Quiz-4 (ADA-2022)

May 4, 2022

1. Consider the following network with s as the source and t as the sink and the capacities of the edges are given as in Figure 1. What is the value of a maximum (s,t)-flow in this network?

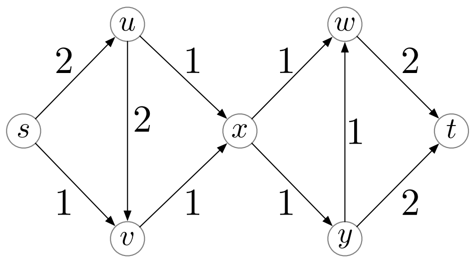

Figure 1: Question 1

   1. 1
   2. 2
   3. 0
   4. None of the above

2. In the network of Question 1, which edge will have zero flow for every possible maximum $s-t$ flow in the network ?

   1. Only (u,v)
   2. Only (y,w)
   3. Both (u,v) and (y,w)
   4. None of the above

3. Consider the network G in Figure 2. Suppose that a flow f with $f(s,a_1) = f(a_1,a_2) = f(a_2,b_1) = f(b_1,t) = 2$ has been sent. Then what will be the residual capacity of the edge (b_1,t) in G_f?

   1. 3
   2. 2
   3. 1
   4. None of the above

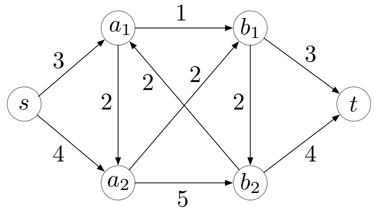

Figure 2: Question 3

4. What will be the residual capacity of the reverse edge (t, b_1) in G_f as defined in the previous question?
    1. 3
    2. 2
    3. 1
    4. There will not be such an arc in G_f
5. Suppose that f is a feasible but not maximum s, t-flow in a network with nonnegative capacity. Then, which of the following statements is true?
    1. The residual graph G_f can have a path P^* from s to t with $\min\{\mathit{capacity}(e) : e \in P^*\} > 0$ that is not there at all in the original graph.
    2. Residual graph G_f will never have a path from s to t that was not a path from s to t in the original graph G.
    3. The residual graph G_f will always have a path P^* from s to t with $\min\{\mathit{capacity}(e) : e \in P^*\}$ that is not there at all in the original graph.
    4. None of the above
6. In the network of Figure 2, what is the capacity of a minimum (s, t)-cut?
    1. 3
    2. 8
    3. 7
    4. 10
7. Suppose that G is a flow-network with source s and sink t. Consider an edge (u, v). Construct a new flow-network G' with source s and sink t as follows. Create a new vertex x, remove (u, v) and add edges (u, x) and (v, x) with capacities same as the capacity of (u, v). Then what happens to the value of a maximum (s, t)-flow of G'
    1. The maximum (s, t)-flow value of G and G' remain unchanged.
    2. The maximum (s, t)-flow value of G will always be smaller than the maximum (s, t)-flow value of G'.
    3. The maximum (s, t)-flow value of G will always be higher than the maximum (s, t)-flow value of G'.

    4. The maximum (s,t)-flow value of G' can increase or remain unchanged compared to the maximum (s,t)-flow value of G depending on the cases.

8. You are given a flow network $G = (V, E)$ with source s, sink t and a maximum (s,t)-flow f. Modify the network by increasing the capacity of a specific edge e by 1 and consider the new network G'.

    1. The value of a maximum (s,t)-flow and the value of a maximum (s,t)-flow in G' remain unchanged.
    2. The value of a maximum (s,t)-flow in G will always increase by 1.
    3. The value of a maximum (s,t)-flow in G can either increase by 1 or remain unchanged
    4. The value of maximum (s,t)-flow in G' can either decrease or increase by 1.

9. What is the worst case asymptotic runtime of Ford Fulkerson's algorithm on a graph where the capacity of every edge is 2 ? (Only the tightest possible answer gets credit)

    1. $O(m+n)$
    2. $O(mn)$
    3. $O(m^2)$
    4. $O(nm)$

10. Consider a flow network $G = (V, E)$ with source s and sink t and a maximum flow $f^\star$. We say an edge is *saturated* if the $f^\star_e = c_e$ for all edges $e \in E$, that is they are carrying the maximum amount of flow that they possibly can. Now suppose we modify the graph G by increasing the capacities of all the saturated edges by 1. Then the max-flow in the new graph

    1. Always increases by 1
    2. Always increases by an amount equal to the number of saturated edges in $f^\star$
    3. Always remains the same
    4. None of the above