

Question 1:

Part 1:

I have used

findChessboardCorners() function to find the chess board corners,
cornerSubPix() for refining the pixel coordinates and calibratCamera() to get intrinsic camera
matrix, lens distortion, rvec and tvec.

The intrinsic matrix is given below.

intrinsic matrix:

```
[[715.29687544 0. 477.48751653]
 [ 0. 537.4991797 268.14595421]
 [ 0. 0. 1. ]]
```

Part 2:

calibrateCamera(): to find the rotation and translation vectors.

Rotation matrices for the images

cv2.Rodrigues(): to find rotation matrices

For image : 1

```
[[ -0.12704071 -0.97100582 -0.2025052 ]
 [ 0.9737439 -0.08320793 -0.21189446]
 [ 0.18890071 -0.22410742 0.95608179]]
```

For image : 2

```
[[ 0.0904718 -0.90845547 0.40807293]
 [ 0.97539591 -0.00188763 -0.22045239]
 [ 0.20104147 0.4179774 0.88593297]]
```

For image : 3

```
[[ 0.04950886 -0.96611099 -0.2533346 ]
 [ 0.98008528 -0.00184385 0.1985685 ]
 [-0.19230632 -0.25812041 0.94677987]]
```

For image : 4

```
[[ 0.00853827 -0.99975635 -0.02035519]
 [ 0.99996106 0.00849109 0.00240336]
 [-0.00222993 -0.02037492 0.99978992]]
```

For image : 5

```
[[ 0.12423905 -0.94575249 0.30019475]
 [ 0.97648946 0.1702476 0.1322274 ]
 [-0.17616183 0.2767092 0.94466874]]
```

For image : 6

```
[[ -0.00387368 -0.99997727 0.00551841]
 [ 0.99998859 -0.00385819 0.00281566]
 [-0.00279431 0.00552925 0.99998081]]
```

For image : 7

```
[[ -0.01315933 -0.99986847 0.00948067]
 [ 0.97602399 -0.01078419 0.21739565]]
```

[-0.21726481 0.01211415 0.97603752]]
For image : 8
[[0.01129213 -0.95794927 0.28671535]
[0.99925788 0.02137067 0.03204665]
[-0.03682636 0.28614069 0.95747967]]
For image : 9
[[-0.01870594 -0.97728212 -0.21111549]
[0.99967796 -0.02190305 0.01281544]
[-0.01714837 -0.21080777 0.97737711]]
For image : 10
[[-0.0567126 -0.9026198 -0.42668627]
[0.98706309 0.01350467 -0.15976258]
[0.14996713 -0.43022682 0.8901768]]
For image : 11
[[-0.0012174 -0.91961321 -0.3928232]
[0.97291841 -0.09188871 0.21209959]
[-0.23114561 -0.38192671 0.89482048]]
For image : 12
[[0.11498665 -0.88865535 0.44392537]
[0.97911409 0.17682078 0.10034947]
[-0.16767133 0.42311473 0.890427]]
For image : 13
[[-0.01117077 -0.99993072 0.00370938]
[0.97788416 -0.01169912 -0.20881979]
[0.20884872 0.00129466 0.9779471]]
For image : 14
[[0.01640851 -0.98136832 0.191434]
[0.9868831 -0.01485682 -0.16075141]
[0.16060044 0.19156067 0.96825204]]
For image : 15
[[0.03425522 -0.94533988 0.32428244]
[0.94916208 0.13236582 0.28560575]
[-0.31291841 0.2980131 0.90181498]]
For image : 16
[[-0.08706249 -0.95143738 -0.29527452]
[0.96959592 -0.01288426 -0.24437215]
[0.2287004 -0.30757262 0.92363153]]
For image : 17
[[-0.00584655 -0.9920168 -0.12597017]
[0.9573742 -0.04193319 0.28579058]
[-0.28879139 -0.1189297 0.94997645]]
For image : 18
[[0.04851548 -0.94320558 0.32864795]
[0.96715823 -0.03783011 -0.25134407]
[0.24950192 0.33004865 0.9103937]]
For image : 19
[[0.02959403 -0.94627072 0.3220185]
[0.95694361 0.11988972 0.26435845]
[-0.28876137 0.30033011 0.90907574]]
For image : 20
[[-0.01154688 -0.95630866 -0.29213081]
[0.95849036 -0.09382198 0.26924649]

```
[-0.28489104 -0.27689561  0.91769598]]  
For image : 21  
[[-0.01029687 -0.99994369 -0.00256839]  
 [ 0.97138441 -0.0093932 -0.23732697]  
 [ 0.23728948 -0.00493862  0.97142643]]  
For image : 22  
[[-0.02410187 -0.99938587 -0.02543592]  
 [ 0.99959432 -0.02447753  0.0145621 ]  
 [-0.01517576 -0.02507462  0.99957039]]  
For image : 23  
[[-0.02791459 -0.99959592  0.00536428]  
 [ 0.94200003 -0.02450994  0.3347166 ]  
 [-0.33444987  0.01439663  0.94230357]]  
For image : 24  
[[-0.01399606 -0.94773247  0.31875895]  
 [ 0.99956689 -0.00500801  0.02899919]  
 [-0.02588713  0.31902677  0.94739209]]  
For image : 25  
[[-0.02576652 -0.94696431 -0.32030403]  
 [ 0.99931588 -0.03290292  0.01688707]  
 [-0.02653039 -0.31964978  0.94716427]]
```

Translation vectors for the images

```
For image : 1  
[[ 4.4804786 ]  
 [-1.23215097]  
 [10.74360978]]  
For image : 2  
[[ 3.07101543]  
 [-1.43776466]  
 [ 8.56259329]]  
For image : 3  
[[ 4.13074271]  
 [-1.97798392]  
 [10.73332697]]  
For image : 4  
[[ 3.46407814]  
 [-1.9364719 ]  
 [ 9.43126268]]  
For image : 5  
[[ 2.42557049]  
 [-2.10121951]  
 [ 9.38774274]]  
For image : 6  
[[ 3.24117788]  
 [-1.7649118 ]  
 [ 8.96358383]]  
For image : 7  
[[ 3.62224279]  
 [-2.02204913]  
 [ 9.92684873]]
```

For image : 8

[[3.17921194]

[-1.373576]

[9.33357675]]

For image : 9

[[4.43136085]

[-1.90016664]

[10.38085173]]

For image : 10

[[3.8410576]

[-1.65886947]

[11.19311647]]

For image : 11

[[4.11934104]

[-1.41918053]

[11.71069848]]

For image : 12

[[2.12132297]

[-1.93943594]

[9.00536141]]

For image : 13

[[3.86321452]

[-1.77393219]

[9.48058565]]

For image : 14

[[3.16754169]

[-1.46140951]

[9.08492318]]

For image : 15

[[2.36595624]

[-2.04139749]

[9.89139603]]

For image : 16

[[3.03860594]

[-2.00492354]

[11.60803583]]

For image : 17

[[3.55744361]

[-1.67078102]

[10.61415989]]

For image : 18

[[3.28198041]

[-1.58761645]

[9.66581742]]

For image : 19

[[2.5592349]

[-2.15559828]

[9.73608884]]

For image : 20

[[3.67635176]

[-1.77322857]

[11.63690553]]

```
For image : 21
[[ 3.71402306]
 [-1.51771843]
 [ 9.48156698]]
For image : 22
[[ 3.6723898 ]
 [-1.6182402 ]
 [10.21373416]]
For image : 23
[[ 3.62038613]
 [-1.3746773 ]
 [10.39916766]]
For image : 24
[[ 2.84408625]
 [-1.61397959]
 [ 9.34796952]]
For image : 25
[[ 3.55435529]
 [-1.39135741]
 [11.95467446]]
```

Part 3:

calibrateCamera(): to obtain distortion coefficients

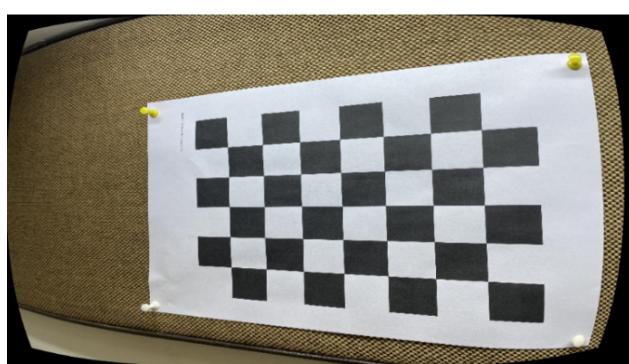
\Distortion matrix:

Below are the radial distortion coefficients:

```
[[ 0.02265107 -0.25819242  0.        0.00181977  0.91198486]]
```

getOptimalNewCameraMatrix(): to refine the matrix

undistort(): to transform an image for radial and tangential distortion





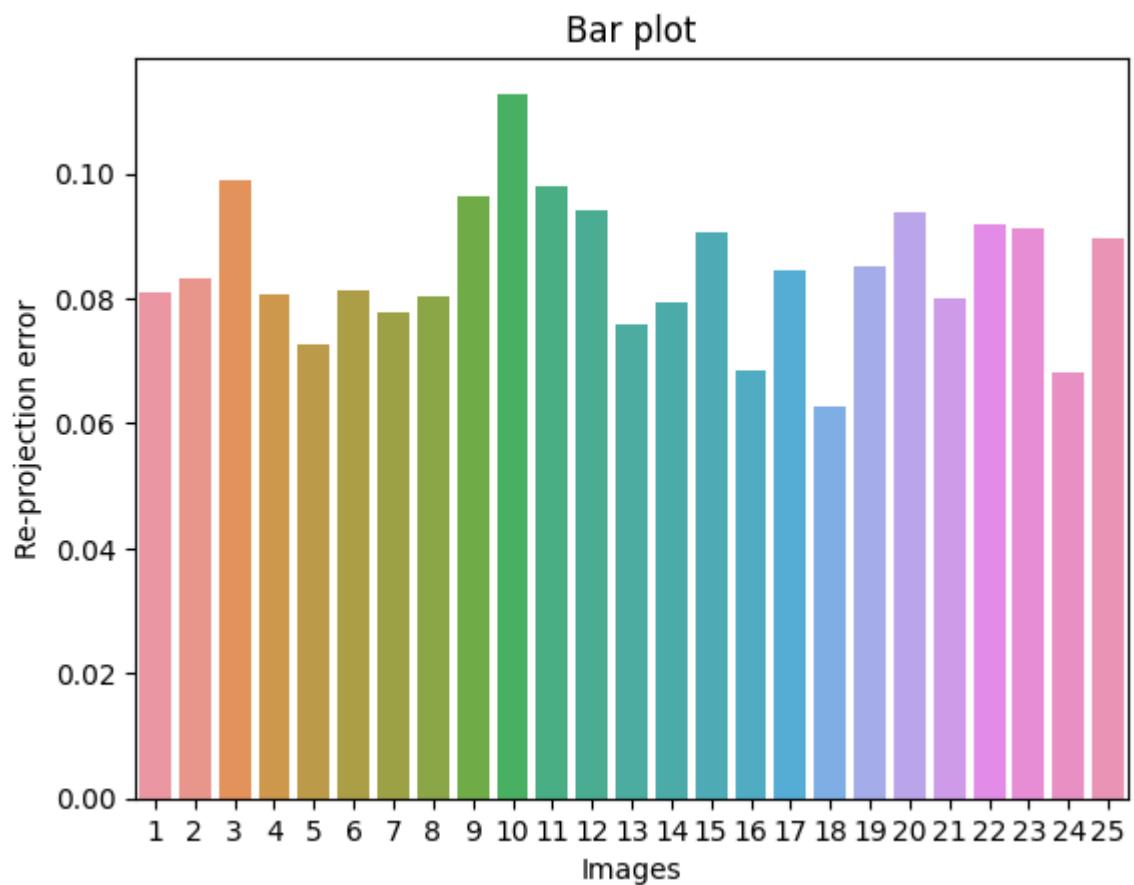
Part 4:

Re-projection error mean : 0.0847430777268711

Re-projection errors:

[0.08101602485305368, 0.08311379060603129, 0.098877481179955, 0.08070031408167916, 0.07279260964644814, 0.08123344260353452, 0.07769547646490807, 0.08020875154210626, 0.09638897534067106, 0.1127998474683328, 0.09808770914341654, 0.09420301667911245, 0.0759098654384246, 0.07931788085903224, 0.09067455255111442, 0.06837205697469244, 0.08444394978156988, 0.0627843894628323, 0.08519940871983742, 0.0939107142213041, 0.0799462779364633, 0.09180949107758692, 0.09125707682071639, 0.06824730533439337, 0.08958653438456166]

Re-projection error standard deviation : 0.011020949764219962



Part 5:

projectPoints(): to compute re-projection error. Where the points are converted to image points.

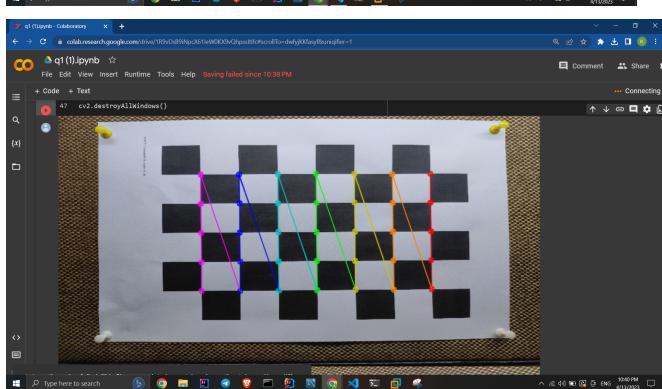
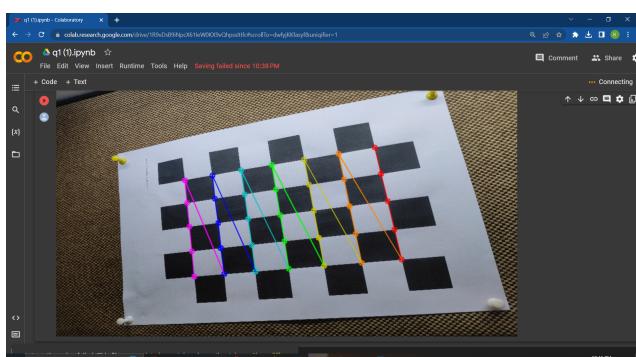
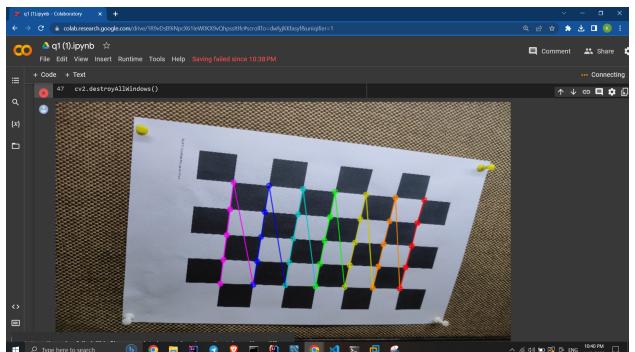
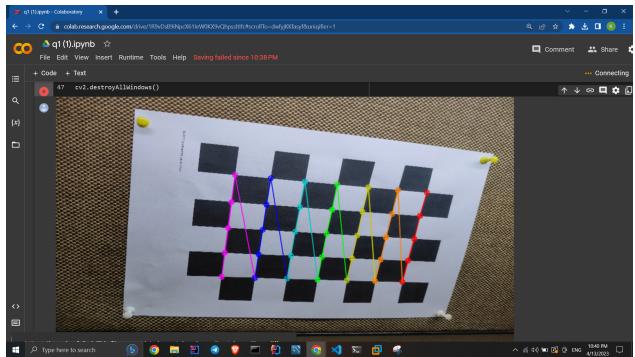
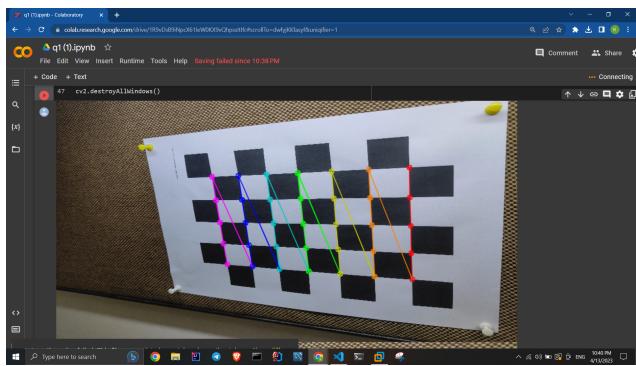
Parameters: object points, rotation vector, translation vector, camera intrinsic matrix and distortion coefficient matrix.

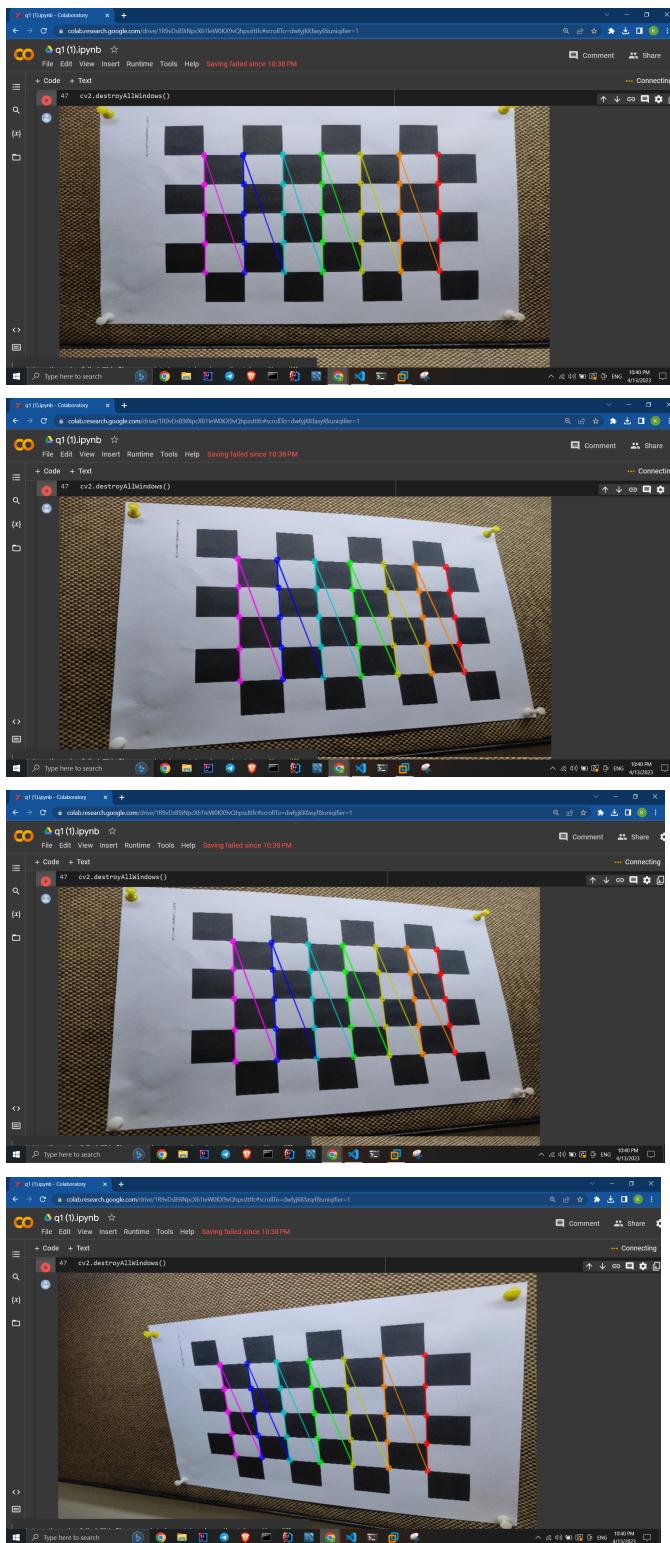
Output: array of image points and jacobian matrix.

To calculate the norm between transformation i have used this output images and original images.

Here this reprojection error tells us how the accuracy of the camera calibration process.

Below are the output images:





Part 6:

We need to convert the rotation vector for each image with it's corresponding rotation matrix. Then the last column of that matrix will be checkerboard plane normal for that image.

Output:

```
[[ -0.2025052 ]  
[ -0.21189446]  
[ 0.95608179]]  
[[ 0.40807293]  
[ -0.22045239]  
[ 0.88593297]]  
[[ -0.2533346 ]  
[ 0.1985685 ]  
[ 0.94677987]]  
[[ -0.02035519]  
[ 0.00240336]  
[ 0.99978992]]  
[[ 0.30019475]  
[ 0.1322274 ]  
[ 0.94466874]]  
[[ 0.00551841]  
[ 0.00281566]  
[ 0.99998081]]  
[[ 0.00948067]  
[ 0.21739565]  
[ 0.97603752]]  
[[ 0.28671535]  
[ 0.03204665]  
[ 0.95747967]]  
[[ -0.21111549]  
[ 0.01281544]  
[ 0.97737711]]  
[[ -0.42668627]  
[ -0.15976258]  
[ 0.8901768 ]]  
[[ -0.3928232 ]  
[ 0.21209959]  
[ 0.89482048]]  
[[ 0.44392537]  
[ 0.10034947]  
[ 0.890427 ]]  
[[ 0.00370938]  
[ -0.20881979]  
[ 0.9779471 ]]  
[[ 0.191434 ]  
[ -0.16075141]  
[ 0.96825204]]  
[[ 0.32428244]  
[ 0.28560575]  
[ 0.90181498]]
```

```
[-0.29527452]
[-0.24437215]
[ 0.92363153]]
[-0.12597017]
[ 0.28579058]
[ 0.94997645]]
[[ 0.32864795]
[-0.25134407]
[ 0.9103937 ]]
[[0.3220185 ]
[0.26435845]
[0.90907574]]
[-0.29213081]
[ 0.26924649]
[ 0.91769598]]
[[-0.00256839]
[-0.23732697]
[ 0.97142643]]
[[-0.02543592]
[ 0.0145621 ]
[ 0.99957039]]
[[0.00536428]
[0.3347166 ]
[0.94230357]]
[[0.31875895]
[0.02899919]
[0.94739209]]
[[-0.32030403]
[ 0.01688707]
[ 0.94716427]]
```

vecs:

```
(array([-0.01045495],
[-0.33506421],
[ 1.66480887]], array([[0.50552998],
[0.16393442],
[1.4916963 ]]), array([[-0.35931826],
[-0.04801644],
[ 1.53124772]]), array([[-0.01779487],
[-0.01415983],
[ 1.56222108]]), array([[0.10557413],
[0.34807797],
[1.40459921]]), array([[0.00213652],
[0.00654495],
[1.57465682]]), array([[-0.16373348],
[ 0.18085325],
[ 1.57598101]]), array([[0.20019362],
[0.25490949],
[1.54202896]]), array([[-0.17925849],
```

```
[-0.15548588],  
[ 1.58474998]], array([[ -0.22343463],  
[-0.47638215],  
[ 1.5610958 ]]), array([[ -0.49849834],  
[-0.13567751],  
[ 1.58818533]]), array([[ 0.23977149],  
[0.45433468],  
[1.3875033 ]]), array([[ 0.16742567],  
[-0.16346135],  
[ 1.57598389]]), array([[ 0.27939684],  
[0.02445218],  
[1.560898 ]]), array([[ 0.00953798],  
[0.48983923],  
[1.45637185]]), array([[ -0.05263187],  
[-0.43635408],  
[ 1.59979167]]), array([[ -0.32815887],  
[ 0.13202014],  
[ 1.5806224 ]]), array([[ 0.46846363],  
[0.06377279],  
[1.53929685]]), array([[ 0.02773727],  
[0.47096429],  
[1.46754344]]), array([[ -0.45661553],  
[-0.00605299],  
[ 1.6009148 ]]), array([[ 0.18537562],  
[-0.19133403],  
[ 1.57252359]]), array([[ -0.03162579],  
[-0.00818649],  
[ 1.59496877]]), array([[ -0.26079729],  
[ 0.27666901],  
[ 1.58080358]]), array([[ 0.23313009],  
[0.27703356],  
[1.56527901]]), array([[ -0.27412807],  
[-0.23929504],  
[ 1.58535394]]))
```

vecs:

```
(array([-0.01045495],  
[-0.33506421],  
[ 1.66480887]], array([[ 0.50552998],  
[0.16393442],  
[1.4916963 ]]), array([[ -0.35931826],  
[-0.04801644],  
[ 1.53124772]]), array([[ -0.01779487],  
[-0.01415983],  
[ 1.56222108]]), array([[ 0.10557413],  
[0.34807797],  
[1.40459921]]), array([[ 0.00213652],  
[0.00654495],  
[1.57465682]]), array([[ -0.16373348],  
[ 0.18085325],  
[ 1.57598101]]), array([[ 0.20019362],  
[0.25490949],
```

```
[1.54202896]], array([[ -0.17925849],  
[-0.15548588],  
[ 1.58474998]], array([[ -0.22343463],  
[-0.47638215],  
[ 1.5610958 ]]), array([[ -0.49849834],  
[-0.13567751],  
[ 1.58818533]], array([[ 0.23977149],  
[0.45433468],  
[1.3875033 ]]), array([[ 0.16742567],  
[-0.16346135],  
[ 1.57598389]], array([[ 0.27939684],  
[0.02445218],  
[1.560898 ]]), array([[ 0.00953798],  
[0.48983923],  
[1.45637185]], array([[ -0.05263187],  
[-0.43635408],  
[ 1.59979167]], array([[ -0.32815887],  
[ 0.13202014],  
[ 1.5806224 ]]), array([[ 0.46846363],  
[0.06377279],  
[1.53929685]], array([[ 0.02773727],  
[0.47096429],  
[1.46754344]], array([[ -0.45661553],  
[-0.00605299],  
[ 1.6009148 ]]), array([[ 0.18537562],  
[-0.19133403],  
[ 1.57252359]], array([[ -0.03162579],  
[-0.00818649],  
[ 1.59496877]], array([[ -0.26079729],  
[ 0.27666901],  
[ 1.58080358]], array([[ 0.23313009],  
[0.27703356],  
[1.56527901]]), array([[ -0.27412807],  
[-0.23929504],  
[ 1.58535394]]))
```

\Error estimate:

0.45913760323847386

Question 2:

(i)

Camera offset: negation of the dot product of camera normal and translation vector.

Camera normal and offset are provided in data.

To find lidar normals and lidar offset, i first found the centroid and then taking diff of points from centroid. Then took the svd decomposition to get lidar normals.

Lidar offset: negation of the dot product of lidar normal and translation vector.

Lidar normal [-0.73737765 0.5739658 0.35614247]

Lidar offset -0.6173963843995338
Lidar normal [-0.43460444 0.89601842 -0.09093933]
Lidar offset 1.6581593500032712
Lidar normal [-0.76477664 0.64422625 0.00944619]
Lidar offset 2.440154424620167
Lidar normal [0.94902101 0.13112718 0.28664401]
Lidar offset -0.5166637933458826
Lidar normal [0.98745566 0.06712949 -0.14291586]
Lidar offset 4.485033369802012
Lidar normal [-0.61872738 0.78293781 -0.06469016]
Lidar offset 2.59786331743304
Lidar normal [-0.97607976 0.21597961 -0.02492227]
Lidar offset 0.12024157412940559
Lidar normal [-0.9493562 -0.22833398 0.21583881]
Lidar offset -0.2863429655511951
Lidar normal [0.95936444 -0.00899235 -0.2820266]
Lidar offset 1.1827325017995012
Lidar normal [0.60371533 -0.79239849 0.08736381]
Lidar offset -0.27591558972753
Lidar normal [-0.95207118 0.1539658 -0.26430097]
Lidar offset 1.503227211795524
Lidar normal [0.75932657 -0.56254883 0.32705042]
Lidar offset -3.0119529682117365
Lidar normal [-0.52528321 0.84450165 0.1043768]
Lidar offset 0.3968984253614216
Lidar normal [0.93288206 -0.2440387 0.26490788]
Lidar offset -1.446075973376584
Lidar normal [0.60407281 -0.77412903 0.18926246]
Lidar offset -2.507059299159562
Lidar normal [-0.9065703 0.30866042 0.28785247]
Lidar offset -1.72362590053854
Lidar normal [-0.40409023 0.88184713 -0.24301591]
Lidar offset 2.5254512198308587
Lidar normal [0.87718352 -0.41582556 0.24007951]
Lidar offset -2.2410852053273342
Lidar normal [-0.47346333 0.7599202 0.44536924]
Lidar offset -1.4118421075881364
Lidar normal [-0.96334688 0.25648046 0.07861659]
Lidar offset -0.6663197888621081
Lidar normal [0.91935323 -0.3607026 0.15710911]
Lidar offset 1.3571817737126528
Lidar normal [0.93809241 -0.22958973 0.25936688]
Lidar offset -1.9877451825345347
Lidar normal [0.72352642 -0.68812782 -0.05467752]
Lidar offset -0.004894921314802292
Lidar normal [0.94151039 0.18949708 0.27865578]
Lidar offset 1.2738537349261618
Lidar normal [0.81878735 0.50763718 -0.26812639]
Lidar offset 3.3563638050211697

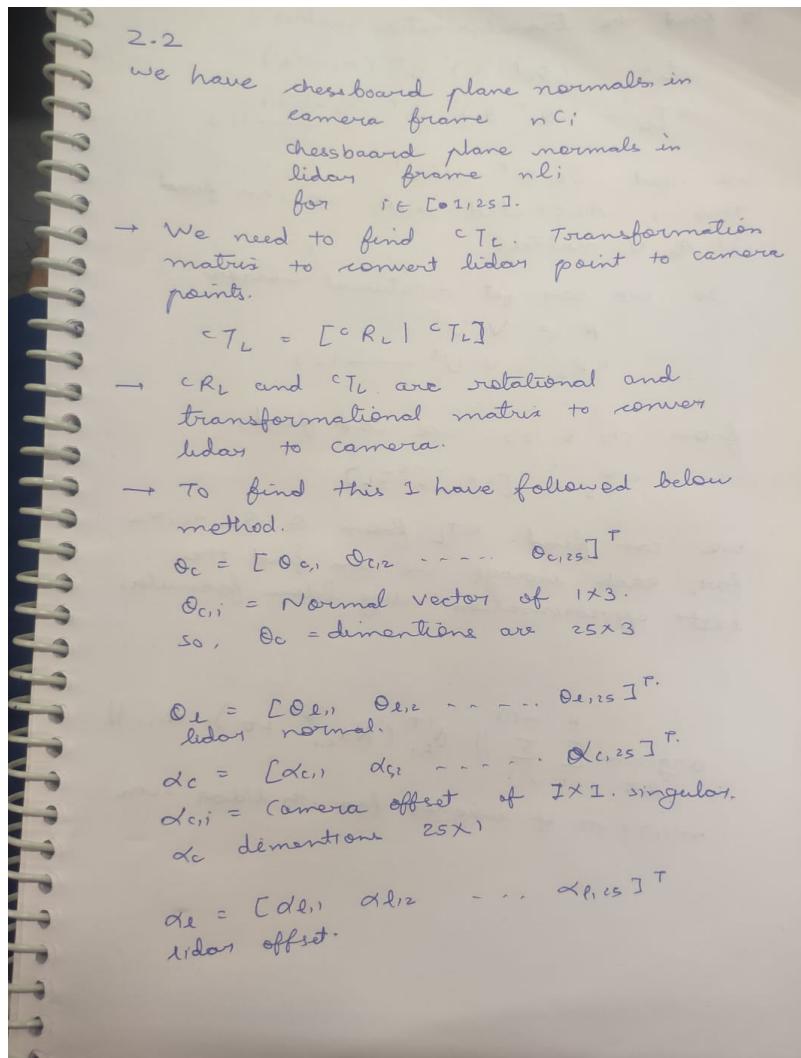
Camera normals

```
[-0.43432021 -0.35965743 -0.82584047]
[-0.81629617  0.09162133 -0.57032105]
[-0.45613391 -0.00554963 -0.88989385]
[ 0.29001348  0.28627248 -0.91320329]
[ 0.27176261 -0.16526395 -0.94806799]
[-0.65671387  0.05477644 -0.75214788]
[-0.03246712 -0.00217473 -0.99947044]
[ 0.39126525 -0.23402562 -0.89002445]
[ 0.16303408 -0.29304993 -0.94209428]
[-0.70422432  0.07305109 -0.70620935]
[ 0.00739864  0.25390736 -0.96720024]
[-0.42402941  0.31924201 -0.84751613]
[-0.73875133 -0.10276552 -0.66609738]
[-0.06913268  0.23786099 -0.96883581]
[-0.65477401  0.164822  -0.73763454]
[-0.13917323 -0.2935912 -0.94574575]
[-0.795691   0.24197359 -0.55526986]
[-0.24936746  0.22474881 -0.94196807]
[-0.68682933 -0.43663579 -0.58104618]
[-0.06925356 -0.07564818 -0.99472675]
[-0.18954575  0.13129915 -0.97305341]
[-0.05275962  0.22970552 -0.9718291 ]
[-0.55917687 -0.07439273 -0.82570391]
[ 0.34835151  0.25517693 -0.90196228]
[ 0.65789388 -0.28542777 -0.69692656]]
```

Camera offset :

```
[3.47754136]
[6.25263754]
[6.54620464]
[8.94983673]
[9.58596338]
[6.27168249]
[4.24353332]
[6.62148444]
[8.00874141]
[3.26695946]
[5.2813932 ]
[4.65765205]
[5.77919912]
[5.25699533]
[4.82438366]
[7.4045124 ]
[5.29235244]
[7.00994496]
[4.82780894]
[7.6113801 ]
[5.20549855]
[4.78574898]
[4.34327318]
[5.80327345]
[5.77199038]]
```

(ii)



To find the transformation matrix,

$$t = (\Omega_c^T \cdot \Omega_c)^{-1} \Omega^T (\alpha_c - \alpha_e)$$

$${}^c t_L = (\Omega_c^T \cdot \Omega_c)^{-1} \Omega_c^T (\alpha_c - \alpha_e) \quad (1)$$

we need $\Omega_c \cdot \Omega_c^T = U S V^T$.

This is Associated SVD. we can find
 U & V matrices.

so we can get rotational matrix

$$R = V \cdot V^T$$

$${}^c R_L = V \cdot V^T \quad (2)$$

from (1) & (2) we can find,

$${}^c T_L = [{}^c R_L] [{}^c t_L]$$

we can find ${}^c T_L$ from $\Omega_c, \Omega_e, \alpha_c, \alpha_e$
for each image. we can find the
best approximations using below formula,

$$\underset{R, t}{\operatorname{argmin}} \sum_{i=1}^n \sum_{j=1}^{m(i)} \| \Omega_{c,i}^T (R \alpha_{e,j} + t) - \alpha_{c,i} \|$$

$m(i)$ = no. of inliers for i th lidar scan

(iii)

I used the above calculation to calculate the transformation matrix.

```
cRL [[ 0.6984812 -0.22088819 -0.68068526]
```

```
[ 0.30438785 -0.76913844  0.56193781]
```

```
[-0.64766662 -0.59969532 -0.46999306]]
```

```
ctl [[-0.50528257]
```

```
[-0.03824241]
```

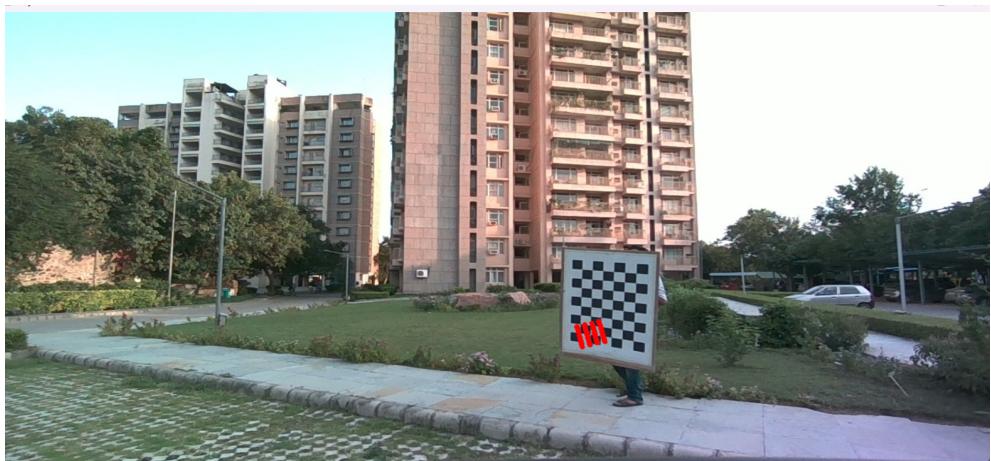
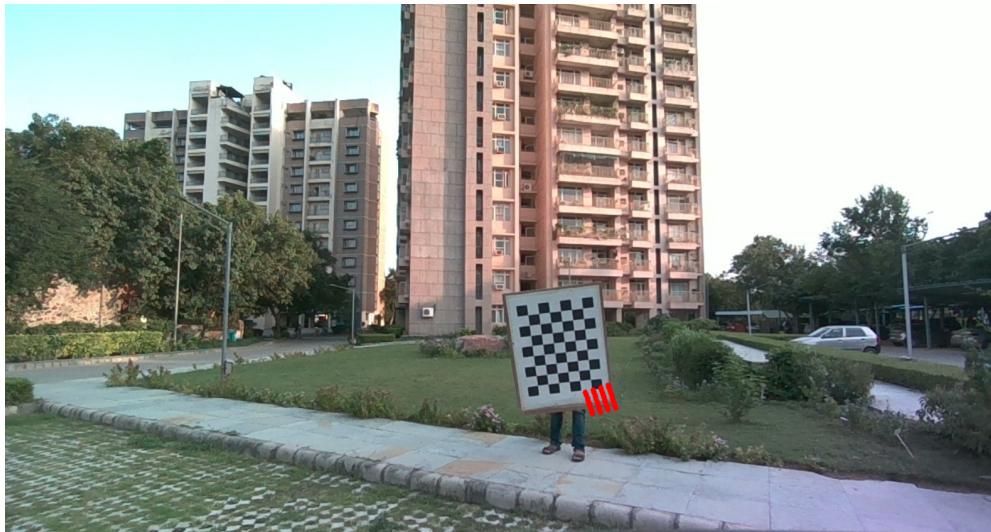
```
[-6.57232298]]
```

```
cTL [[ 0.6984812 -0.22088819 -0.68068526 -0.50528257]
```

```
[ 0.30438785 -0.76913844  0.56193781 -0.03824241]
```

```
[-0.64766662 -0.59969532 -0.46999306 -6.57232298]]
```

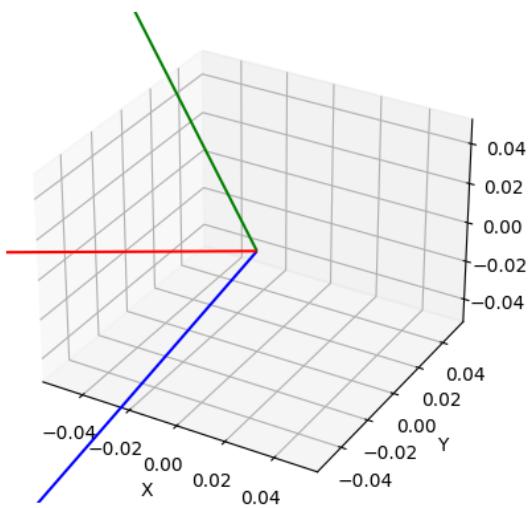
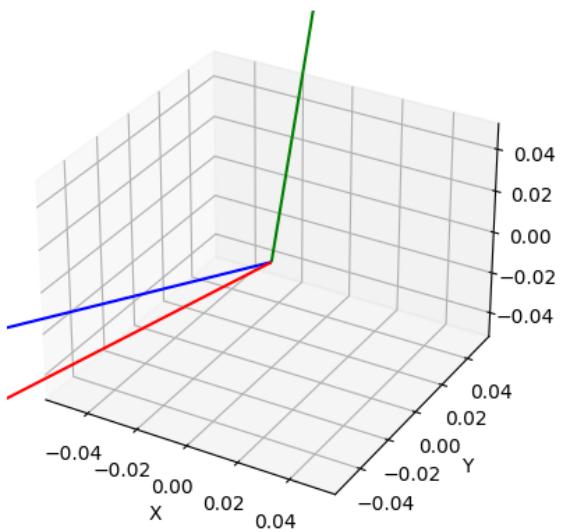
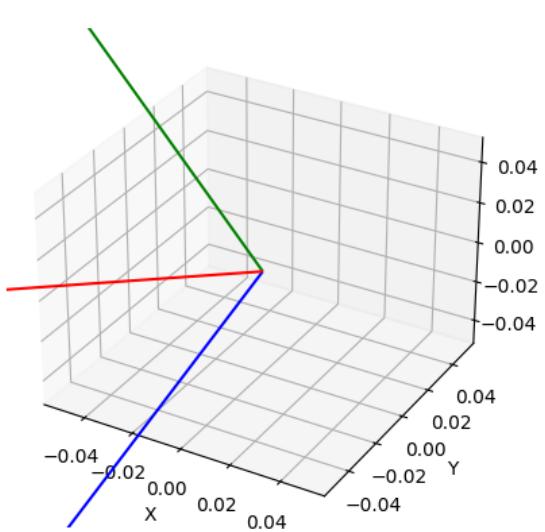
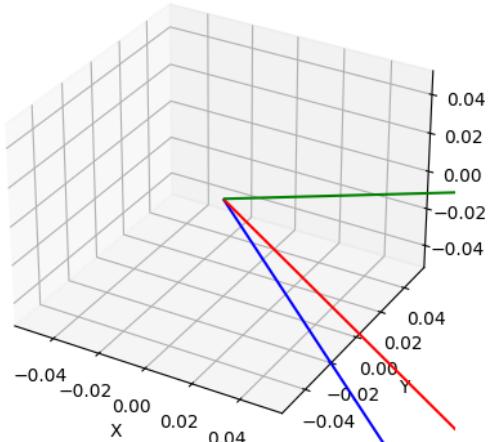
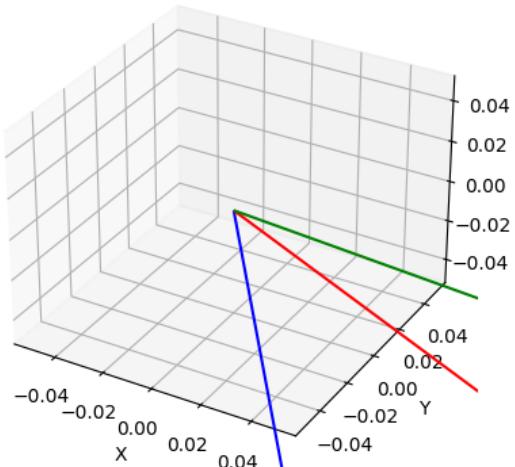
(iv):



(iv):

Just like the question 1, i have used `projectPoints()` function to get the image points from lidar points. Then i calculated the output images and jacobian matrix from intrinsic matrix and distortion matrix.

The projected points are inside for some of the image but not all. Some are on boundary and some are even outside.



average error 0.2696878579001874
standard deviation 0.5497982427658992

