

Non-square matrices as transformations between dimensions

where i lands \rightarrow $\left[\begin{array}{cc} 3 & 1 \\ 4 & 1 \\ 5 & 9 \end{array} \right]$ \leftarrow where j lands

Transformations between dimensions:

$\underbrace{\begin{bmatrix} 2 \\ 7 \end{bmatrix}}_{2d \text{ input}} \rightarrow L(\vec{v}) \rightarrow \underbrace{\begin{bmatrix} 1 \\ 8 \\ 2 \end{bmatrix}}_{3d \text{ output}}$

$\cdot \underbrace{\begin{bmatrix} 2 & 0 \\ -1 & 1 \\ -2 & 1 \end{bmatrix}}$

Column space

\rightarrow The column space of this matrix, i.e. the place where all the vectors land is a 2-D plane slicing through origin of 3-D space. But matrix is still full rank, since no. of dimensions in column space is same as the no. of dimensions of input space.

$\rightarrow \underbrace{\begin{bmatrix} 3 & 1 \\ 4 & 1 \\ 5 & 9 \end{bmatrix}}$ It has geometric interpretation of mapping 2 dimensions to 3 dimensions, since 2 columns indicate that input space has 2 basis vectors and

three rows indicate that landing spots for each of those basis vectors is described with 3 separate coordinates.

3 basis vectors

$$\rightarrow \begin{bmatrix} 3 & 1 & 4 \\ 1 & 5 & 9 \end{bmatrix}_{2 \times 3} \left. \vphantom{\begin{bmatrix} 3 & 1 & 4 \\ 1 & 5 & 9 \end{bmatrix}} \right\} \begin{array}{l} 2 \text{ coordinates for} \\ \text{each landing spot.} \end{array}$$

3 columns indicate that you're starting in space that has 3-basis vectors, so we are starting in 3 dimensions and 2 rows indicate that landing spots for each of those 3 basis vectors is described with only two coordinates, so they land in 2 dimensions.

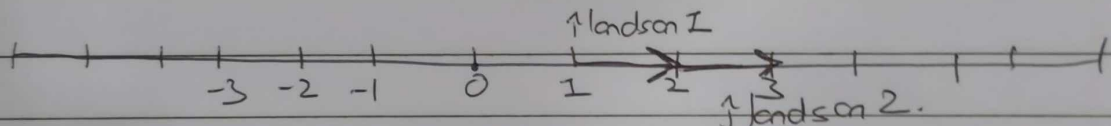
Therefore transformation \rightarrow 3-D space to 2-D plane.

\rightarrow 2-D to 1-D

$$\begin{bmatrix} 2 \\ 7 \end{bmatrix} \rightarrow L(W^*) [1.8]$$

1-d space is a number line.

Transformation matrix $\begin{bmatrix} 1 & 2 \end{bmatrix}$



Transformations take 2-D vectors and give output as numbers.

In transformations, Linearity is maintained as grids are parallel and evenly spaced.

\Rightarrow Linearity can be checked by imagining a line of evenly spaced dots on 2-D grid space, which must remain evenly spaced when mapped on number line.