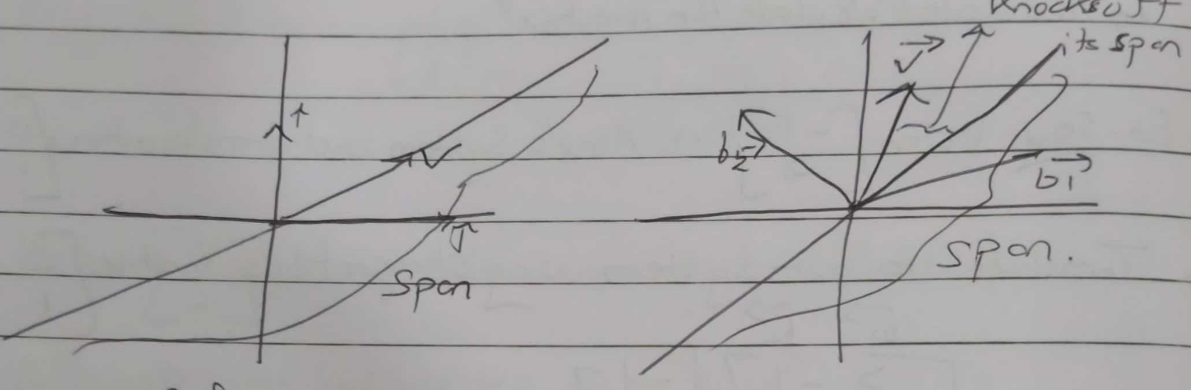
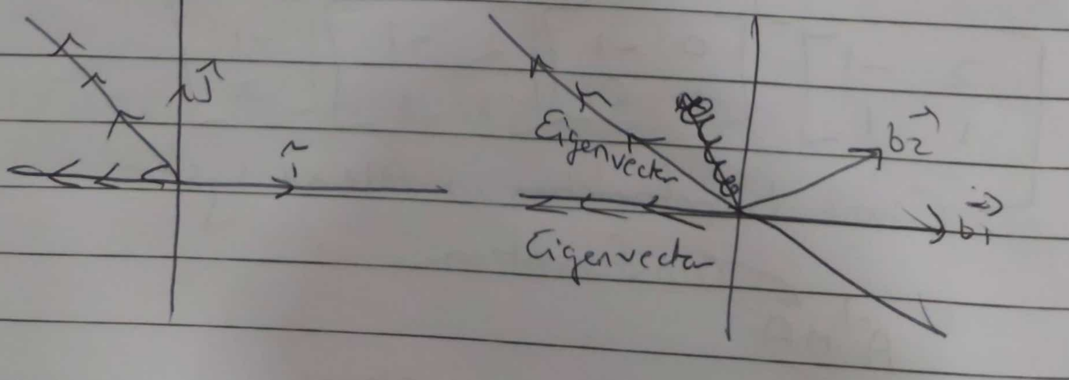
Eigenvectors & Eigenvalues

During a transformation (or change of basis), the original vector in old system ~~[illegible]~~ knocks of its span to form a new vector.



Before

But there are some special vectors (like x-axis $\begin{bmatrix}1\\0\end{bmatrix}$ or $\begin{bmatrix}-1\\1\end{bmatrix}$) which ~~[illegible]~~ after transformation, gets squished or stretched but remain on its span axis. Such vectors are called 'eigenvectors' and the factor by which it stretches or squishes is known as 'eigenvalue'



Computation

Transformation matrix

$$\underbrace{A}_{\text{}}\vec{v} = \lambda\vec{v}$$

Eigenvalue

Eigenvector

Solve for λ and $\vec{v}$

Matrix vector Multiplication $A\vec{v}$ gives same result as scalar multiplication $\lambda\vec{v}$.

or $A\vec{v} = (\lambda I)\vec{v}$

$(A - \lambda I)\vec{v} = 0$

We need $\det(A - \lambda I) = 0$

Goal is to ~~need~~ find such value of λ which makes $|A - \lambda I| = 0$

Meaning the transformation squishes into a lower dimension

Then find eigen vector by:

$$M\vec{v} = \vec{0} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad \text{where } \vec{v} = \begin{bmatrix} x \\ y \end{bmatrix}$$

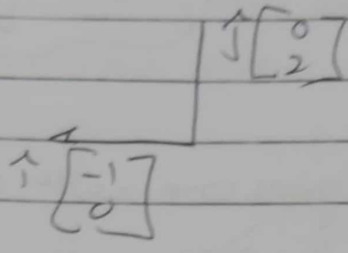
Matrix after finding λ

Complete by Row echelon or Cramers rule.

- If there are no eigen vectors, value of λ comes out to be imaginary.
- It's possible to have one eigen ~~vector~~ value and multiple eigen vectors, if scaling/squishing factor is same but vector ~~doesn't~~ change the orientations.

Eigen bases

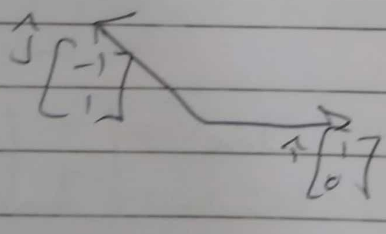
What if our basis vectors are our eigen vectors.



$$\begin{bmatrix}-1 & 0\\0 & 2\end{bmatrix}$$

Eigen values on diagonals, every other value is 0.

→ If our transformation has enough eigen vectors such that we can find a set ~~such~~ which spans the full space, then we can change the co-ordinate system such that these eigen vectors are our basis vectors.



$$\begin{bmatrix}3 & 1\\0 & 2\end{bmatrix}$$

$$\therefore \underbrace{\begin{bmatrix}1 & -1\\0 & 1\end{bmatrix}^{-1}\begin{bmatrix}3 & 1\\0 & 2\end{bmatrix}\begin{bmatrix}1 & -1\\0 & 1\end{bmatrix}}_{\text{Change of basis}} = \underbrace{\begin{bmatrix}3 & 0\\0 & 2\end{bmatrix}}_{\text{Diagonal matrix with 3, 2 as eigen value's}}$$

A set of vectors (basis vectors) which are also eigen ~~vector~~ vectors are called eigen basis.

- If we want to compute $\begin{bmatrix} 3 & 1 \\ 0 & 2 \end{bmatrix}^{100}$, we should change to an eigen basis, compute 100^{th} power in that system, then convert back to our system.

- A shear transformation, which don't have enough eigen vectors to span the whole space won't be able to do this.

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