CentraleSupélec

MASTER DSBI
DECISION MODELING
PRACTICAL WORKS

AN ANALYSIS OF A RANKING OF CHILDREN'S DIAPERS

The French magazine *60 millions de consommateurs* in its edition of *August 2018, number 540*, has published, after comparative tests, a ranking of several brands of diapers for children. The results of their investigations are given by Figure 1 below :

We try to analyze this ranking made with two criteria *The Performance* and *The Composition* with their respective weights $w_p = 60\%$ and $w_c = 40\%$ [1]. All these information are given in the following Table 1.

	Performance	Composition	Global score (/20)
A- Joone	+++	+++	17
B- Pamp. Prem	++	++	14.5
C- Pamp. Baby	+	+++	12.5
D- Naty	+	+++	12.5
E- Pamp. Activ.	+	+	12.5
F- Carref. Baby	++	+	12.5
G- Lupilu	++	−	12
H- Mots d'enfants	+	−	12
I- Love & Green	++	−	9.5
K- Lotus Baby	++	−−	9.5
L- Pommette	++	−−	9.5
M- Lillydoo	+	−−	6.5
	$w_p = 60\%$	$w_c = 40\%$	

Very good :	$+++ \in [17, 20]$
Good :	$++ \in [13, 16.5]$
Acceptable :	$+ \in [10, 12.5]$
Insufficient :	$- \in [7, 9.5]$
Very Insufficient :	$-- \in [0, 6.5]$

TABLE 1 – A performance table of children's diapers

1 The overall score (/20) of each diaper could be explain by a weighted sum ?

We associate to each qualitative evaluation a real number which will be computed trough a suitable linear program. Hence for the diaper a (diaper "Joone"), if we assume that its score is obtained by using a weighted sum f, then there exist non-negative real numbers $U_{1a}(+++) \in [17, 20]$ and $U_{2a}(+++) \in [17, 20]$ such that :

$$f(a) = 0.6\, U_{1a}(+++) + 0.4\, U_{2a}(+++) = 17. \tag{1}$$

More generally, given a product x, if its global score is assumed to be obtain by a weighted sum f, then its evaluation is given by the following equation

$$f(x) = 0.6\, U_{1x}(\alpha_{1x}) + 0.4\, U_{2x}(\alpha_{2x}) \tag{2}$$

1. We assume that the evaluations on each criterion are correctly obtained by the French magazine.

where $U_{ix}(\alpha_{ix})$ is the non-negative number associated to the qualitative evaluation α_{ix} of x on the criterion i (the qualitative value α_{ix} being given by the French magazine).

To determine if the ranking produced by the magazine is compatible with a weighted sum, it is sufficient to solve the following linear program PL_1 :

$$
\begin{aligned}
&\max \quad \epsilon && (3)\\
&\text{Such that}\\
&\quad 0.6\ U_{1a}(\text{+++}) + 0.4\ U_{2a}(\text{+++}) = f^a && (4)\\
&\quad 0.6\ U_{1b}(\text{++}) + 0.4\ U_{2b}(\text{++}) = f^b && (5)\\
&\quad 0.6\ U_{1c}(\text{+}) + 0.4\ U_{2c}(\text{+++}) = f^c && (6)\\
&\quad 0.6\ U_{1d}(\text{+}) + 0.4\ U_{2d}(\text{+++}) = f^d && (7)\\
&\quad 0.6\ U_{1e}(\text{+}) + 0.4\ U_{2e}(\text{+}) = f^e && (8)\\
&\quad 0.6\ U_{1f}(\text{++}) + 0.4\ U_{2f}(\text{+}) = f^f && (9)\\
&\quad 0.6\ U_{1g}(\text{++}) + 0.4\ U_{2g}(-) = f^g && (10)\\
&\quad 0.6\ U_{1h}(\text{+}) + 0.4\ U_{2h}(-) = f^h && (11)\\
&\quad 0.6\ U_{1i}(\text{++}) + 0.4\ U_{2i}(-) = f^i && (12)\\
&\quad 0.6\ U_{1j}(\text{++}) + 0.4\ U_{2j}(--) = f^j && (13)\\
&\quad 0.6\ U_{1k}(\text{++}) + 0.4\ U_{2k}(--) = f^k && (14)\\
&\quad 0.6\ U_{1l}(\text{+}) + 0.4\ U_{2l}(--) = f^l && (15)\\
&\quad \epsilon \geq 0 && (16)\\
(\text{PL}_1) \quad &\quad f^a \geq f^b + \epsilon; f^b \geq f^c + \epsilon; f^c = f^d && (17)\\
&\quad f^d = f^e; f^e = f^f; f^f \geq f^g + \epsilon; && (18)\\
&\quad f^g = f^h; f^h \geq f^i + \epsilon; f^i = f^j && (19)\\
&\quad f^j = f^k; f^k \geq f^l + \epsilon && (20)\\
&\quad 17 \leq U_{1a}(\text{+++}) \leq 20; 17 \leq U_{2a}(\text{+++}) \leq 20 && (21)\\
&\quad 17 \leq U_{2c}(\text{+++}) \leq 20; 17 \leq U_{2d}(\text{+++}) \leq 20 && (22)\\
&\quad 13 \leq U_{1b}(\text{++}) \leq 16.5; 13 \leq U_{2b}(\text{++}) \leq 16.5; 13 \leq U_{1f}(\text{++}) \leq 16.5 && (23)\\
&\quad 13 \leq U_{1g}(\text{++}) \leq 16.5; 13 \leq U_{1i}(\text{++}) \leq 16.5 && (24)\\
&\quad 13 \leq U_{1j}(\text{++}) \leq 16.5; 13 \leq U_{1k}(\text{++}) \leq 16.5 && (25)\\
&\quad 10 \leq U_{1c}(\text{+}) \leq 12.5; 10 \leq U_{1d}(\text{+}) \leq 12.5; 10 \leq U_{1e}(\text{+}) \leq 12.5 && (26)\\
&\quad 10 \leq U_{1e}(\text{+}) \leq 12.5; 10 \leq U_{1f}(\text{+}) \leq 12.5; 10 \leq U_{1h}(\text{+}) \leq 12.5 && (27)\\
&\quad 10 \leq U_{1l}(\text{+}) \leq 12.5 && (28)\\
&\quad 7 \leq U_{2g}(-) \leq 9.5; 7 \leq U_{2h}(-) \leq 9.5; 7 \leq U_{2i}(-) \leq 9.5 && (29)\\
&\quad 0 \leq U_{2j}(--) \leq 6.5; 0 \leq U_{2k}(--) \leq 6.5; 0 \leq U_{2l}(--) \leq 6.5 && (30)
\end{aligned}
$$

where

- The objective function (see (3)) maximizes the value of ϵ which ensure to have strict preferences between some diapers. Hence,

if the optimal solution is $\epsilon > 0$, then the linear find a weighted sum model compatible with the preferences of the magazine. If the program is not feasible or the optimal solution is $\epsilon = 0$, then there is no weighted sum model compatible with the preferences of the magazine.

- The constraints (4) to (15) correspond to the evaluation of each diaper according to the Equation 2 above.
- The ranking produced by the French magazine is given by the constraints (17) to (20).
- The constraints (21) to (30) ensure the compatibility of the numerical value $U_{ix}(\alpha_{ix})$ (associated to the qualitative value α_{ix}) with the scale given by the magazine.

1. Compute a python function `CheckAdditiveModel_1` checking if the ranking produced by the magazine, with their given global score, i.e.

$$f^a = 17,\ f^b = 14.5,\ f^c = 12.5,\ f^d = 12.5,\ f^e = 12.5,$$
$$f^f = 12.5,\ f^g = 12,\ f^h = 12,\ f^i = 9.5,\ f^j = 9.5,$$
$$f^k = 9.5,\ f^l = 6.5.$$

   is compatible with a weighted sum model.

2. In this question, we do not take into account the precise global score given by the magazine.

   2.1 Compute a python function `CheckAdditiveModel_2` checking if the ranking produced by the magazine is compatible with a weighted sum model.

   2.2 Remove in the linear program PL_1 the constraints $f^d = f^e$ (18), $f^i = f^j$ (19) and $17 \leq U_{2c}(\text{+++})$. We keep the ranking given by the magazine.

   Compute a python function `CheckAdditiveModel_3` returning the existence of the weighted sum model and the global score of each diaper when the objective function is

   i. $\max f^a$

   ii. $\min f^a$

   iii. $\max f^l$

   iv. $\min f^l$

   Are some diapers improved their global score compared to their global score given by the magazine ?

3. Now we do not consider both the ranking of the diapers and their global score given by the magazine. We remove also the constraint $17 \leq U_{2c}(\text{+++})$, related to the diaper $c\text{-}Pamp.Baby$.

   3.1 Determine the maximal and minimal global score obtained by each diaper by solving the corresponding linear program PL_1.

   3.2 Compare the rankings obtained with the original ranking given by the magazine ? You can compute the python function `CompareRankings` returning for instance the Kendall rank correlation coefficient (Kendall's tau coefficient) of two rankings.

   `https://www.statisticshowto.datasciencecentral.com/kendalls-tau/` `https://en.wikipedia.org/wiki/Kendall_rank_correlation_coefficient`

FIGURE 1 – A ranking of children's diapers published by the French magazine *60 millions de consommateurs*