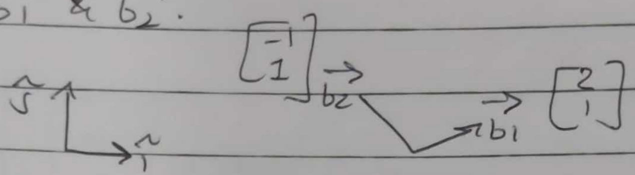
Change of basis

- In our coordinate system, we consider basis vectors as $\hat{i}$ and $\hat{j}$ for all vector operations.

- But what if we use different set of basis vectors say $\vec{b_1}$ & $\vec{b_2}$.



- Vector $\begin{bmatrix} 3 \\ 2 \end{bmatrix}$ in our basis vector grid will be different when basis vectors are $\vec{b_1}$ & $\vec{b_2}$.

- $\begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$

  $\underline{x}$ y is in our coordinate system.

→ Multiplying any vector (in our grid) with $\begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix}$ (in this case) will give us the vector in the changed grid

→ Multiplying any vector (in changed grid) with $\begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix}^{-1}$ (in this case) will give us vector in our grid.

• <u>Rotating 90° anticlockwise</u>

In our basis system, we get  after rotating 90° ↺

In changed basis system ($\vec{b_1}$ & $\vec{b_2}$), When we rotate 90° ↺, it will change the rotated vector. So, Now how to translate/calculate the matrix?

Ex. Say vector $\begin{bmatrix}-1\\2\end{bmatrix}$ in Alex's System and transformation $= \begin{bmatrix}0 & -1\\1 & 0\end{bmatrix}$

→ Translate into our system, using change of base matrix. $\begin{bmatrix}2 & -1\\1 & 1\end{bmatrix}$

$$\begin{array}{cc} \vec{b_1} & \vec{b_2} \\ \end{array}$$
$$\begin{bmatrix}2 & -1\\1 & 1\end{bmatrix}\cdot\begin{bmatrix}-1\\2\end{bmatrix}$$

This gives $\begin{bmatrix}-1\\2\end{bmatrix}$ in our system.

→ Applying rotation in our system.

$$\begin{bmatrix}0 & -1\\1 & 0\end{bmatrix}\begin{bmatrix}2 & -1\\1 & 1\end{bmatrix}\begin{bmatrix}-1\\2\end{bmatrix}$$

→ Converting transformation back to their system.

$$\begin{bmatrix}2 & -1\\1 & 1\end{bmatrix}^{-1}\begin{bmatrix}0 & -1\\1 & 0\end{bmatrix}\begin{bmatrix}2 & -1\\1 & 1\end{bmatrix}\begin{bmatrix}-1\\2\end{bmatrix}$$

Give's transformation in Alex's system.

Transformation.

$A^{-1}MA$:

shifting between systems.