Q1 Solution

Q.1.

$$y(x) = \cos^2 x + 4$$

$$y' \cot x - 2y + 8 = -2\cos x \cdot \sin x \cdot \frac{\cos x}{\sin x} - 2(\cos^2 x + 4) + 8$$

$$= -2\cos^2 x - 2\cos^2 x - 8 + 8$$

$$= -4\cos^2 x \neq 0$$

$\boxed{\text{Ans. No}}$

Q.2.

$$y'(t) = k\ln(2)\, y(t) \Rightarrow \frac{y'}{y} = k\ln 2\, dt$$

$$\Rightarrow \ln y = \ln 2 \cdot tk + C \Rightarrow y = e^{kt \cdot \ln 2 + C}$$

$$\Rightarrow y = e^C e^{\ln 2\, tk} \Rightarrow y = C \cdot 2^{kt}$$

$$\Rightarrow \frac{C}{2} = C \cdot 2^{kt} \Rightarrow 2^{kt} = 2^{-1}$$

$$\Rightarrow kt = -1 \Rightarrow k = -\frac{1}{t} = -\frac{1}{6/24} = -4$$

$$\Rightarrow \boxed{-k = 4}$$

Q.3. Eliminate other options by looking at vectors close to x & y-axes.

$\boxed{\text{Ans. } y' = \sin^2 x}$

Q.4.

$$y' = (y - x)^2$$

$$x_1 = x_0 + h = 0 + 0.1 = 0.1$$

$$y_1 = y_0 + (y_0 - x_0)^2 h = 0 + 0 = 0$$

$$x_2 = x_1 + h = 0.1 + 0.1 = 0.2$$

$$y_2 = y_1 + (y_1 - x_1)^2 h = 0 + (0.1)^2 \times 0.1 = (0.1)^3$$

$$= 10^{-3} \Rightarrow A = 1,\ B = 3 \Rightarrow \boxed{A + B = 4}$$

Q.5.

$$y' = 1+4y^2 \Rightarrow \int \frac{dy}{1+4y^2} = \int dx$$

$$y = \frac{1}{2}\tan\theta$$
$$dy = \frac{1}{2}\sec^2\theta\, d\theta$$
$$1+4y^2 = 1+\tan^2\theta = \sec^2\theta$$

$$\Rightarrow \frac{1}{2}\theta = x + C$$

$$\Rightarrow \frac{\tan^{-1}(2y)}{2} = x + C$$

$$\Rightarrow \tan^{-1}(2y) = 2x + C$$

$$\Rightarrow y = \frac{1}{2}\tan(2x+C)$$

$$y(1)=0 \Rightarrow 0 = \frac{1}{2}\tan(2+C) \Rightarrow 2+C = 0 \Rightarrow C=-2$$

So, $y = \frac{1}{2}\tan(2(x-1))$

$$\Rightarrow y\left(\frac{\pi}{8}+1\right) = \frac{1}{2}\tan\left(\frac{\pi}{4}\right) = \frac{1}{2} = \boxed{0.5}$$