Q3 Solution

Q.1. $x^2 y'' + Axy' + By = 0$

Let $y = x^m$ be a solution then

$m(m-1) + Am + B = 0 \Rightarrow m^2 + (A-1)m + B = 0$

Since $y = (a + b\ln x)x^3$ is the general solution

$\Rightarrow m = 3$ is a double root of

$$m^2 + (A-1)m + B = 0 = (m-3)^2$$
$$= m^2 - 6m + 9$$

$\Rightarrow A - 1 = -6$ & $B = 9$

$\Rightarrow A = -5$ & $B = 9$

$\Rightarrow \boxed{A + B = 4}$

Q.2. You can have $t \cdot e^{2t}$ as a particular solution of a mass-spring system only when it is $\boxed{\text{critically-damped}}$

Q.3. For a damped mass-spring system with oscillating solution, it must be under-damped, i.e.

$$c^2 - 4mk < 0$$

$$\Rightarrow c^2 < 4mk = 4 \times 1.75 = 7$$

Since c is a prime number $\boxed{c = 2}$

Q.4. When $t = 0$, $y(0) = 1$, $y'(0) = 3$

$\Rightarrow C\cos(d) = 1$, $C\omega\sin(d) = 3$

$\Rightarrow \tan(d) = \dfrac{\sin(d)}{\cos(d)} = \dfrac{3}{\omega} = \dfrac{3}{\sqrt{\frac{k}{m}}}$

$\Rightarrow \tan(d) = \dfrac{3}{\sqrt{90/10}} = \dfrac{3}{3} = 1 \Rightarrow \boxed{d = 45}$

Q.5. $(D^2 + aI)\sin(0.4x) = 0$

$\Rightarrow (-(0.4)^2 + a)\sin(0.4x) = 0$

$\Rightarrow \boxed{a = 0.16}$

Q.6. General solution of $y'' + 4y' + 13y = 0$ is $y(x) = e^{-2x}(C_1\cos(3x) + C_2\sin(3x))$

$\Rightarrow y$ is always $\boxed{\text{oscillating}}$.

But may be bounded $(y = 0)$ or unbounded $(y = e^{-2x}\cos(3x))$.

Q.7. One root is $1+i$ then other is $1-i$. So, Char. eq is.

$(\lambda - (1+i))(\lambda - (1-i)) = 0$

$\Rightarrow \lambda^2 - \lambda + \cancel{i\lambda} - \lambda - \cancel{i\lambda} + 1 + 1 = 0$

$\Rightarrow \lambda^2 - 2\lambda + 2 = 0$

So, ODE is $\boxed{y'' - 2y' + 2 = 0}$.