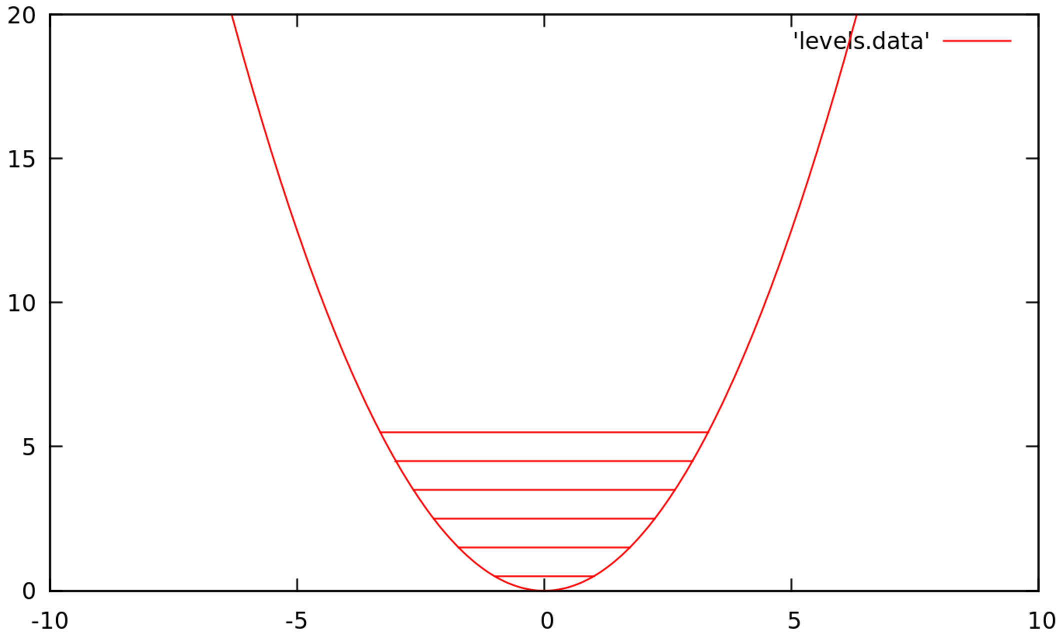
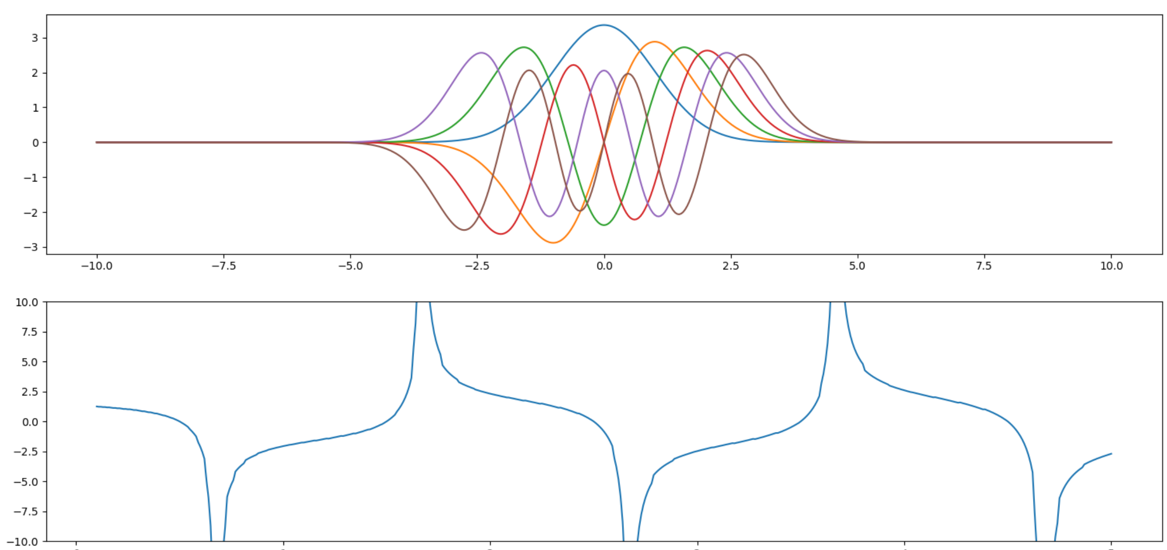
<u>*Homework 11 Solutions:*</u> <u>*Hirak Bandyopadhyay*</u>

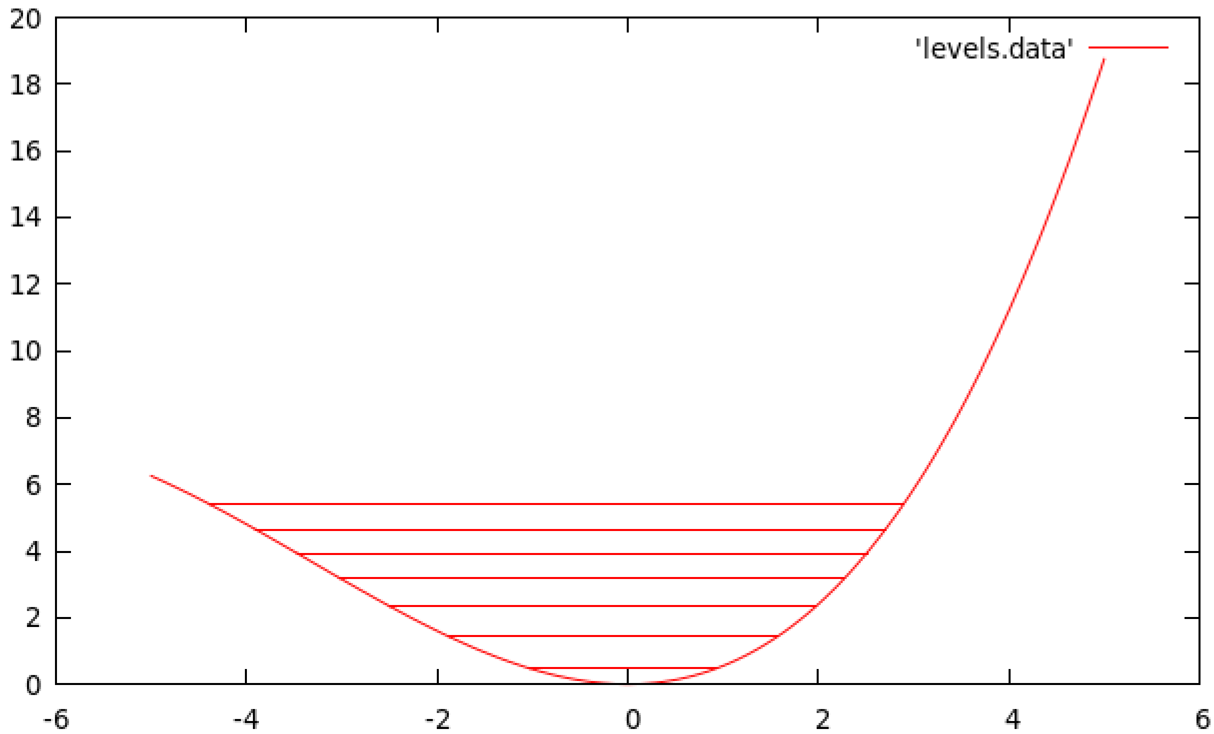
Problem 1:

In this problem, we are required to add small cubic and quartic terms to the SHO potential. 1^{st} let's see the levels and states for simple harmonic oscillator.

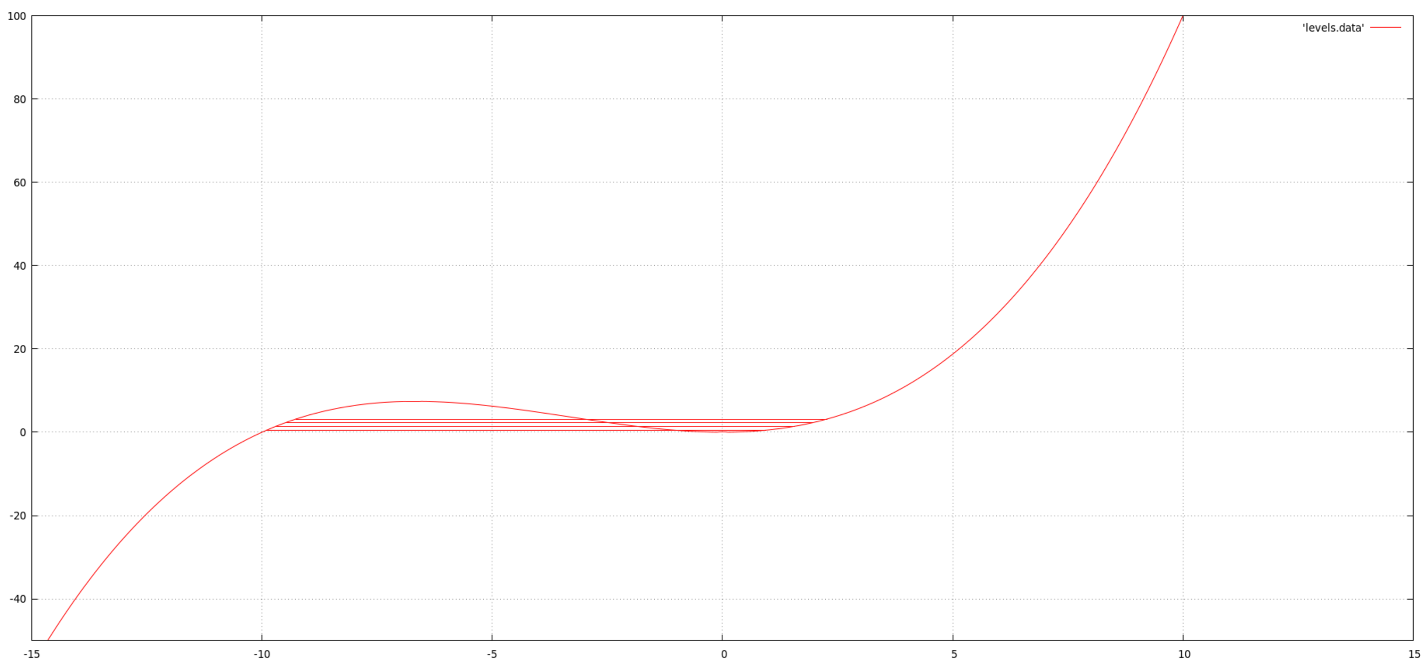




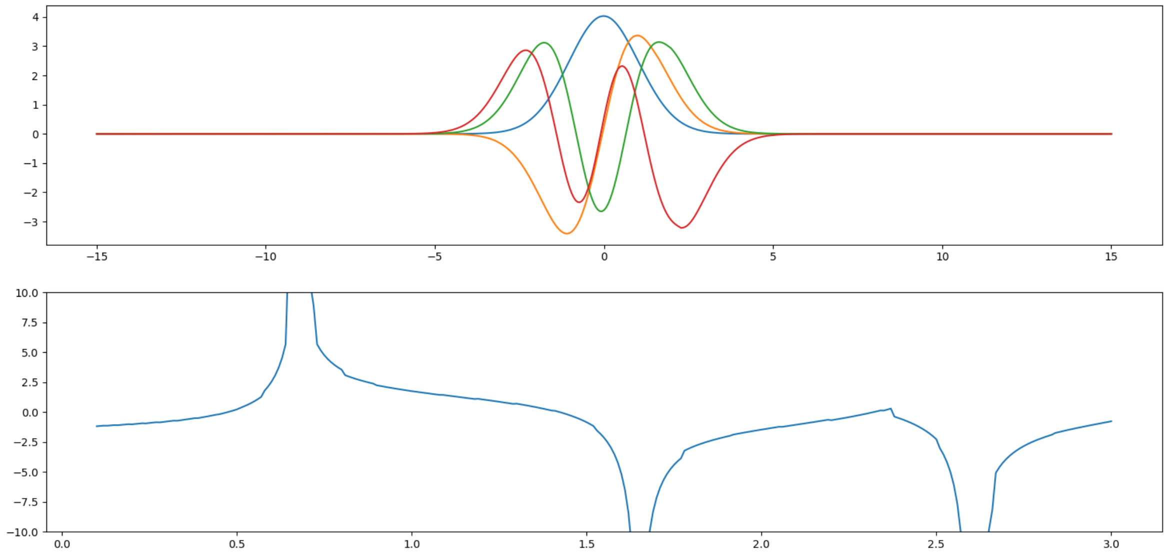
Now, we observe that, when we add large cubic terms, the wave functions are not there, because there cannot be any bound states. If we add small cubic terms, depending on the cubic weight, there can be infinite and finite number of bound states. The examples are hereby,



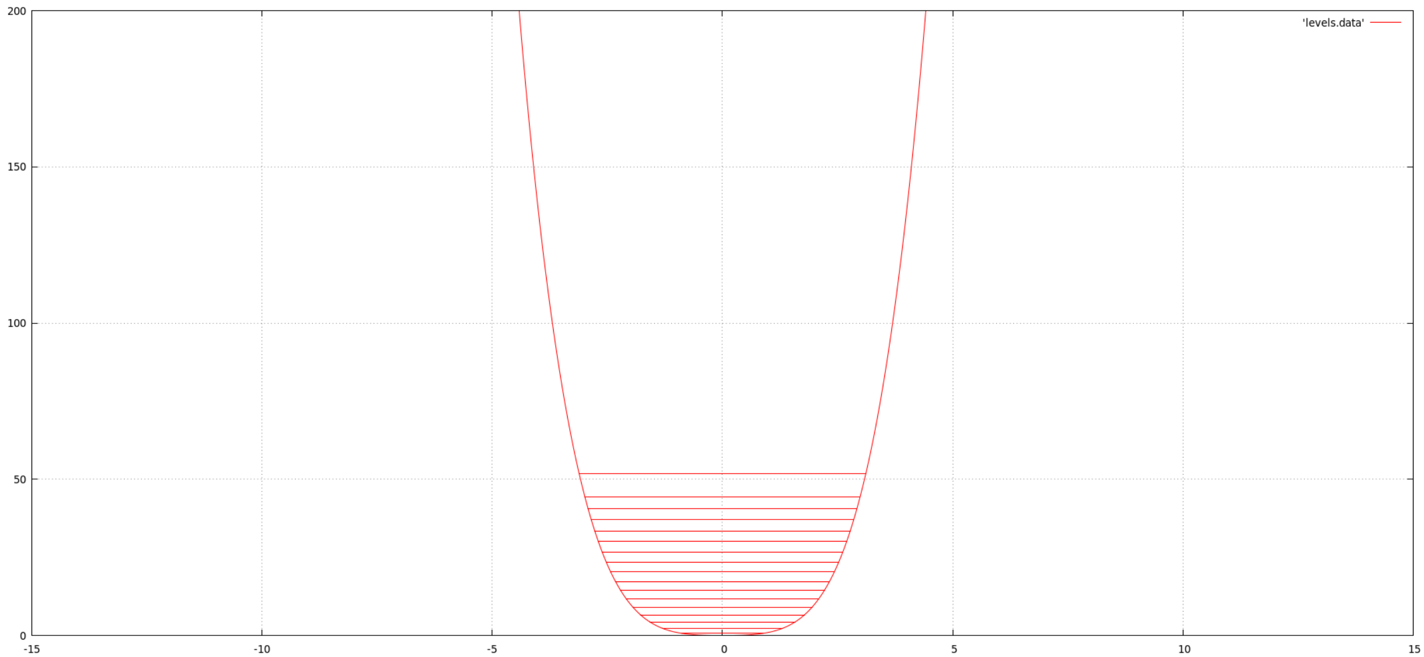
This happens when the cubic term is very small, we can get infinite bound states if boundaries are appropriate.

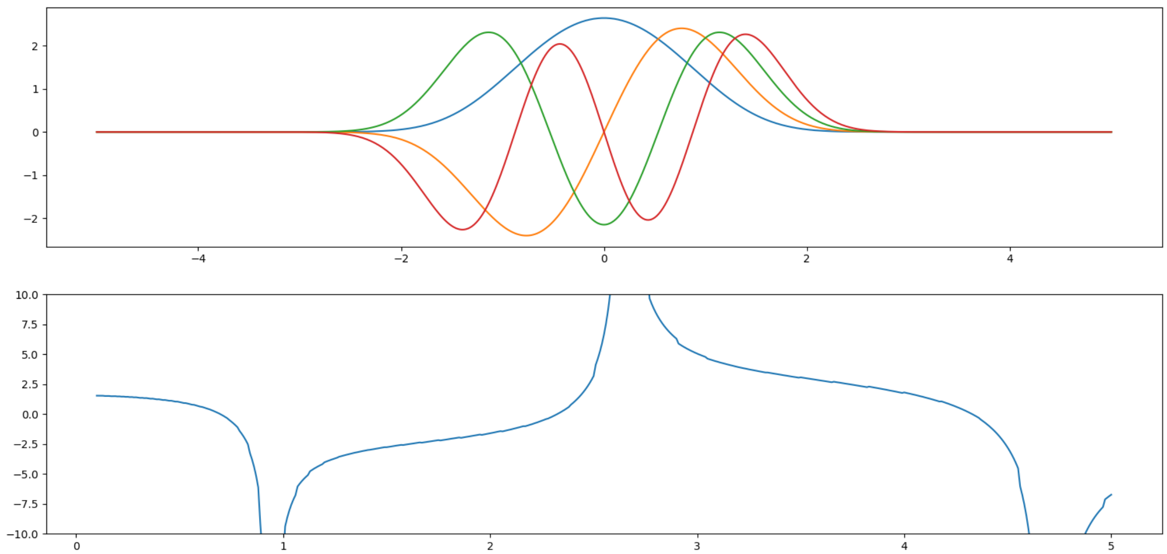


This happens when we have cubic term relatively large, where, in that small region we can contain some states, the number of which is finite. The states are given below,



Now, for quartic potentials, we can see that the number of states that can be contained is infinite, just like the SHO potential, but the more the weightage of the quartic term, the less is the separation between energies of the two states.

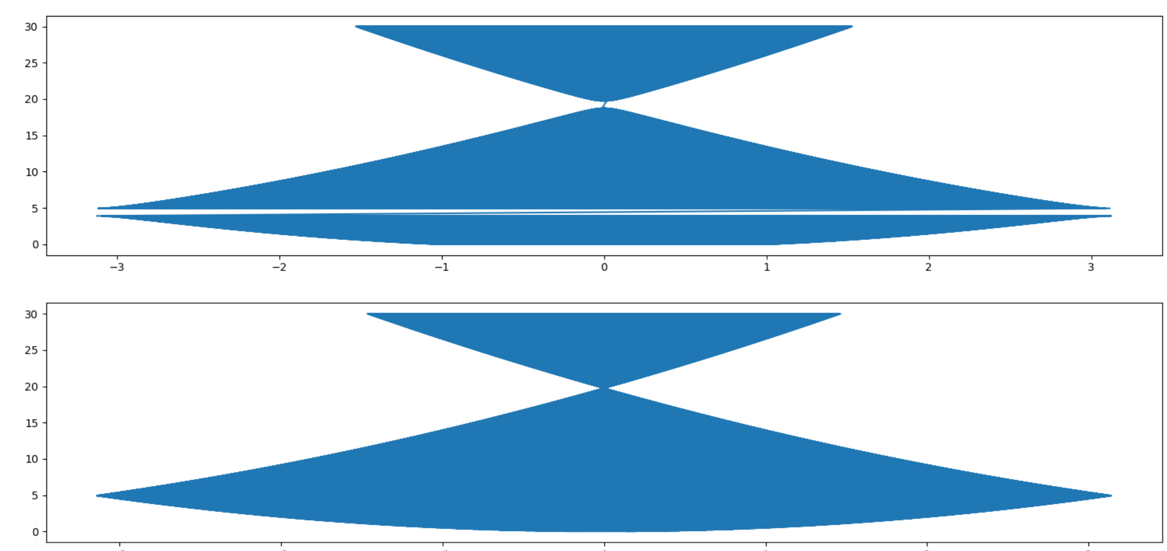


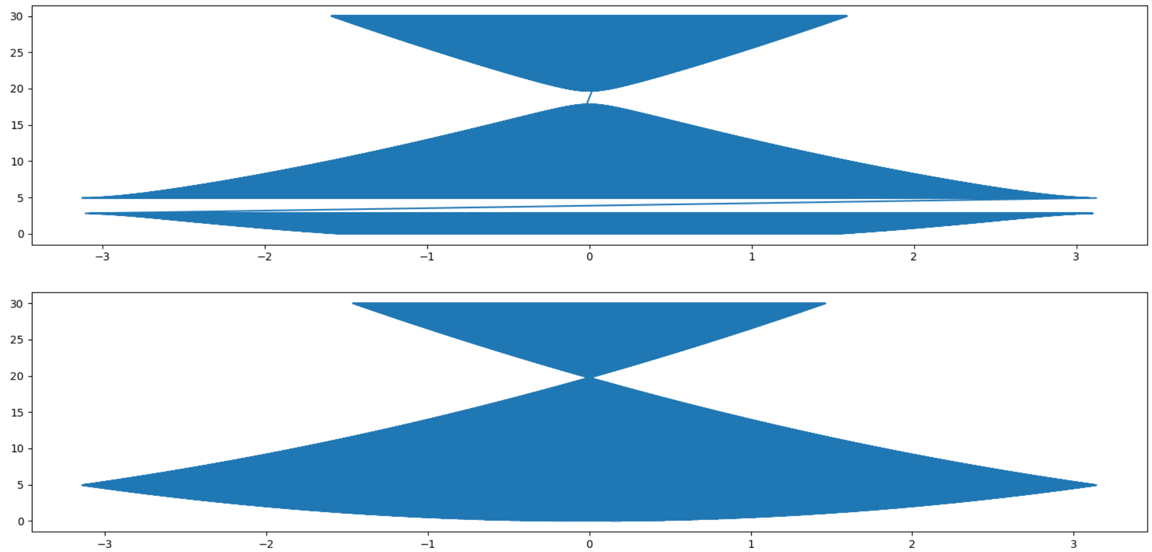


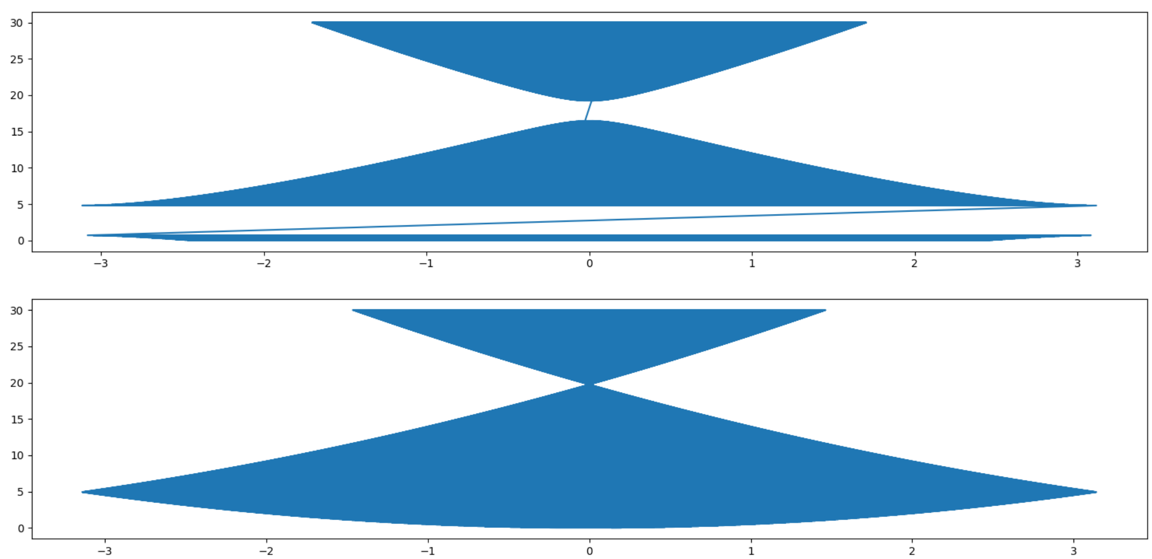
I have shown some excited states for the quartic term.

Problem 2:

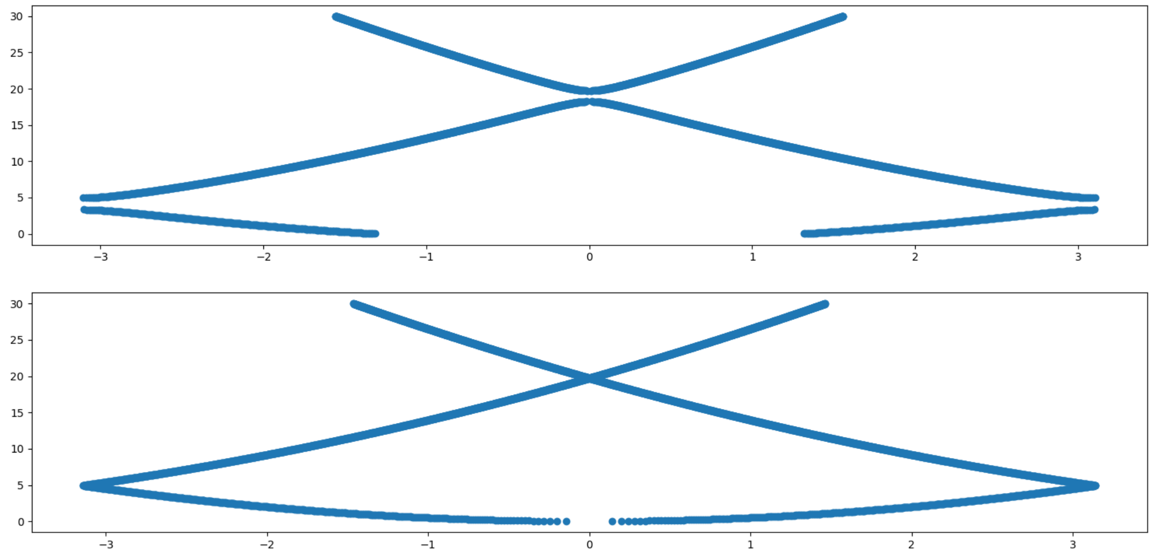
In this problem we have to study the KP potential and find out how the band structure changes when we change the depth and width of the potential. We observe that when we change the depth, depending on the same width, the band gap becomes more and more prominent. This happens because we can see the transitions and tunneling becomes more integers multiples of the eigenfunctions, so the band gap increases.







Now, if we change the width of the potential, we can observe that the modes in k space becomes more wide spread and can accommodate more number of states if the width of the well becomes more than one multiple of the separation of the wells, that means the potentials overlap and create an effective potential. At the integer multiple of width of a given depth and separation, we observe the band gap to be highest in one mode, whereas if the multiplicity changes, we find the bands to be overlaping and more widespread over k space.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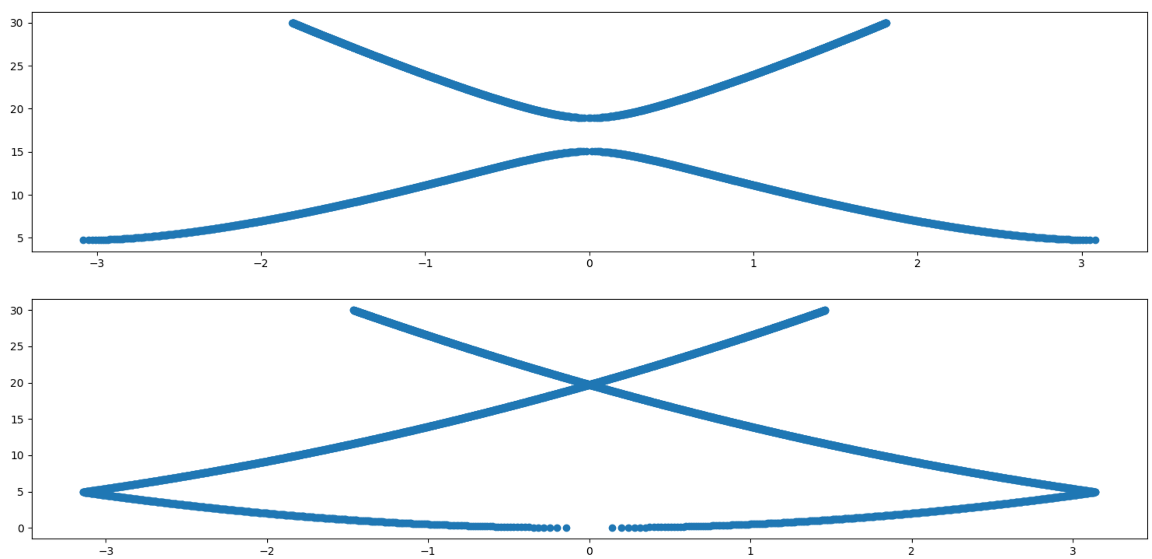
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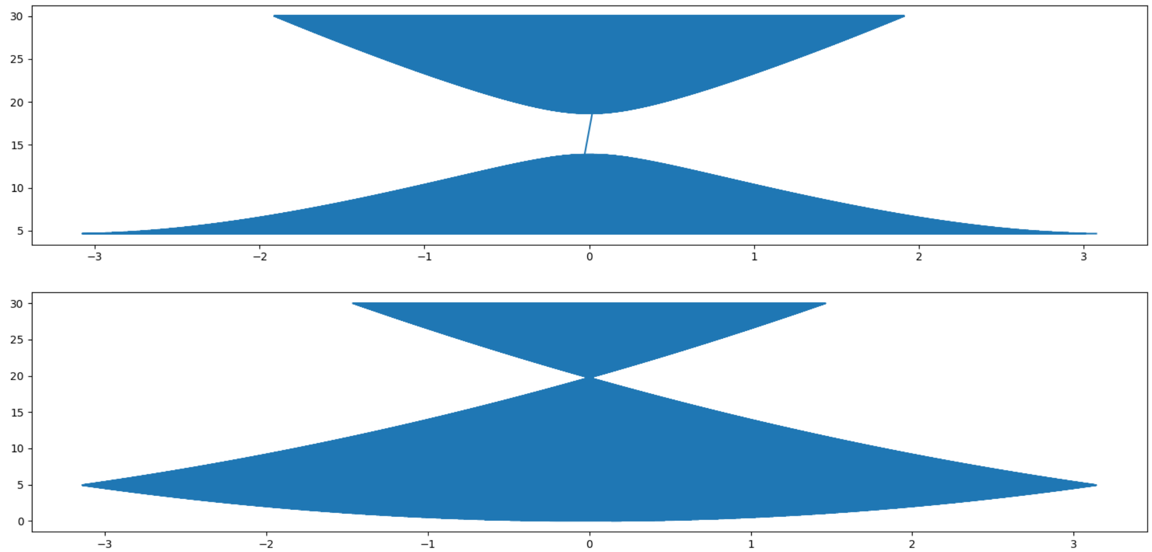
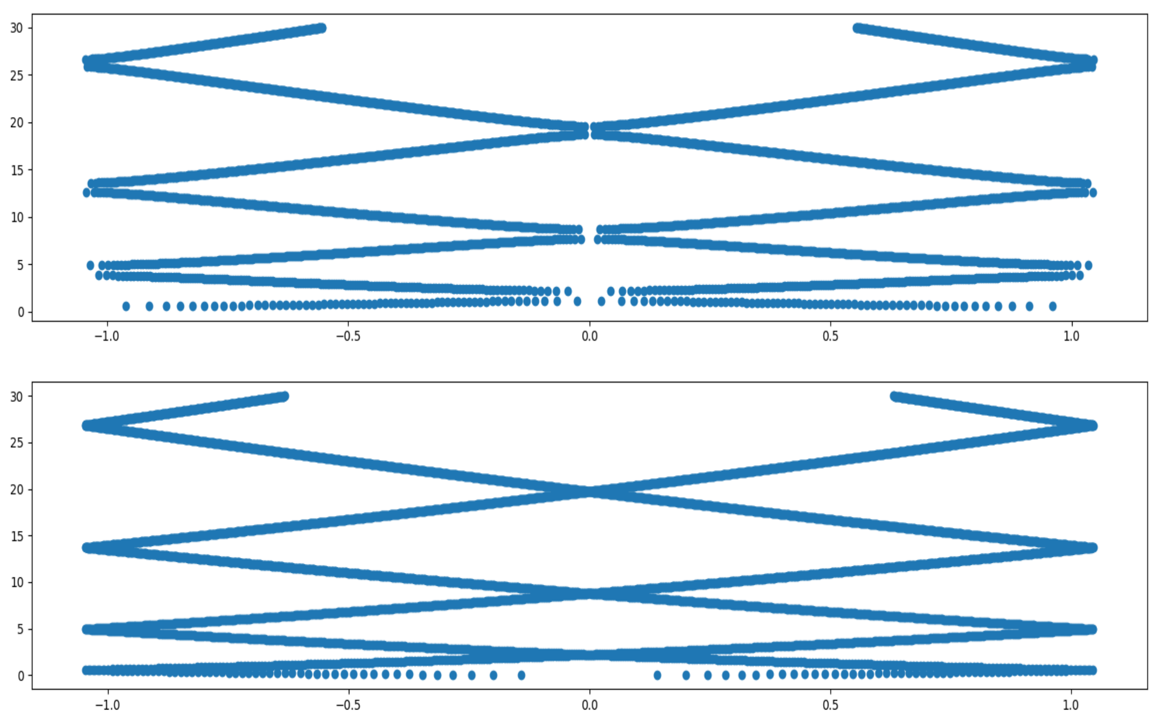


We see that if we change the width, then the band gap is increased.

Now, for multiplicity of the levels, we can see,



This happens because, now we can accommodate more than one brillouin zone which gives us the multiplicity of the wave functions.