remember we have assumed truth does not matter

Propositional Logic

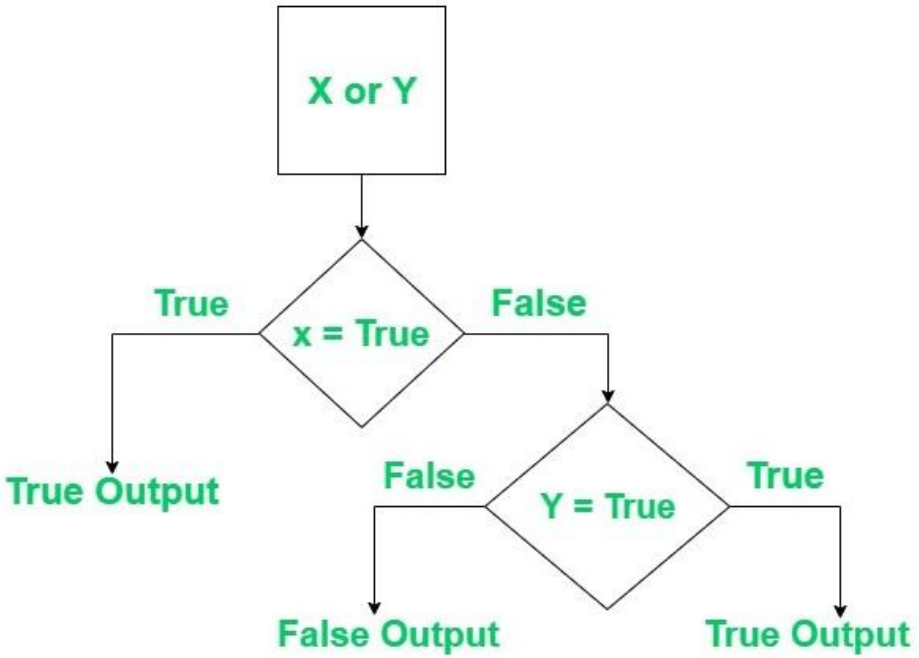
NOW we are concerned with the TRUTH



Propositional Logic

Truth Tables & Compound Propositions

- Think about your careers and when you need to make a decision.
  - Engineering – which material works
  - Chemistry – series of reactions
  - Law – offense and punishment
  - Physics – machine learning
  - Medicine – diagnosis
  - Economics – monetary policy or Sales
  - Agriculture – breed combinations and yield



Propositional Logic - Truth Tables

Make a table to determine all possible True/False combinations

Number of different simple propositions	Number of lines in truth table
1	2
2	4
3	8
4	16
5	32
6	64

$$L = 2^n$$

Where L is number of lines
2 is number of possible answers (T/F)
and n is number of simple propositions or terms

Propositional Logic - Truth Tables

Make a table to determine all possible True/False combinations

Number of different simple propositions	Number of lines in truth table
1	2
2	4
3	8
4	16
5	32
6	64

$$L = 2^n$$

If I pass the class and do not get a good grade on the quiz, then I will be respected by my parents.

Propositional Logic - Truth Tables

Make a table to determine all possible True/False combinations

p
T
F

Half lines T Half F

p = passing the class
q = n/a
r = n/a
s = n/a

$$L = 2^n$$

L = number of lines
2 = possible answers (T/F)
n = number of propositions/terms

Propositional Logic - Truth Tables

Make a table to determine all possible True/False combinations

p	q
T	T
T	F
F	T
F	F

Half lines T Half F
Half Ts T Half F

p = passing the class
q = good grade on the quiz
r = n/a
s = n/a

$$L = 2^n$$

L = number of lines
2 = possible answers (T/F)
n = number of propositions/terms

Propositional Logic - Truth Tables

Make a table to determine all possible True/False combinations

p	q	r
T	T	T
T	T	F
T	F	T
T	F	F
F	T	T
F	T	F
F	F	T
F	F	F

Half lines T Half F
Half Ts T Half F
Half Ts T Half F

p = passing the class
q = good grade on the quiz
r = respected by my parents
s = n/a

$$L = 2^n$$

L = number of lines
2 = possible answers (T/F)
n = number of propositions/Terms

Propositional Logic - Truth Tables

ADD 1 more column for each operator

p	q	r	~q	p · ~q	(p · ~q) ⊃ r
T	T	T			
T	T	F			
T	F	T			
T	F	F			
F	T	T			
F	T	F			
F	F	T			
F	F	F			

p = passing the class
q = good grade on the quiz
r = respected by my parents

If I pass the class and do not get a good grade on the quiz, then I will be respected by my parents.

$$(p \cdot \sim q) \supset r$$

Propositional Logic - Truth Tables

ADD 1 more column for each operator

p	q	r	~q	p · ~q	(p · ~q) ⊃ r
T	T	T			
T	T	F			
T	F	T			
T	F	F			
F	T	T			
F	T	F			
F	F	T			
F	F	F			

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Propositional Logic - Truth Tables

Negation

- Will be opposite of the proposition

p	$\sim p$
T	
F	

Ex.

- McDonalds does not make hamburgers. → ~M
- Starbucks does not make hamburgers. → ~S

Propositional Logic - Truth Tables

Conjunction

- True if both options are true

p	q	$p \bullet q$
T	T	
T	F	
F	T	
F	F	

Ex.

- Ferrari and Maserati make sports cars. F · M
- Ferrari and GMC make sports cars. F · G
- GMC and Jeep make sports cars. G · J

```
/*Drug use improvement after treatment*/
        if (freq1_a=3) and (freq1_d in(1,2)) then IMPROVED = 1;
        else if (freq1_a =2) and (freq1_d=1) then IMPROVED = 1;
        else if (freq1_a =1) and (freq1_d=1) then IMPROVED = 1;
        else if (freq1_a = -9) or (freq1_d = -9) then delete;
        else IMPROVED = 0;
```

Propositional Logic - Truth Tables

Disjunction

- True if either option is true

p	q	p ∨ q
T	T	
T	F	
F	T	
F	F	

Ex.

- Either Murakami or Kawabata are novelists. M v K
- Either Murakami or Suga are novelists. M v S
- Either Ichiro or Murakami are novelists. I v M
- Either Ichiro or Suga are novelists. I v S

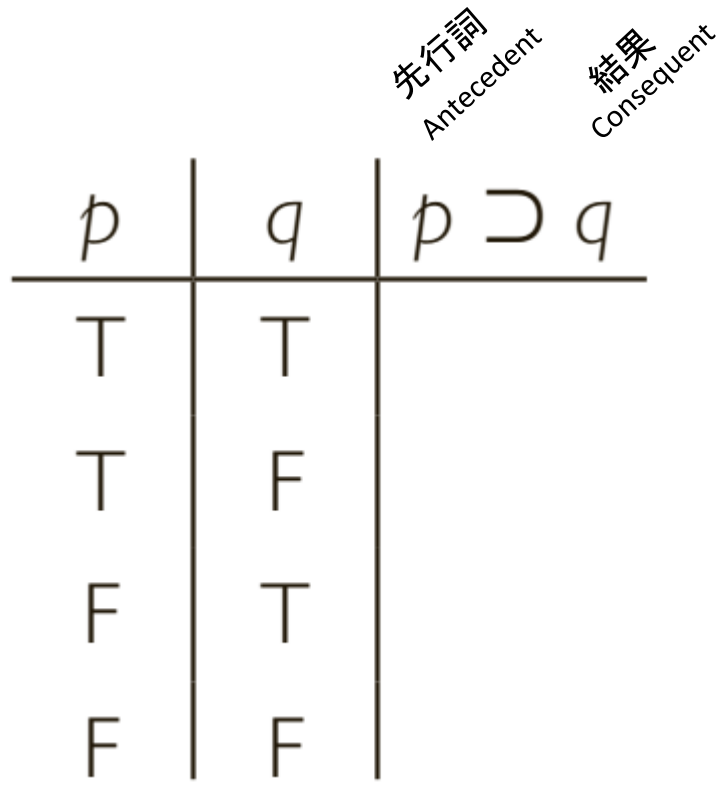
Propositional Logic - Truth Tables

Conditional

- Antecedent (先行詞) and Consequent (結果)
- True when Consequent = T
- Why? → sometimes result is true without cause
- True when Antecedent and Consequent = F
- Result is false & cause is false = rule must be true

Ex.

- If I work hard, then I will get a raise.



Propositional Logic - Truth Tables

Biconditional - (if and only if)

- True if both Antecedent and Consequent = T
- True if both Antecedent and Consequent = F

Ex.

- If and only if you get a D or higher, then you will pass my class.

先行詞 Antecedent　結果 Consequent

p	q	$p \equiv q$
T	T	
T	F	
F	T	
F	F	

Propositional Logic - Truth Tables

Logic Operators

∧ or & and	**↔ or ≡** iff	**⊥** false, contradiction
∨ or	**|** nand	**⊢** entails, proves
¬ or ~ not	**∀** Universal Quantification	**⊨** entails, therefore
→ implies	**∃** Existential quantification	**∴** therefore
⊃ implies, superset	**⊤** true, tautology	**∵** because

Propositional Logic

Simple Compound Propositions

p	q	~q	p · q	p ∨ q	p ⊃ q	p ≡ q
T	T	F	T	T	T	T
T	F	T	F	T	F	F
F	T	F	F	T	T	F
F	F	T	F	F	T	T

Complex Propositions

$$(p \cdot \sim q) \supset r$$

Lines: $L = n^2$
$= 8$

Columns = n + # of operators
= 3 + 3 = 6

Propositional Logic - Truth Tables

Solve for proposition combinations

p	q	r	~q	p ▪ ~q	(p ▪ ~q) ⊃ r

p = passing the class
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r = respected by my parents

If I pass the class and do not get a good grade on the quiz, then I will be respected by my parents.

$$(p \cdot \sim q) \supset r$$

Propositional Logic - Truth Tables

Solve for proposition combinations

p	q	r	~q	p ▪ ~q	(p ▪ ~q) ⊃ r
T					
T					
T					
T					
F					
F					
F					
F					

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Propositional Logic - Truth Tables

Solve for proposition combinations

p	q	r
T	T	
T	T	
T	F	
T	F	
F	T	
F	T	
F	F	
F	F	

~q	p ▪ ~q	(p ▪ ~q) ⊃ r

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~ ▪ ∨ ⊃ ≡

Propositional Logic - Truth Tables

Solve for proposition combinations

p	q	r	~q	p ▪ ~q	(p ▪ ~q) ⊃ r
T	T	T			
T	T	F			
T	F	T			
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F	T	F			
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$$(p \cdot \sim q) \supset r$$

~ · ∨ ⊃ ≡

Propositional Logic - Truth Tables

Solve for each operator

p	q	r
T	T	T
T	T	F
T	F	T
T	F	F
F	T	T
F	T	F
F	F	T
F	F	F

~q	p · ~q	(p · ~q) ⊃ r

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$$(p \cdot \sim q) \supset r$$

Propositional Logic - Truth Tables

Solve for each operator

p	q	r	~q	p ▪ ~q	(p ▪ ~q) ⊃ r
	T		F		
	T		F		
	F		T		
	F		T		
	T		F		
	T		F		
	F		T		
	F		T		

p = passing the class
q = good grade on the quiz
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~ ▪ ∨ ⊃ ≡

Propositional Logic - Truth Tables

Solve for each operator

p	q	r	~q	p ▪ ~q	(p ▪ ~q) ⊃ r
T	T	T	F		
T	T	F	F		
T	F	T	T		
T	F	F	T		
F	T	T	F		
F	T	F	F		
F	F	T	T		
F	F	F	T		

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~ ▪ ∨ ⊃ ≡

Propositional Logic - Truth Tables

Solve for each operator

p	q	r	~q	p ▪ ~q	(p ▪ ~q) ⊃ r
T			F	F	
T			F	F	
T			T	T	
T			T	T	
F			F	F	
F			F	F	
F			T	F	
F			T	F	

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Solve for each operator

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T	T	T	F	F	
T	T	F	F	F	
T	F	T	T	T	
T	F	F	T	T	
F	T	T	F	F	
F	T	F	F	F	
F	F	T	T	F	
F	F	F	T	F	

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If I pass the class and do not get a good grade on the quiz, then I will be respected by my parents.

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Propositional Logic - Truth Tables

Solve for each operator

p	q	r
		T
		F
		T
		F
		T
		F
		T
		F

~q	p ▪ ~q	(p ▪ ~q) ⊃ r
	F	T
	F	T
	T	T
	T	F
	F	T
	F	T
	F	T
	F	T

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Propositional Logic - Truth Tables

Solve for each operator

p	q	r	~q	p ▪ ~q	(p ▪ ~q) ⊃ r
T	T	T	F	F	T
T	T	F	F	F	T
T	F	T	T	T	T
T	F	F	T	T	F
F	T	T	F	F	T
F	T	F	F	F	T
F	F	T	T	F	T
F	F	F	T	F	T

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