Quiz-5 (ADA-2022)

April 19, 2022

Name: Roll No: Section:

1. The runtime (choose the best one) for detecting cycle in an undirected connected graph $G = (V, E)$ is

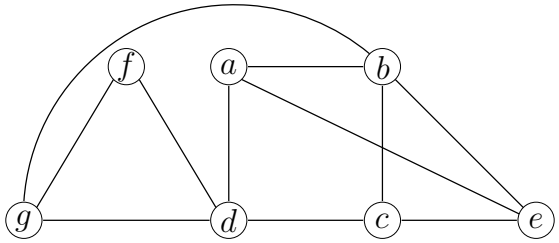
   ☐ $O(|E|)$ ☐ $O(|V|)$ ☐ $O(|V| + |E|)$ ☐ $O(|V| \times |E|)$

2. Suppose we run DFS from some source s in a undirected connected graph $G = (V, E)$. Then, for every back edge (u, v) which of the following is/are true?

   A. $\text{dep}[u] > \text{dep}[v]$ B. $\text{arr}[u] > \text{arr}[v]$ C. $\text{arr}[u] < \text{arr}[v]$ D. $\text{dep}[u] < \text{dep}[v]$

   ☐ only (A) ☐ only (B) ☐ Both (A) and (C) ☐ Both (B) and (D)

3. In the given graph, run DFS from vertex a. When having multiple choices for which vertex to visit next, always visit the vertex first in the alphabetical order. Then, the order in which DFS(a) visits vertices is:

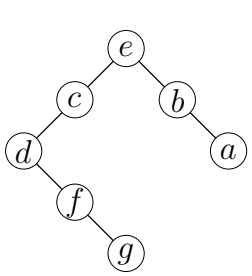


   ☐ a, b, c, d, e, f, g ☐ a, b, c, d, f, g, e ☐ a, b, c, e, d, f, g ☐ None of the above

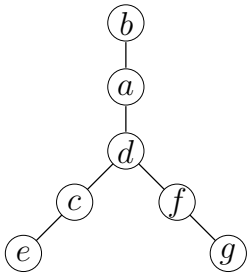
4. In the above DFS, which of the following is a back edge

   ☐ e, c ☐ f, d ☐ d, c ☐ None of the above

5. Consider a DFS on the graph given in question 3 ignoring the restriction of alphabetical order. Which of the following is/are valid DFS tree(s)?



(i)



(ii)

   ☐ Only (i) ☐ Only (ii) ☐ Both (i) and (ii) ☐ None of the above

6. You are given a directed graph $G = (V, E)$ with edge lengths, and a source vertex s. However it is given that all the negative edges are outgoing form s. Say there is also no negative cycle. Then, running Dijkstra from s

   - ☐ Will always find the shortest path from s to all vertices reachable form s
   - ☐ Will find the shortest path from s to neighbours of s but not necessarily to other vertices
   - ☐ Will find the shortest path form s to itself but not necessarily to other vertices
   - ☐ None of the above

7. You are given a directed graph $G = (V, E)$ with edge lengths, and a source vertex s. Let $d(v)$ be the shortest path distance from s to v. Say there are no negative cycles. Then, the relationship $d(v) \leq d(u) + l(u, v)$ necessarily holds for all edges (u, v) whenever

   - ☐ All edge lengths are non-negative
   - ☐ Edge lengths may be negative but v is not the source s
   - ☐ Edge lengths may be negative and v may be any vertex
   - ☐ None of the above

   p.s.: By default we are giving +1 to everyone for this question.

8. You are given an undirected graph $G = (V, E)$ with **positive integer distinct edge lengths**. Consider a minimum spanning tree (MST) T of G. Consider the shortest path P form a source vertex s to a destination vertex v. Now, suppose we decrease length of every edge by 1. Then

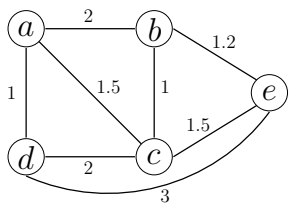
   - ☐ T necessarily continues to be an MST and P necessarily continues to be a shortest path from s to v
   - ☐ T necessarily continues to be an MST and P is not necessarily now a shortest path
   - ☐ T is not necessarily now an MST and P necessarily continues to be a shortest path
   - ☐ None of the above

9. Consider $G' = (V, E)$ to be a modified version of the graph G given in the previous question with only the following changes to the edge lengths : $\forall e \in E$, edge length $l_e \to l_e^2$. Then

   - ☐ T is necessarily an MST of G'
   - ☐ T is not necessarily an MST of G'
   - ☐ T might be an MST of G'
   - ☐ T is the only possible MST of G'

   p.s.: Any of the two options is correct. +1 for attempting either one of them.

10. Consider the following undirected graph G



Which of the following edge(s) is/are necessarily in every MST of G?

☐ ac, be ☐ ad, ce ☐ Only bc ☐ dc, be