Inverse Matrices, Column space and nullspace

Linear system of equations

$$2x + 3y + 5z = -3$$
$$4x + 0y + 8z = 0$$
$$x + 3y + 0z = 2$$

$$\underbrace{\begin{bmatrix} 2 & 3 & 5 \\ 4 & 0 & 8 \\ 1 & 3 & 0 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x \\ y \\ z \end{bmatrix}}_{\vec{x}} = \underbrace{\begin{bmatrix} -3 \\ 0 \\ 2 \end{bmatrix}}_{\vec{v}}$$

This means that the vector $\vec{x}$ after transformation by $\vec{A}$ lands on $\vec{v}$.

- When $|A| \neq 0$ (Area not squished to line or plane), we get 1 vector which will land on $\vec{v}$. To find that vector, we apply reverse transformation from $\vec{v}$ to $\vec{x}$. For this we use inverse of matrix A, denoted as A^{-1}!

- If A denotes clockwise rotation by $90°$, A^{-1} denotes anticlockwise rotation by $90°$
- If A is a rightward shear that pushes $\hat{j}$ 1 unit to right, A^{-1} will be a leftward shear that pushes it 1 unit to left.

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$$AA^{-1} = I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$A\vec{x} = \vec{v}$

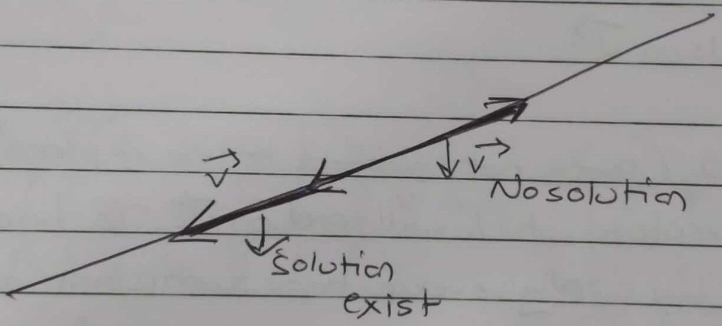
$A^{-1}A\vec{x} = A^{-1}\vec{v}$

$\vec{x} = A^{-1}\vec{v}$. To find $\vec{x}$.

- If $|A| = 0$

  The space is squished to line or point, and therefore we cannot use inverse transformation to make its space again same.

  Still solution can exist if $\vec{v}$ is on same squished line (after transformation).



☆ Rank

- When output of transformation is a line, we say it has a rank of 1.
- When output of transformation lands on a 2-D plane, we say it has rank of 2.
- "Rank" → No. of dimensions of output.

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- Highest possible rank of 2x2 matrix = 2
  3x3 matrix = 3

- For 3x3 matrix,
  if rank = 3, It is undeformed
  if rank = 2, It has squished into a plane
  if rank = 1, It has squished into a line

Column space

- It's the span of columns of a matrix i.e span of $\hat{i}$ and $\hat{j}$ axis or columns.

- Rank is the no. of dimensions in a column space.

$\begin{bmatrix} 0 \\ 0 \end{bmatrix}$ is always included in column space, i.e origin is always included.

- For matrices that aren't full rank, (which squishes after transformation), you can have bunch of vectors that land on 0.

Null space / Kernel

- If a 2-D transformation or 3-D transformation squishes to lower dimensions, there's a full line of vectors that squishes onto the origin.
- This set of vectors that lands on origin is called null space of matrix. It's a space of all vectors that become null i.e land on zero vector.
- In linear equations, $A\vec{x} = \vec{v}$, when $\vec{v} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$, null space gives all possible solutions to equation.