

ENDSEM ANSWERS

Q7
(2+2+2)

1.

$$e^{-y^3/3 + f(x)}$$

2.

$$a^2/b^2$$

3.

$$-1/\pi^2$$

Q8
(1x15)

1 F 2 T 3 T 4 T 5 F

6 T 7 F 8 T 9 F 10 F

11 F 12 F 13 T 14 F 15 F

Q9
(2+2+2)

1.

$$sY_1 = -4Y_2$$

$$sY_2 - 1 = 3Y_1$$

2.

$$Y_1 = -\frac{4}{s^2+12}, Y_2 = \frac{s}{s^2+12}$$

3.

$$y_1 = -\frac{4}{\sqrt{12}} \sin(t\sqrt{12})$$

$$y_2 = \cos(t\sqrt{12})$$

Q10

1.

$$s^2 Y - 8s + 4Y = e^{-\pi s}$$

(2+1+2)

2.

$$Y = \frac{8s + e^{-\pi s}}{s^2 + 4}$$

3.

$$y = 8 \cos 2t + \frac{1}{2} \sin 2t u(t-\pi)$$

Q 11

(2+2+2)

$$1. \quad s^2 Y + 6sY + 8Y = \frac{1}{s+3} - \frac{1}{s+5}$$

$$2. \quad Y = \frac{2}{(s+2)(s+3)(s+4)(s+5)} \\ = \frac{A}{s+2} + \frac{B}{s+3} + \frac{C}{s+4} + \frac{D}{s+5}$$

$$A = 1/3, \quad B = -1, \quad C = 1, \quad D = -1/3$$

$$3. \quad y = \frac{1}{3} e^{-2t} - e^{-3t} + e^{-4t} - \frac{1}{3} e^{-5t}$$

Q 12

(1+2+1+2+1)

$$1. \quad x = t - 1.5$$

$$y'' + 3y' - 4y = 6e^{2x}, \quad y(0) = 4, \quad y'(0) = 5$$

$$2. \quad s^2 Y + 3sY - 4Y - 4s - 17 = \frac{6}{s-2}$$

$$3. \quad Y = \frac{4s-7}{(s-1)(s-2)}$$

$$4. \quad Y = \frac{A}{s-1} + \frac{B}{s-2}, \quad A = 3, \quad B = 1$$

$$\Rightarrow \mathcal{L}^{-1}(Y) = 3e^x + e^{2x}$$

$$5. \quad y = 3e^{t-1.5} + e^{2t-3}$$

Q 13

(1+1+2+3)

$$1. \quad y' = \sum_{m=1}^{\infty} m a_m x^{m-1}, \quad y'' = \sum_{m=2}^{\infty} m(m-1) x^{m-2}$$

$$2. \quad \sum_{m=2}^{\infty} m(m-1) x^{m-2} - \sum_{m=1}^{\infty} m a_m x^{m-1}$$

$$m=2$$

$$m=1$$

$$-x^2 \sum_{m=0}^{\infty} a_m x^m = 0$$

$$3. \quad x^0: 2a_2 - a_1 = 0 \quad \dots \textcircled{1}$$

$$x^1: 6a_3 - 2a_2 = 0 \quad \dots \textcircled{2}$$

$$x^2: 12a_4 - 3a_3 - a_0 = 0 \quad \dots \textcircled{3}$$

$$x^3: 20a_5 - 4a_4 - a_1 = 0 \quad \dots \textcircled{4}$$

$$4. \quad \textcircled{1} \Rightarrow a_2 = a_1/2$$

$$\textcircled{2} \Rightarrow a_3 = \frac{a_2}{3} = a_1/6$$

$$\textcircled{3} \Rightarrow a_4 = \frac{a_3}{4} + \frac{a_0}{12} = \frac{a_1 + 2a_0}{24}$$

$$\textcircled{4} \Rightarrow a_5 = \frac{a_4}{5} + \frac{a_1}{20} = \frac{7a_1 + 2a_0}{120}$$

$$y = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4 + a_5 x^5.$$

Q 14

(1+2+1
+1+2+1)

$$1. \quad Y' = \begin{pmatrix} 4 & 1 \\ 2 & 3 \end{pmatrix} Y + \begin{pmatrix} 0.6t \\ -2.5t \end{pmatrix} \\ = AY + F(t)$$

$$2. \quad \lambda = 2 \rightarrow \begin{pmatrix} -1 \\ 2 \end{pmatrix} \quad \lambda = 5 \rightarrow \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$3. \quad \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = c_1 \begin{pmatrix} -1 \\ 2 \end{pmatrix} e^{2t} + c_2 \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{5t}$$

$$4. \quad Y_p = \begin{pmatrix} a \\ b \end{pmatrix} + \begin{pmatrix} c \\ d \end{pmatrix} t$$

$$\dots \dots \dots \dots \dots$$

$$5. \quad a = -.241, b = .534, c = -.43, a = 1.12$$

$$6. \quad \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = c_1 \begin{pmatrix} -1 \\ 2 \end{pmatrix} e^{2t} + c_2 \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{5t} \\ + \begin{pmatrix} -.241 \\ .534 \end{pmatrix} + \begin{pmatrix} -.43 \\ 1.12 \end{pmatrix} t$$

Q15

(1+2+1
+1+2+1)

$$1. \quad Y' = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} Y + \begin{pmatrix} 1 \\ -3 \end{pmatrix} e^{3t} \\ = AY + F(t)$$

$$2. \quad \lambda = 1 \rightarrow \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \quad \lambda = -1 \rightarrow \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

$$3. \quad \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = c_1 \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^t + c_2 \begin{pmatrix} 1 \\ -1 \end{pmatrix} e^{-t}$$

$$4. \quad \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} c_1 e^t + c_2 e^{-t} \\ c_1 e^t - c_2 e^{-t} \end{pmatrix} = \begin{pmatrix} e^t & e^{-t} \\ e^t & -e^{-t} \end{pmatrix} \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} \\ = X(t) \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}$$

$$5. \quad Y_p = X(t) \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

$$u_1 = -e^{2t}/2, \quad u_2 = e^{4t}/2$$

$$6. \quad \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = c_1 \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^t + c_2 \begin{pmatrix} 1 \\ -1 \end{pmatrix} e^{-t} \\ + \begin{pmatrix} 0 \\ -1 \end{pmatrix} e^{3t}$$

Q16

(1+1+1+1
+3+3)

$$1. \quad y_1' = y_2 \\ y_2' = 9y_1 - y_1^3$$

$$2. \quad \begin{pmatrix} y_1' \\ y_2' \end{pmatrix} = \begin{pmatrix} y_2 \\ 9y_1 - y_1^3 \end{pmatrix}$$

3. critical points $\rightarrow (0,0), (3,0), (-3,0)$

4. $Y' = \begin{bmatrix} 0 & 1 \\ q - y_1^2 & 0 \end{bmatrix}$

5. $(0,0) : Y' = \begin{bmatrix} 0 & 1 \\ q & 0 \end{bmatrix} Y$

$(3,0) : Y' = \begin{bmatrix} 0 & 1 \\ -18 & 0 \end{bmatrix} Y$

$(-3,0) : Y' = \begin{bmatrix} 0 & 1 \\ -18 & 0 \end{bmatrix} Y$

6. $(0,0) : \lambda^2 - q = 0 \Rightarrow \lambda = \pm \sqrt{q}$ (Node)

$(3,0) : \lambda^2 + 18 = 0 \Rightarrow \lambda = \pm \sqrt{18}i$ (center)

$(-3,0) : \lambda^2 + 18 = 0 \Rightarrow \lambda = \pm \sqrt{18}i$ (center)