Introduction to Probability – E2

Examination date: 2025/07/24, Thursday 2nd period
Examination time: 80 minutes
Examination venue: 14, Yoshida-South Campus Bldg. No. 4 ("4 共 14")

Whilst the exam will primarily focus on analysis of example applications (similar to the examples sheets), you should also be familiar with the technical background.

You will need to answer all questions.

Things that it might help to review when preparing for the exam:

1. Basic probability
    - Probability distribution on a discrete (countable) space.
    - Conditional probability
    - Independence
    - Law of total probability
    - Expectation of a discrete random variable
    - Probability generating functions and their properties

2. Markov chains
    - Basic definition, Markov property
    - Graphical representation of a Markov chain
    - Transition probabilities via matrix multiplication
    - Class structure (closed/open communicating classes)
    - Hitting times and absorption probabilities, first-step decomposition
    - Stopping times and strong Markov property
    - Applications of probability generating functions to Markov chains
    - Invariant distributions, conditions for existence and uniqueness
    - **Convergence to equilibrium (not examinable!)*

3. Important examples
    - Two state Markov chain
    - Simple random walk, gambler's ruin, birth and death chains