

TERM PROJECT REPORT

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CHE 312: HEAT TRANSFER AND ITS APPLICATIONS

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at

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PROBLEM STATEMENT:

An aluminium heat sink is to be used to maintain the surface temperature of a 15 mm x 15 mm CPU below 80°C. The CPU has a peak power requirement of 7.5 W. The base of the heat sink can be assumed to be of the same area and at the same temperature as the CPU surface, and parallel straight rectangular vertical fins are to be attached to this base. Determine the number of fins and the length of each fin that minimizes aluminium required if each fin has to be 2 mm thick.

I. INTRODUCTION

Convection is the mode of energy transfer between a solid surface and the adjacent liquid or gas that is in motion, and it involves the combined effects of conduction and fluid motion. The faster the fluid motion, the greater the convection heat transfer. In the absence of any bulk fluid motion, heat transfer between a solid surface and the adjacent fluid is by pure conduction. The presence of bulk motion of the fluid enhances the heat transfer between the solid surface and the fluid, but it also complicates the determination of heat transfer rates. Convection is classified as natural (or free) and forced convection, depending on how the fluid motion is initiated. In forced convection, the fluid is forced to flow over a surface or in a pipe by external means such as a pump or a fan. In natural convection, any fluid motion is caused by natural means, such as the buoyancy effect, which manifests itself as the rise of warmer fluid and the fall of the cooler fluid. Convection is also classified as external and internal, depending on whether the fluid is forced to flow over a surface or in a pipe.

II. NATURAL CONVECTION

Natural Convection, also called free convection, occurs as a result of density differences between the heated fluid and cold fluid, which bring about convection currents so as to transfer the heat from one point to another. Applications of natural convection are in many chemical industries and food industries. Also, solar applications of heating fluid work on natural convection mechanism. Several experimental and numerical studies on natural convection in nanofluids have used different geometries.

III. FORCED CONVECTION

Forced convection heat transfer refers to a process in which fluid flow generated by external means, such as a fan or an externally imposed pressure gradient, extracts or imparts heat from or to objects at temperatures different from the flow temperatures. Forced convection is commonly used in applications such as cooling of electronic components by fans, heating of homes using forced air blowers, cooling of oil and coolants in vehicles using externally forced air, etc.

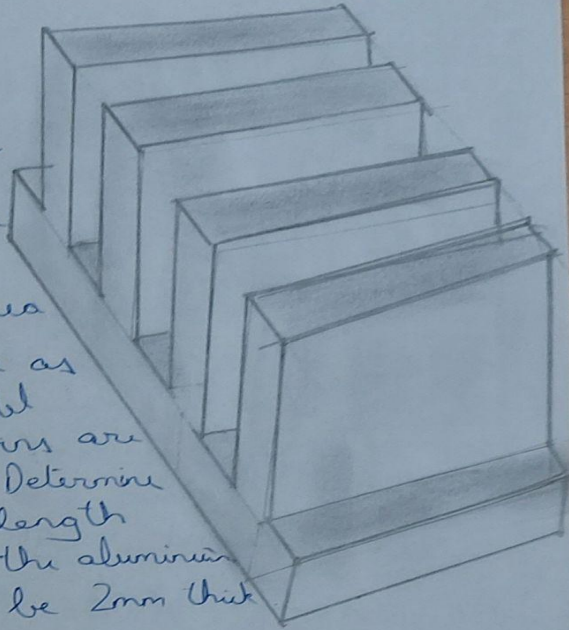
IV. APPROACH

For the aforesaid mentioned question, initially, we tried implementing the concept of natural convection.

L = length of each fin, n = number of fins

NATURAL CONVECTION BASED APPROACH -

Q An aluminium heat sink is to be used to maintain the surface temperature of a $15\text{mm} \times 15\text{mm}$ CPU below 80°C . The CPU has a peak power requirement of 7.5W . The base of the heat sink can be assumed to be of the same area and at the same temperature as the CPU surface, and parallel straight rectangular vertical fins are to be attached to this base. Determine the number of fins and the length of each fin that minimizes the aluminium required, if each fin has to be 2mm thick.



Soln

Considering it to be a case of Natural Convection
Data given :

$$\textcircled{Q} (\text{Peak Power}) = 7.5\text{W}$$

$$t = 2 \times 10^{-3}\text{m}$$

$$W = 15 \times 10^{-3}\text{m}$$

$$T_s = 80^\circ\text{C}$$

$$T_\infty = 25^\circ\text{C} \text{ (room temperature)}$$

$$\frac{T_s + T_\infty}{2} = \underline{52.5^\circ\text{C}}$$

$$S = \frac{W}{N} - t$$

(spacing
b/w two
fins)

$$\therefore (\rho_{\text{air}})_{52.5^\circ\text{C}} = 1.084\text{kg/m}^3$$

Data taken from Yunus A. Cengel for air at 52.5°C

$$h_c = 0.733225$$

$$\nu = 19.5 \times 10^{-6}$$

$$(k)_{\text{air at } 52.5^\circ\text{C}} = 28 \times 10^{-3} \frac{\text{W}}{\text{mK}}$$

$$\beta = \frac{2}{T_s + T_\infty} = \frac{2}{325.5}$$

For ~~forced~~ ^{natural} convection

$$\dot{Q} = Nh_1(T_s - T_\infty)LW + h_2[(0.015)^2 - Nx \times W](T_s - T_\infty)$$

$$h_1 = \frac{k}{s} \left[\frac{576}{(Ra_s \frac{s}{L})^2} + \frac{2.873}{(Ra_s \frac{s}{L})^{0.5}} \right]^{-\frac{1}{2}}$$

$$Ra_s = \frac{g\beta(T_s - T_\infty)s^3 \rho_r}{\nu^2}$$

Since the above equations are quite complex we used MATLAB, the code is attached with the report.

The two terms involved in the expression of \dot{Q} comes from the part of finned and unfinned region both.

The results that we obtained in the aforesaid mentioned case with natural convection ~~are~~ were quite abnormal as the length of the fin that we got was of the order of ~~the~~ very large insignificant figures.

So we decided to consider the case of forced convection with a CPU fan installed ~~at~~ with typical flow rate of air as $20 \frac{\text{ft}^3}{\text{min}}$.

The screenshot shows the MATLAB R2021a web interface. The top bar includes navigation icons and the URL 'matlab.mathworks.com'. The main toolbar has tabs for HOME, PLOTS, APPS, EDITOR, PUBLISH, FILE VERSIONS, and VIEW. Below the toolbar is a search bar and a set of icons for file operations (New, Open, Save, Go To, Find, Bookmark), code editing (Refactor, Run, Run and Advance, Run to End), and execution (Run, Step, Stop). The workspace on the left shows the current folder and a list of files: f.m, specvol.m, vpequilm.m, rhs.m, simple.m, rate1.m, run_rate1.m, Computer_Assignment_2.m, che312.m, and slv.m. The editor window displays the following MATLAB code:

```
1 function q = che312(1)
2     n = 7;
3     g = 9.8;
4     Pr = 0.733225;
5     nu = 19.5e-6;
6     ts = 273 + 80;
7     tinf = 273 + 25;
8     t = 0.002;
9     w = 0.015;
10    k = 240;
11    beta = 2/(ts + tinf);
12    s = w/n - t;
13    if(1 < 0)
14        q = che312(0.0001);
15    else
16        Ra = g*beta*(ts - tinf)*(1^3)*Pr/(nu ^ 2);
17        h = (k/s)*( 576/((Ra * s/l)^2) + 2.873/((Ra * s/l)^0.5) )^(-0.5);
18        Ac = w * w - n * w * t;
19        p = 2*(w - n*t) + 2*n*w;
20        lc = Ac/p;
21        alpha = 2.109e-4;
22        Ra2 = g*beta*(ts - tinf)* lc^3/(nu * alpha);
23        if(Ra2 < 10^9)
24            Nu = 0.59*Ra2^(1/4);
25        else
26            Nu = 0.1*Ra2^(1/3);
27
28        h2 = Nu*k/lc;
29        q = n*h*(ts - tinf)*l*w + h2*(0.015*0.015-n*t*w)*(ts - tinf) - 7.5;
30    end
31    %x = fsolve(@che312, 0.1)
32    %end
```

The Command Window shows the result of the calculation:

```
ans =
0.1677
>>
```

The status bar at the bottom indicates the file is 'che312.m' at line 22, column 48, using UTF-8 encoding and CRLF line endings.

As mentioned in the solution, the results we received were quite abnormal as the length of the fin that we got were of very large orders of magnitude. So we decided to consider the case of forced convection with a 120mm diameter CPU fan installed with a typical air flow rate of 20 ft³/min (0.836 m/sec).

FORCED CONVECTION BASED APPROACH

The solution for forced convection is as follows:

$$T_s = 273 + 80 = 353 \text{ K}$$

$$T_\infty = 273 + 25 = 298 \text{ K} \leftarrow \begin{array}{l} \text{Ambient} \\ \text{Temperature} \end{array}$$

$$Q = 7.5 \text{ W} \leftarrow \text{peak power requirement}$$

$$\beta = \frac{1}{\left(\frac{T_s + T_\infty}{2}\right)} ; \frac{T_s + T_\infty}{2} = 325.5 \text{ K} = 52.5^\circ \text{C}.$$

$$W (\text{width of each fin}) = 0.015 \text{ m}$$

$$t (\text{thickness of each fin}) = 0.002 \text{ m}$$

$$u = 0.836 \text{ m/s}$$

$$\rho = 1.084 \text{ kg/m}^3$$

μ, β are considered at 52.5°C .

$$W = 0.015 \text{ m}$$

$$\mu = 1.97 \times 10^{-5} \text{ kg/m.s.}$$

$$A_s, \text{ Surface area of each fin} = L \times W$$

For estimating the average flow velocity of air in a CPU, we used standard air flow rates for 120 mm diameter fans.

$$\text{Typical flow rate of air, } V_{\text{air}} = 20 \text{ ft}^3/\text{min}$$

From this, we get an estimate of air velocity as

$$u = \frac{V_{\text{air}} \times 4}{\pi (0.12)^2} = 0.836 \text{ m/s}.$$

$$\text{Reynold's number, } Re = \frac{\rho u L}{\mu} = \frac{1.084 \times 0.836 \times L}{1.974 \times 10^{-5}}$$

$$\Rightarrow Re = 45908 L$$

Since, critical value of Re for a flat plate is 5×10^5 , that implies that for the flow to be turbulent, we should have

$$L > 10 \text{ m, which is highly impractical for a CPU fin.}$$

So, to begin with, we can safely assume the flow of air to be laminar over the fins.

For the flow of air over the unfinned horizontal plate region:

$$u = 0.836 \text{ m/s}$$

$$\rho = 1.084 \text{ kg/m}^3$$

$$w = 0.015 \text{ m}$$

$$\eta = 1.97 \times 10^{-5} \text{ kg/m.s}$$

$$\Rightarrow Re_2 = \frac{\rho u w}{\eta} = 688.6 < 5 \times 10^5; \text{ so this flow is also laminar.}$$

For laminar flow over a flat plate (forced convection), we have

$$Nu = \frac{hL}{k} = 0.664 Re^{0.5} Pr^{1/3}$$

$$\Rightarrow h = \frac{k}{L} \times 0.664 Re^{0.5} Pr^{1/3} \text{ (for flow parallel to fins)}$$

$$\text{Similarly, } h_2 = \frac{k}{w} \times 0.664 Re_2^{0.5} Pr^{1/3} \left[\begin{array}{l} \text{for flow over the unfinned region} \\ \text{of the horizontal base of the heat} \\ \text{Sink} \end{array} \right]$$

We have,

$$Q_s = h \times 2nLw \times (T_s - T_\infty) + h_2 \times (Cw \times w - n \times w \times t) (T_s - T_\infty)$$

Where n is the number of fins that we use.

Since, we have all other data except L in the above expression, we can easily solve for it using a MATLAB program.

The screenshot displays the MATLAB R2021a web interface. The top navigation bar includes tabs for HOME, PLOTS, APPS, EDITOR, PUBLISH, FILE VERSIONS, and VIEW. A search bar is located on the right. Below the navigation bar is a toolbar with icons for file operations (New, Open, Save), navigation (Go To, Find, Bookmark), code editing (Refactor, Run, Run and Advance, Run to End), and execution (Run, Step, Stop). The main workspace shows a list of open files: f.m, specvol.m, vpequil.m, rhs.m, simple.m, rate1.m, run_rate1.m, Computer_Assignment_2.m, che312.m, and slv.m. The 'slv.m' file is currently active, displaying the following MATLAB code:

```
1 function q = slv(1)
2     n = 3;
3     g = 9.8;
4     Pr = 0.733225;
5     nu = 19.5e-6;
6     ts = 273 + 80;
7     tinf = 273 + 25;
8     t = 0.002;
9     w = 0.015;
10    k = 28e-3;
11    rho = 1.084;
12    v = 0.836;
13    eta = 1.974e-5;
14    beta = 2/(ts + tinf);
15    s = w/n - t;
16    Re = rho*v*L/eta;
17    Nu = 0.664*(Re^0.5)*Pr^(1/3);
18    h2 = (k/w)*(Pr^(1/3))*0.664*(688.6^(0.5));
19    h = k*Nu/L;
20    if (1 < 0)
21        q = slv(0.0001);
22    else
23        q = n*h^2*L*w*(ts - tinf) + h2*(ts-tinf)*(w*w - n*w*t) - 7.5;
24    end
25    % if(1 < 0)
26    %     q = che312(1);
27    % else
28    %     Ra = g*beta*(ts - tinf)*(L^3)*Pr/(nu ^ 2);
```

Below the code editor is the Command Window, which shows the execution of the following commands and their results:

```
>> fzero('slv', 1)
ans =
    0.1677
>>
```

The status bar at the bottom indicates the file encoding is UTF-8, line endings are CRLF, and the current cursor position is at line 14, column 23.

We implemented the aforesaid MATLAB code for the case of forced convection and the results that we obtained are listed in the table below.

RESULTS

We got the following results for n and L using the MATLAB program for forced convection:

n	L (in meters)	Volume = $n \cdot L \cdot w \cdot t$ (in m^3)
1	1.4697	4.4091e-5
2	0.3724	2.2344e-5
3	0.1677	1.5093e-5
4	0.0956	1.1472e-5
5	0.0618	9.27e-6
6	0.0436	7.848e-6
7	0.0325	7.392e-6

Since the width of the plate is 15mm and the thickness of each fin is 2 mm, we can use at most 7 fins on the heat sink. For n varying from 1 to 7, we can see from the above table that the volume of aluminium required is minimum for $n = 7$ and $L = 0.0325 \text{ m} = 32.5\text{mm}$.

CONCLUSIONS

So after implementing both the case of natural and forced convection we obtained the results that are mentioned above.

With the mentioned assumption, the results obtained in the case of forced convection are comparable to that in practical situations.

The final answer that we obtained is:

$$\underline{N(\text{Number of fins}) = 7}$$

$$\underline{\text{Volume of additional aluminium used for fins in this case} = 7.392 \times 10^{-6} \text{ m}^3}$$

References

BOOKS REFERRED

1. YUNUS A. CENGEL | AFSHIN J. GHAJAR
2. Incropera's Principles of Heat and Mass Transfer, Wiley India Edition

WEB

<https://www.engineeringtoolbox.com/>

THANKING YOU