

## Q2 Solution

Q.1.

$$x^2 y'' - 5xy' + 9y = 0, \quad y_1 = x^3$$

$$u(x) = \int \frac{1}{(x^3)^2} e^{-\int -\frac{5}{x} dx} dx = \int \frac{1}{x^6} e^{5 \ln x} dx$$

$$= \int \frac{1}{x^6} x^5 dx = \int \frac{1}{x} dx = \ln x$$

$$\boxed{y_2 = (x^3)(\ln x)}$$

Q.2.

$$3(y+1)dx - 2x dy = 0, \quad IF = \frac{y+1}{x^4}$$

$$M = 3(y+1) \cdot \frac{y+1}{x^4} = \frac{3(y+1)^2}{x^4}$$

$$N = -2x \cdot \frac{y+1}{x^4} = \frac{-2(y+1)}{x^3}$$

$$\frac{\partial M}{\partial y} = \frac{3}{x^4} \cdot 2(y+1) = \frac{6(y+1)}{x^4}$$
$$\frac{\partial N}{\partial x} = \frac{-2 \cdot -3(y+1)}{x^4} = \frac{6(y+1)}{x^4} \quad \left. \vphantom{\frac{\partial M}{\partial y}} \right\} \text{equal}$$

$$\Rightarrow \boxed{\text{Yes, } \frac{y+1}{x^4} \text{ is an integrating factor}}$$

Q.3.

$$y' = 1 + y^2, \quad y(0) = 0$$

$$R = \{(x, y) : |x| < 5, |y| < 3\}$$

$$f(x, y) = 1 + y^2 \Rightarrow |f(x, y)| = 1 + y^2 \leq 1 + 9 = 10$$

$$\text{Then, } \boxed{\alpha = \min \left\{ 5, \frac{3}{10} \right\} = \frac{3}{10} = 0.3}$$

Q.4.

$$y' + ky = e^{-kx}, \quad k > 0$$

$$IF = e^{\int k dx} = e^{kx}$$

$$\frac{d}{dx} (e^{kx} y) = 1 \Rightarrow e^{kx} y = x + C \Rightarrow y = \frac{x+C}{e^{kx}}$$
$$\Rightarrow \boxed{\lim_{x \rightarrow \infty} f(x) = \lim_{x \rightarrow \infty} \frac{x+C}{e^{kx}} = 0}$$

Q.5.

$$xy' + y = 0 \Rightarrow y' = -\frac{y}{x}$$

ODE for OTs is

$$\Rightarrow \boxed{y^2 = x^2 + C}$$

$$y' = \frac{x}{y} \Rightarrow y dy = x dx$$