



AN ANALYSIS OF A RANKING OF CHILDREN'S DIAPERS

The French magazine *60 millions de consommateurs* in its edition of August 2018, number 540, has published, after comparative tests, a ranking of several brands of diapers for children. The results of their investigations are given by Figure 1 below :

We try to analyze this ranking made with two criteria *The Performance* and *The Composition* with their respective weights $w_p = 60\%$ and $w_c = 40\%$ ¹. All these information are given in the following Table 1.

	Performance	Composition	Global score (/20)
A- Joone	+++	+++	17
B- Pamp. Prem	++	++	14.5
C- Pamp. Baby	+	+++	12.5
D- Naty	+	+++	12.5
E- Pamp. Activ.	+	+	12.5
F- Carref. Baby	++	+	12.5
G- Lupilu	++	—	12
H- Mots d'enfants	+	—	12
I- Love & Green	++	—	9.5
K- Lotus Baby	++	--	9.5
L- Pommette	++	--	9.5
M- Lillydoo	+	--	6.5
	$w_p = 60\%$	$w_c = 40\%$	

Very good : $+++ \in [17, 20]$

Good : $++ \in [13, 16.5]$

Acceptable : $+ \in [10, 12.5]$

Insufficient : $- \in [7, 9.5]$

Very Insufficient : $-- \in [0, 6.5]$

TABLE 1 – A performance table of children's diapers

1 The overall score (/20) of each diaper could be explain by a weighted sum ?

We associate to each qualitative evaluation a real number which will be computed trough a suitable linear program. Hence for the diaper a (diaper "Joone"), if we assume that its score is obtained by using a weighted sum f , then there exist non-negative real numbers $U_{1a}(+++)$ and $U_{2a}(+++)$ such that :

$$f(a) = 0.6 U_{1a}(+++)+0.4 U_{2a}(+++)=17. \quad (1)$$

More generally, given a product x , if its global score is assumed to be obtain by a weighted sum f , then its evaluation is given by the following equation

$$f(x)=0.6 U_{1x}(\alpha_{1x})+0.4 U_{2x}(\alpha_{2x}) \quad (2)$$

1. We assume that the evaluations on each criterion are correctly obtained by the French magazine.

where $U_{ix}(\alpha_{ix})$ is the non-negative number associated to the qualitative evaluation α_{ix} of x on the criterion i (the qualitative value α_{ix} being given by the French magazine).

To determine if the ranking produced by the magazine is compatible with a weighted sum, it is sufficient to solve the following linear program PL_1 :

$$\max \quad \epsilon \quad (3)$$

Such that

$$0.6 U_{1a}(+++) + 0.4 U_{2a}(+++) = f^a \quad (4)$$

$$0.6 U_{1b}(++) + 0.4 U_{2b}(++) = f^b \quad (5)$$

$$0.6 U_{1c}(+) + 0.4 U_{2c}(+++) = f^c \quad (6)$$

$$0.6 U_{1d}(+) + 0.4 U_{2d}(+++) = f^d \quad (7)$$

$$0.6 U_{1e}(+) + 0.4 U_{2e}(+) = f^e \quad (8)$$

$$0.6 U_{1f}(++) + 0.4 U_{2f}(+) = f^f \quad (9)$$

$$0.6 U_{1g}(++) + 0.4 U_{2g}(-) = f^g \quad (10)$$

$$0.6 U_{1h}(+) + 0.4 U_{2h}(-) = f^h \quad (11)$$

$$0.6 U_{1i}(++) + 0.4 U_{2i}(-) = f^i \quad (12)$$

$$0.6 U_{1j}(++) + 0.4 U_{2j}(--) = f^j \quad (13)$$

$$0.6 U_{1k}(++) + 0.4 U_{2k}(--) = f^k \quad (14)$$

$$0.6 U_{1l}(+) + 0.4 U_{2l}(--) = f^l \quad (15)$$

$$\epsilon \geq 0 \quad (16)$$

(PL_1)

$$f^a \geq f^b + \epsilon; f^b \geq f^c + \epsilon; f^c = f^d \quad (17)$$

$$f^d = f^e; f^e = f^f; f^f \geq f^g + \epsilon; \quad (18)$$

$$f^g = f^h; f^h \geq f^i + \epsilon; f^i = f^j \quad (19)$$

$$f^j = f^k; f^k \geq f^l + \epsilon \quad (20)$$

$$17 \leq U_{1a}(+++) \leq 20; 17 \leq U_{2a}(+++) \leq 20 \quad (21)$$

$$17 \leq U_{2c}(+++) \leq 20; 17 \leq U_{2d}(+++) \leq 20 \quad (22)$$

$$13 \leq U_{1b}(++) \leq 16.5; 13 \leq U_{2b}(++) \leq 16.5; 13 \leq U_{1f}(++) \leq 16.5 \quad (23)$$

$$13 \leq U_{1g}(++) \leq 16.5; 13 \leq U_{1i}(++) \leq 16.5 \quad (24)$$

$$13 \leq U_{1j}(++) \leq 16.5; 13 \leq U_{1k}(++) \leq 16.5 \quad (25)$$

$$10 \leq U_{1c}(+) \leq 12.5; 10 \leq U_{1d}(+) \leq 12.5; 10 \leq U_{1e}(+) \leq 12.5 \quad (26)$$

$$10 \leq U_{1e}(+) \leq 12.5; 10 \leq U_{1f}(+) \leq 12.5; 10 \leq U_{1h}(+) \leq 12.5 \quad (27)$$

$$10 \leq U_{1l}(+) \leq 12.5 \quad (28)$$

$$7 \leq U_{2g}(-) \leq 9.5; 7 \leq U_{2h}(-) \leq 9.5; 7 \leq U_{2i}(-) \leq 9.5 \quad (29)$$

$$0 \leq U_{2j}(--) \leq 6.5; 0 \leq U_{2k}(--) \leq 6.5; 0 \leq U_{2l}(--) \leq 6.5 \quad (30)$$

where

- The objective function (see (3)) maximizes the value of ϵ which ensure to have strict preferences between some diapers. Hence,

if the optimal solution is $\epsilon > 0$, then the linear find a weighted sum model compatible with the preferences of the magazine. If the program is not feasible or the optimal solution is $\epsilon = 0$, then there is no weighted sum model compatible with the preferences of the magazine.

- The constraints (4) to (15) correspond to the evaluation of each diaper according to the Equation 2 above.
 - The ranking produced by the French magazine is given by the constraints (17) to (20).
 - The constraints (21) to (30) ensure the compatibility of the numerical value $U_{ix}(\alpha_{ix})$ (associated to the qualitative value α_{ix}) with the scale given by the magazine.
1. Compute a python function `CheckAdditiveModel_1` checking if the ranking produced by the magazine, with their given global score, i.e.

$$\begin{aligned} f^a &= 17, f^b = 14.5, f^c = 12.5, f^d = 12.5, f^e = 12.5, \\ f^f &= 12.5, f^g = 12, f^h = 12, f^i = 9.5, f^j = 9.5, \\ f^k &= 9.5, f^l = 6.5. \end{aligned}$$

is compatible with a weighted sum model.

2. In this question, we do not take into account the precise global score given by the magazine.
 - 2.1 Compute a python function `CheckAdditiveModel_2` checking if the ranking produced by the magazine is compatible with a weighted sum model.
 - 2.2 Remove in the linear program PL_1 the constraints $f^d = f^e$ (18), $f^i = f^j$ (19) and $17 \leq U_{2c}(+++)$. We keep the ranking given by the magazine.

Compute a python function `CheckAdditiveModel_3` returning the existence of the weighted sum model and the global score of each diaper when the objective function is

- i. $\max f^a$
- ii. $\min f^a$
- iii. $\max f^l$
- iv. $\min f^l$

Are some diapers improved their global score compared to their global score given by the magazine ?

3. Now we do not consider both the ranking of the diapers and their global score given by the magazine. We remove also the constraint $17 \leq U_{2c}(+++)$, related to the diaper *c-Pamp.Baby*.

- 3.1 Determine the maximal and minimal global score obtained by each diaper by solving the corresponding linear program PL_1 .
- 3.2 Compare the rankings obtained with the original ranking given by the magazine ? You can compute the python function `CompareRankings` returning for instance the Kendall rank correlation coefficient (Kendall's tau coefficient) of two rankings.






<https://www.statisticshowto.datasciencecentral.com/kendalls-tau/> https://en.wikipedia.org/wiki/Kendall_rank_correlation_coefficient

✓ Les résultats de notre essai

Couches-culottes

- +++ Très bon 20 à 17
- ++ Bon 16,5 à 13
- + Acceptable 12,5 à 10
- Insuffisant 9,5 à 7
- 0 Très insuffisant 6,5 à 0

Les pourcentages entre parenthèses expriment le poids de chaque critère dans la notation finale.

					
• Label bio	Non	Non	Non	FSC/AB Vincotte UE	Non
• Prix indicatif	64,90 € ⁽¹⁾ 162 couches	12,60 € 50 couches	15,60 € 50 couches	18,90 € 50 couches	12,30 € 46 couches
• Prix pour une couche	0,40 €	0,25 €	0,31 €	0,38 €	0,27 €
Performances (60 %)	+++	++	+	+	+
• Tenue	+++	++	++	++	++
• Absorption	+++	++	+	+	+
• Protection contre l'humidité	+++	++	+	-	+
Composition (40 %)	+++	++	+++	+++	+
Pesticides					
• Résidu du glyphosate	+++	+++	+++	+++	+++
• Pesticides organochlorés	+++	+++	+++	+++	+
Autres molécules toxiques potentielles					
• Dioxines	+++	+++	+++	+++	+++
• Composés organiques volatils (COV)	+++	++	+++	+++	+++
• Composés organiques halogénés adsorbables (AOX)	+++	+++	+++	+++	+++
• Allergènes	+++	+++	+++	+++	+++
NOTE GLOBALE (100 %)	17/20	14,5/20	12,5/20 ⁽²⁾	12,5/20 ⁽²⁾	12,5/20

(1) Livraison comprise dans le prix. (2) L'appréciation globale ne peut pas être supérieure à l'appréciation sur les performances. (3) Le fabricant indique que cette référence est en fin de commercialisation. (4) L'appré-

						
Non	FSC	PEFC	FSC	FSC	FSC/Nordic ecolabel	Non
10,80 € 56 couches	7,30 € 56 couches	8,90 € 50 couches	19,65 € 52 couches	19 € 58 couches	9 € 32 couches	12 € 33 couches
0,19 €	0,13 €	0,18 €	0,38 €	0,33 €	0,28 €	0,36 €
++	++	+	++	++	++	+
++	++	++	++	++	++	+
++	++	++	++	++	++	++
++	++	+	++	+	+	+
+	-	-	-	-	-	-
+++	+++	+++	-	-	-	-
+	+++	+++	+++	+++	+++	-
+++	+++	+++	+++	+++	+++	+++
++	++	-	+++	+++	+++	++
+++	-	+++	+++	+++	-	+++
+++	+++	+++	+++	+++	+++	+++
12,5/20	12/20	12/20	9,5/20 ⁽⁴⁾	9,5/20 ⁽⁴⁾	9,5/20 ⁽⁴⁾	6,5/20 ⁽⁴⁾

ciation globale ne peut pas être supérieure à "insuffisante" ou "Très insuffisante" en fonction de la composition du produit.

FIGURE 1 – A ranking of children's diapers published by the French magazine 60 millions de consommateurs