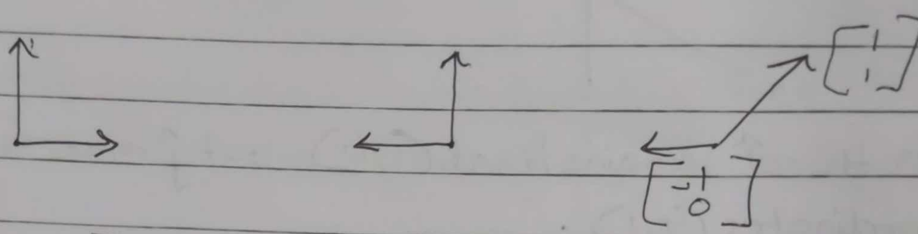
Matrix multiplication as composition

Performing two linear transformations continuously is known as "Composition"

Eg. Composition of rotation and shear



$$\begin{bmatrix} 1 & -1 \\ 1 & 0 \end{bmatrix} \rightarrow \text{Composition.}$$

$$\Rightarrow \underbrace{\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}}_{\text{Shear}} \left(\underbrace{\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}}_{\text{Rotation.}} \begin{bmatrix} x \\ y \end{bmatrix} \right)$$

$$= \underbrace{\begin{bmatrix} 1 & -1 \\ 1 & 0 \end{bmatrix}}_{\text{Composition}} \begin{bmatrix} x \\ y \end{bmatrix}$$

Read right to left $\longleftarrow$

$$\underbrace{\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}}_{\text{shear}} \underbrace{\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}}_{\text{Rotation}} = \underbrace{\begin{bmatrix} 1 & -1 \\ 1 & 0 \end{bmatrix}}_{\text{Composition}}$$

Eg.

$$\underbrace{\begin{bmatrix} 0 & 2 \\ 1 & 0 \end{bmatrix}}_{M_2} \underbrace{\begin{bmatrix} 1 & -2 \\ 1 & 0 \end{bmatrix}}_{M_1} = \begin{bmatrix} ? & ? \\ ? & ? \end{bmatrix}$$

First here

$$\begin{bmatrix} 0 & 2 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = 1 \begin{bmatrix} 0 \\ 1 \end{bmatrix} + 1 \begin{bmatrix} 2 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 & ? \\ 1 & ? \end{bmatrix}$$

Then 2^{nd} column.

$$\begin{bmatrix} 0 & 2 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} -2 \\ 0 \end{bmatrix} = -2 \begin{bmatrix} 0 \\ 1 \end{bmatrix} + 0 \begin{bmatrix} 2 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ -2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2 & 0 \\ 1 & -2 \end{bmatrix}$$

General

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} ? & ? \\ ? & ? \end{bmatrix}$$

i) $\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} e \\ g \end{bmatrix} = e \begin{bmatrix} a \\ c \end{bmatrix} + g \begin{bmatrix} b \\ d \end{bmatrix} = \begin{bmatrix} ea + gb \\ ec + gd \end{bmatrix}$

ii) $\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} f \\ h \end{bmatrix} = f \begin{bmatrix} a \\ c \end{bmatrix} + h \begin{bmatrix} b \\ d \end{bmatrix} = \begin{bmatrix} fa + bh \\ cf + dh \end{bmatrix}$

$$\Rightarrow \begin{bmatrix} ae + bg & af + bh \\ ce + dg & cf + dh \end{bmatrix}$$

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* Order Matters

Rotation then shear $\neq$ shear then rotation

$M_1 M_2 \neq M_2 M_1$

Matrix multiplication is associative

$A(BC) = (AB)C$ ✓