Introduction to Probability: Exercise sheet 1

These questions are adapted from Grinstead/Snell 'Introduction to Probability'.

1. A six-sided die is loaded so that the probability of each face turning up is proportional to the number of dots on that face. (E.g. A six is three times more likely than a two.) What is the probability of getting an even number in one throw?

2. For a bill to come before the president of the USA, it must pass both the House of Representatives and the Senate. Assume that, of the bills presented to these two bodies, 60% pass the House of Representatives, 80% pass the Senate, and 90% pass at least one of the two. Calculate the probability that the next bill presented to the two groups will come before the president. NB. You may assume a frequentist position, namely that past proportions reflect the probabilities of the various outcomes on the subsequent trial.

3. From a deck of five cards numbered $1, 2, 3, 4, 5$, respectively, a card is drawn at random and replaced. This is done three times. What is the probability that the card numbered 1 was drawn exactly 2 times, given that the sum of the numbers on the three draws is 6?

4. Consider the following information:

   - In London, half the days have some rain.
   - The weather forecaster is correct $2/3$ of the time, i.e. the probability that it rains, given that rain is predicted, and that probability that it does not rain, given that rain is not predicted, are both equal to $2/3$.
   - When rain is forecast, Mr. Pickwick takes his umbrella. When rain is not forecast, he takes it with probability $1/3$.
   - Conditional on the prediction of the forecaster, the decision of Mr. Pickwick is independent of whether it rains or not.*

   Find:

   (a) the probability that rain is predicted;

   (b) the probability that Mr. Pickwick takes his umbrella;

   (c) i. the probability that Mr. Pickwick has no umbrella, given that it rains;

   ii. the probability that it doesn't rain, given that he brings his umbrella.

   * *To say that, conditional on event C, the event A is independent of the event B means that*

$$\mathbf{P}(A \cap B | C) = \mathbf{P}(A | C)\, \mathbf{P}(B | C)\,.$$