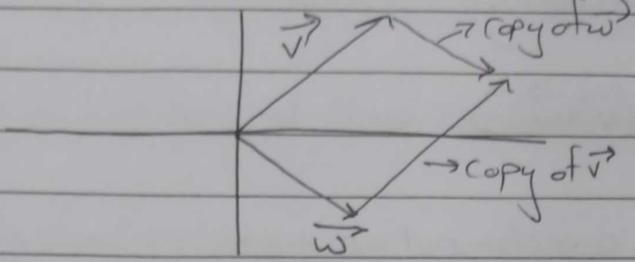
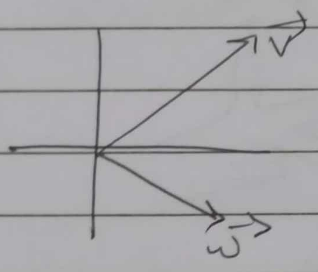
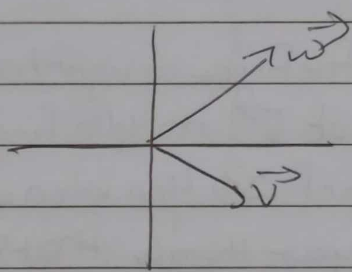
Crossproducts



The vectors $\vec{v}$, $\vec{w}$, copy of $\vec{v}$ and copy of $\vec{w}$ encloses or forms a parallelogram.

$\vec{v} \times \vec{w}$ = Area of this parallelogram
↓
crossproduct

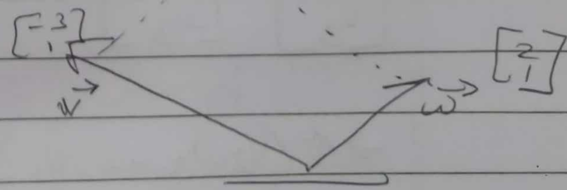




$\vec{v} \times \vec{w}$ = −Area of parallelogram $\vec{v} \times \vec{w}$ = +Area of //gm

$$\boxed{\vec{v} \times \vec{w} = -\vec{w} \times v}$$

- $\hat{i} \times \hat{j}$ = +ve order
  ∴ when $\hat{i}$ is at right of $\hat{j}$ → its positive
  or anticlockwise → +ve   clockwise → −ve



→ $\vec{v} \times \vec{w} = \det\left(\begin{bmatrix} -3 & 2 \\ 1 & 1 \end{bmatrix}\right)$

$= -3 \cdot 1 - 2 \cdot 1 = \underline{\underline{-5}}$

Since $\vec{v}$ is on left of $\vec{w}$, it's negative and area is $\underline{\underline{5}}$

- when 2 vectors are perpendicular, $\vec{v} \times \vec{w}$ is bigger since are of parallelogram is maximum.

- $(3\vec{v}) \times \vec{w} = (3(\vec{v} \times \vec{w}))$

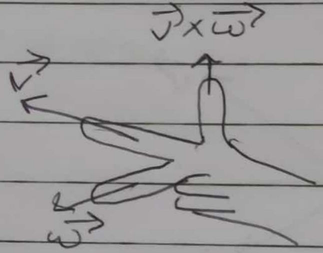
- Cross product is a vector, not a number.

  $$\vec{v} \times \vec{w} = \underbrace{\vec{p}}_{\text{vector}}$$

  & direction of $\vec{p}$ is $\perp$ to ||gm formed by $\vec{v}$ & $\vec{w}$.

- The $\perp$ direction of $\vec{p}$ is denoted by right hand rule.
  (as there can be 2 $\perp$ directions)

→ Right hand rule:



Put fore finger of right hand in dirn of $\vec{v}$, middle finger in dirn of $\vec{w}$, then when you point at your thumb, that's dirn of cross product

Calculation by formula

$$\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} \times \begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix} = \det\left(\begin{bmatrix} \hat{i} & v_1 & w_1 \\ \hat{j} & v_2 & w_2 \\ \hat{k} & v_3 & w_3 \end{bmatrix}\right)$$

$$\rightarrow \hat{i}\underbrace{(v_2w_3 - w_2v_3)}_{\text{Some number}} + \hat{j}\underbrace{(v_3w_1 - v w_3)}_{\text{Some number}} + \hat{k}\underbrace{(v_1w_2 - v_2w_1)}_{\text{Some number}}$$

Cross product in light of Linear transformations.

Process:

1) Define a 3-d to 1-d linear transformation in terms of $\vec{v}$ and $\vec{w}$.
2) Find its dual vector
3) Show that this dual is $\vec{v} \times \vec{w}$

It will clear the ~~[illegible]~~ connection between computation and geometry of cross product.

Not real (3D) Cross product:

$$\vec{u} \times \vec{v} \times \vec{w} = \det\left(\begin{bmatrix} u_1 & v_1 & w_1 \\ u_2 & v_2 & w_2 \\ u_3 & v_3 & w_3 \end{bmatrix}\right)$$

$$\vec{u} = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} \quad \vec{v} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} \quad \vec{w} = \begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix}$$

This gives volume of Parallelopiped

Real 3-D cross product:

$\vec{v} \times \vec{w} = \vec{p}$ Resultant is a vector of a Cross product

$$f\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \det\left(\begin{array}{c} \begin{array}{ccc} & \vec{v} & \vec{w} \end{array} \\ \begin{bmatrix} x & v_1 & w_1 \\ y & v_2 & w_2 \\ z & v_3 & w_3 \end{bmatrix} \end{array}\right)$$

Above function is linear, we can think of duality.

$$\overset{\vec{p}}{\begin{bmatrix} p_1 \\ p_2 \\ p_3 \end{bmatrix}} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \det\left(\begin{array}{c} \begin{array}{ccc} & \vec{v} & \vec{w} \end{array} \\ \begin{bmatrix} x & v_1 & w_1 \\ y & v_2 & w_2 \\ z & v_3 & w_3 \end{bmatrix} \end{array}\right)$$

$$\begin{aligned} p_1 \cdot x + p_2 \cdot y + p_3 \cdot z = \; & x\,\overbrace{(v_2 w_3 - v_3 w_2)}^{p_1} \\ & + y\,\overbrace{(v_3 w_1 - v_1 w_3)}^{p_2} \\ & + z\,\overbrace{(v_1 w_2 - v_2 w_1)}^{p_3} \end{aligned}$$

Plugging in p_1, p_2, p_3 is same as plugging in $\hat{i}\hat{j}\hat{k}$. It gives us a sense of plugging in a vector.

- Geometric interpretation:

$$\vec{p} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = (\text{Length of projection}) \times (\text{Length of } \vec{p})$$

$$\text{Area of parallellopiped} = (\text{Area of //gm}) \times \left(\text{component of } \begin{bmatrix} x \\ y \\ z \end{bmatrix} \perp \text{ to } \vec{v} \,\&\, \vec{w}\right)$$

It is same as taking dot product between $[x\ y\ z]$ and vector $\perp$ to $\vec{v}$ & $\vec{w}$ with length equal to area of //gm.

PAGE No.	
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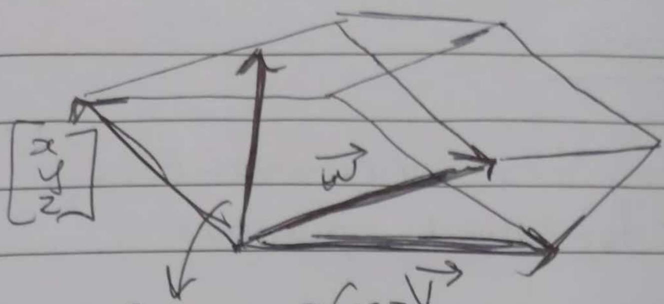

Component of $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ on $\perp$ to $\vec{v}$ & $\vec{w}$