Abstract vectorspaces

Describing derivatives with matrix

Basis for polynomials:

$1, x, x^2, x^3 \ldots$

$b_0(x) = 1$

$b_1(x) = x$

$b_2(x) = x^2$

$b_3(x) = x^3$

$\vdots$

∞

A polynomial x^2+3x+5 will be described by coordinates,

$$\begin{matrix} 5\cdot 1 \\ +3\cdot x \\ +1x^2 \\ +0x^3 \\ +0x^4 \\ \vdots \end{matrix} \quad \begin{bmatrix} 5 \\ 3 \\ 1 \\ 0 \\ 0 \\ \vdots \\ \infty \end{bmatrix}$$

$$\frac{d}{dx}(1x^3+5x^2+4x+5) = 3x^2+10x+4$$

$$= \begin{bmatrix} 0 & 1 & 0 & 0 & \cdots \\ 0 & 0 & 2 & 0 & \cdots \\ 0 & 0 & 0 & 3 & \cdots \\ 0 & 0 & 0 & 0 & \cdots \\ \vdots & & & & \end{bmatrix} \begin{bmatrix} 5 \\ 4 \\ 5 \\ 1 \\ \vdots \end{bmatrix} = \begin{bmatrix} 1\cdot 4 \\ 2\cdot 5 \\ 3\cdot 1 \\ 0 \\ \vdots \end{bmatrix}$$

This is possible because derivative is linear.

How to construct derivative matrix?

Take derivative of each basis fn and put the coordinates of results in each column.

$$\frac{d\, b_0(x)}{dx} = \frac{d(1)}{dx} = 0 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\frac{d\, b_1(x)}{dx} = \frac{d(x)}{dx} = 1 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\frac{d\, b_2(x)}{dx} = \frac{d}{dx}(x^2) = 2x = \begin{bmatrix} 0 \\ 2 \\ 0 \\ 0 \end{bmatrix}$$

$$\frac{d\, b_3(x)}{dx} = \frac{d}{dx}(x^3) = 3x^2 = \begin{bmatrix} 0 \\ 0 \\ 3 \\ 0 \end{bmatrix}$$

$$\therefore \begin{bmatrix} 0 & 1 & 0 & 0 & \cdots \\ 0 & 0 & 2 & 0 & \cdots \\ 0 & 0 & 0 & 3 & \cdots \\ 0 & 0 & 0 & 0 & \cdots \\ & & \vdots & & \end{bmatrix}$$

Linear algebra Concepts	Alternate names when applied to functions
• Linear transformations	Linear operations
• Dot products	Inner products
• Eigenvectors	Eigen functions.

This set of vectorish things like arrows, fns etc. are called vector spaces.

- Rules for vector addition and scaling.

There are 8 rules/axioms that any vector space must specify.

1. $\vec{u} + (\vec{v} + \vec{w}) = (\vec{u} + \vec{v}) + \vec{w}$
2. $\vec{v} + \vec{w} = \vec{w} + \vec{v}$
3. There is a vector 0 such that $0 + \vec{v} = \vec{v}$ for all $\vec{v}$.
4. For every vector $\vec{v}$, there is a vector $-\vec{v}$, so that $\vec{v} + (-\vec{v}) = 0$
5. $a(b\vec{v}) = (ab)\vec{v}$
6. $1\vec{v} = \vec{v}$
7. $a(\vec{v} + \vec{w}) = a\vec{v} + a\vec{w}$
8. $(a+b)\vec{v} = a\vec{v} + b\vec{v}$.

In additions to these axioms, every vectorish thing must obey its basic rule of scaling and additivity.

Axioms are interface between the person who discovered the results and others who want to apply the results to new sorts of vector spaces. So we form our results abstractly. (i.e in terms of axioms.)