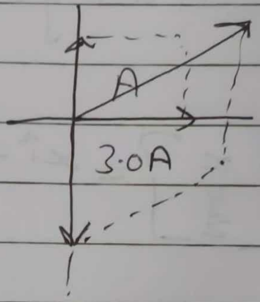
The Determinants

The special scaling factor, by which a linear transformation changes any area is called the 'Determinant' of that transformation.

- For eg. determinant of a transformation would be 3, if that transformation increases area of region by 3.

$$\det\left(\begin{bmatrix} 0.0 & 2.0 \\ -1.5 & 1.0 \end{bmatrix}\right) = 3.0$$

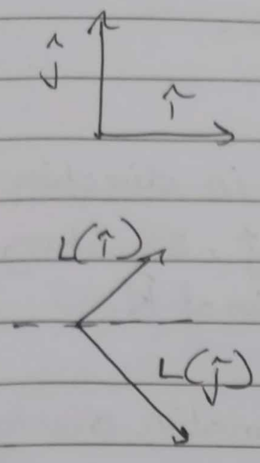


- Determinant of transformation would be 0.5, if it squishes down all areas by factor of 1/2.

- Determinant of 2-D transformation is 0, if it squishes all of space into a line, or single point

$$\det\left(\begin{bmatrix} 4 & 2 \\ 2 & 1 \end{bmatrix}\right) = 0$$

- The full concept of determinant allows for negative values, but what does scaling an area by negative amount mean? This has to do with idea of orientation.

→ 

- In starting positions, $\hat{j}$ hat ~~is~~ is to the left of $\hat{i}$, if after transformation, $\hat{j}$ is now on right of $\hat{i}$, orientation of space has been inverted.
- Whenever, orientation of space has been inverted, determinant will be -ve.
- The absolute value still gives us the scaling factor.

→ Why -ve determines orientation flipping?

As value of det ~~i~~ decreases, $\hat{i}$ comes closer to $\hat{j}$.
when det = 0, $\hat{i}$ overlaps $\hat{j}$
When $\hat{i}$ moves to Left of $\hat{j}$, det = -ve, orientation changes and det increases again in magnitude.

In 3-D

In 3-D, determinants gives us the volume that is squished or scaled.
It gives us volume of a parallelopiped = $\det\left(\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}\right)$

when det = 0, It means all space is squished to a single plane, line or point.
This means that,

$$\det\left(\begin{bmatrix} 1.0 & 0.0 & 1.0 \\ 0.5 & 1.0 & 1.5 \\ 1.0 & 0.0 & 1.0 \end{bmatrix}\right) = 0$$

Columns must be linearly dependent

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What does $\det(M) < 0$ mean?

Right hand rule

Point the forefinger of your right hand in direction of $\hat{i}$, your middle finger in direction of $\hat{j}$, so when you point your thumb up, it's in direction of $\hat{k}$.

If you can still do that after transformation, orientation is not changed, and det is +ve

IF after transformation, if it only makes sense to do with your left hand, orientation is flipped, and det is -ve.

$$\det\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = ad - bc$$

$$\det\left(\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}\right) = a\det\left(\begin{bmatrix} e & f \\ h & i \end{bmatrix}\right) - b\det\left(\begin{bmatrix} d & f \\ g & i \end{bmatrix}\right) + c\det\left(\begin{bmatrix} d & e \\ g & h \end{bmatrix}\right)$$

$$\det(M_1 \cdot M_2) = \det(M_1)\det(M_2)$$