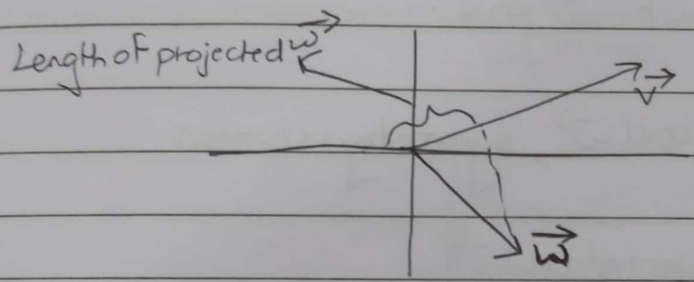
DOT PRODUCTS AND DUALITY

$$\begin{bmatrix} 1 \\ 2 \end{bmatrix} \cdot \begin{bmatrix} 3 \\ 4 \end{bmatrix} = 1\cdot 3 + 2\cdot 4 = 3 + 8 = \underline{11}$$

Dot product

Geometric interpretation:

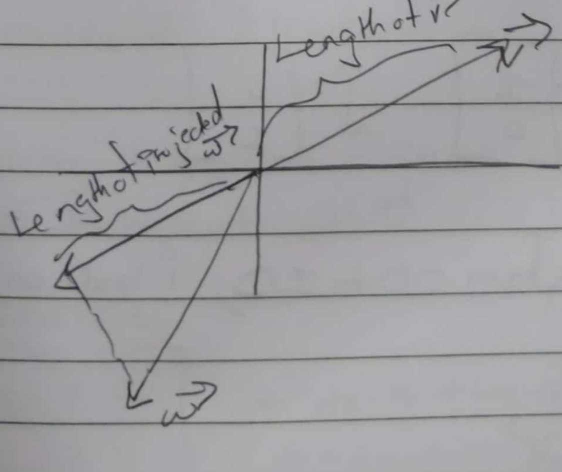


- Projection of $\vec{w}$ is pointing in same dim as $\vec{v}$

  $\vec{v}\cdot\vec{w} > 0$

- Projection of $\vec{w}$ is pointing in opposite dim as $\vec{v}$

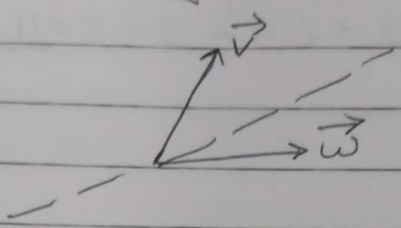
  $\vec{v}\cdot\vec{w} < 0$



- If $\vec{v} \perp \vec{w}$, $\vec{v}\cdot\vec{w} = 0$

Why $\vec{v}.\vec{w} = \vec{w}.\vec{v}$?

If $\text{length}(\vec{v}) = \text{length}(\vec{w})$



There is an axis of symettry due of which proj of $\vec{v}$ on $\vec{w}$ is same as proj of $\vec{w}$ on $\vec{v}$.

- If we have $2\vec{v}$ and $\vec{w}$, symmetry is broken.

I] If $\vec{w}$ is projected on $2\vec{v}$
II] If $2\vec{v}$ is projected on $\vec{w}$

Both have same effect,

$$(2\vec{v}).\vec{w} = 2(\vec{v}.\vec{w})$$

Duality

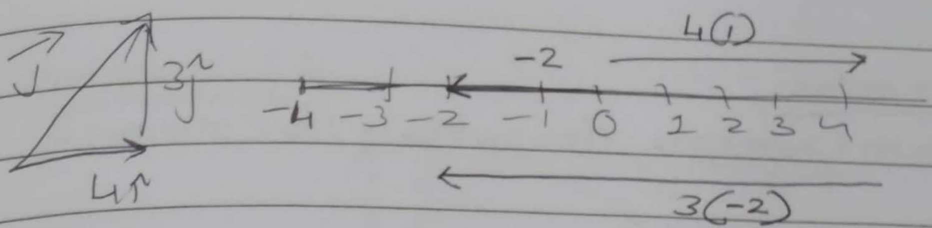
- $\vec{v} = \begin{bmatrix} 4 \\ 3 \end{bmatrix}$ $\hat{i} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ $\hat{j} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

After transformation from 2D to 1D, $\hat{i}$ lands on 1 and $\hat{j}$ lands on -2

$$\vec{v} = 4\hat{i} + 3\hat{j}$$

After transformation,

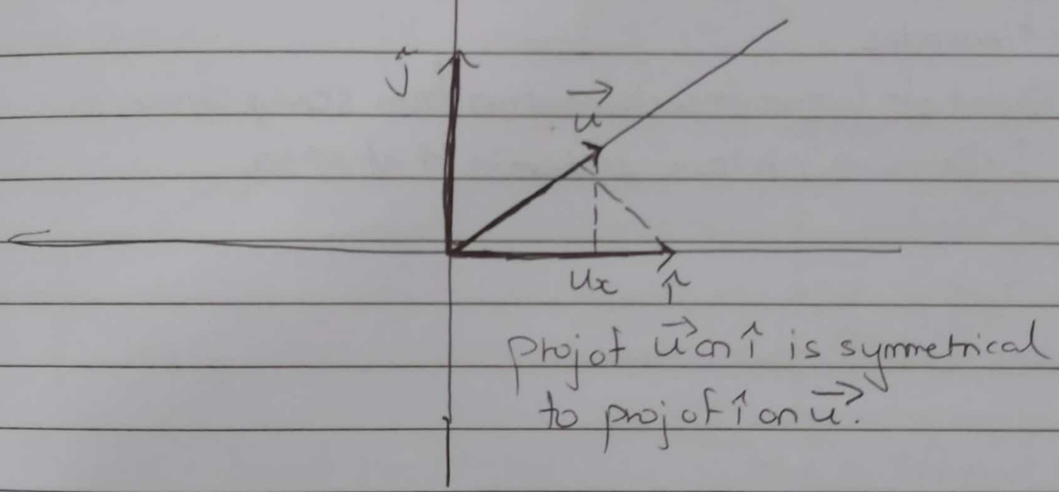
$$L(\vec{v}) = 4(1) + 3(-2) = \underline{\underline{-2}}$$



$\therefore$ Matrix multiplication of $\begin{bmatrix} 1 & -2 \end{bmatrix}\begin{bmatrix} 4 \\ 3 \end{bmatrix} = 4-6 = -2$

is same as taking their dot product $\begin{bmatrix} 1 \\ -2 \end{bmatrix}\begin{bmatrix} 4 \\ 3 \end{bmatrix} = 1\cdot 4 + 3\cdot -2 = -2$

- Consider 2-D vector space being squished onto a number line



$\hat{i}$ lands on u_x & $\hat{j}$ lands of u_y

For an arbitrary vector in 2-D space, its equivalent to taking its dot product with $\hat{u}$.

$$\overbrace{\begin{bmatrix} u_x & u_y \end{bmatrix}}^{\text{Transformation}}\overbrace{\begin{bmatrix} x \\ y \end{bmatrix}}^{\text{vector}} = u_x\cdot x + u_y\cdot y$$

$$\begin{bmatrix} u_x \\ u_y \end{bmatrix}\cdot\begin{bmatrix} x \\ y \end{bmatrix} = u_x\cdot x + u_y\cdot y$$

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Non-unit vectors

First we take projections, then multiply its components by 3.

$[3u_x \; 3u_y]$

* Whenever you have a 2-D to 1-D linear transformation, applying transformation is same as taking dot product with that vector.

This is called duality

- Dual of a vector is the linear transformation that it encodes
- Dual of linear transformation from some space to 1-D is a certain vector in that space