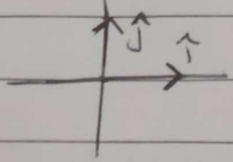
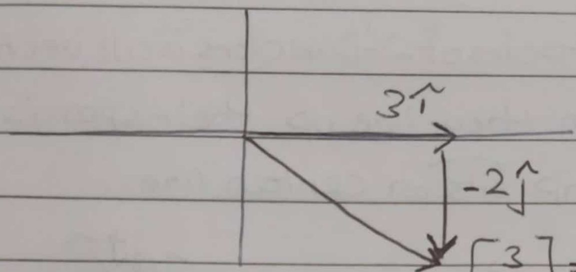
Linear combination, span and Basis vector



$\hat{i} \rightarrow$ unit vector in x-direction

$\hat{j} \rightarrow$ unit vector in y-direction

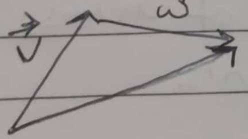


$\begin{bmatrix} 3 \\ -2 \end{bmatrix} = 3\hat{i} + (-2)\hat{j}$

- $\hat{i}$ and $\hat{j}$ are the "basis vectors" of the xy coordinate system.
- Any time you describe vectors numerically, it depends on implicit choise of what basis vector you are using.

✱ Linear combination of $\vec{v}$ and $\vec{w}$: $a\vec{v} + b\vec{w}$

a, b → scalars



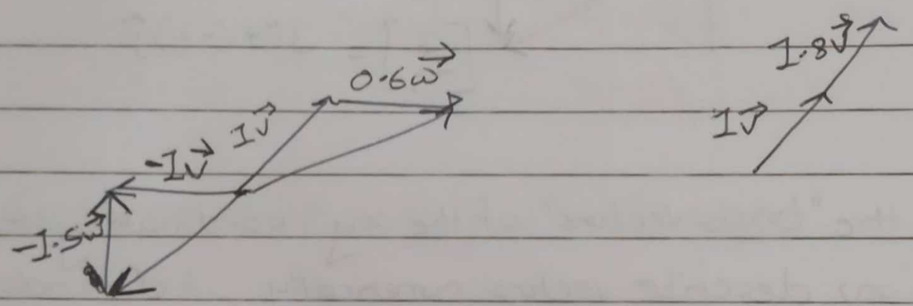
- If one vector remains fixed, and other changes, ~~it can~~ the resultant vector's tip moves in a line (straight)
- If both vectors changes, the resultant vector covers the entire space
- If both vectors are 0 vectors, Resultent vector is a point, stuck at origin.

★ The span of $\vec{v}$ and $\vec{w}$ is the set of all their linear combinations

$$a\vec{v} + b\vec{w}$$

Let a and b vary over all real numbers

The span of most pairs of 2-D vecters is all vecters of 2-D space, but when they line up, their span is all vecters whose tip sits on certain line.



★ Vecters vs Points

→ If you are thinking of vecter on its own, think of it as an arrow.

→ If you are dealing with collection of vecters, it is convinient to think of them as points.

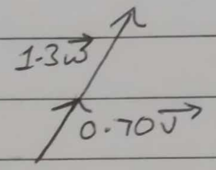
For span example, span of most pairs of vectors ends up being entire infinite sheet of 2-D space,
If they line up, their span is just a line

* Linear combination of 3-vectors $\vec{u}$, $\vec{v}$ and $\vec{w}$,

$$a\vec{v} + b\vec{w} + c\vec{u}$$

For span, let these constants vary.

In case when third vector is sitting on span of first two, or when 2 vectors happen to line up,



- We want some terminology to describe fact that at least one of these vectors is reductant, not adding anything to our span.
  Whenever this happens, where you have multiple vectors and you could remove one without reducing the span, the relevant terminology is to say that they are "Linearly dependent."

  OR

  One of the vectors can be expressed as linear combination of others since its already in span of others.

- If each vector really does add another dimension to span, they are said to be "Linearly independent"

  Linearly independent : $\vec{u} \neq a\vec{v} + b\vec{w}$

  For all values of a and b.

** Basis of vector space is a set of linearly independent vectors that span the full space