Units and Measurement

⇨ Physical Quantity (PQ)
The quantities which can be measured by an instrument & which describes the laws of physical world.

$$PQ = n \times u$$

$n = numerical\ value\ (number\ of\ times\ of\ unit)$
$u = unit\ (a\ comparison)$

> Ex – Take an elephant and a goat and tell the weight of elephant in terms of weight of goat, so we can say that:
>
> Weight of elephant = $150 \times (weight\ of\ goat)$
>
> In this example,
>
> weight of goat = unit
> 150 = numerical value

⇨ Types of Physical Quantities (PQ)
1. Fundamental PQ
2. Derived PQ
3. Supplementary PQ

1. Fundamental PQ:- which do not depend on any other physical quantity.

PQ	S.I. Unit	Symbol
Length	Meter	m
Mass	Kilogram	kg
Time	Seconds	S
Temperature	Kelvin	K
Electric Current	Ampere	A
Luminous Intensity	Candela	Cd
Amount of Substance	Mole	mol

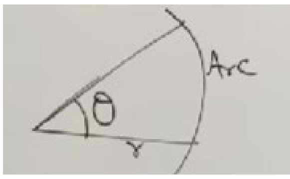
2. Derived PQ:- which are made up of Fundamental PQ.

Examples:
- Area = m^2
- Volume = m^3
- Velocity = $\frac{displacement}{time}$ $(\frac{m}{s})$ | $\frac{length}{time}$

- Force = $Mass \times Acceleration$
- Mass = N = Kg $\frac{m}{s^2}$
- Work = Joule = Kg $\frac{m^2}{s^2}$

3. Supplementary PQ:
   1. Angle
   2. Solid Angle

   1. Angle:



$$\theta = \frac{Arc}{Radius}$$

unit = radian
symbol = rad

$360^o = 2\pi rad$
$180^o = \pi rad$

   2. Solid Angle: 3D

$\theta = \frac{A}{r^2}$ | Actual Formula = $\frac{dA}{r^2}$

Unit = Steredian
Symbol = sd

⇨ Dimensions
Dimensions are the powers to which the fundamental units are raised in order to express the derived unit of a quantity.

1. Length = L
2. Mass = M
3. Time = T
4. Temperature = K or θ

5. Electric Current = A
6. Luminous Intensity = C
7. Amount of Substance = mol

⇨ Dimensions of Physical Quantities

1. Area = m^2 $[L^2]$ = $[M^0L^2T^0]$
2. Volume = m^3 $[L^3]$ = $[M^0L^3T^0]$
3. Mass Density = $\frac{Kg}{m^3} = \frac{M}{L^3}$ $[ML^{-3}]$ = $[M^1L^{-3}T^0]$
4. Velocity = $\frac{Displacement}{Time}$ $[L^1T^{-1}]$ = $[M^0L^1T^{-1}]$
5. Acceleration = $\frac{Change\ in\ Velocity}{Change\ in\ Time} = \frac{m}{s^2}$ $[L^1T^{-2}]$ = $[M^0L^1T^{-2}]$
6. Force = $Mass \times Acceleration = kg \times \frac{m}{s^2}$ = $[M^1L^1T^{-2}]$
7. Momentum = $Mass \times Velocity = kg \times \frac{m}{s}$ = $[M^1L^1T^{-1}]$
8. Work = $Force \times Displacement \times Cos\theta$ = $[M^1L^2T^{-2}]$
9. Power = $\frac{Work}{Time} = \frac{[M^1L^2T^{-2}]}{T}$ = $[M^1L^2T^{-3}]$
10. Energy = Joule = $[M^1L^2T^{-2}]$
11. Kinetic Energy = $\frac{1}{2} \times mv^2$ $[M^1 \frac{L^2}{T^2}]$ = $[M^1L^2T^{-2}]$
12. Impulse = $Force \times \Delta t$ = $[M^1L^1T^{-1}]$
13. Angular Velocity (ω) = $\frac{Velocity}{Radius} = \frac{L}{T \times L}$ = $[M^0L^0T^{-1}]$
14. Frequency = $\frac{1}{Time}$ = $[M^0L^0T^{-1}]$
15. Angular Frequency (ω) = $2\pi f$ = $[M^0L^0T^{-1}]$
16. Angular Acceleration (α) = $\frac{\Delta\omega}{\Delta t}$ = $[M^0L^0T^{-2}]$
17. Torque = $F \times distance \times Sin\theta$ = $[M^1L^2T^{-2}]$
18. Moment of Inertia (I) = mr^2 = $[M^1L^2T^0]$
19. Angular Momentum = $mvrsin\theta$ = $[M^1L^2T^{-1}]$
20. Stress = $\frac{Force}{Area} = \frac{MLT^{-2}}{L^2}$ = $[M^1L^{-1}T^{-2}]$
21. Strain = $\frac{\Delta l}{l} = \frac{m}{m}$ = $[M^0L^0T^0]$
22. Yang's Modules of Elasticity = $\frac{Stress}{Strain}$ = $[M^1L^{-1}T^{-2}]$
23. Surface Tension (T) = $\frac{Force}{length}$ = $[M^1L^0T^{-2}]$
24. Electric Charge (Q) = $I \times T$ = $[M^0L^0A^1T^1]$
25. Electric Potential (V) = $\frac{W}{Q}$ = $[M^1L^2A^{-1}T^{-3}]$
26. Electrical Resistance (R) = $\frac{\Delta V}{I}$ = $[M^1L^2A^{-2}T^{-3}]$
27. Capacitance (C) = $\frac{Q}{V} = \frac{A^1T^1}{M^1L^2A^{-1}T^{-3}}$ = $[M^{-1}L^{-2}A^2T^4]$
28. Temperature Gradient = $\frac{\Delta Temperature}{Length}$ = $[M^0L^{-1}K^1T^0]$

⇨ Dimensions of Some Constants

1. Refractive Index (μ) = $\frac{C}{v} = \frac{\frac{l}{t}}{\frac{l}{t}}$ = $[M^0L^0T^0]$
2. Uni. Gravitational Cons. (G) = $\frac{Fr^2}{m1m2}$ = $[M^{-1}L^3T^{-2}]$
3. Planck's Constant (h) = $\frac{E}{f} = \frac{M^1L^2T^{-2}}{T^{-1}}$ = $[M^1L^2T^{-1}]$

4. Boltzmann Constant (K) = $\frac{Pressure \times Volume}{Temperature}$ = $\frac{F}{Area} \times \frac{Volume}{Temp}$ = $\frac{M^1L^1T^{-2}}{L^2} \times \frac{L^3}{Kelvin}$

$[M^1L^2T^{-2}K^{-1}]$

5. Coefficient of Viscosity (∩) = $\frac{F(Force)\Delta Z(Change\ in\ distance)}{A(Area)\Delta Velocity}$ = $\frac{M^1L^1T^{-2}}{L^2} \times \frac{L}{LT^{-1}}$

$[M^1L^{-1}T^{-1}]$

6. Molar Specific Heat Capacity (C) or (Cp), (Cv) = $\frac{Q(Heat\ Energy)}{n(Num.of\ Moles)\Delta Tempeature}$

= $\frac{M^1L^2T^{-2}}{mol} \times \frac{1}{K}$ $[M^1L^2T^{-2}mol^{-1}K^{-1}]$

Note –

- Two supplementary PQ, that is, Angle(radian) & Solid Angle(steredian) have units but no dimensions.
- Refractive Index have no unit and no dimension.
- Strain have no unit and no dimension.
- No PQ have dimensions but no unit.

⇨ Principal of Homogeneity

The dimensions of each term of an equation must be same.

Or,

We can add & subtract similar physical quantities.

Q1. Find the dimensions of :

- $a \& b\ in\ V = a + bt$

Sol.

$V - a - bt = 0$

$\boxed{M^0L^1T^{-1} = a}$

$M^0L^1T^{-1} = b \times T$

$\boxed{M^0L^1T^{-2} = b}$

- $A, B, C, D, E \quad in \quad x = A + Bt + Ct^2 + \frac{Dt^3}{E+t}$

Sol.

$x - A - Bt - Ct^2 - \frac{Dt^3}{E+t} = 0$

$\boxed{A = L}$

$B \times T = L$

$\boxed{B = LT^{-1}}$

$C \times T^2 = L$

$\boxed{C = LT^{-2}}$

$$\frac{D \times T^3}{E + T} = L$$

$$\frac{D \times T^3}{T} = L$$

$$\boxed{D = LT^{-2}}$$

$$\boxed{E = T}$$

- $\frac{a}{b} \quad in \quad P = \frac{a - t^2}{bx}$

Sol.

$$P = \frac{T^2}{b \times L}$$

$$P = \frac{Force}{Area} = \frac{MLT^{-2}}{L^2}$$

$$P = ML^{-1}T^{-2}$$

$$ML^{-1}T^{-2} = \frac{T^2}{b \times L}$$

$$b = \frac{T^2}{ML^{-1}T^{-2} \times L}$$

$$b = M^{-1}T^4$$

$$\frac{a}{b} = \frac{T^2}{M^{-1}T^4}$$

$$\frac{a}{b} = \frac{T^{-2}}{M^{-1}}$$

$$\boxed{\frac{a}{b} = MT^{-2}}$$

- $\alpha \quad in \quad P = P\sigma e^{-\alpha t^2}$

$$\alpha \times T^2 = M^0L^0T^0$$

$$\boxed{\alpha = M^0L^0T^{-2}}$$

- $\beta \quad in \quad P = \frac{\alpha}{\beta} e^{\frac{-\alpha z}{k\theta}}$

$$\frac{\alpha z}{k\theta} = M^0L^0T^0$$

$$\frac{\alpha \times L}{ML^2T^{-2}\theta^{-1} \times \theta} = M^0L^0T^0$$

$$\alpha = MLT^{-2}$$

$$P = \frac{\alpha}{\beta}$$

$$\frac{Force}{Area} = \frac{\alpha}{\beta}$$

$$\frac{MLT^{-2}}{L^2} = \frac{MLT^{-2}}{\beta}$$

$$\boxed{\beta = M^0L^2T^0}$$

- $A, \omega \,\&\, k \quad in \quad Y = A\,Sin(\omega t - kx)$

$$\omega \times T = M^0L^0T^0$$

$$\boxed{\omega = M^0L^0T^{-1}}$$

$$k \times L = M^0L^0T^0$$

$$\boxed{k = M^0L^{-1}T^0}$$

$$Y = A$$

$$\boxed{A = M^0L^1T^0}$$

⇨ Applications of Dimensional Analysis

1. Checking the correctness of equation:- For a correct equation dimension of each term on LHS = Dimension of Each term on RHS

Example:

- $S = ut + \frac{1}{2}at^2$

  $S = L$

  $ut = \frac{L}{T} \times T = L$

  $\frac{1}{2}at^2 = \frac{L}{T^2} \times T^2 = L$

- $T = 2\pi\sqrt{\frac{l}{g}}$

  $T = T$

  $2\pi\sqrt{\frac{l}{g}} = \sqrt{\frac{L}{\frac{L}{T^2}}} = T$

- $h = \frac{r\rho g}{2SCos\theta}$

  $h = L$

  $\frac{L \times \frac{M}{L^3} \times \frac{L}{T^2}}{\frac{MLT^{-2}}{L}} = \frac{\frac{M}{LT^2}}{MT^{-2}}$

  $\frac{M}{ML} = L^{-1}$[Wrong Formula Spotted] -> Correct Formula: $h = \frac{2SCos\theta}{r\rho g}$

2. Conversion of Units:

$PQ = n \times u$

Formula:

$n1u1 = n2u2$

$\downarrow\downarrow \quad \downarrow \quad \downarrow$

1 m = 100 cm

Q.1. Convert 1N(Force S.I.) to Dyne(Force C.G.S.)

Sol.

$F = MLT^{-2}$

$1N = 1Kg\frac{m}{s^2}$

$\_\, dyne = \_\, g\frac{cm}{s^2}$

N	D
$n1 = 1$	n2 = __
$M1 = Kg$	M2 = g
$L1 = m$	L2 = cm
$T1 = s^2$	T2 = s^2
$U1 = M1L1T1^{-2}$	$U2 = M2L2T2^{-2}$

Applying formula,

$1 \times M1L1T1^{-2} = n2 \times M2L2T2^{-2}$

$\left(\frac{M1}{M2}\right)\left(\frac{L1}{L2}\right)\left(\frac{T1}{T2}\right)^{-2} = n2$

$\left(\frac{Kg}{g}\right)\left(\frac{m}{cm}\right)\left(\frac{s^2}{s^2}\right)^{-2} = n2$

$1000 \times 100 = n2$

$\boxed{10^5 = n2}$

Q.2. The value of $G = 6.67 \times 10^{-11}N\frac{m^2}{kg^2}$ in S.I. unit. Find it’s value in C.G.S unit.

$G = \frac{Kgm}{s^2} \times \frac{m^2}{kg^2}$

$G = m^3 kg^{-1} s^{-2}$

$G = M^{-1} L^3 T^{-2}$

$\_dyne \ = \ \_\frac{cm^2}{g^2}$

SI

n1 = 6.67×10^{-11}

M1 = kg

L1 = m

T1 = s

U1 = $M1^{-1} L1^3 T1^{-2}$

CGS

n2 = _

M2 = g

L2 = cm

T2 = s

U2 = $M2^{-1} L2^3 T2^{-2}$

Applying formula,

$6.67 \times 10^{-11} \times M1^{-1} L1^3 T1^{-2}$ = $n2 \times M2^{-1} L2^3 T2^{-2}$

$$6.67 \times 10^{-11} \times \left(\frac{kg}{g}\right)^{-1} \left(\frac{m}{cm}\right)^3 \left(\frac{s}{s}\right)^{-2} = n2$$

$$6.67 \times 10^{-11} \times 10^{-3} \times 10^6 = n2$$

$$6.67 \times 10^{-11} \times 10^3 = n2$$

$$\boxed{6.67 \times 10^{-8} = n2}$$

⇒ Derive the relation between physical quantities.

Q.1. Force 'F' on a particle depends upon:
i) Mass 'm' of particle
ii) Acceleration 'a' of particle

Find the expression for F.

Sol.⇒ $F \propto m^x . a^y$ $\quad\quad m = M$

$F \propto K m^x a^y$ $\quad\quad a = LT^{-2}$

$F = K m^x a^y$ $\quad\quad F = MLT^{-2}$

$\Rightarrow MLT^{-2} = M^x (LT^{-2})^y$

$\Rightarrow M^1 = M^x, \quad L^1 = L^y, \quad T^{-2} = T^{-2y}$

$\Rightarrow 1 = x, \quad y = 1,$

$\Rightarrow$ $F \propto m^1 a^1$

$\Rightarrow F = ma$ (Derived)

Q.2. Time period 'T' of a simple pendulum depends upon mass of bob 'm', acceleration due to gravity 'g', length of pendulum 'l'

Find expression for 'T'.

Sol. ⇒ $T \propto m^x g^y l^z$ $\quad\quad m = M$

$T = K m^x g^y l^z$ $\quad\quad g = LT^{-2}$

$l = L$

$\Rightarrow M^0L^0T^1 = M^x (LT^{-2})^y L^z$

$\Rightarrow M^0L^0T^1 = M^x L^{(y+z)} T^{-2y}$

$\Rightarrow$ $0 = x$, $0 = y+z$, $1 = -2y$

$\Rightarrow -z = y$ $\qquad -\frac{1}{2} = y$

$\Rightarrow +z = +\frac{1}{2}$

$\Rightarrow z = \frac{1}{2}$

$\Rightarrow T \propto m^0 g^{-\frac{1}{2}} l^{\frac{1}{2}}$

$\Rightarrow T = \sqrt{\frac{l}{g}}$ (Derived)

Q.3. Centripetal force 'F' acting on a particle of mass 'm' moving in a circle of radius 'r' ~~with~~ with velocity 'v' ~~ate~~ depends upon 'm', 'v' and 'r'. Find expression for F.

Sol. $\Rightarrow$ $F \propto m^x v^y r^z$

$\Rightarrow F = K m^x v^y r^z$

$m = M$

$v = LT^{-1}$

$r = L$

$F = MLT^{-2}$

$\Rightarrow MLT^{-2} = M^x (LT^{-1})^y L^z$

$\Rightarrow MLT^{-2} = M^x L^{(y+z)} T^{-y}$

$\Rightarrow 1 = x$, $y+z = 1$, $+2 = +y$

$\Rightarrow y = 1 - z$

$\Rightarrow 2 = 1 - z$

$\Rightarrow 2 - 1 = -z$

$\Rightarrow 1 = -z$

$\Rightarrow -1 = z$

$F \propto m^1 v^2 r^{-1}$

$\Rightarrow F = \frac{mv^2}{r}$ (Derived)

Q.4. Energy 'E' of a particle of mass 'm' executing simple ~~harmo~~ harmonic motion (SHM) with ~~amplitude~~ amplitude 'A' & angular frequency ω depends upon m, A and ω. Find the ~~expr~~ expression for E?

$\Rightarrow E \propto m^x A^y \omega^z$

$\Rightarrow E = K m^x A^y \omega^z$

$m = M$

$A = L$

$\omega = T^{-1}$

$E = ML^2T^{-2}$

$\Rightarrow ML^2T^{-2} = M^x L^y T^{-1z}$

$\Rightarrow x = 1, \quad y = 2, \quad +z = +2$

$\Rightarrow E = m^1 A^2 \omega^2$

$\Rightarrow E = mA^2\omega^2$ (Derived)

Q.5. If Force 'F', velocity 'v' and time 'T' are fundamental qualities. Find dimensions of energy 'E'.

Sol. $\Rightarrow E \propto K F^x v^y T^z$

$\Rightarrow E = F^x v^y T^z$

$F = MLT^{-2}$

$V = LT^{-1}$

$T = T$

$E = ML^2T^{-2}$

$\Rightarrow ML^2T^{-2} = (MLT^{-2})^x (LT^{-1})^y T^z$

$\Rightarrow ML^2T^{-2} = M^x L^{(x+y)} T^{(-2x-y+z)}$

$\Rightarrow x = 1, \quad x+y = 2, \quad -2 = -2x-y+z$

$\Rightarrow$

$\Rightarrow x+y=2$

$\Rightarrow 1+y=2$

$\Rightarrow y=1$

$\Rightarrow -2 = -2\times1 -2\times1 + z$

$\Rightarrow \cancel{-2} = \cancel{-2} -2+2$

$\Rightarrow 2 = z$

$\Rightarrow z = 2$

$\Rightarrow E = F^1 v^1 T^2$

$\Rightarrow E = FVT^2$

$\Rightarrow ML^2T^{-2} = (MLT^{-2})^x (LT^{-1})^y T^z$

$\Rightarrow ML^2T^{-2} = M^x L^{(x+y)} T^{(-2x-y+z)}$

$\Rightarrow 1 = x, \quad x+y=2, \quad -2 = -2x-y+z$

$\Rightarrow x+y=2$

$\Rightarrow 1+y=2$

$\Rightarrow y=1$

$-2 = -2\times1 - 1 + z$

$\Rightarrow \cancel{-2} = \cancel{-2} - 1 + z$

$\Rightarrow 1 = z$

$\Rightarrow E = F^1 V^1 T^1$

$E = FVT$

Q.6. If Energy 'E', Velocity 'v' and Time 't' are fundamental quantities. Find dimensions of surface Tensions.

$\Rightarrow S \propto E^x V^y T^z$

$\Rightarrow S = K E^x V^y T^z$

$S = MT^{-2}$

$E = ML^2T^{-2}$

$V = LT^{-1}$

$T = T$

$\Rightarrow MT^{-2} = (ML^2T^{-2})^x (LT^{-1})^y T^z$

$\Rightarrow MT^{-2} = M^x L^{(2x+y)} T^{(-2x-y+z)}$

$\Rightarrow 1 = x$, $0 = 2x+y$, $-2 = -2x-y+z$

$\Rightarrow 0 = 2x+y$

$\Rightarrow 0 = 2+y$

$\Rightarrow -2 = y$

$-\cancel{2} = -\cancel{2} + 2 + z$

$-2 = z$

$\Rightarrow S = E^1 v^{-2} T^{-2}$

$\Rightarrow S = E v^{-2} T^{-2}$

$\Rightarrow$ Limitations of Dimensional Analysis

* Can't find value of proportionality constant
* Can't be applied in expression involving trigonometric ratio (sinθ, cosθ, etc) & exponential forms.
* Can't be applied in expressions involving proportionality constant which has dimensions.
* Can't be applied in expression involving two or more terms seperated by '+'.

~~DPP-I~~ DPP's

1) $Q = K \dfrac{A(\theta_2 - \theta_1)t}{d}$

$\Rightarrow K = \dfrac{Qd}{A(\theta_2 - \theta_1)t}$

$\Rightarrow K = \dfrac{ML^2T^{-2} \times L}{L^2 \theta \times T}$

$\Rightarrow K = \dfrac{M\cancel{L^3}T^{-2}}{\cancel{L^2}\theta T}$

$\Rightarrow K = \cancel{MT^{-3}\theta^{-1}}\ MLT^{-3}\theta^{-1}$

Q = Heat Energy
A = Area
$\theta_2 - \theta_1$ = Temp diff.
t = Time
d = distance

$\cancel{1)}\ S \cancel{=}\ \cancel{\dagger}\ \dfrac{\rho g r h}{2}$

$\Rightarrow S = \dfrac{\frac{F}{A} \times \frac{m}{s^2} \times r \times h}{2}$

$\Rightarrow S = \dfrac{\frac{MLT^{-2}}{L^2} \times \frac{\cancel{ML}}{T^2} \times L \times L}{2}$

$\Rightarrow S = \dfrac{ML^{-1}T^{-2} \times LT^{-2} \times L^2}{2}$

$\Rightarrow S = \dfrac{ML^2T^{-4}}{2}$

S = Surface Tension
ρ = density
g = Acceleration due to gravity
r = radius
h = height

1) $S = \dfrac{\rho g r h}{2}$

$\cancel{\Rightarrow S = \frac{M}{V} \times \frac{m}{S^2} \times r \times h}$

$\cancel{\Rightarrow S = MLT^{-1} \; ML^{-1}T \times LT^{-2} \times L \times L}$

$\cancel{\Rightarrow S = ML^2T^{-1}}$

$\Rightarrow S = \frac{M}{V} \times \frac{m}{S^2} \times r \times h$

$\Rightarrow S = ML^{-3} \times LT^{-2} \times L \times L$

$\Rightarrow S = MT^{-2}$

iii) Find the dimensions of the Potential and then the Capacitance.

$\Rightarrow E = Vit$

$\Rightarrow V = \frac{E}{IT}$

E = Energy
I = Current
T = Time

$\Rightarrow V = \frac{ML^2T^{-2}}{IT}$

$\Rightarrow V = ML^2T^{-3}I^{-1}$

$\Rightarrow Q = CV$

$\Rightarrow C = \frac{Q}{V}$

Q = Charge
V = Potential

$\Rightarrow C = \frac{IT}{ML^2T^{-3}I^{-1}}$

$\Rightarrow C = M^{-1}L^{-2}\cancel{T^{2}}\,T^4 I^2$

14) Check the correctness of the equation, $s = ut + \frac{at^2}{2}$,

where u is the initial velocity, v is the final velocity, a is the acceleration, s is the displacement and t is the time in ~~which~~ in which the change occurs.

$$\Rightarrow s = ut + \frac{at^2}{2}$$

$$\Rightarrow L = \cancel{v}\, LT^{-1} \times T + \frac{LT^{-2} \times T^2}{\cancel{2}}$$ Verified

$$\Rightarrow L = L + L$$

~~$\Rightarrow L = 2L$~~ ~~$\Rightarrow$~~

~~$\Rightarrow L = L$~~

v) $F = \frac{mv^2}{r}$

F = Force
m = Mass
v = Velocity
r = radius

~~$\Rightarrow F$~~

$$\Rightarrow MLT^{-2} = \cancel{MLT^{-2}}\ \frac{M \times \left(\frac{L}{T}\right)^2}{L}$$

$$\Rightarrow MLT^{-2} = \frac{M \times \frac{L^2}{T^2}}{L}$$

$$\Rightarrow MLT^{-2} = \frac{ML^2T^{-2}}{L}$$

$$\Rightarrow MLT^{-2} = \cancel{MLT^{-1}}\ MLT^{-2}$$ Verified

vi) Check the correctness of the equation, ~~check~~ when the frequency of vibration 'n' of a string of length 'l' having mass per unit length 'm' kept under tension 'F' is given by:

$$n = \frac{1}{2l}\sqrt{\frac{F}{m}}$$

n = Frequency
l = Length
F = Tension Force
m = $\frac{\text{Mass}}{\text{Length}}$

$$\Rightarrow T^{-1} = \frac{1}{L}\sqrt{\frac{\cancel{M}LT^{-2}}{\cancel{M}L^{-1}}}$$

$$\Rightarrow T^{-1} = \frac{1}{L}\sqrt{\frac{L^2}{T^2}}$$

$$\Rightarrow T^{-1} = \frac{1}{\cancel{L}} \times \frac{\cancel{L}}{T}$$

$$\Rightarrow T^{-1} = \frac{1}{T}.$$

$$\Rightarrow T^{-1} = T^{-1}$$ Verified

vii) Check the correctness of the equation, when the rate of ~~flow~~ flow of a liquid having a coefficient of viscosity 'n' through a capillary tube of length 'l' and radius 'a' under true pressure head 'p' is given by:

$$\frac{dV}{dt} = \frac{\pi p a^4}{8 l n}$$

V = Volume
t = Time
p = Pressure
a = radius
l = Length
n = Coefficient of visco

$$\Rightarrow \frac{L^3}{T} = \frac{\frac{F}{A} \times L^4}{L \times ML^{-1}T^{-1}}$$

$$\Rightarrow \frac{L^3}{T} = \frac{\cancel{ML^2T}\ MLT^{-2}}{\cancel{L^2}} \times \cancel{L^4}\ \cancel{L^2}\ L^2$$

$$\Rightarrow \frac{L^3}{T} \Rightarrow \frac{\cancel{M}L^3T^{-2}}{\cancel{M}T^{-1}}$$

$$\Rightarrow \frac{L^3}{T} \Rightarrow L^3T^{-1}$$

$$\Rightarrow \frac{L^3}{T} = \frac{L^3}{T} \quad \text{Verified}$$

Q. VIII) Check the correctness of the equation, when the periodic time 'T' of Vibration of the magnet of the moment of Inertia 'I', magnetic moment, 'M' vibrating in magnetic induction 'B' is given by:

$$T = 2\pi\sqrt{\frac{I}{MB}}$$

I = Moment of Inertia
M = Magnetic ~~Induction~~ moment
B = Magnetic Induction

$$\Rightarrow T = \sqrt{\frac{\cancel{ML^2}}{\cancel{AL^2} \times \cancel{M}T^{-2}\cancel{A^{-1}}}}$$

$$\Rightarrow T = \sqrt{\frac{1}{T^{-2}}}$$

$$\Rightarrow T = \sqrt{T^2}$$

$$\Rightarrow T = T \quad \text{Verified}$$

$I = mr^2 = ML^2$

$M = IA = AL^2$

$B = \frac{F}{IL} = \frac{M\cancel{L}T^{-2}}{A \times \cancel{L}} = MT^{-2}A^{-1}$

ix) Check the correctness of the equation, when the terminal velocity 'v' of a small sphere of radius 'a' and density 'ρ' falling through a liquid of density 'σ' and coefficient of ~~viscocity~~ viscosity 'η' is given by?

$$v = \cancel{2ga^2(\sigma - \rho)} \quad \frac{2ga^2(\sigma-\rho)}{9\eta}$$

v = Terminal velocity
g = acc. due to gravity
a = radius
ρ, σ = density

$$v = \frac{\cancel{L}T^{-2} \times \cancel{L^2} \times \frac{M}{\cancel{L^3}}}{ML^{-1}T^{-1}}$$

$$v = \cancel{\frac{T^{-2} \times M}{ML^{-1}}} \quad \frac{T^{-2} \times \cancel{M}}{\cancel{M}L^{-1}T^{-1}}$$

$$LT^{-1} = LT^{-1}$$ Verified

x) A force 'F' is given by $F = at + bt^2$, where 't' is the time. Find the dimensional formula of 'a' and 'b'.

$$F = at + bt^2$$

F = Force = MLT^{-2}
t = Time = T

$\Rightarrow MLT^{-2} = aT + bT^2$
$\Rightarrow MLT^{-2} = aT$
$\Rightarrow \frac{MLT^{-2}}{T} = a$
$\Rightarrow MLT^{-3} = a$

$\Rightarrow MLT^{-2} = bT^2$
$\Rightarrow \frac{MLT^{-2}}{T^2} = b$
$\Rightarrow MLT^{-4} = b$

xi) In an equation $\left(P + \frac{a}{V^2}\right)(V - b) = RT$, where P is the pressure, V is the volume, T is the temperature and 'a', 'b' and 'R' are constants. What is the dimensional formula of $\frac{a}{b}$?

$$\left(P + \frac{a}{V^2}\right)(V - b) = RT$$

$\Rightarrow P = \frac{a}{V^2}$	$V = b$	$\frac{a}{b} = \frac{ML^{\cancel{5}}T^{-2}}{\cancel{L^3}}$
$\Rightarrow \frac{F}{A} = \frac{a}{(L^3)^2}$	$L^3 = b$	$= ML^2T^{-2}$
$\Rightarrow \frac{MLT^{-2}}{L^2} = \frac{a}{(L^3)^2}$		
$\Rightarrow \frac{ML^{\cancel{6}}T^{-2}}{\cancel{L^2}} = a$		
~~$\Rightarrow ML^2T^{-2} = a$~~		
$\Rightarrow ML^5T^{-2} = a$		

xi) A force is given in terms of distance x and time t by F
$F = A \sin Ct = B \cos Dx$. Then what are the dimensions of $\frac{A}{B}$ and $\frac{C}{D}$?

$\Rightarrow F = A \sin Ct$

~~$\Rightarrow F$~~ $\Rightarrow Ct = M^0L^0T^0$

$\Rightarrow C = \frac{M^0L^0T^0}{T}$

$\Rightarrow C = T^{-1}$

$\Rightarrow F = B\cos Dx$

$\Rightarrow Dx = M^0L^0T^0$

$\Rightarrow D = \frac{M^0L^0T^0}{L}$

$\Rightarrow D = L^{-1}$

$\Rightarrow \cancel{A}\ F = A \sin Ct$

$\Rightarrow F = A$

$\Rightarrow \cancel{ML^2} = MLT^{-2} = A$

$\Rightarrow F = B \cos Dx$

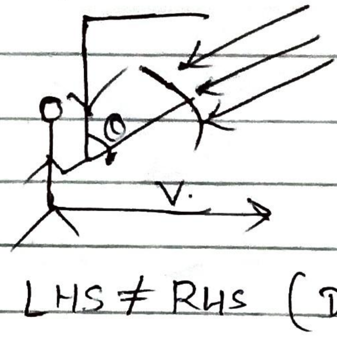
$\Rightarrow F = B$

$\Rightarrow MLT^{-2} = B$

$\Rightarrow \frac{A}{B} = \frac{ML\cancel{T^{-2}}}{\cancel{MLT^{-2}}} \quad M^0L^0T^0$

$\Rightarrow \frac{C}{D} = \frac{\cancel{T^{-1}}}{\cancel{D^{-1}}} \quad \frac{T^{-1}}{L^{-1}} = LT^{-1}$

xii) A man walking in rain with speed v must slant his umbrella forward making an angle θ ~~to~~ with ~~vertical~~ the vertical. A student derives the following relation between θ and v: $\tan\theta = v$ and checks that the relation ~~between~~ has a correct limit: as $v \to 0$, $\theta \to 0$, as expected, ~~(we are assuming)~~ Do you think this relation can be correct? If not, guess the correct relation.



$\underbrace{\tan\theta}_{M^0L^0T^0} = \underbrace{V}_{LT^{-1}}$

LHS $\neq$ RHS (Dimensionally)

No, it is not correct. The correct relation can be:

$\tan\theta = \frac{V_R}{V_R}$

XIII) The air bubble formed by explosion inside water perform ~~oscilla~~ oscillations with time period T which depends on pressure (p), density (ρ) and on energy due to explosion (E), Establish relation between T, p, E, and ρ.

$T \propto K p^{x} E^{y} \rho^{z}$

$T = K p^{x} E^{y} \rho^{z}$

$p = \frac{F}{A} = \frac{MLT^{-2}}{L^2}$

$p = ML^{-1}T^{-2}$

$E = ML^{2}T^{-2}$

$\rho = \frac{M}{V} = \frac{M}{L^3}$

$\Rightarrow ML^{-3}$

$\Rightarrow T = (ML^{-1}T^{-2})^{x} (ML^{2}T^{-2})^{y} (ML^{-3})^{z}$

$\Rightarrow T = M^{(x+y+z)} L^{(-x+2y-3z)} T^{(-2x-2y)}$

$\Rightarrow$

$\Rightarrow 0 = x+y+z$

$\Rightarrow x = -y-z$

$\Rightarrow x = -4z-z$

$\Rightarrow x = -5z$

$\Rightarrow x = -5 \times \frac{1}{2}$

$\Rightarrow x = -\frac{5}{2}$

$\Rightarrow -x+2y-3z = 0$

$\Rightarrow -y-z+2y-3z = 0$

$\Rightarrow y-4z = 0$

$\Rightarrow y = 4z$

$\Rightarrow y = \cancel{4} \times \frac{1}{\cancel{2}}$

$\Rightarrow y = 2$

$\Rightarrow -2x-2y = 1$

$\Rightarrow 10z - 8z = 1$

$\Rightarrow 2z = 1$

$\Rightarrow z = \frac{1}{2}$

$\Rightarrow \cancel{T = K p^{-\frac{5}{2}} E^{2} \rho^{\frac{1}{2}}}$

$\Rightarrow x+y+z = 0$

$\Rightarrow -\frac{1}{2}+z = 0$

$\Rightarrow z = \frac{1}{2}$

$\Rightarrow -x+2y-3z = 0$

$\Rightarrow -x+2y-\frac{3}{2} = 0$

$\Rightarrow -x+2y = \frac{3}{2}$

$\Rightarrow -2x-2y = 1$

$\Rightarrow -2(x+y) = 1$

$\Rightarrow x+y = -\frac{1}{2}$

$\Rightarrow \cancel{x+y+z=0}$

$\Rightarrow \cancel{x=-y-z}$

$\Rightarrow \cancel{x+2y+3z}$

$\Rightarrow x+y+z=0$

$\Rightarrow -x+2y-3z=0$

$3y-2z=0$

$\Rightarrow 3y=2z$

$\Rightarrow y=\frac{2z}{3}$

$\Rightarrow \cancel{x+y+} x+\frac{2z}{3}+z=0$

$\Rightarrow x+\frac{2z+3z}{3}=0$

$\Rightarrow x+\frac{5z}{3}=0$

$\Rightarrow \frac{3x+5z}{3}=0$

$\Rightarrow 3x=-5z$

$\Rightarrow x=\frac{-5z}{3}$

$\Rightarrow -2x-2y=1$

$\Rightarrow -2\left(\frac{-5z}{3}\right)-2\times\left(\frac{2z}{3}\right)=1$

$\Rightarrow \frac{10z}{3}-\frac{4z}{3}=1$

$\Rightarrow \frac{6z}{3}=1$

$\Rightarrow 2z=1$

$\Rightarrow z=\frac{1}{2}$

$\Rightarrow x=\frac{-5}{3}\times\frac{1}{2}$

$=\frac{-5}{6}$

$\Rightarrow y=\frac{2}{3}\times\frac{1}{2}$

$\Rightarrow y=\frac{1}{3}$

$\Rightarrow T=Kp^{-\frac{5}{6}}E^{\frac{1}{3}}\rho^{\frac{1}{2}}$

xiv) The velocity v of a particle depends upon time 't' according to the equation.

$v=\sqrt{ab}+bt+\frac{c}{d+t}$

$\Rightarrow v = bt$

$\Rightarrow LT^{-1} = bT$

$\Rightarrow \frac{LT^{-1}}{T} = b$

$\Rightarrow LT^{-2} = b$

$\Rightarrow d = T$

$\Rightarrow v = \frac{c}{T}$

$\Rightarrow LT^{-1} = \frac{c}{T}$

$\Rightarrow L = c$

$\Rightarrow LT^{-1} = \sqrt{aLT^{-2}}$

$\Rightarrow LT^{-1} = \sqrt{\frac{aL}{T^2}}$

$\Rightarrow L = \sqrt{aL}$

$\Rightarrow \sqrt{aL} = L$

$\Rightarrow aL = (L)^2$

$\Rightarrow aL = L^2$

$\Rightarrow a = \frac{L^2}{\cancel{L}}$

$\Rightarrow a = L$

$\boxed{\text{S.I. units}}$

b

$\Rightarrow LT^{-2} = ms^{-2}$

$= \frac{m}{s^2}$

c

$\Rightarrow L = m$

a

~~$\Rightarrow LT^2 = ms^2$~~

$\Rightarrow L = m$

xv) An artificial satellite is revolving around a planet of mass M and Radius R, in a circular orbit of radius r, Determine its time period T.

$T = \frac{k}{R}\sqrt{\frac{r^3}{g}}$ where k is a ~~d~~ dimensionless constant

$\Rightarrow T = \frac{1}{L}\sqrt{\frac{L^3}{LT^{-2}}}$

$\Rightarrow T = \frac{1}{L}\sqrt{\frac{L^2}{T^{-2}}}$

$\Rightarrow T = \frac{1}{L}\sqrt{L^2T^2}$

$\Rightarrow T = \frac{1}{\cancel{L}} \times \cancel{L}T$

$\Rightarrow T = T$

$\Rightarrow T = \dfrac{K}{R}\sqrt{\dfrac{r^3}{g}}$ where K is dimensionless constant

$\Rightarrow T \propto K \cancel{M^a R^d} M^a R^b r^c g^d$

$\Rightarrow T = K M^a R^b r^c g^d$

$\Rightarrow T = M^a L^b L^c (LT^{-2})^d$

$\Rightarrow T = M^a L^{(b+c+d)} T^{-2d}$

$\Rightarrow a = 0$, $\Rightarrow b+c+d = 0$, $1 = -2d$

$\cancel{\Rightarrow b = -c-d}$ $\qquad -\dfrac{1}{2} = d$

$\Rightarrow b+c - \dfrac{1}{2} = 0$

$\Rightarrow b+c = \dfrac{1}{2}$

$\Rightarrow$ Kepler's law states that,

$T^2 \propto r^3$

$\Rightarrow T = r^{\frac{3}{2}}$

So, $c = \dfrac{3}{2}$

$\Rightarrow b + \dfrac{3}{2} = \dfrac{1}{2}$

$\Rightarrow b = \dfrac{1}{2} - \dfrac{3}{2}$

$\Rightarrow b = \dfrac{-2}{2}$

$\Rightarrow b = -1$

$\Rightarrow T = K M^0 R^{-1} r^{\frac{3}{2}} g^{-\frac{1}{2}}$

$\Rightarrow T = \dfrac{K}{R}\sqrt{\dfrac{r^3}{g}}$

xvi) The speed of sound in a ~~gas~~ gas v might plausibly depend on the pressure p, density ρ, and the volume V on the gas. Use the dimensional analysis to determine the exponents x, y and z in the formula.

$v = C p^x \rho^y V^z$

C is dimensionless ~~k~~ constant

$v = \frac{m}{s} = LT^{-1}$

$\Rightarrow LT^{-1} = \left(\frac{F}{A}\right)^x \left(\frac{M}{V}\right)^y V$

$\Rightarrow LT^{-1} = (ML^{-1}T^{-2})^x (ML^{-3})^y L^{3z}$

$\Rightarrow LT^{-1} = M^{(x+y)} L^{(-x-3y+3z)} T^{-2x}$

$\Rightarrow -1 = -2x$

$\Rightarrow \frac{-1}{-2} = x$

$\Rightarrow x = \frac{1}{2}$

$\Rightarrow 0 = x + y$

$\Rightarrow -y = x$

$\Rightarrow -y = \frac{1}{2}$

$\Rightarrow y = -\frac{1}{2}$

$\Rightarrow -x - 3y + 3z = 1$

$\Rightarrow -\frac{1}{2} - 3\times\left(-\frac{1}{2}\right) + 3z = 1$

$\Rightarrow -\frac{1}{2} + \frac{3}{2} + 3z = 1$

$\Rightarrow \frac{2}{2} + 3z = 1$

$\Rightarrow 1 + 3z = 1$

$\Rightarrow 3z = 1 - 1$

$\Rightarrow 3z = 0$

$\Rightarrow z = 0$

$\Rightarrow v = C p^{\frac{1}{2}} \rho^{-\frac{1}{2}} V^0$

$\Rightarrow v = C p^{\frac{1}{2}} \rho^{-\frac{1}{2}}$

xvii) What variable could not influence the velocity if it is proposed that the velocity depends ~~upon~~ on a diameter?

a) length $\rightarrow L$
b) gravity $\rightarrow LT^{-2}$
c) rotational speed $(\omega) \Rightarrow \frac{v}{r} \rightarrow \frac{\cancel{L}T^{-1}}{\cancel{L}} \rightarrow T^{-1}$

d) Coefficient of viscosity $\rightarrow ML^{-1}T^{-1}$

$$v \propto l^a g^b \omega^c n^d$$

$$\Rightarrow v = K l^a g^b \omega^c n^d$$
$$\Rightarrow v = L^a (LT^{-1})^b (T^{-1})^c (ML^{-1}T^{-1})^d$$
$$\Rightarrow LT^{-1} = L^{(a+b-d)} M^d T^{(-b-c-d)}$$

$$\Rightarrow a+b-d = 1, \quad -b-c-d = -1, \quad \boxed{d=0}$$

That means n (Coefficient of visocity does not influence the velocity if it depends on a diameter.

xviii) If v stands for velocity of sound, E is elasticity and d is the density, then find x in the equation:

$$v = \left(\frac{d}{E}\right)^x$$

Elasticity $= \frac{\text{Stress}}{\text{Strain}}$

Stress $= \frac{F}{A}$

Strain $= \frac{\Delta l}{l}$

Elasticity $= ML^{-1}T^{-2}$

$$\Rightarrow LT^{-1} = \left(\frac{\cancel{M}L^{-3}}{\cancel{M}L^{-1}T^{-2}}\right)^x$$

$\Rightarrow LT^{-1} = (L^{-2}T^{2})x \longrightarrow LT^{-1} = (L^{-2}T^{2})^{-\frac{1}{2}}$

$\Rightarrow \frac{LT^{-1}}{L^{-2}T^{2}} = x \qquad \Rightarrow LT^{-1} = LT^{-1}$

$\Rightarrow L^{3}T^{-3} = x$

xix) Force of viscosity F acting on a spherical body moving through a fluid depends upon its ~~viscosity (v),~~ velocity (v), radius (r) and coefficient of ~~viscosity~~ viscosity 'n' of the fluid. Using method of dimensions, obtain an expression for 'F'.

$\Rightarrow F \propto K v^{x} r^{y} n^{z}$

$\Rightarrow F = K v^{x} r^{y} n^{z}$

$n = ML^{-1}T^{-1}$

$v = LT^{-1}$

$r = L$

$\Rightarrow MLT^{-2} = (LT^{-1})^{x} L^{y} (ML^{-1}T^{-1})^{z}$

$\Rightarrow MLT^{-2} = \cancel{L^{(x+y)}} M^{z} L^{(x+y-z)} T^{(-x-z)}$

$\Rightarrow z = 1$

$\Rightarrow x+y-z = 1$

$\Rightarrow x+y-1 = 1$

$\Rightarrow x+y = 2$

$\Rightarrow 1+y = 2$

$\Rightarrow y = 2-1$

$\Rightarrow y = 1$

$\Rightarrow -x-z = -2$

$\Rightarrow -x-1 = -2$

$\Rightarrow -x = -2+1$

$\Rightarrow x = +1$

$\Rightarrow x = 1$

$\Rightarrow F = K v^{1} r^{1} n^{1}$

$\Rightarrow F = K v r n$

xx) A spherical ball of mass 'm' and radius 'r' ~~is~~ is allowed to fall in a medium of viscosity n. The time in which the velocity of the body increases from zero to 0.63 times the terminal velocity is called time constant (t). Dimensionally t can be represented by:

$\Rightarrow t \propto K m^x r^y \eta^z$

$\Rightarrow t = K m^x r^y \eta^z$

$\Rightarrow T = M^x L^y (ML^{-1}T^{-1})^z$

$\Rightarrow T = M^{(x+z)} L^{(y-z)} T^{-z}$

$\Rightarrow x+z=0$	$\Rightarrow y-z=0$	$\Rightarrow 1=-z$
$\Rightarrow x=-z$	$\Rightarrow y=z$	$\Rightarrow -1=z$
$\Rightarrow x=-(-1)$	$\Rightarrow y=-1$	
$\Rightarrow x=1$		

$\Rightarrow t = K m^1 r^{-1} \eta^{-1}$

$\Rightarrow t = K m r^{-1} \eta^{-1}$

$\Rightarrow t = K \dfrac{m}{r\eta} \longrightarrow K = \dfrac{1}{6\pi} \longrightarrow t = \dfrac{m}{6\pi\eta r}$

xxi) A length scale 'l' depends on the permittivity ε of a dielectric material, Boltzman's constant K_B, the absolute temperature T, the number ~~of~~ per unit volume n of certain charged ~~pa~~ particles, and the charge q ~~is~~ carried by each of the particles. Which of the following expression(s) for l are dimensionally correct?

~~By law G~~

$\Rightarrow PV = K_B T$

$\Rightarrow ML^{-1}T^{-2} \times L^3$

$\Rightarrow ML^2T^{-2}$

$\Rightarrow$ By coulomb's law:

$\Rightarrow F = \frac{1}{4\pi\varepsilon_0} \times \frac{q^2}{r^2}$

$\Rightarrow \frac{q^2}{\varepsilon_0} = 4\pi r^2 F$

$\Rightarrow r^2 F = L^2 \times MLT^{-2}$

$= ML^3T^{-2}$

a) $l = \sqrt{\frac{nq^2}{\varepsilon_0 K_B T}} \rightarrow l = \sqrt{\frac{L^{-3} \times \cancel{M}\cancel{L^3}\cancel{T^{-2}}}{\cancel{M}\cancel{L^2}\cancel{T^{-2}}}} \Rightarrow \sqrt{L^{-2}}$

$\Rightarrow \frac{1}{L}$ Not equal

b) $l = \sqrt{\frac{\varepsilon_0 K_B T}{nq^2}}$ $\rightarrow L = \sqrt{\frac{1}{L^{-2}}} \Rightarrow L$

Equal

c) $l = \sqrt{\frac{q^2}{\varepsilon_0 n^{\frac{2}{3}} K_B T}}$

d) $l = \sqrt{\frac{q^2}{\varepsilon_0 n^{\frac{1}{3}} K_B T}}$

$\rightarrow L = \sqrt{\frac{\cancel{M}\cancel{L^3}\cancel{T^{-2}}}{L^{-2} \times \cancel{M}\cancel{L^2}\cancel{T^{-2}}}} \Rightarrow \sqrt{\frac{L}{L^{-2}}} \Rightarrow \sqrt{L^3}$

Not equal

$\rightarrow L = \sqrt{\frac{\cancel{M}\cancel{L^3}\cancel{T^{-2}}}{L^{-1} \times \cancel{M}\cancel{L^2}\cancel{T^{-2}}}} \Rightarrow \sqrt{\frac{L}{L^{-1}}} \Rightarrow L$

Equal

(b) and (d) are correct

xxii) Plank's constant h, speed of light c and gravitational constant G are used to form a unit of length L and a unit of mass M. Then, the correct options are:-

$\Rightarrow$ ~~$L \propto h$~~ $L \propto k h^x c^y G^z$ and $M \propto k h^x c^y G^z$

$\Rightarrow L = h^x c^y G^z$

$\Rightarrow L = (ML^2T^{-1})^x (LT^{-1})^y (M^{-1}L^3T^{-2})^z$

$\Rightarrow L = M^{(x-z)} L^{(2x+y+3z)} T^{(-x-y-2z)}$

$\Rightarrow x - z = 0$

$\Rightarrow x = z$

$\Rightarrow 2x+y+3z = 1$

$\Rightarrow 2x+y+3x = 1$

$\Rightarrow 5x+y = 1$

$\Rightarrow -x-y-2z = 0$

$\Rightarrow -x-y-2x = 0$

$\Rightarrow -3x-y = 0$

$\Rightarrow$ ~~$2x+y+3z=1$~~

~~$-x-y-2z=0$~~

~~$x-z=1$~~

~~$x=z+1$~~

$\Rightarrow 5x+y = 1$

$-3x-y = 0$

$2x = 1$

$x = \frac{1}{2}$

$z = \frac{1}{2}$

$\Rightarrow -3 \times \frac{1}{2} - y = 0$

$\Rightarrow -\frac{3}{2} - y = 0$

$\Rightarrow -\frac{3}{2} = y$

$\Rightarrow h = \frac{E}{f}$

$= \frac{ML^2T^{-2}}{T^{-1}}$

$= ML^2T^{-1}$

$\Rightarrow c = \frac{m}{s} = LT^{-1}$

$\Rightarrow F = \frac{Gm_1m_2}{r^2}$

$\Rightarrow G = \frac{Fr^2}{m_1m_2}$

$= \frac{\cancel{M}LT^{-2} \times L^2}{\cancel{M} \times M}$

$= \frac{L^3T^{-2}}{M}$

$= M^{-1}L^3T^{-2}$

$\Rightarrow L = k h^{\frac{1}{2}} c^{-\frac{3}{2}} G^{\frac{1}{2}}$

$\Rightarrow L \propto \sqrt{h}$ and $L \propto \sqrt{G}$ (c) and (d) correct

Now,

$$M = M^{(x-z)} L^{(2x+y+3z)} T^{(-x-y-2z)}$$

$\Rightarrow x - z = 1$

$\Rightarrow x = 1 + z$

$2x + y + 3z = 0$

$\Rightarrow 2 + 2z + y + 3z = 0$

$\Rightarrow 2 + 5z + y = 0$

$\Rightarrow \cancel{5z + y +} \; \cancel{y+5}$

$\Rightarrow y + 5z + 2 = 0$

$\Rightarrow -x - y - 2z = 0$

$\Rightarrow -1 - z - y - 2z = 0$

$\Rightarrow \cancel{-3} - 1 - 3z - y = 0$

$\Rightarrow -y - 3z - 1 = 0$

$$\Rightarrow \begin{array}{r} y + 5z + 2 = 0 \\ -y - 3z - 1 = 0 \\ \hline 2z + 1 = 0 \end{array}$$

$\Rightarrow 2z = -1$

$\Rightarrow z = -\frac{1}{2}$

$x = 1 + \left(-\frac{1}{2}\right)$

$\Rightarrow x = \frac{2-1}{2}$

$\Rightarrow x = \frac{1}{2}$

$-y - 3 \times \left(-\frac{1}{2}\right) - 1 = 0$

$\Rightarrow -y + \frac{3}{2} - 1 = 0$

$\Rightarrow -y = 1 - \frac{3}{2}$

$\Rightarrow -y = \frac{2-3}{2}$

$\Rightarrow +y = +\frac{1}{2}$

$\Rightarrow M = k h^{\frac{1}{2}} c^{\frac{1}{2}} G^{\frac{-1}{2}}$

$\Rightarrow M \propto \sqrt{h}$ and $M \propto \sqrt{c}$ (a) is also correct

➪ Significant Figures

Q. 2.0 Km = ______?

a) 2000m

b) $2.0 \times 10^3 m$

c) $0.2 \times 10^4 m$ (Correct)

d) All of the above

Significant figures tells us the numbers of digits in a measured value, we are confident of.

More Significant figure = More accurate measurement

➪ Rules for calculating significant figures

1. All non-zero digits are significant.

    1234 -> 4 S.F.

    243 -> 3 S.F.

2. Trapped zero:- Zeroes trapped between two non-zero digits are significant.

    1.004 -> 4 S.F.

3. Initial zeroes/Leading zeroes are never significant.

    0.001 -> 1 S.F.

    0.0043 -> 2 S.F.

    0.304 - > 3 S.F.

4. Ending zeroes/Trading zeroes are significant if they appear after decimal.

    2.00 -> 3 S.F.

    4.0000 - > 5 S.F.

    0.003020 -> 4 S.F.

    200 - > 1 S.F.

    1400 -> 2 S.F.

Q. Find number of significant digits in:

    => 20.00 -> 4 S.F.

    => 1310.000 -> 7 S.F.

    => 131000 -> 3 S.F.

5. Order of magnitude is never significant.

    $2.1 \times 10^3 \rightarrow 2\ S.F.$

$2.010 \times 10^6 \rightarrow 4\, S.F.$

$200 = 2 \times 10^2$

$1400 = 1.4 \times 10^3$

6. Pure numbers or constants have infinite significant figures.

Q. Find Significant Figures of:

=> 212 -> 3 S.F.

=> 2.120 -> 4 S.F.

=> 2.0042 -> 5 S.F.

=> 310.020 -> 6 S.F.

=> 310.00 -> 5 S.F.

=>3100 -> 2 S.F.

=> 4.20 -> 3 S.F.

=> 0.004 -> 1 S.F.

7. While changing units, Number of S.F. remains same.

=> 1.5m = 150 cm

=> 2.0km = 2.0×10^3

⇨ Round off to two Significant figures.

- 3.72 = 3.7
- 3.76 = 3.8
- 3.75 = 3.8 (If digit before '5' is odd, then increase.
- 3.65 = 3.6 If not, then decrease.)
- 4.352 = 4.4
- 4.350 = 4.4
- 4.450 = 4.4

⇨ Calculations in Significant figures.
The result of addition, subtraction, multiplication & division of measured values cannot be more accurate than the least accurate measurement.

a+b+c = R

Addition & Subtraction: The result of addition & subtraction must have the same number of decimal places as present in the value with least decimal places.

Q. Find the results:

- 2.24 + 0.3 = 2.54 = 2.5
- 0.331 + 1.12 + 0.03 = 1.481 = 1.48
- 1.4Kg + 1.32g = 1400 + 1.32 = 1401.32 = 1401g = 1.4Kg
- $5.8 \times 10^4 - 1.5 \times 10^3$ = 3.3

Multiplication & Division: The result is rounded off to same number of S.F. as present in the value with least S.F.

Q. Find the results:

- $2.1 \times 2.1 = 4.41 = 4.4$
- $142.06 \times 0.23 = 32.6728 = 33$
- $\frac{0.90}{4.25} = 0.21126 = 0.21$

Error Analysis

However hard we try, there is always some difference between the measured value and true value (Actual value) of a quantity, which gives birth to error.

* Absolute error :- The Magnitude of ~~differ~~ difference between the true value (Actual value) & the measured value of a quantity.

Let a quantity be measured 'n' times.
The measurements are $x_1, x_2, x_3 \ldots\ldots x_n$

True / Mean Value x_m or $x = \dfrac{x_1 + x_2 + x_3 \ldots x_n}{n}$

Absolute error : $\Delta x_1 = x - x_1$
$\Delta x_2 = x - x_2$
$\Delta x_3 = x - x_3$
$\Delta x_n = x - x_n$

[Can be positive or negative]

* Mean Absolute error :

$$\Delta x = \frac{|\Delta x_1| + |\Delta x_2| + |\Delta x_3| + \ldots\ldots + |\Delta x_n|}{n}$$

$| \, |$ ⇒ Mod (Converts any value to positive)

⇒ All physical quantities are written as : $x \pm \Delta x$

* Relative Error / Fractional Error : $\dfrac{\Delta x}{x} = \dfrac{\text{Mean absol. error}}{\text{Mean value}}$

* Percentage error : Fractional error $\times 100$

$\Rightarrow \frac{\Delta x}{x} \times 100$

Q.1. The difference measurements of length of a rod are 4.2, 4.5, 4.4, 4.3, 4.6 (in c.m.)

Calculate :

a) Mean value $= \frac{4.2+4.5+4.4+4.3+4.6}{5}$

$= \cancel{\frac{26.4}{5}} \quad \frac{\cancel{22}}{\cancel{5}} \; 4.4$

b) Absolute error $= 4.4 - 4.2 = 0.2$

$4.4 - 4.5 = -0.1$

$4.4 - 4.4 = 0$

$4.4 - 4.3 = 0.1$

$4.4 - 4.6 = -0.2$

c) Mean absolute error $= \frac{0.2+0.1+0+0.1+0.2}{5}$

$= \frac{\cancel{0}.6}{\cancel{5}} \quad 0.12 = 0.1$

) Fractional error $= \cancel{\frac{0.12}{4.4}} = \cancel{\frac{0.12 \times 10}{100 \times 44}} = \cancel{\frac{3}{110}}$

$\frac{0.1}{4.4} = \frac{1}{44}$

e) % error $= \frac{1}{\cancel{44}} \times \cancel{100}$ $= \frac{25}{11}$ 2.272%

* Propogation of error

(I) Addition & Substraction

Addition :

$x = a + b$

$\Delta x = \Delta a + \Delta b$

Result $\rightarrow x \pm \Delta x$

Errors are always added in addition & subtraction.

Subtraction :

$x = a - b$

$\Delta x = \Delta a + \Delta b$

Result $\rightarrow x \pm \Delta x$

Q2. 

$R_1 = 2 \pm 0.2\,\Omega$ $\quad R_2 = 4 \pm 0.4\,\Omega$

Find equivalent resistance :

$\Rightarrow$ Equiv. ~~resr~~ resistance $= R_1 + R_2$

$= \cancel{2 \pm 0.2 \quad 4 \pm 0.4} \; 2 + 4$

$= \cancel{6 \pm 0.6\,\Omega} \; 6\,\Omega$

Error $= \Delta R_1 + \Delta R_2$

$= 0.2 + 0.4$

$= 0.6\,\Omega$

Result $\rightarrow 6 \pm 0.6\,\Omega$

(II) Multiplication & Division

Multiplication:

$x = ab$

$\Rightarrow \dfrac{\Delta x}{x} = \dfrac{\Delta a}{a} + \dfrac{\Delta b}{b}$

$\Rightarrow \Delta x = \left(\dfrac{\Delta a}{a} + \dfrac{\Delta b}{b}\right) x$

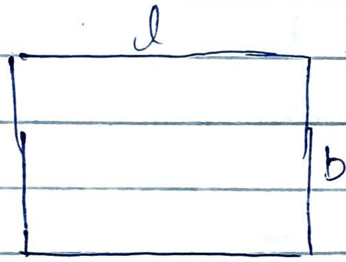
Division:

$x = \dfrac{a}{b}$

$\dfrac{\Delta x}{x} = \dfrac{\Delta a}{a} + \dfrac{\Delta b}{b}$

$\Delta x = \left(\dfrac{\Delta a}{a} + \dfrac{\Delta b}{b}\right) x$

Q.3.



$l = 4 \pm 0.4\,m$

$b = 2 \pm 0.2\,m$

Find the area with proper error limits.

$\Rightarrow$ Area $= lb$

$\cancel{\frac{0.6}{2}} = 4 \times 2 = 8\,m^2$

~~Error = 0.4 × 0.2 = ...~~

Error $= \dfrac{\Delta A}{A} = \dfrac{\Delta l}{l} + \dfrac{\Delta b}{b}$

$= \dfrac{\Delta A}{8} = \dfrac{0.4}{4} + \dfrac{0.2}{2}$

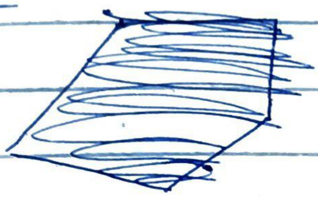
$\Rightarrow \dfrac{\Delta A}{8} = \dfrac{0.4 + 0.4}{4}$

$\Rightarrow \Delta A = \dfrac{0.8}{\cancel{4}} \times \cancel{8}^{2}$

$\Rightarrow \Delta A = \cancel{0.16}\ 1.6\,m^2$

Area = $A \pm \Delta A = (8 \pm 1.6)\, m^2$

~~Q.5.~~



Q.4. There is a cube with mass = 6 ± 0.3 Kg and
volume = 2 ± 0.2 Kg

Find it's density with proper error limits.

$\Rightarrow$ Density = $\frac{M}{V} = \frac{\cancel{6}}{\cancel{2}}\ 3$ Kg/m^3

$\Rightarrow$ Error = $\frac{\Delta D}{D} = \frac{\Delta m}{m} + \frac{\Delta V}{V}$

$\Rightarrow \frac{\Delta D}{3} = \frac{0.3}{6} + \frac{0.2}{2}$

$\Rightarrow \Delta D = \left(\frac{0.3 + 0.6}{\cancel{8}\,6}\right) \times 3$

$\Rightarrow \Delta D = \frac{\cancel{0.9}}{\cancel{6}\ \cancel{2}} \times \cancel{3}\quad 0.45$ Kg/m^3

Density = $D \pm \Delta D = (3 \pm 0.45)$ Kg/m^3

* Quantities raised to some power

$x = \frac{a^p b^q}{c^r}$

Trick:

$$\frac{\Delta x}{x} = p\frac{\Delta a}{a} + q\frac{\Delta b}{b} + r\frac{\Delta c}{c}$$

$\frac{\Delta x}{x} \downarrow$ F.e in x; $\frac{\Delta a}{a} \downarrow$ F.e. in a; $\frac{\Delta b}{b} \downarrow$ F.e. in b; $\frac{\Delta c}{c} \downarrow$ F.e in c

$$\frac{\Delta x}{x}\times 100 = p\left(\frac{\Delta a}{a}\times 100\right) + q\left(\frac{\Delta b}{b}\times 100\right) + r\left(\frac{\Delta c}{c}\times 100\right)$$

$\downarrow$ %.e. in x; $\downarrow$ %.e. in a; $\downarrow$ %.e in b; $\downarrow$ %.e. in c

Q. In an experiment, the percentage of error occured in the measurement of physical quantities A, B, C and D are 1%, 2%, 3% and 4% respectively. Then the maximum percentage of error in the measurement x. Where,

$$x = \frac{A^2 B^{\frac{1}{2}}}{C^{\frac{1}{3}} D^3}$$

$$\frac{\Delta x}{x}\times 100 = 2\left(\frac{\Delta A}{A}\times 100\right) + \frac{1}{2}\left(\frac{\Delta B}{B}\times 100\right) + \frac{1}{3}\left(\frac{\Delta C}{C}\times 100\right) + 3\left(\frac{\Delta D}{D}\times 100\right)$$

$$= 2\times 1 + \frac{1}{\cancel{2}}\times \cancel{2} + \frac{1}{\cancel{3}}\times\cancel{3} + 3\times 4$$

$$= 2 + 1 + 1 + 12$$

$$= 16\%$$

Q. The density of a material in the shape of a cube is determined by measuring three sides of the cube and its mass. If the relative errors in measuring the mass and length are respectively 1.5%

and 1%, the maximum error in determining the density is:-

$\Rightarrow$ ~~Density = Mass / Volume~~

$\frac{\Delta m}{m} \times 100 = 1.5\%$

$\frac{\Delta l}{l} \times 100 = 1\%$

$\Rightarrow d = \frac{m}{v}$

$\Rightarrow d = \frac{m}{l^3}$

$\Rightarrow$ ~~$\frac{\Delta d}{d} \times 100 = \frac{\Delta m}{m}$~~

$\Rightarrow \frac{\Delta d}{d} \times 100 = \frac{\Delta m}{m} \times 100 + 3\left(\frac{\Delta l}{l} \times 100\right)$

$\Rightarrow \quad = 1.5\% + 3 \times 1$

$= 1.5 + 3$

$= 4.5\%$

DPP's

Q.1 The error in the measurement of a sphere is 1%. Find the error in the measurement of Volume.

$\Rightarrow \frac{\Delta r}{r} \times 100 = 1\%$

$\Rightarrow$ Volume of sphere $= \frac{4}{3}\pi r^3$

$\frac{\Delta V}{V} \times 100 = \cancel{\frac{4}{3} \times \frac{22}{7}} \times 3\left(\frac{\Delta r}{r} \times 100\right)$

$= 3 \times 1$

$= 3\%$

Q.2 Given $R_1 = 5.0 \pm 0.2\ \Omega$ and $R_2 = 10.0 \pm 0.1\ \Omega$. What is the total resistance in parallel with possible ~~error~~ % error?

$\Rightarrow$ Eq. resistance in parallel $= \frac{1}{R_1} + \frac{1}{R_2}$

$= \frac{1}{5.0} + \frac{1}{10.0}$

$= \cancel{\frac{10.0\ 2+1}{10.0}} = \cancel{\frac{3}{10.0}}$

$= \cancel{\frac{10}{3}}\ \frac{2+1}{5.0 \times 10.0}$

$= \frac{5.0 \times 10.0}{2+1}$

$=$

$\Rightarrow$ Equivalent resistance $= \frac{1}{R_1} + \frac{1}{R_2}$

$$\Rightarrow \frac{1}{R_e} = \frac{R_2 + R_1}{R_1 R_2}$$

$$\Rightarrow R_e = \frac{R_1 R_2}{R_2 + R_1}$$

$$\Rightarrow \frac{\Delta R}{R} = \frac{\Delta R_1}{R_1} + \frac{\Delta R_2}{R_2} + \frac{\Delta(R_1 + R_2)}{R_1 + R_2}$$

$$\Rightarrow \frac{\Delta R_e}{R} = \frac{0.2}{5} + \frac{0.1}{10} + \frac{0.3}{15}$$

$$\Rightarrow \frac{\Delta R_e}{R} \times 100 = \frac{0.2}{\cancel{5}} \times \cancel{100}^{20} + \frac{0.1}{\cancel{10}} \times \cancel{100}^{10} + \frac{\cancel{0.3}}{\cancel{15}_{\cancel{5}}} \times \cancel{100}^{\cancel{20}\,2}$$

$$= 4 + 1 + 2 = 7\%.$$

$$\Rightarrow R_e = \frac{R_1 R_2}{R_1 + R_2} = \frac{5 \times 10}{5 + 10} = \frac{\cancel{50}}{\cancel{15}} \; \frac{\cancel{10}}{\cancel{3}} \; 3.33\,\Omega$$

$$\Rightarrow 3.3\,\Omega$$

Q.3. The length of one rod is 2.53 ~~cm~~ cm and the other is 1.27 cm. The least count of the measuring instrument is 0.01 cm. If the two rods are put together end to end, find the combined length.

$\Rightarrow$ Combined length = length of one rod + length of other rod

L.C of ~~in~~ measuring instrument = 0.01 cm ; L.C. = Absolute error

$\Rightarrow l = a + b$

$= 2.53 + 1.27$

$= 3.80$ cm

$\Rightarrow \Delta l = 0.01 + 0.01$

$= 0.02$ cm

$\Rightarrow l = (3.80 \pm 0.02)$ cm

Q 4. In a simple pendulum, experiment for determination of acceleration due to gravity (g), time taken for 20 oscillations is measured by using a watch of 1 second least count. The mean value of time taken comes out to be 30 s. The length of pendulum is measured by using a meter ~~scall~~ scale of least count 1 mm and ~~have~~ the value obtained 55.0 cm. The percentage in the determination ~~of~~ of g is close to :-

a) 0.7 %   b) 6.8 %   c) 3.5 %   d) 0.2 %

$\Rightarrow$ Time taken = 30 s

Error = ~~1s~~ 1s

~~Actual time take 30~~ Time taken = (30 ± 1) s

$\Rightarrow$ Length of pendulum = 55.0 cm ~~0.550 m~~

Error = ~~0.01 cm~~ 0.1 cm = ~~0.001 m~~ ~~0.0010 m~~ ~~0.0010 m~~

~~Length~~ = ~~(55.0 ± 0.01) cm~~ ~~(0.550 ± 0.0010) m~~

Length = (55.0 ± 0.1) cm

$\Rightarrow \cancel{g = \frac{\text{displacement}}{\text{time}}} = \cancel{\frac{(0.550 \pm 0.001)m}{(30 \pm 1)s}}$

$\Rightarrow t = 2\pi\sqrt{\frac{l}{g}}$

$$\boxed{\Delta T = \frac{\Delta t}{n}}$$

$\Rightarrow t^2 = 4\pi^2 \frac{l}{g}$

$\Rightarrow g = \frac{4\pi^2 l}{t^2}$

$\Rightarrow \frac{\Delta g}{g} \times 100 = \frac{\Delta l}{l} \times 100 + 2\left(\frac{\Delta t}{t} \times 100\right)$

$= \left(\frac{0.1}{55} \times 100\right) + 2\left(\frac{1}{30} \times 100\right)$

$= \frac{2}{11} + \frac{20}{3} = \frac{6 + 220}{33} = \frac{226}{33}$ 6.84%

$\Rightarrow$ 6.8%

Q.5. The period of oscillation of a simple pendulum is $T = 2\pi\sqrt{\frac{l}{g}}$, Measured value of L is 20.0 cm Known to 1mm accuracy and time for 100 oscillations of the pendulum is found to be 90s using a wrist watch of 1s resolution. The accuracy in the determination of g is :-

$\Rightarrow L = 20.0$ cm

Absolute error :- 1mm $\Rightarrow$ 0.1 cm

$L = (20.0 \pm 0.1)$ cm

Time taken for 100 oscillations : 90 s

Absolute error = 1s
Time taken = (90 ± 1)s

$$\Rightarrow T = 2\pi\sqrt{\frac{l}{g}}$$

$$\Rightarrow T^2 = \frac{l}{g}$$

$$\Rightarrow g = \frac{l}{T^2}$$

$$\Rightarrow \frac{\Delta g}{g}\times 100 = \left(\frac{\Delta l}{l}\times 100\right) + 2\left(\frac{\Delta T}{T}\times 100\right)$$

$$= \cancel{20.0}\left(\frac{0.1}{2\cancel{0}}\times 1\cancel{00}\right) + 2\left(\frac{1}{9\cancel{0}}\times 1\cancel{0}\right)$$

$$= \frac{1}{2} + \frac{20}{9} = \frac{9+40}{18} = \frac{49}{18}\ 2.72\%$$

$$= 3\%.$$

Q.6. The ~~number~~ observations are shown in the table.
Least count for length = 0.1cm,
Least count for time = 0.1s

Student	Length of pendulum (cm)	Number of oscillations	Total time for n oscillations	Time period (s)
I	64.0 cm	8	128.0	16.0
II	64.0 cm	4	64.0	16.0
III	20.0 cm	4	36.0	9.0

If E_I, E_{II}, E_{III} are the percentage errors in g. i.e. $\frac{\Delta g}{g}\times 100$ for students I, II and III, respectively

$\Rightarrow E_1 = L = 64.0\,cm$

A.E = 0.1 cm $\Rightarrow (64.0 \pm 0.1)\,cm$

Time = 128.0 s

A.E = 0.1 s $\Rightarrow (128.0 + 0.1)\,s$

$\Rightarrow T = 2\pi\sqrt{\frac{l}{g}}$

$\Rightarrow T = \sqrt{\frac{l}{g}}$

$\Rightarrow T^2 = \frac{l}{g}$

$\Rightarrow g = \frac{l}{T^2}$

$\Rightarrow \frac{\Delta g}{g} \times 100 = \frac{\Delta l}{l} \times 100 + 2\left(\frac{\Delta T}{T} \times 100\right)$

$= \left(\frac{0.1}{\cancel{64}_{16}} \times \cancel{100}^{25}\right) + 2\left(\frac{0.1}{\cancel{128}_{32}} \times \cancel{100}^{25}\right)$

$= \frac{2.5}{16} + \frac{5}{32}$

$= \frac{5+5}{32} = \frac{\cancel{10}\ 5}{\cancel{32}\ 16}$

0.312%

$\Rightarrow E_2 = L = 64.0\,cm$

A.E = 0.1 cm $\Rightarrow (64.0 \pm 0.1)\,cm$

Time = 64.0 s

A.E = 0.1 s $\Rightarrow (64.0 \pm 0.1)\,s$

$\Rightarrow g = \frac{l}{T^2}$

$\Rightarrow \cancel{g =} \frac{\Delta g}{g} \times 100 = \frac{\Delta l}{l} \times 100 + 2\left(\frac{\Delta T}{T} \times 100\right)$

$= \frac{0.1}{\cancel{64}_{16}} \times \cancel{100}^{25} + 2\left(\frac{0.1}{\cancel{64}_{16}} \times \cancel{100}^{25}\right)$

$\Rightarrow \frac{2.5}{16} + \frac{5}{16} \Rightarrow \frac{7.5}{16}$

$\Rightarrow \frac{\cancel{75}\ \cancel{25}}{\cancel{16} \times \cancel{10}}$

0.7024

12.5

$\frac{\cancel{85}\ \cancel{75}\ 25}{16 \times \cancel{10}\ 2}$

$\Rightarrow E_3 = \cancel{L = 20.0\,cm}$ $L = (20.0 \pm 0.1)\,cm$

$T = (36.0 \pm 0.1)\,s$

$$\Rightarrow \frac{\Delta g}{g} \times 100 = \frac{\Delta l}{l} \times 100 + 2\left(\frac{\Delta T}{T} \times 100\right)$$

$$= \frac{0.1}{\cancel{20}} \times \overset{5}{\cancel{100}} + 2\left(\frac{0.1}{\underset{9}{\cancel{36}}} \times \overset{25}{\cancel{100}}\right)$$

$$= 0.5 + \frac{5}{9} \Rightarrow \frac{4.5 + 5}{9} = \frac{\cancel{9.5}}{\cancel{9}}\ 1.05\%$$

$\cancel{\Rightarrow b)}$ Correct option (b) E_1 is minimum

Q.7. A student uses a simple pendulum of exactly 1m length to determine e.g. acceleration due to gravity. He uses a stop watch with the least count of 1s for this and records 40 s for 20 oscillations. For this observation, which of the following statements is/are true?

a) Error ΔT in measuring T, the time period, is 0.05 s
b) " " " " ", " " ", is 1 s
c) Percentage error in the determination of g is 5%.
d) Percentage error in the determination of g is 2.5%.

$$\Rightarrow \Delta T = \frac{\Delta t}{n} = \frac{1}{20} = 0.05\,s$$

Option (a) is true

Now,

$\Rightarrow$ Length = 1m
Time = $(40 \pm 1)s$

$\Rightarrow g = \frac{l}{T^2}$

$$\Rightarrow \frac{\Delta g}{g} \times 100 = \frac{\Delta l}{l} \times 100 + 2\left(\frac{\Delta T}{T} \times 100\right)$$

$$= \frac{0}{1} \times 100 + 2\left(\frac{1}{40} \times 100\right)$$

$$= 2\left(\frac{1}{\cancel{40}_{2}} \times \cancel{100}^{5}\right)$$

$$= \cancel{2} \times \frac{5}{\cancel{2}} \Rightarrow 5\%.$$

Option (c) is also true

$\Rightarrow$ Option (a) and (c) are correct

~~Q.17.~~ Q. 8. In an experiment to determine the acceleration due to gravity 'g', the formula ~~use~~ used for the time period of a periodic motion $T = 2\pi\sqrt{\frac{7(R-r)}{5g}}$. The values of ~~r~~ R and r are measured to be (60 ± 1) mm and (10 ± 1) mm, respectively. In five consecutive measurements, the time period is found to be 0.52 s, 0.56 s, 0.57 s, 0.54 s, and 0.59 s. The least count of the watch used for the measurement of time period is ~~ae~~ 0.01 s. ~~Which of~~ Which of the following statement(s) is/are true?

a) The error in the measurement of r is 10%.
b) The error in the measurement of T is 3.57%.
c) The error in the measurement of T is 2%.
d) The error in the measurement of g is 11%.

$\Rightarrow \frac{\Delta r}{r} \times 100 \Rightarrow \frac{1}{1\cancel{0}} \times 10\cancel{0} \Rightarrow 10\%$.

Option (a) is correct

$\Rightarrow T_m = \frac{0.52+0.56+0.57+0.54+0.59}{5}$

$= \frac{\cancel{2.78}}{\cancel{5}}$ $0.556 \Rightarrow 0.56\,s$

$\Delta T_1 = 0.56-0.52 = 0.04\,s$
$\Delta T_2 = 0.56-0.56 = 0.00\,s$
$\Delta T_3 = 0.56-0.57 = -0.01\,s$
$\Delta T_4 = 0.56-0.54 = 0.02\,s$
$\Delta T_5 = 0.56-0.59 = -0.03\,s$

$\Rightarrow \frac{0.04+0+0.01+0.02+0.03}{5} \Rightarrow \frac{0.1}{5} \Rightarrow \frac{1}{5\times10} = \frac{1}{\cancel{50}} \Rightarrow 0.02\,s$

$\Rightarrow \Delta T = 0.02\,s$

$\Rightarrow \frac{\Delta T}{T} \times 100 \Rightarrow \frac{0.02}{0.56} \times 100 \Rightarrow \frac{\cancel{2}\times 1\cancel{00}^{\cancel{50}\,25}}{\cancel{56}_{\cancel{28},\cancel{14},7}} \Rightarrow \frac{\cancel{25}}{\cancel{7}}\ 3.57\%$.

option (b) is also correct

$$\Rightarrow T^2 = 4\pi^2 \frac{7(R-r)}{5g}$$

$$\Rightarrow g = 4\pi^2 \frac{7(R-r)}{5T^2}$$

$$\Rightarrow \frac{\Delta g}{g} \times 100 = \frac{\Delta(R-r)}{R-r} + 2\left(\frac{\Delta T}{T}\right)$$

$$\cancel{\Rightarrow \frac{\Delta g}{g} \times 100 = \frac{(1-1)}{60-10} + 2\left(\frac{0.02}{0.56}\right)}$$

$$\cancel{= \frac{0}{50} + 2\left(\frac{0.02}{0.56}\right)}$$

$$\cancel{= 2\left(\frac{2 \times 100}{56 \times 100}\right)}$$

$$\cancel{\Rightarrow 2 \times \frac{1}{28} \Rightarrow \frac{1}{14}}$$

$$\Rightarrow R-r = (50 \pm 2)\ \text{cm}$$

$$\cancel{\Rightarrow \frac{\Delta g}{g} \times 100 = \frac{2}{50} + 3.57}$$

$$\cancel{= \frac{1}{25} + \frac{357}{100}}$$

$$\cancel{= \frac{4+357}{100} \Rightarrow \frac{361}{100}}$$

$\Rightarrow \frac{\Delta g}{g} \times 100 = \frac{2}{\cancel{56}} \times \cancel{100}^{2} + \cancel{3.57}\ 2 \times 3.57$

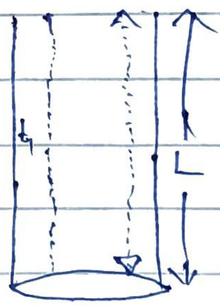
$= \cancel{4 + 3.57}\ 4 + 7$

$= 11\%$

Option (d) is also correct

Q. 9. A person measures the depth of ~~the~~ a well by measuring the time interval between dropping a stone and receiving the sound of impact ~~ti~~ with the bottom of the well. The error in his measurement of ~~Tim~~ $\Delta T = 0.01s$ and he ~~the~~ measures the ~~depth~~ depth of the well to be $L = 20m$. Take the ~~acct~~ acceleration due to gravity $g = 10 m/s^2$ and the velocity of the sound is $300 ms^{-1}$. Then the fractical error in the measurement, $\frac{\delta L}{L}$ is closest to:-

a) 1%
b) 5%
c) 3%
d) 0.2%



$t_1: \quad s = ut + \frac{1}{2}at^2$

$\Rightarrow L = \cancel{0 \times t +}\ 0 + \frac{1}{2} \times g \times t^2$

$\Rightarrow t^2 = \frac{2L}{g}$

$\Rightarrow t = \sqrt{\frac{2L}{g}} \Rightarrow \sqrt{\frac{2 \times 2\cancel{0}}{1\cancel{0}}} \Rightarrow \sqrt{4} \Rightarrow 2$ seconds

t_2: $T = \dfrac{\cancel{S}\ \text{Distance}}{\text{Speed}} = \dfrac{2\cancel{0}}{30\cancel{0}} = \dfrac{1}{15}$ seconds

$\Rightarrow$ ~~$t_1 + t_2 = 2 + \frac{1}{15} = \frac{30+1}{15} = \frac{31}{15}$ seconds~~

$\Rightarrow t_1 = \sqrt{\dfrac{2l}{g}}$

$\Rightarrow \dfrac{\Delta t_1}{t_2} = \dfrac{2\, l^{\frac{1}{2}}}{g^{\frac{1}{2}}}$

$\Rightarrow \dfrac{\Delta t_1}{\cancel{\Delta} t_1} = \cancel{\dfrac{1}{2}\left(\dfrac{\Delta l}{l} + \dfrac{\Delta g}{g}\right)}\ \dfrac{1}{2}\dfrac{\Delta l}{l} + \dfrac{1}{2}\dfrac{\Delta g}{g}$

$\Rightarrow \dfrac{\Delta t_1}{t_1} = \dfrac{1}{2} \times \dfrac{\Delta l}{\cancel{20}\, l}$

$\Rightarrow \Delta t_1 = \dfrac{1}{\cancel{2}} \times \dfrac{\Delta l}{\cancel{20}\, l\ \cancel{10}} \times \cancel{2}$

$\Rightarrow \Delta t_1 = \cancel{\dfrac{\Delta t}{20}}\ \dfrac{\Delta l}{l}$

$\dfrac{\Delta t_2}{t_2} = \dfrac{\Delta l}{l} + \dfrac{\Delta v}{v}$

$\dfrac{\Delta t_2}{t_2} = \dfrac{\Delta l}{l}$

$\Rightarrow \dfrac{\Delta t_2}{\cancel{t_2}} = \dfrac{\Delta l}{l} \times t_2$

$\Rightarrow \Delta t_2 = \dfrac{\Delta l}{l} \times \dfrac{1}{15}$

$\Rightarrow \Delta t = \Delta t_1 + \Delta t_2$

$0.01 = \dfrac{\Delta l}{l} + \dfrac{\Delta l}{l} \times \dfrac{1}{15}$

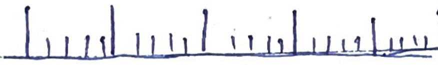
$0.01 = \dfrac{\Delta 15 l + \Delta l}{15 l} = \dfrac{\Delta 16 l}{15 l}$

$\dfrac{\Delta l}{l} = 0.01 \times \dfrac{15}{16}$

$\dfrac{\Delta l}{l} \times 100 = 0.01 \times \dfrac{15}{16} \times 100$

$= \dfrac{15}{16} \simeq 1\%$

To measure length of an object, we use normal scale which is marked in cm.

 cm $\qquad \frac{1\text{ cm}}{10} = 0.1$ cm

Smallest measurement which can be made = 0.1 cm
↓
Least count (LC) of an istrument

It cannot give precision upto second decimal place

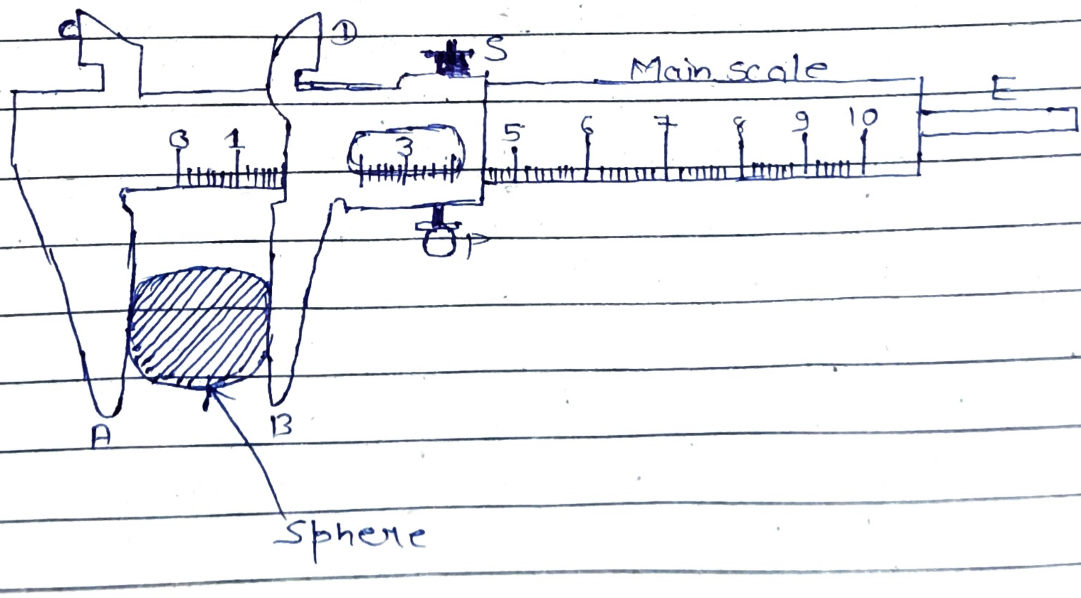
In 1631, Piere Vernier deviced
→ Vernier Callipers

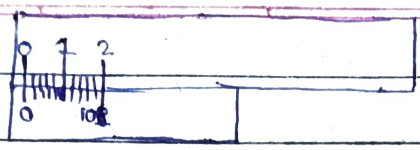
Uses → i) to measure length, breadth or height of small regular size objects.

ii) To ~~m~~ measure diameter (or radius) of small spherical or cylindrical body.

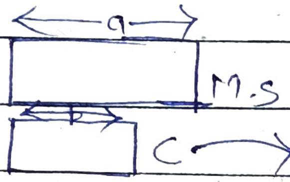
iii) to measure depth of beaker, glass, ~~to~~ hole

iv) To measure internal diameter of hollow cylinder.





Generally each division of V.S is smaller than each division of M.S



$LC = 1M.S.D - 1V.S.D$

→ Smallest Measurement ~~which can~~ be made ⇒ (LC)

Let 'n' ~~vernier~~ V.S.D are equal to 'm' M.S.D

$n\,V.S.D = m\,M.S.D$

$1V.S.D = \frac{m}{n}\,M.S.D$

$LC = \cancel{M\,MSD}\ 1MSD - 1VSD$

$= 1MSD - \frac{m}{n}\cancel{VSD}\,MSD$

$= MSD\left(1 - \frac{m}{n}\right)$

⇒ Ordinary Vernier Calliper:

$1MSD = \frac{1cm}{10} = 0.1cm$

$10VSD = 9MSD$

$LC = MSD\left(1 - \frac{m}{n}\right) = 0.1\left(1 - \frac{9}{10}\right)$

$\Rightarrow 0.1\left(\frac{10-9}{10}\right)$

$\Rightarrow 0.1 \times \frac{1}{10}$

$\Rightarrow 0.01\ cm$

Q.1. In a vernier callipers, the main scale is marked in mm & 20 vernier scale divisions are equal to 16 main scale divisions. Calculate least count.

a) 0.02 mm b) 0.05 mm c) 0.1 mm d) 0.2 mm

$\Rightarrow MSD = \cancel{0.2\ mm}\ 0.02\ mm$

$\Rightarrow LC = MSD\left(1-\frac{m}{n}\right)$

$= 0.02\left(1-\frac{16}{20}\right)$

$= 0.02\left(\frac{20-16}{20}\right)$

$\Rightarrow 0.02 \times \frac{\cancel{4}}{\cancel{20}_{5}}$

$\Rightarrow \frac{0.02}{5}$

$\Rightarrow \frac{\cancel{2}}{5\times\cancel{100}_{50}} \Rightarrow \frac{1}{\cancel{250}}\ 0.004\ mm$

$1 MSD = 1mm$

$\Rightarrow LC = MSD\left(1-\frac{m}{n}\right)$

$= 1\left(1-\frac{16}{20}\right)$

$= 1\left(\frac{20-16}{20}\right)$

$\Rightarrow 1\left(\frac{\cancel{4}}{\cancel{20}}\right)$

$\Rightarrow 1 \times \frac{1}{5} \Rightarrow \frac{1}{5}\ 0.2\ mm$

(d) 0.2 mm (Ans)

Q.2 'N' divisions of Main scale are equal to 'N+1' divisions of Vernier scale. If each division on main scale measures 'a' units. Find least count.

⇒ $1\,MSD = a$

⇒ $N\,MSD = (N+1)\,VSD$

⇒ $LC = MSD\left(1 - \frac{m}{n}\right)$

$= \cancel{a\left(1 - \frac{(N+1)}{N}\right)}$

$= \cancel{a\left(\frac{N-(N+1)}{N}\right)}$

$= \cancel{a\left(\frac{N-N-1}{N}\right)}$

$= a\left(\frac{-1}{-N}\right) \cancel{\Rightarrow \frac{-a}{N}}$

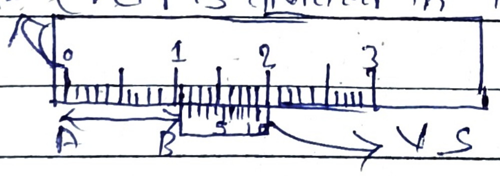
⇒ $a\left(1 - \frac{N}{(N+1)}\right)$

⇒ $a\left(\frac{\cancel{N}+1-\cancel{N}}{N+1}\right)$

⇒ $\frac{a}{N+1}$

⇒ Reading of Vernier Callipers

M.S (1 CM is divided in 10 divisions)



$10\,VSD = 9\,MSD$

$LC = 0.1\left(1 - \frac{9}{10}\right) \Rightarrow 0.01\ cm$

~~A + B~~

1 cm < AB < 1.1 cm

Total Reading = M.S.R. + V.S.R

M.S.R. ↓ Reading on M.S just ~~before~~ ~~V.S~~ zero of V.S

V.S.R ↓ LC × Coinciding division of VS

$\Rightarrow$ 1 cm + 0.01 × 5

$\Rightarrow$ 1 + 0.05 $\Rightarrow$ 1.05 cm

Q.3 In a Vernier Callipers, 10 VSD = 9 MSD & 1cm on main scale is divided into 10 parts while measuring the length of a line, the zero of VS is just ahead 1.8 cm mark & 4th division of V.S. coincides with a main scale division. The length of line is :-

a) 1.800 cm b) 1.840 cm c) 1.804 cm d) 1.844 cm

Total Reading = M.S.R + V.S.R

= 1.8 + LC × 4

= 1.8 + ~~LC ×~~ 0.01 × 4

$\Rightarrow$ LC = ~~0.01~~ 0.01 cm

$\Rightarrow$ 1.8 + 0.04

$\Rightarrow$ ~~1.84 cm~~

~~(b) Ans~~

Q.4. LC = 0.01 cm

Zero Ahead = 12.2 cm

Coincides on = ~~12.3 cm~~ ~~12.3 cm~~ 7

⇒ Total Reading = 12.2 + 0.01 × ~~8~~ 7
= 12.2 + ~~0.03~~ 0.7
= ~~12.23~~ 12.24

Q.5. The diameter of a cylinder is measured using a vernier calliper with no-zero error. It is found that the zero of V.S. lies between 5.10 cm & 5.15 cm of the M.S. The V.S. has 50 divisions equivalent to 2.45 cm. The ~~24~~ 24th division of the V.S. exactly coincides with one of the main scale division. The diameter of cylinder.

a) 5.112 cm b) 5.124 cm c) 5.136 cm d) 5.148 cm

⇒ $\frac{\cancel{245}^{49}}{50 \times \cancel{100}_{20}}$ ⇒ $\frac{49}{1000}$ ⇒ ~~0.49 cm~~ 0.049 cm

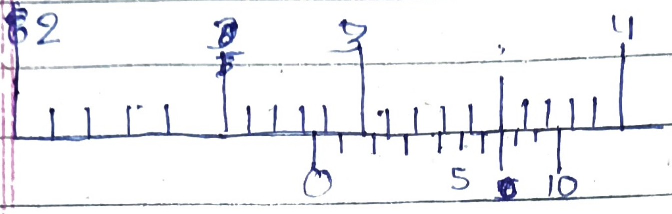
⇒ ~~LC = 0.049~~ 1 MSD = 5.15 − 5.10 = 0.05 cm
⇒ ~~0.049 × 24 =~~

⇒ LC = 1 MSD − 1 VSD
= 0.05 − 0.049
= 0.001 cm

⇒ Total diameter = MSR + VSR
= 5.10 + 0.001 × 24
= 5.10 + 0.024
= 5.124 cm

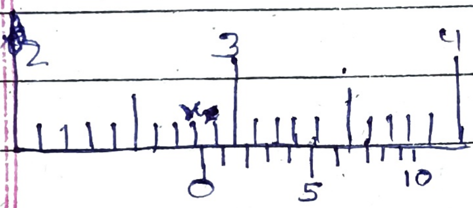
Q.6.

C_1



10 VSD = 9 MSD

C_2



10 VSD = 11 MSD

Reading of C_1 & C_2 are :

a) 2.87, 2.87 b) 2.87, 2.83 c) 2.85, ~~2.82~~ 2.82

d) 2.87, 2.86

⇒ C1 :-

$1 MSD = 0.1\ cm$

$LC = MSD\left(1 - \frac{m}{n}\right)$

$= 0.1\left(1 - \frac{9}{10}\right) \Rightarrow 0.1\left(\frac{10-9}{10}\right)$

$\Rightarrow 0.1 \times \frac{1}{10}$

$\Rightarrow 0.01\ cm$

⇒ Total reading = MSR + VSR

$= 2.8 + 0.01 \times 7$

$= 2.8 + 0.07$

$= 2.87$

Q2:

$\Rightarrow$ MSD = 0.1 cm

$\Rightarrow LC = 0.1\left(1 - \frac{11}{10}\right) \Rightarrow 0.1\left(\frac{10-11}{10}\right)$

$= 0.1 \times \frac{-1}{10}$

$\Rightarrow -0.01$ cm

$\Rightarrow$ Total Reading = MSR + VSR

$= 2.8 + (-0.01 \times 7)$

$= 2.8 - 0.07$

$= 2.73$

$\Rightarrow$ Total Reading = $2.8 + x$

$\Rightarrow 1\,VSD = \cancel{\frac{11}{10}}^{1.1}\,MSD \Rightarrow \cancel{1.1\text{ cm}}\ 1.1 \times 0.1$

$\Rightarrow \frac{1.1 \times 1}{10} \Rightarrow 0.11$ cm

$\Rightarrow 8\,MSD - x = \cancel{7x}\ 7\,VSD$

$\Rightarrow 8 \times 0.1 - x = 7 \times 0.11$

$\Rightarrow 0.8 - x = 0.77$

$\Rightarrow 0.8 - 0.77 = x$

$\Rightarrow 0.03$ cm $= x$

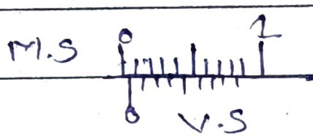
$\Rightarrow$ Total reading $= 2.8 + 0.03$

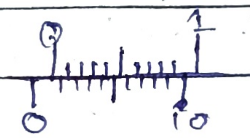
$= 2.83$ cm

$\Rightarrow$ (B) Correct answer

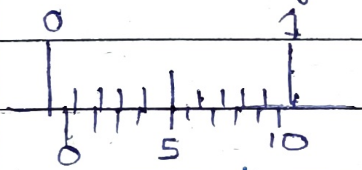
⇒ Zero Error: When zero of ~~Ve~~ V.S & zero of M.S does not coincide, then the instrument is said to have a zero error.

+ve zero error | -ve zero error



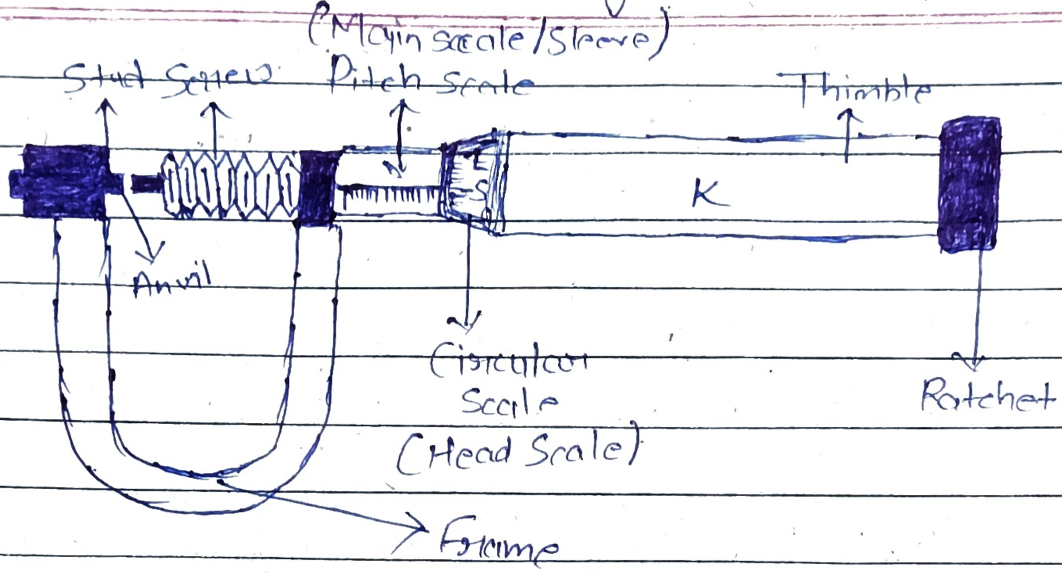


⇒ Correct reading = Total Reading - Zero Error



⇒ ~~...~~ zero error = LC × Coinciding div of V.S

Screw Gauge



* Pitch :- Distance between two ~~con~~ consecutive threads. or,
The distance covered by screw in one complete ~~rota~~ rotation of the circular scale is called pitch.

$$\text{Least Count} = \frac{\text{Pitch}}{\text{No. of Circular Scale Divisions}}$$

Q.1. Two full turns of the ~~circular~~ circular scale of a gauge cover a distance of 1mm on scale. The total number of divisions ~~a~~ on circular scale is 50. Find the least count.

⇒ 1 full rotation = $\frac{1}{2}$ 0.5 mm

$$\Rightarrow LC = \frac{0.5}{50} \Rightarrow \frac{5}{50 \times 10_2} = \frac{1}{100} \Rightarrow 0.01 \text{ mm}$$

Q2. The least count of the main scale of a screw gauge is ~~1mm~~ 1mm. The minimum ~~no~~ number of divisions on its circular circular scale required to measure 5µm diameter of a wire is:

$\Rightarrow$ Pitch = 1mm
LC = 5 μm

$\Rightarrow$ $LC = \frac{Pitch}{n}$

$\Rightarrow$ $0.005 = \frac{1}{n}$

$\Rightarrow$ $n = \frac{1}{0.005}$

$\Rightarrow$ $n = \frac{1 \times 10\cancel{00}}{\cancel{5}}$ 200

$\Rightarrow$ n = 200

$\Rightarrow$ Finding Reading

$\Rightarrow$ Reading = MSR + CSR
= Reading on ~~main~~ MS just before zero on CS + LC X Coinciding division of CS ~~with~~ with base line.

Q.3. Circular Scale has 100 divisions. Calculate diameter of wire.

$\Rightarrow$ Reading = MSR + CSR
= $\cancel{0.5}$ 5 + CSR

$\Rightarrow$ $LC = \frac{\cancel{0.1}\,1}{100} \rightarrow \frac{1}{\cancel{100 \times 10}} \rightarrow$ ~~0.001 mm~~ 0.01 mm

$\Rightarrow$ $\cancel{0.5}\,5 + 0.0\cancel{0}1 \times 34 \Rightarrow \cancel{0.5}\,5 + 0.34 \Rightarrow$ ~~0.534~~ 5.34 mm
$\Rightarrow$ 0.534 cm

Q.4. The thimble of ~~a~~ a screw has 50 divisions and spindle advances 1 mm when the screw is turned through two rotations. When the ~~screw~~ screw is used to measure the diameter of wire, the reading on sleeve is found out to be 0.5 mm and reading on thimble is found 27 divisions. What is the diameter of wire in cm?

a) 0.027 cm
b) 0.005 cm
c) 0.077 cm
d) 0.035 cm

→ Divisions of ~~CS = 50~~ ~~CS~~ CS = 50
One rotation advancement = $\frac{\cancel{1}}{\cancel{2}}$ 0.5 mm

~~Reading = 0.5 mm~~ MSR = 0.5 mm
CSR = LC × 27

→ $LC = \frac{0.5}{50} \Rightarrow \frac{5}{50 \times 10} \Rightarrow \frac{1}{100} \Rightarrow 0.01$ mm

→ CSR = 0.01 × 27
= 0.27 mm

→ Reading = 0.5 + 0.27
= 0.77 mm
= 0.077 cm

→ Zero Error & Correct Reading

If zero of circular scale does not coincide with base line (at initial condition before starting measurement, when spindle and stud touch each other), then it is said ~~to~~ that there is a zero error in screw gauge.

→ Two types of zero Error ~~are~~ :-

a) Positive Zero Error
b) Negative Zero Error

Correct Reading = Reading − Zero Error

a) Positive Zero Error :- When zero of CS is below base line.

+ve ~~zero~~ zero error = +LC × Coinciding division of CS with base line.

b) ~~Correct Read~~ ~~ite~~ Negative Zero :- When zero of CS is above the base line.

-ve Zero error = −LC × Coinciding div. of CS with BL

Q.5 The Pitch ~~on~~ and the number of divisions, on the circular scale ~~for~~ for a given ~~a~~ screw ~~gauge~~

Date / /
Page No.

gauge are 0.5 mm 100, respectively. When the screw gauge is fully tightened without any object, the zero of it's circular scale lies 3 divisions below the mean line.

The reading of the MS and ~~the~~ the CS for a thin sheet are 5.5 mm and 48 respectively, the thickness of this sheet is :-

a) 5.950 mm
b) 5.725 mm
c) 5.755 mm
d) 5.740 mm

$\Rightarrow$ $LC = \frac{0.5}{100} \Rightarrow 0.005$ mm

Zero error $= +0.005 \times 3$
$= 0.015$ mm

$\Rightarrow$ Reading $= (MSR + CSR) - \text{Zero error}$
$= (5.5 + LC \times 48) - 0.015$
$= (5.5 + 0.005 \times 48) - 0.015$
$= (5.5 + 0.240) - 0.015$
$= 5.740 - 0.015$
$= 5.725$ mm

Q6. A screw gauge with a pitch of 0.5 mm and a ~~it~~ circular scale with 50 divisions is used to measure the thickness of a ~~thin sheet~~ thin sheet of aluminium. Before starting the measurement, it is found that when two Jaws of the screw gauge are brought in contact, the 45th division coincides with the main scale line and ~~the~~ that the zero of the main scale is barely visible. What is the thickness of the sheet, if the MSR is 0.5 mm and the 25th division ~~co~~ coincides with the main scale line?

a) 0.75 mm
b) 0.80 mm
c) 0.70 mm
d) 0.50 mm

$\Rightarrow$ $LC = \frac{0.5}{50} \Rightarrow \frac{5}{50 \times 10^{2}} \Rightarrow \frac{1}{100} \Rightarrow 0.01$ mm

$\Rightarrow$ Zero error $= -0.01 \times (50-45)$
$= -0.01 \times 5$
$= -0.05$ mm

$\Rightarrow$ Reading $= (MSR + CSR) - (-0.05)$
$= (0.5 + 0.01 \times 25) + 0.05$
$= 0.5 + 0.25 + 0.05$
$=$ ~~0.30 + 0.05~~ $0.75 + 0.05$
$=$ ~~0.35~~ 0.80 mm