

Quiz-4 (ADA-2022)

May 4, 2022

1. Consider the following network with s as the source and t as the sink and the capacities of the edges are given as in Figure 1. What is the value of a maximum (s, t) -flow in this network?

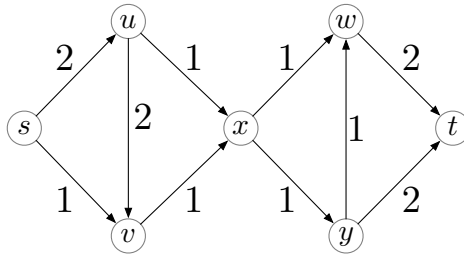


Figure 1: Question 1

1. 1
2. 2
3. 0
4. None of the above

Answer: Option 2

2. In the network of Question 1, which edge will have zero flow for every possible maximum $s - t$ flow in the network ?
 1. Only (u, v)
 2. Only (y, w)
 3. Both (u, v) and (y, w)
 4. None of the above

Answer: Option 4

3. Consider the network G in Figure 2. Suppose that a flow f with $f(s, a_1) = f(a_1, a_2) = f(a_2, b_1) = f(b_1, t) = 2$ has been sent. Then what will be the residual capacity of the edge (b_1, t) in G_f ?
 1. 3
 2. 2

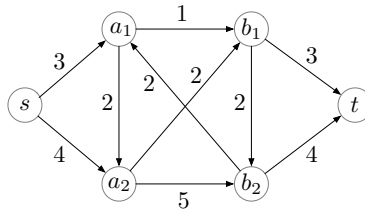


Figure 2: Question 3

3. 1
4. None of the above

Answer: Option 3

4. What will be the residual capacity of the reverse edge (t, b_1) in G_f as defined in the previous question?
 1. 3
 2. 2
 3. 1
 4. There will not be such an arc in G_f

Answer: Option 2

5. Suppose that f is a feasible but not maximum s, t -flow in a network with nonnegative capacity. Then, which of the following statements is true?
 1. The residual graph G_f can have a path P^* from s to t with $\min\{\text{capacity}(e) : e \in P^*\} > 0$ that is not there at all in the original graph.
 2. Residual graph G_f will never have a path from s to t that was not a path from s to t in the original graph G .
 3. The residual graph G_f will always have a path P^* from s to t with $\min\{\text{capacity}(e) : e \in P^*\}$ that is not there at all in the original graph.
 4. None of the above

Answer: Option 1.

6. In the network of Figure 2, what is the capacity of a minimum (s, t) -cut?
 1. 3
 2. 8
 3. 7
 4. 10

Answer: Option 3

7. Suppose that G is a flow-network with source s and sink t . Consider an edge (u, v) . Construct a new flow-network G' with source s and sink t as follows. Create a new vertex x , remove (u, v) and add edges (u, x) and (v, x) with capacities same as the capacity of (u, v) . Then what happens to the value of a maximum (s, t) -flow of G'
1. The maximum (s, t) -flow value of G and G' remain unchanged.
 2. The maximum (s, t) -flow value of G will always be smaller than the maximum (s, t) -flow value of G' .
 3. The maximum (s, t) -flow value of G will always be higher than the maximum (s, t) -flow value of G' .
 4. The maximum (s, t) -flow value of G' can increase or remain unchanged compared to the maximum (s, t) -flow value of G depending on the cases.

Answer: Option 1. **Note:** There is ambiguity in this question. We wanted to say (u, x) and (x, v) since we work with directed graphs here. Everyone gets 1 point for this one.

8. You are given a flow network $G = (V, E)$ with source s , sink t and a maximum (s, t) -flow f . Modify the network by increasing the capacity of a specific edge e by 1 and consider the new network G' .
1. The value of a maximum (s, t) -flow and the value of a maximum (s, t) -flow in G' remain unchanged.
 2. The value of a maximum (s, t) -flow in G' will always increase by 1.
 3. The value of a maximum (s, t) -flow in G' can either increase by 1 or remain unchanged
 4. The value of maximum (s, t) -flow in G' can either decrease or increase by 1.

Answer: Option 3.

9. What is the worst case asymptotic runtime of Ford Fulkerson's algorithm on a graph where the capacity of every edge is 2 ? (Only the tightest possible answer gets credit)
1. $O(m + n)$
 2. $O(mn)$
 3. $O(m^2)$
 4. $O(nm)$

Answer: Option 4.(or 2)

10. Consider a flow network $G = (V, E)$ with source s and sink t and a maximum flow f^* . We say an edge is *saturated* if the $f_e^* = c_e$ for all edges $e \in E$, that is they are carrying the maximum amount of flow that they possibly can. Now suppose we modify the graph G by increasing the capacities of all the saturated edges by 1. Then the max-flow in the new graph
1. Always increases by 1
 2. Always increases by an amount equal to the number of saturated edges in f^*
 3. Always remains the same
 4. None of the above

Answer: Option 4.