

LOGIC I

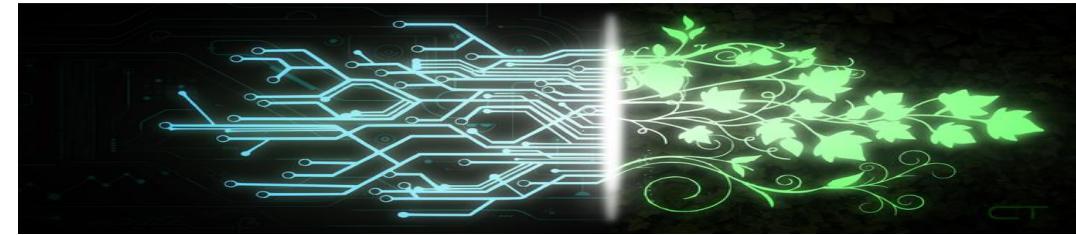
FORMAL LOGIC: CATEGORICAL PROPOSITIONS

Dr. Ethan SAHKER, PhD

Categorical Propositions カテゴリの命題

- A proposition (statement) relating to 2 groups is a categorical proposition.
- The groups are named by the subject term 主語 and the predicate term 述語.
- The proposition claims that *all*, *part*, or *none* of the subject group is included or excluded from the predicate group.

All/some/none of A is/is not B.



EXAMPLES:

1. **All** reality TV stars hope for recognition.
2. **No** junk foods belong in school cafeterias.
3. **Many** of today's unemployed have given up on finding work.
4. **Not all** romances have a happy ending.

Categorical Propositions

Every complete sentence contains two parts: a **subject** and a predicate.

Subject term: what (or whom) the sentence is about

Predicate term: tells something about the subject

EXAMPLES:

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1. All **reality TV stars** hope for **recognition**.
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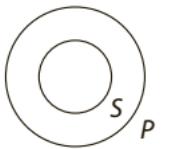


Categorical Propositions

4 Types of Standard-Form Categorical Propositions

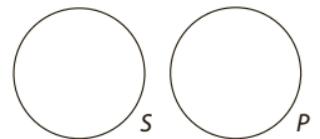
1. Claim that the whole subject group is included in the predicate group

- All S are P



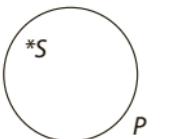
2. Claim that the whole subject group is excluded from the predicate group

- No S are P



3. Claim that part of the subject group is included in the predicate group

- Some S are P



4. Claim that part of the subject group is included in the predicate group

- Some S are not P



Categorical Propositions

Analyzing Standard Form Categorical Propositions

Quantifier: all, some, none

Subject Term: group, class, thing, idea

Copula: are, are not (linking term)

Predicate Term: action, state, descriptor, thing, class, group, idea



Quantifier: all

Subject Term: members of the JMA

Copula: are

Predicate Term: people holding degrees...

All members of the Japan Medical Association are people holding degrees from recognized academic institutions.

Categorical Propositions

EXAMPLES:

1. Every reality TV star hopes for recognition.
2. No junk foods belong in school cafeterias.
3. Many of today's unemployed have given up on finding work.
4. Not all romances have a happy ending.

Quantifier: all, some, none - **many, lots, total, completely, a few, a couple, zero**

Subject Term: group, class, thing, idea **[noun]**

Copula: are, are not **[to be verbs & verbs] – have, exist, feel,**

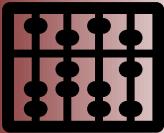
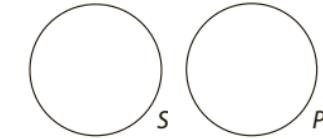
Predicate Term: action, state, descriptor, thing, class, group, idea **[noun, verb, adjective]**

Categorical Propositions



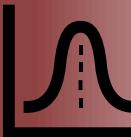
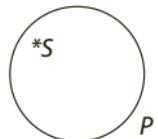
Quality: Negative or Positive

- affirms or denies group membership
- all S are P = affirming S = Positive
- no S are P = denies S = Negative



Quantity: Universal or Particular

- all versus some of the group members
- no S are P = every member included = Universal
- some S are P = some members included = Particular



Distribution: Distributed or Undistributed

- If every P or every S is something it is distributed, else undistributed
- All S are P = S distributed
- some S are not P = P distributed

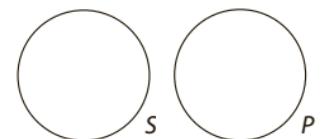
Categorical Propositions

4 Types of Standard-Form Categorical Propositions

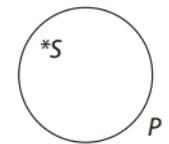
1. Claim that the whole subject group is included in the predicate group
 - All S are P = Universal Affirmative – S Distributed



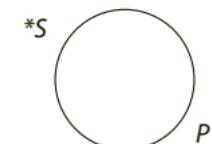
2. Claim that the whole subject group is excluded from the predicate group
 - No S are P = Universal Negative – S & P Distributed



3. Claim that part of the subject group is included in the predicate group
 - Some S are P = Particular Affirmative – S & P Undistributed



4. Claim that part of the subject group is excluded from the predicate group
 - Some S are not P = Particular Negative – P Distributed



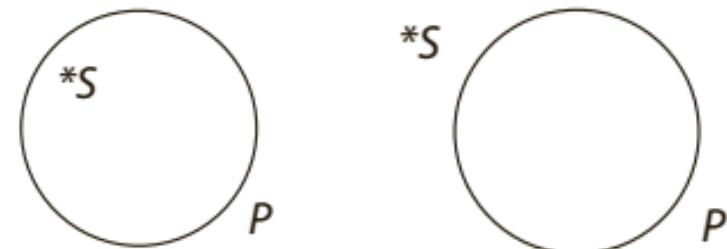
Categorical Propositions

Real life example

Race: Black, White, Asian, Native American

Ethnicity: Hispanic/Latino

Raceeth: New variable from combining both



S

P

Some Asians are Latino

(Peru, Chile, Brazil & Argentina)

Some Asians are not Latino

(Japan)

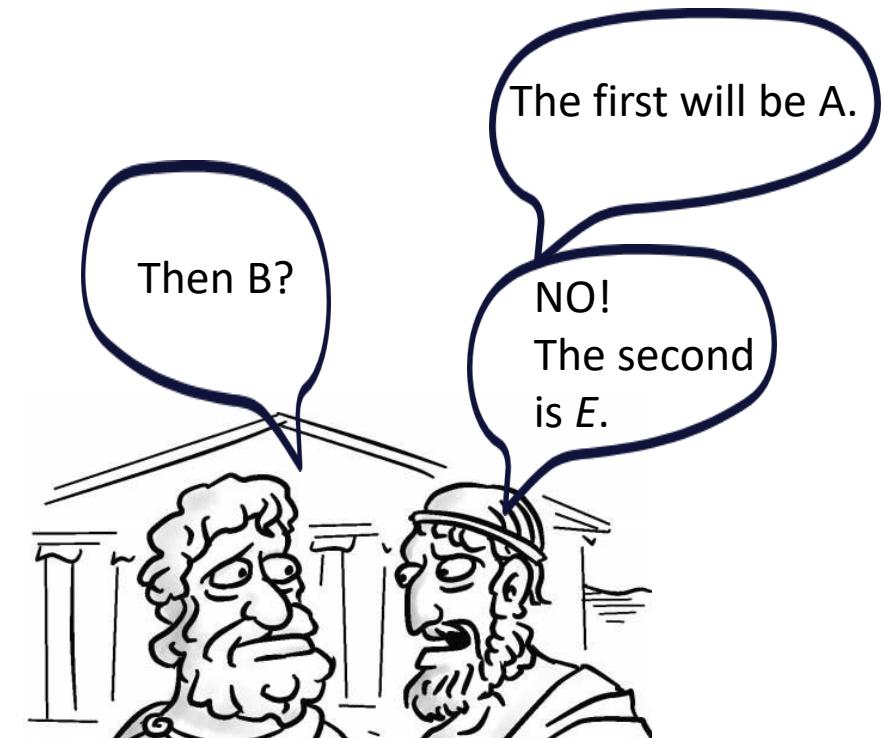
/*Race Ethnicity = raceeth*/

```
if (RACE = white and ETHNIC = non-latino) then raceeth = 1; /*white non-latino*/
else if (ETHNIC ne . and ETHNIC ne non-latino) then raceeth = 2; /*latino*/
else if (RACE = black and ETHNIC = non-latino) then raceeth = 3; /*black non- latino */
else if ((RACE = asian and ETHNIC = non-latino) then raceeth = 4; /*asian non-lat*/
if raceeth = . then delete;
```

Categorical Propositions

Old men decided the 4 forms are represented as A, E, I, and O

- All S are P = Universal Affirmative = A
- No S are P = Universal Negative = E
- Some S are P = Particular Affirmative = I
- Some S are not P = Particular Negative = O



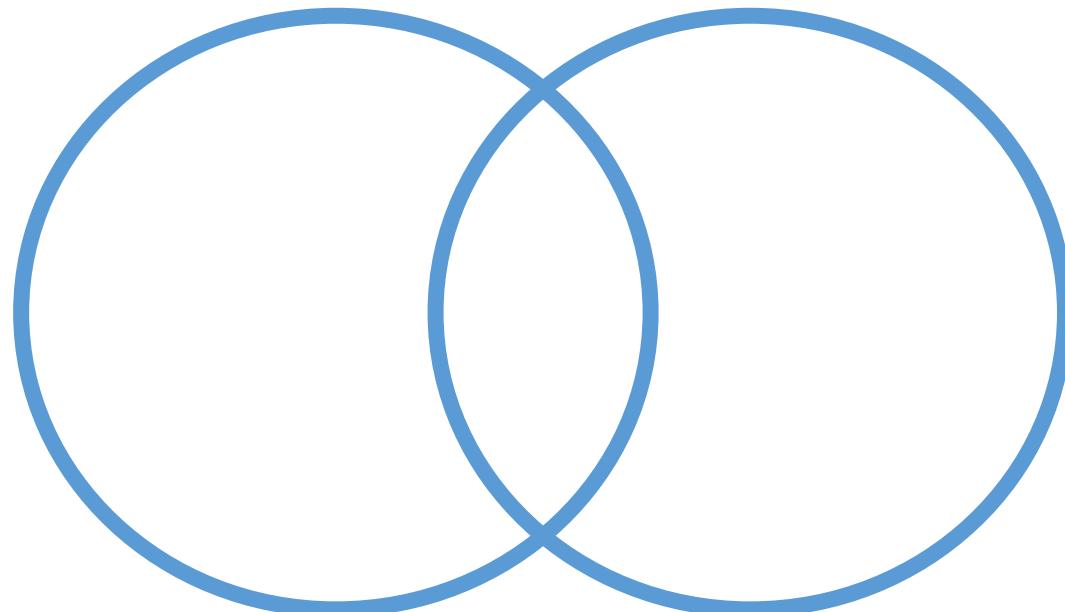
Categorical Propositions

OVERVIEW

Proposition	Letter name	Quantity	Quality	Terms distributed
All S are P.	A	universal	affirmative	S
No S are P.	E	universal	negative	S and P
Some S are P.	I	particular	affirmative	none
Some S are not P.	O	particular	negative	P

Categorical Propositions

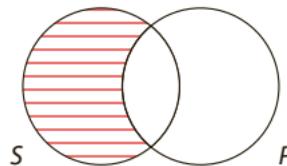
Venn Diagrams



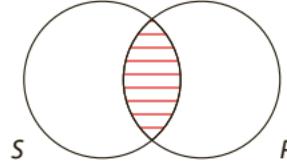
Categorical Propositions

Venn Diagrams

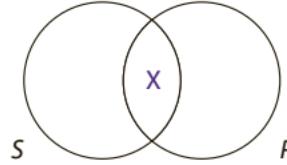
A: All S are P.



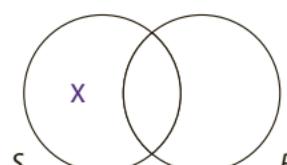
E: No S are P.



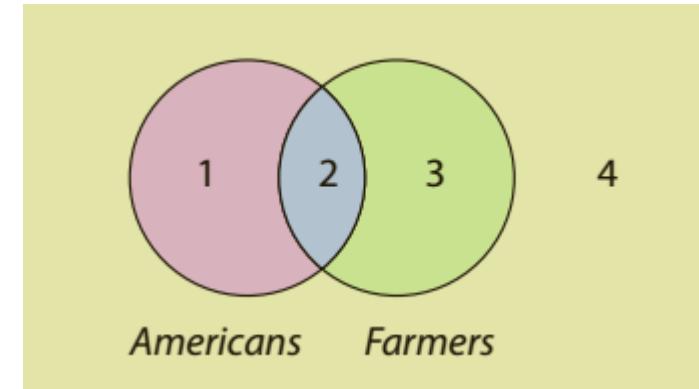
I: Some S are P.



O: Some S are not P.



Shading = emptiness
X = existence



1 = American but not a farmer

2 = American *and* a farmer

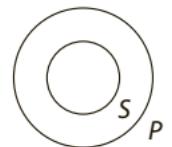
3 = A farmer but not American

4 = neither a farmer nor American

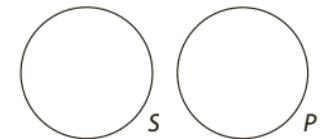
Categorical Propositions

Proposition	Letter name
All S are P.	A
No S are P.	E
Some S are P.	I
Some S are not P.	O

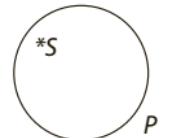
All hinoki are evergreen trees.



No hinoki are evergreen.



Some hinoki trees are evergreen.



All hinoki are not evergreen.



Categorical Propositions

The Traditional Square of Opposition

Existence Matters – Aristotelian View

Contradictory: opposite truth value

Contrary: at least one is false (not both true)

Subcontrary: at least one is true (not both false)

Subalternation: truth flows downward, falsity flows upward

Categorical Propositions

The Traditional Square of Opposition

Existence Matters – Aristotelian View

Contradictory: opposite truth value

Contrary: at least one is false (not both true)

T	F
F	T

Subcontrary: at least one is true (not both false)

Subalternation: truth flows downward, falsity flows upward

Categorical Propositions

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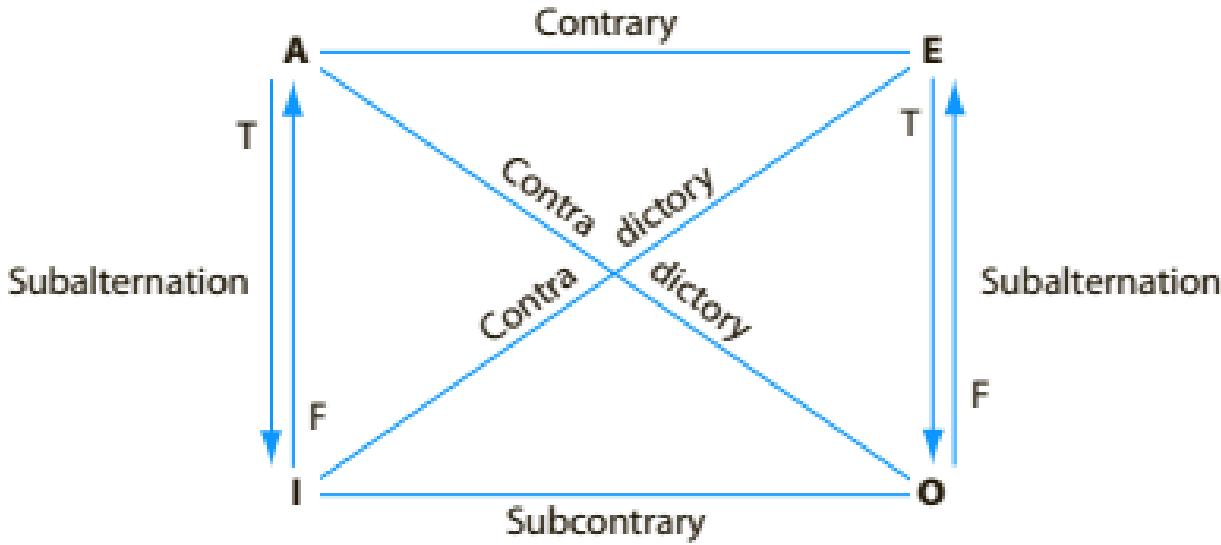
Subalternation: truth flows downward, falsity flows upward

F
↑
T

Categorical Propositions

The Traditional Square of Opposition

Proposition	Letter name
All S are P.	A
No S are P.	E
Some S are P.	I
Some S are not P.	O



A: All hinoki are evergreen trees.

E: No hinoki are evergreen trees.

I: Some hinoki are evergreen trees.

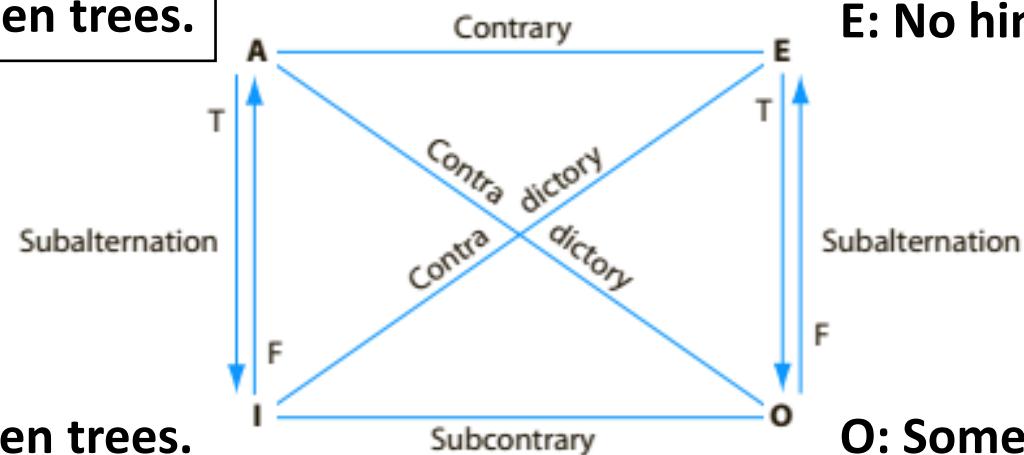
O: Some hinoki are not evergreen trees.

Categorical Propositions

The Traditional Square of Opposition

Contrary: at least one is false (not both true)

A: All hinoki are evergreen trees.



E: No hinoki are evergreen trees.

I: Some hinoki are evergreen trees.

Subalternation: truth flows downward, falsity flows upward

O: Some hinoki are not evergreen trees.

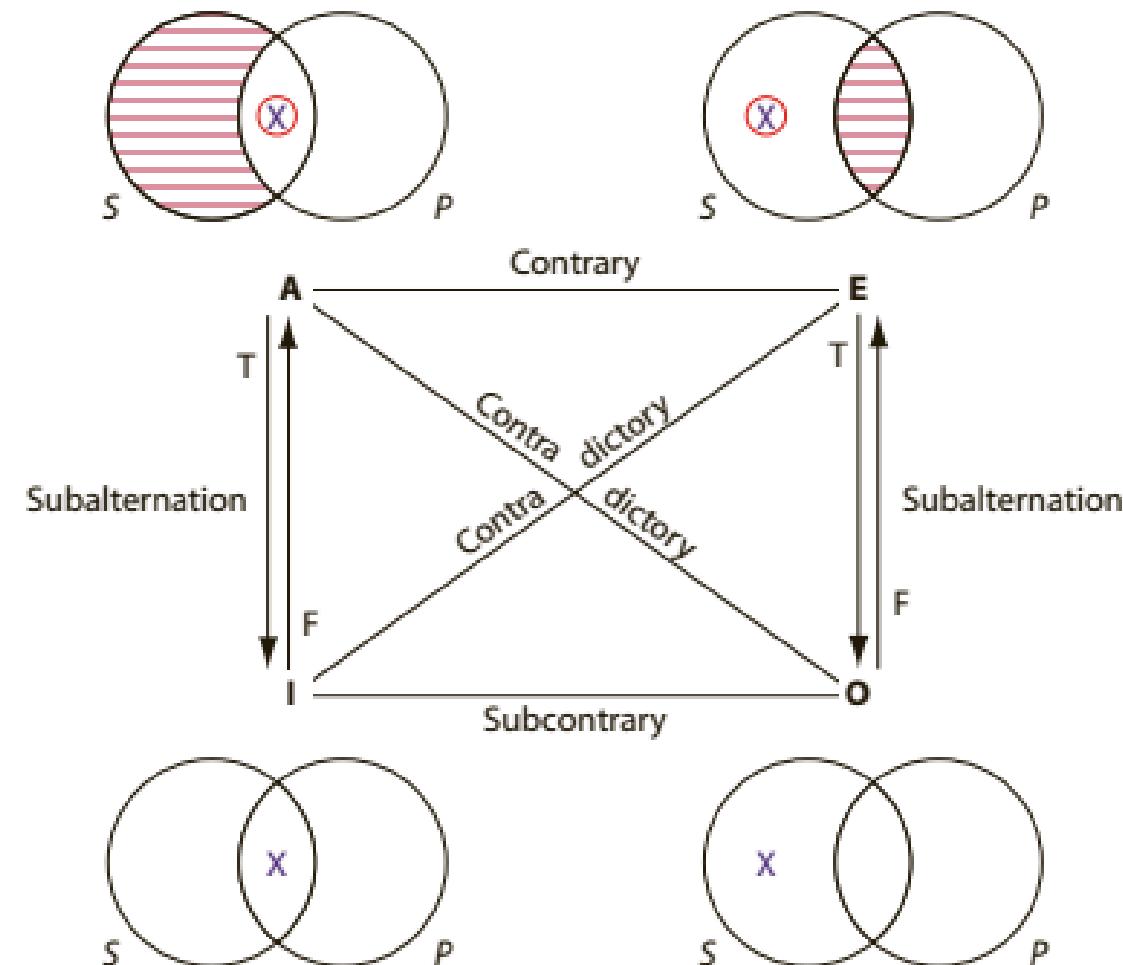
Contradictory: opposite truth value

T	F
F	T

Subcontrary: at least one is true (not both false)

Categorical Propositions

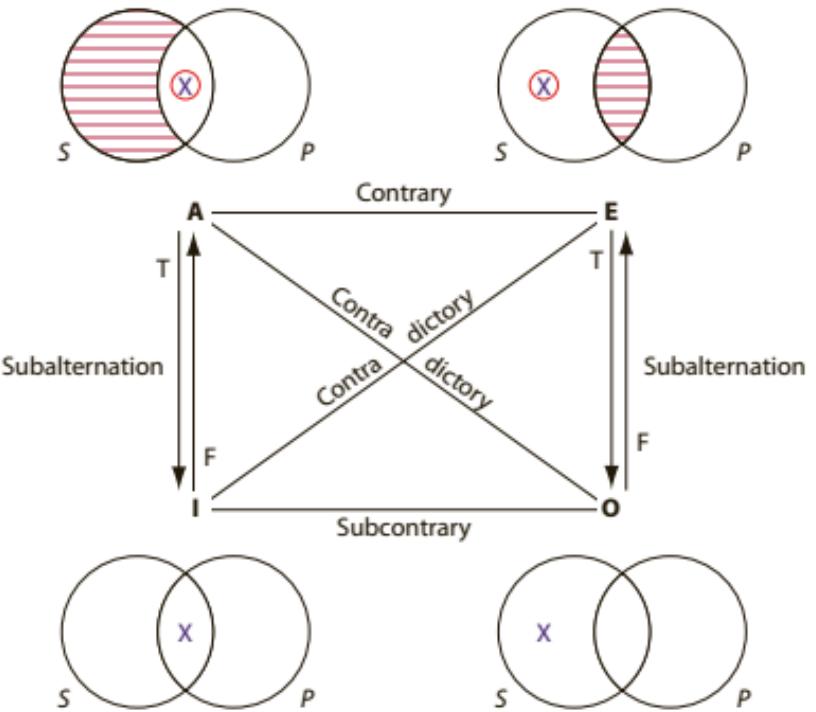
The Traditional
Square of
Opposition
Venn Diagrams



Categorical Propositions

The Traditional Square of Opposition

____ are ____
____ are ____
____ are ____
____ are ____



Categorical Propositions

Existential Issue

T or F

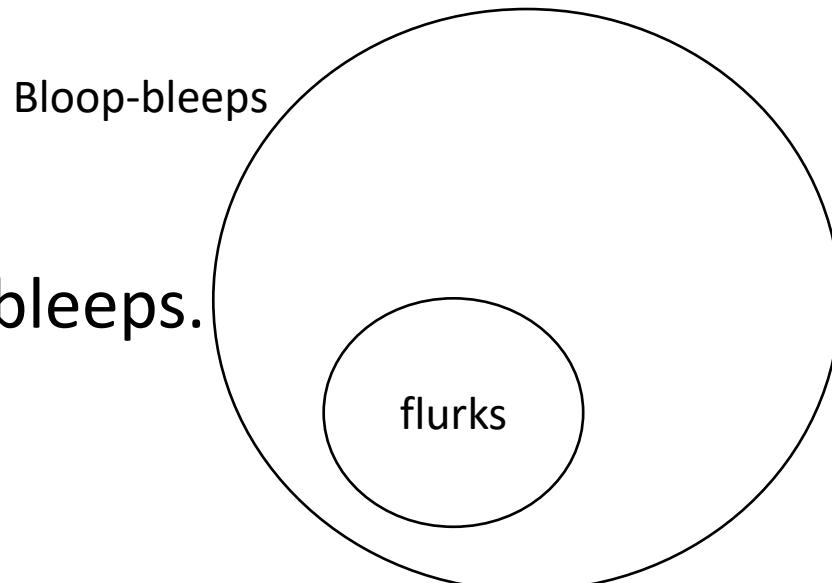
Reality does not matter.

We only assume **logical or possible truths**.

This is deduction

T
F

All flurks are Bloop-bleeps.



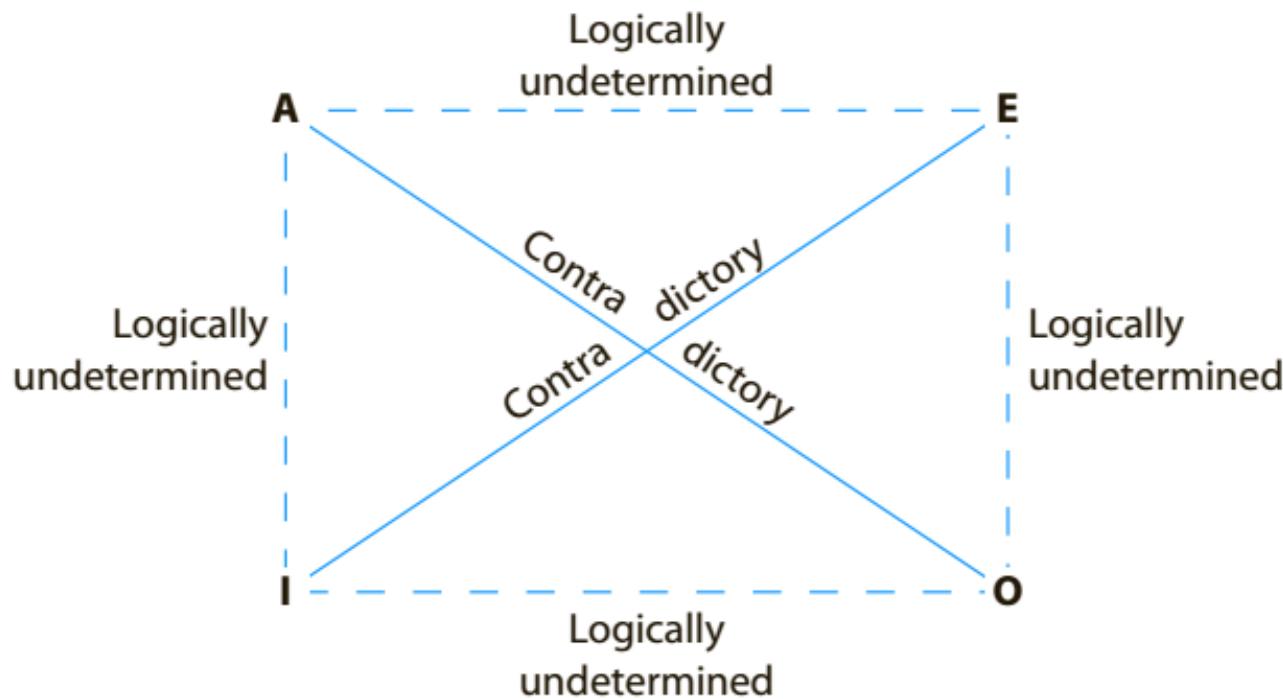
Categorical Propositions



Categorical Propositions

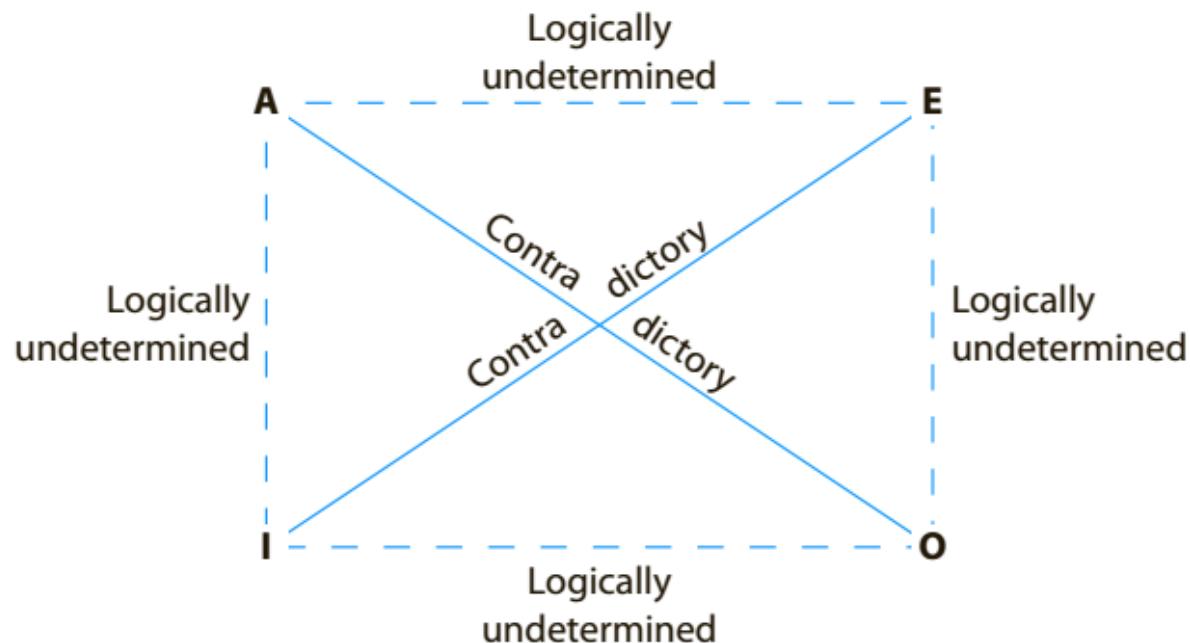
The Modern Square of Opposition

Existence does not matter – Boolean View



Categorical Propositions

The Modern Square of Opposition



Proposition	Letter name
All S are P.	A
No S are P.	E
Some S are P.	I
Some S are not P.	O

A: All Bigfoot are happy.

E: No Bigfoot are happy.

I: Some Bigfoot are happy.

O: Some Bigfoot are not happy.

Categorical Propositions

The Modern Square of Opposition

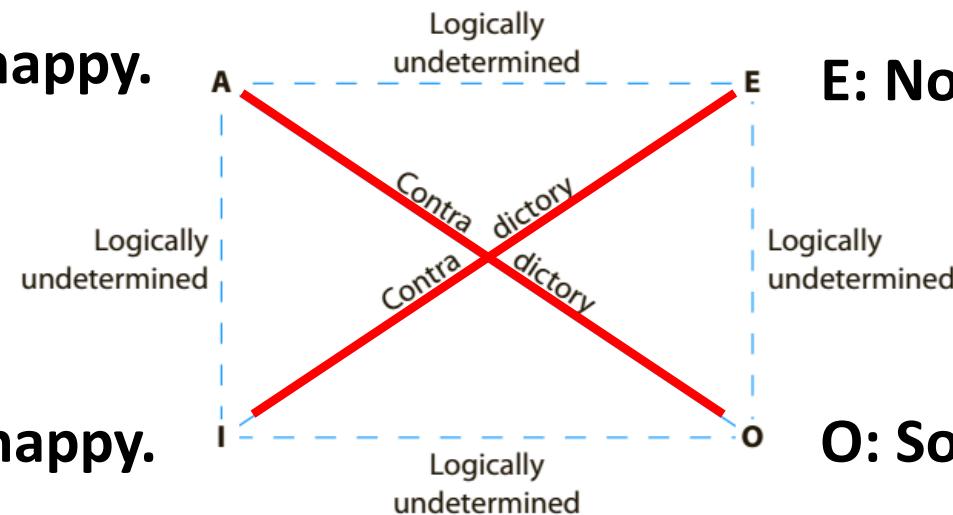
A	O	E	I
T	T	T	T
T → F	F	T → F	F
F	T	F	T
F	F	F	F

A: All Bigfoot are happy.

E: No Bigfoot are happy.

I: Some Bigfoot are happy.

O: Some Bigfoot are not happy.



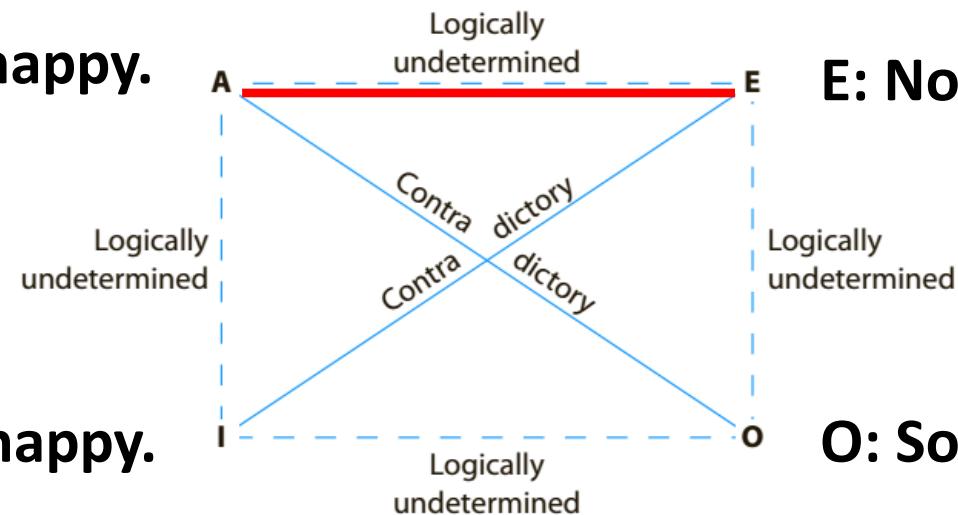
Categorical Propositions

The Modern Square of Opposition

- Universal statements (**A** and **E**) do not imply that **S** exists.
- Only particular statements (**I** and **O**) carry existential import, implying **S** exists.
- Remember in the Modern Square, existence doesn't matter.
- Both **A** & **E** statements can be true because there are no bigfoots



A: All Bigfoot are happy.



E: No Bigfoot are happy.

I: Some Bigfoot are happy.

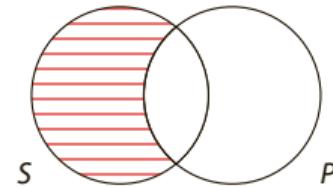
O: Some Bigfoot are not happy.

Categorical Propositions

The Modern Square of Opposition

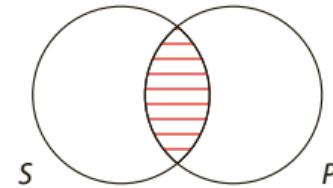
A: All Bigfoot are happy.

A: All S are P .



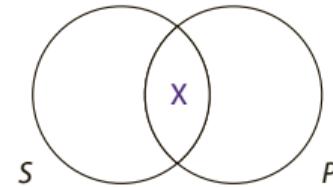
E: No Bigfoot are happy.

E: No S are P .



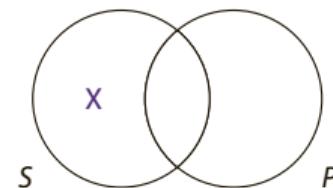
I: Some Bigfoot are happy.

I: Some S are P .



O: Some Bigfoot are not happy.

O: Some S are not P .



Categorical Propositions

Conversion, Obversion, & Contraposition

Evaluating through logical substitution

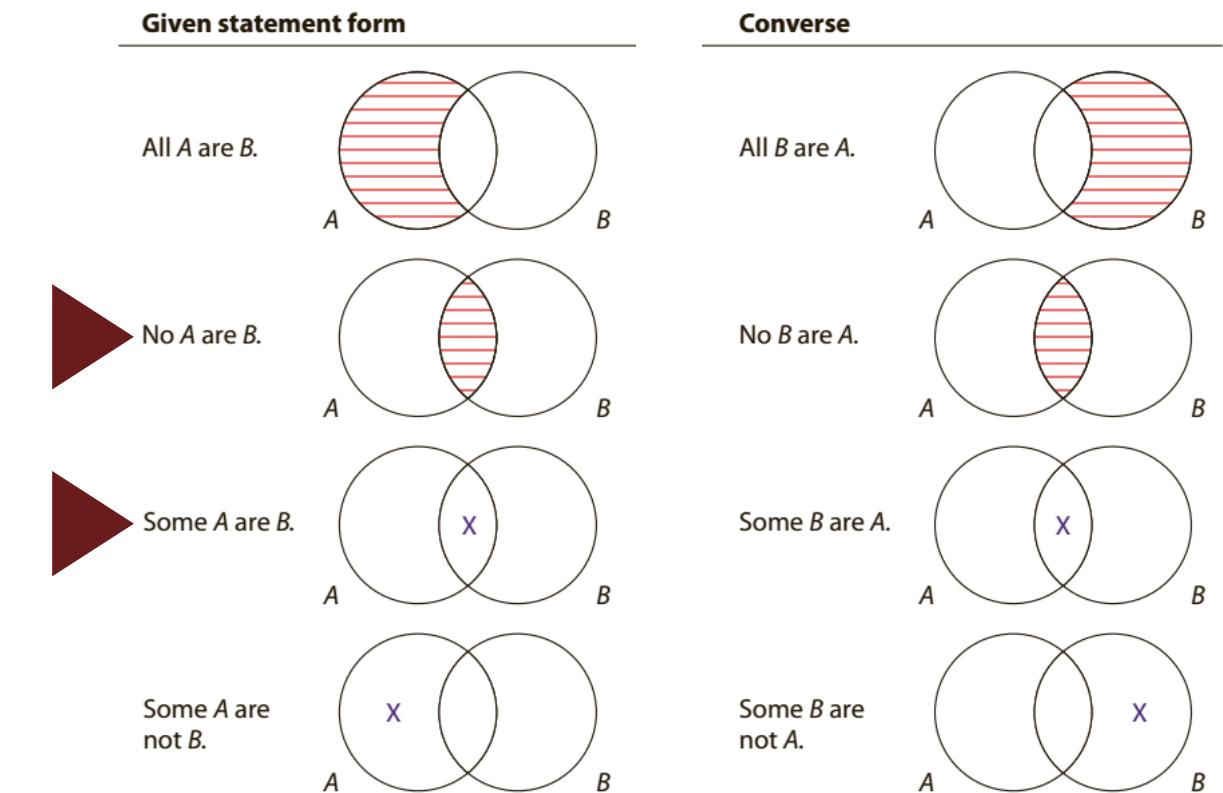
If statements are true after substitution, then the statements are valid and well constructed.

Categorical Propositions

Conversion

Switching the subject with the predicate

Notice the **E** and **I** statements:
They are identical and logically equivalent

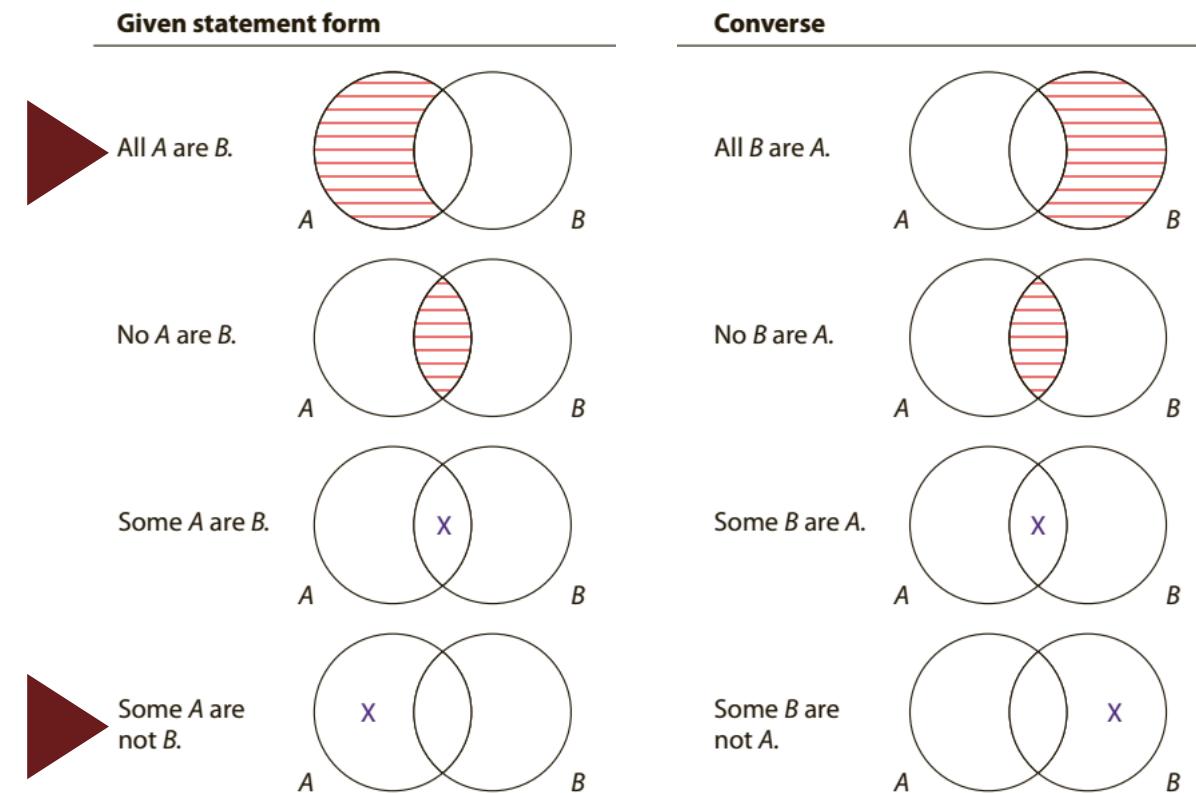


Categorical Propositions

Conversion

Switching the subject with the predicate

The **A** and **O** statements:
They are obviously different



Categorical Propositions

Conversion

We can infer truths through conversion for E and I Statements.

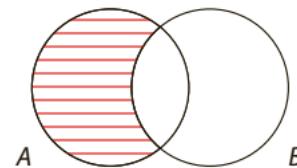
If no ocean is red,
then nothing red is an ocean

If some oceans are red,
then some red things are oceans

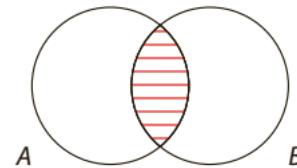
These statements are **VALID**

Given statement form

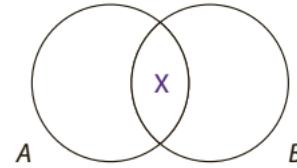
All A are B.



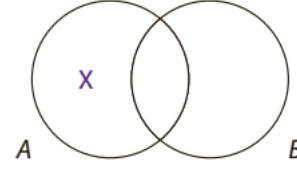
No A are B.



Some A are B.

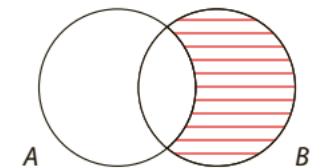


Some A are not B.

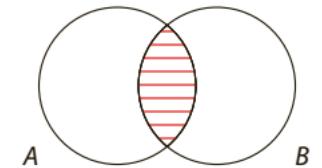


Converse

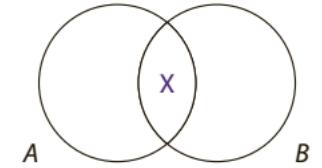
All B are A.



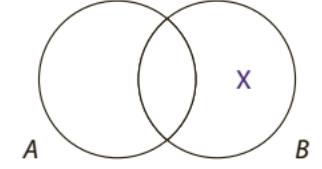
No B are A.



Some B are A.



Some B are not A.



Categorical Propositions

Conversion

A and O statements are illicit conversions

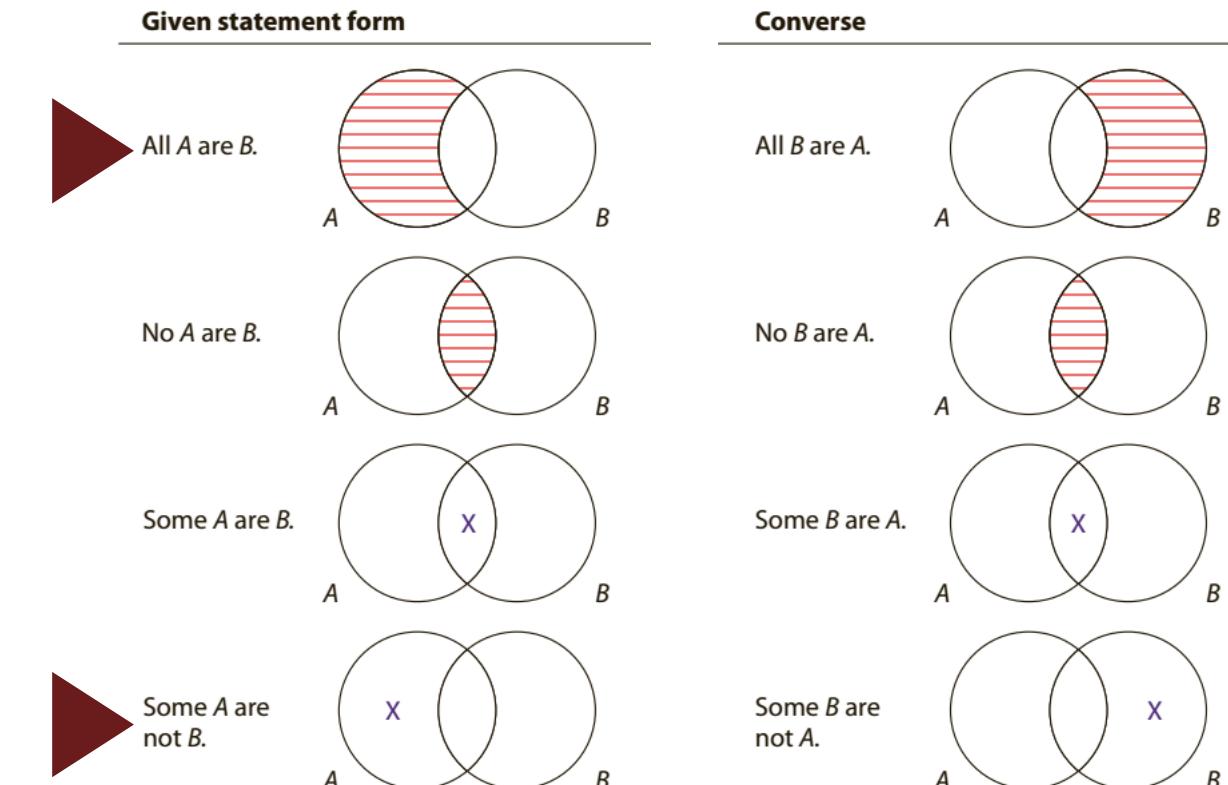
All dogs are animals.

Therefore, all animals are dogs.

Some animals are not dogs.

Therefore, some dogs are not animals.

These statements are **NOT VALID**



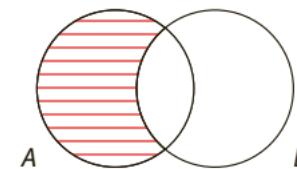
Categorical Propositions

Conversion

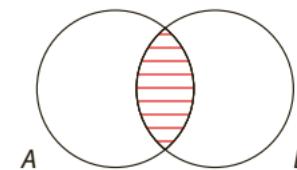
_____ are _____
_____ are _____
_____ are _____
_____ are _____

Given statement form

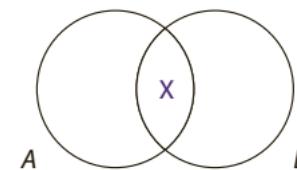
All A are B.



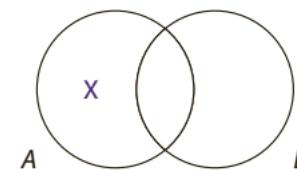
No A are B.



Some A are B.

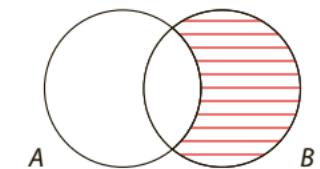


Some A are not B.

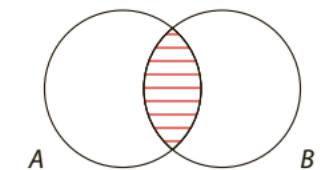


Converse

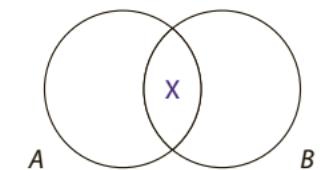
All B are A.



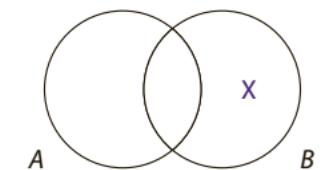
No B are A.



Some B are A.



Some B are not A.

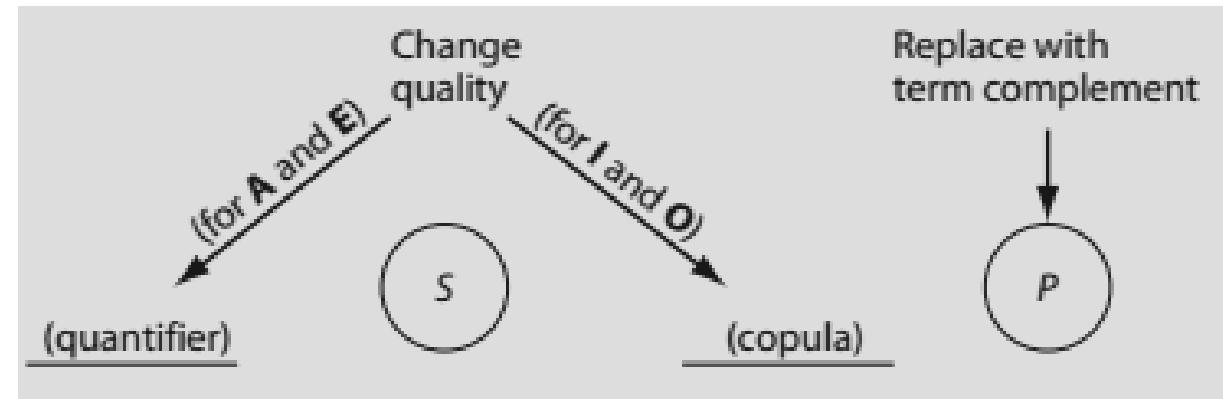


Categorical Propositions

Obversion

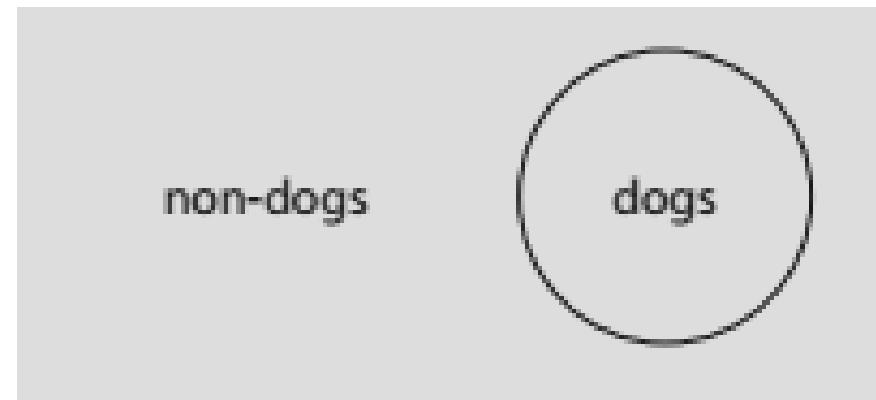
1. Changing the quality

- All → None
- Are → Not



2. Replacing the predicate

- Class compliment: NON-class



Categorical Propositions

Obversion

1. Changing the quality
2. Replacing the predicate

All Kyodai students are smart.



No Kyodai students are non-smart



No Kyodai students are smart



All Kyodai students are non-smart.



VALID

Given statement form	Obverse
All A are B.	No A are non-B.
No A are B.	All A are non-B.
Some A are B.	Some A are not non-B.
Some A are not B.	Some A are non-B.

Categorical Propositions

Obversion

1. Changing the quality
2. Replacing the predicate

Some Kyodai students are smart.

Some Kyodai students are not non-smart

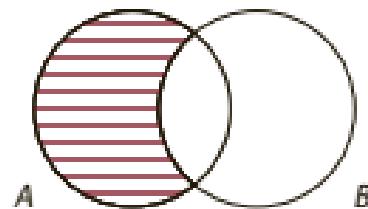
Some Kyodai students are not smart

Some Kyodai students are non-smart.

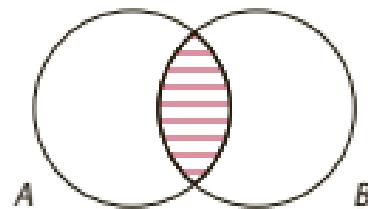
VALID

Given statement form

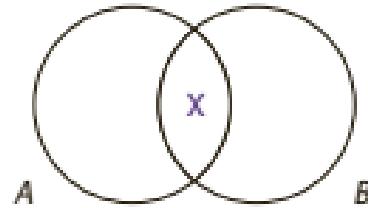
All A are B.



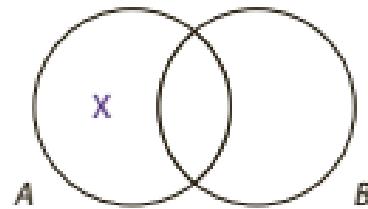
No A are B.



Some A are B.

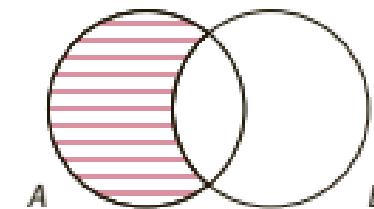


Some A are not B.

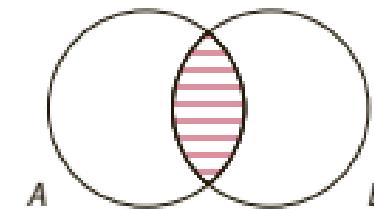


Obverse

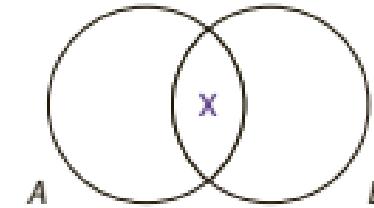
No A are non-B.



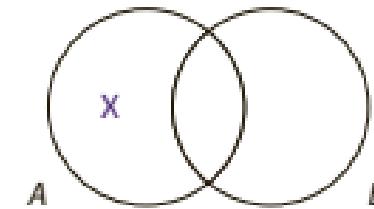
All A are non-B.



Some A are not non-B.



Some A are non-B.



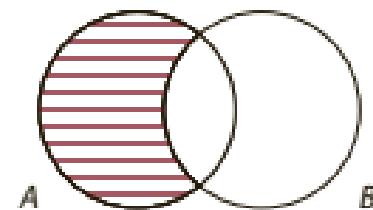
Categorical Propositions

Obversion

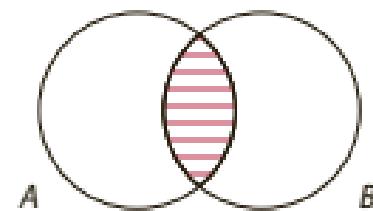
_____ are _____
_____ are _____
_____ are _____
_____ are _____

Given statement form

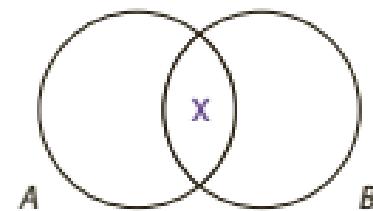
All A are B.



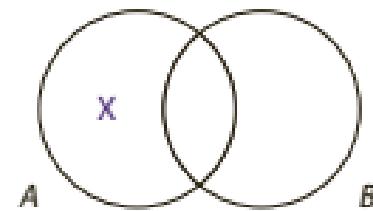
No A are B.



Some A are B.

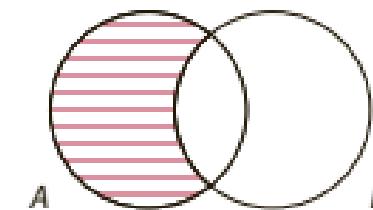


Some A are not B.

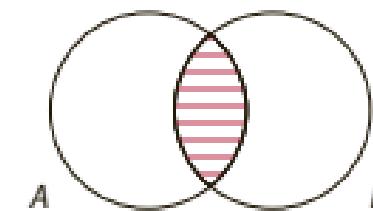


Obverse

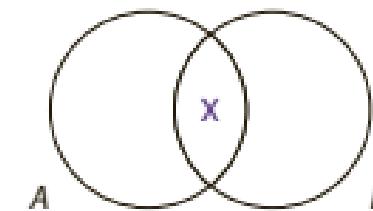
No A are
non-B.



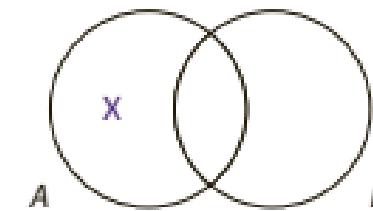
All A are
non-B.



Some A are
not non-B.



Some A are
non-B.



Categorical Propositions

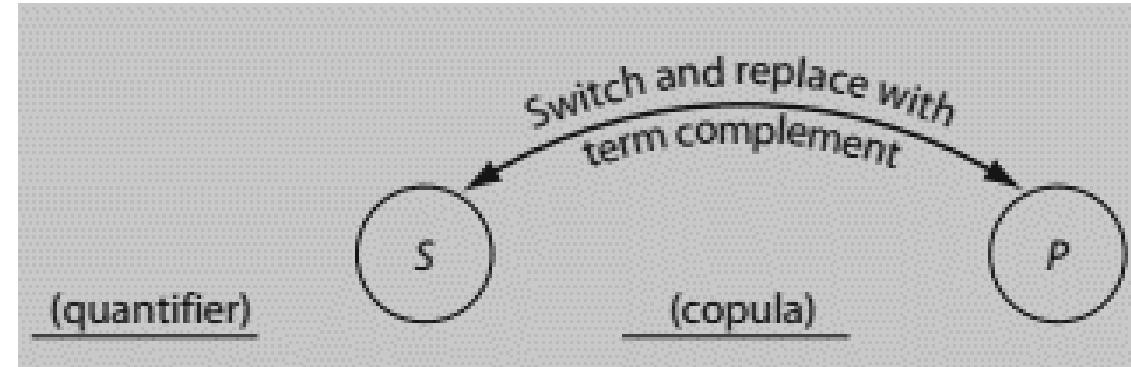
Contraposition

1. Switching subject and predicate

- Class compliment – NON-class

2. Replacing the subject and predicate with their term complements

- All → None
- Are → Not



Categorical Propositions

Contraposition

1. Switching the subject and predicate
2. Replacing the subject and predicate with their compliment

All dogs are animals

All non-animals are non-dogs

Some dogs are not animals

Some non-animals are not non-dogs

VALID

Given statement form	Contrapositive
All A are B .	All non- B are non- A .
No A are B .	No non- B are non- A .
Some A are B .	Some non- B are non- A .
Some A are not B .	Some non- B are not non- A .

Categorical Propositions

Contraposition

1. Switching the subject and predicate
2. Replacing the subject and predicate with their compliment

No dogs are animals

No non-animals are non-dogs

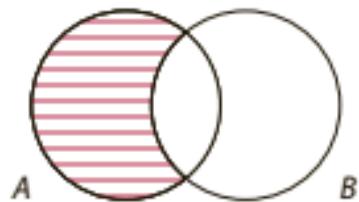
Some dogs are animals

Some non-animals are non-dogs

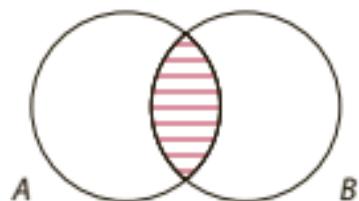
NOT VALID

Given statement form

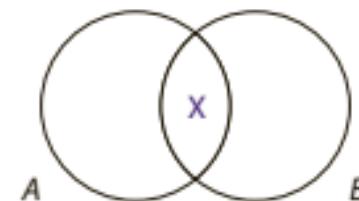
All A are B .



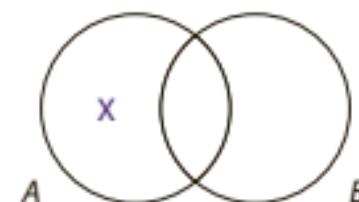
No A are B .



Some A are B .

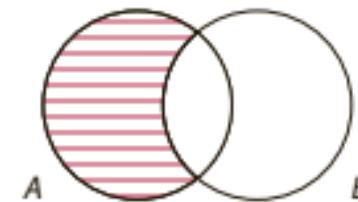


Some A are not B .

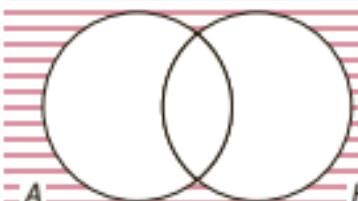


Contrapositive

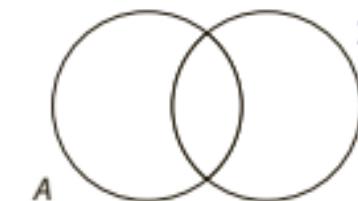
All non- B are non- A .



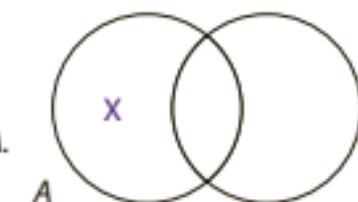
No non- B are non- A .



Some non- B are non- A .



Some non- B are not non- A .



Categorical Propositions

Contraposition

_____ are _____

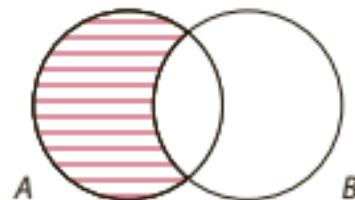
_____ are _____

_____ are _____

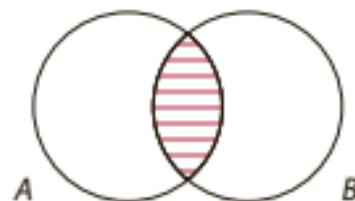
_____ are _____

Given statement form

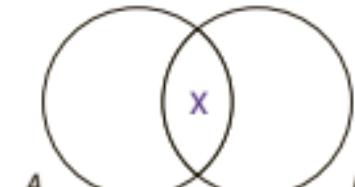
All A are B .



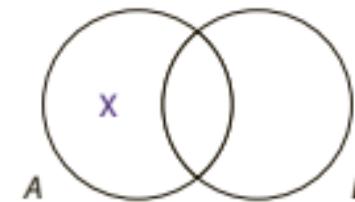
No A are B .



Some A are B .

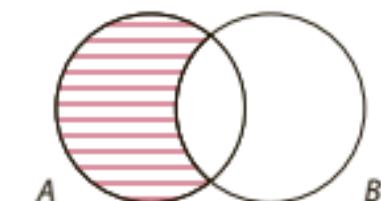


Some A are not B .

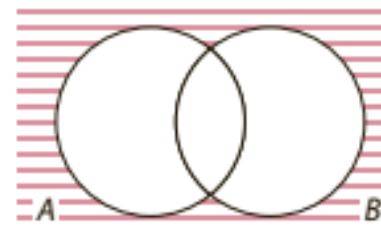


Contrapositive

All non- B are
non- A .



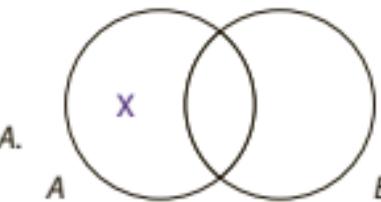
No non- B
are non- A .



Some non- B
are non- A .



Some non- B
are not non- A .



Categorical Propositions

Conversion, Obversion, & Contraposition

These transformations help to:

- **Rewrite arguments** in valid, logically equivalent ways.
- **Detect invalid arguments** that mistakenly assume equivalence.
- **Understand implications** and restatements in natural language reasoning.

Conversion: Switch Subject and Predicate Terms

Given Statement	Converse	Truth Value
E : No A are B.	No B are A.	Same truth value as given statement
I : Some A are B.	Some B are A.	
A : All A are B.	All B are A.	Undetermined truth value
O : Some A are not B.	Some B are not A.	

Obversion: Change Quality; Replace Predicate with Term Complement

Given Statement	Obverse	Truth Value
A : All A are B.	No A are non-B.	Same truth value as given statement
E : No A are B.	All A are non-B.	
I : Some A are B.	Some A are not non-B.	
O : Some A are not B.	Some A are non-B.	

Contraposition: Switch Subject and Predicate Terms; Replace Each with Its Term Complement

Given Statement	Contrapositive	Truth Value
A : All A are B.	All non-B are non-A.	Same truth value as given statement
O : Some A are not B.	Some non-B are not non-A.	
E : No A are B.	No non-B are non-A.	Undetermined truth value
I : Some A are B.	Some non-B are non-A.	

Categorical Propositions

Conversion, Obversion, & Contraposition

For example, obversion helps detect errors like:

- Mistaking non-equivalent forms for logical conclusions.
- Treating similar-sounding statements as interchangeable when they aren't.
- Revealing that an argument is just a restatement of a premise (and therefore not actually a new conclusion).

Obversion gives you a tool to test whether a conclusion is genuinely supported by a premise or just sounds good.

Conversion: Switch Subject and Predicate Terms

Given Statement	Converse	Truth Value
E : No A are B.	No B are A.	Same truth value as given statement
I : Some A are B.	Some B are A.	
A : All A are B.	All B are A.	Undetermined truth value
O : Some A are not B.	Some B are not A.	

Obversion: Change Quality; Replace Predicate with Term Complement

Given Statement	Obverse	Truth Value
A : All A are B.	No A are non-B.	Same truth value as given statement
E : No A are B.	All A are non-B.	
I : Some A are B.	Some A are not non-B.	
O : Some A are not B.	Some A are non-B.	

Contraposition: Switch Subject and Predicate Terms; Replace Each with Its Term Complement

Given Statement	Contrapositive	Truth Value
A : All A are B.	All non-B are non-A.	Same truth value as given statement
O : Some A are not B.	Some non-B are not non-A.	
E : No A are B.	No non-B are non-A.	Undetermined truth value
I : Some A are B.	Some non-B are non-A.	

Categorical Propositions

OVERVIEW

Proposition	Letter name	Quantity	Quality	Terms distributed
All S are P.	A	universal	affirmative	S
No S are P.	E	universal	negative	S and P
Some S are P.	I	particular	affirmative	none
Some S are not P.	O	particular	negative	P

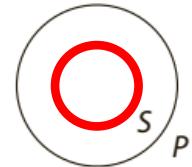
Categorical Propositions

QUALITY QUANTITY	AFFIRMATIVE	NEGATIVE
UNIVERSAL	ALL students are smart	NO students are smart
PARTICULAR	SOME students are smart	SOME students are NOT smart

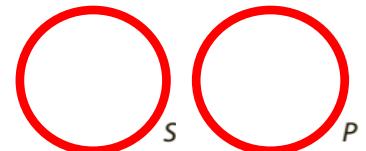
Categorical Propositions

DISTRIBUTION

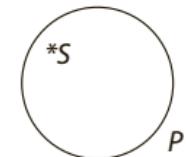
A: All S are P = S Distributed



E: No S are P = S & P Distributed



I: Some S are P = Undistributed



O: Some S are not P = P Distributed

