Drag Coefficient: The Air Pocket Effect

OBJECTIVE:

Study of dynamic forces on a solid particle moving through a liquid and comparing the Drag Coefficient of Objects with and without an Air Pocket.

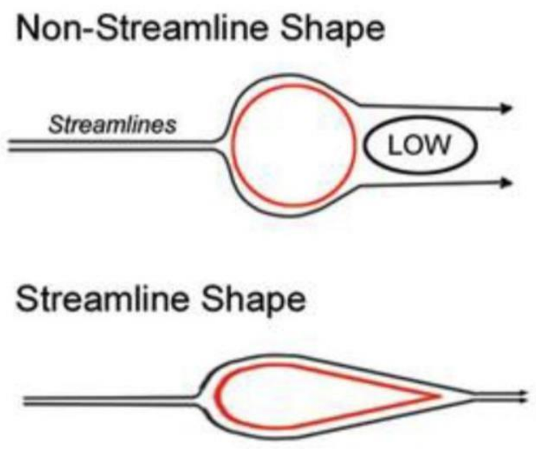
AIM:

To determine the Drag Coefficient **Theory:**

In fluid dynamics, the shape of an object has a significant impact on the forces that act upon it as it moves through a fluid, such as water. In particular, streamlined shapes are designed to minimize drag and turbulence in the fluid flow, while non-streamlined shapes can experience much greater resistance and other forces.

Streamlined shapes, such as the torpedo-shaped object mentioned in the previous question, are designed to have a smooth, curved surface that allows the fluid to flow smoothly around the object with minimal turbulence. This reduces the drag force, which is the force that opposes the motion of the object through the fluid. By minimizing drag, streamlined shapes allow objects to move more efficiently through the fluid, requiring less energy to achieve a given speed.

In contrast, non-streamlined shapes, such as a flat plate, sphere or cube, can experience much greater drag and other forces as they move through a fluid. These shapes create a lot of turbulence in the fluid flow, which increases the drag force and can also create other forces such as lift or side forces. These forces can make it more difficult to move the object through the fluid and can also affect its stability and maneuverability.



Overall, streamlined shapes are preferred for objects that need to move efficiently through a fluid, such as airplanes, boats, and submarines, while nonstreamlined shapes may be more appropriate for other applications where drag and other forces are not as much of a concern.

The difference in densities between a particle and a fluid under a gravitational field is what causes a particle to move through the fluid. We consider the motion of a sphere with a diameter of d_p and a density of $\boldsymbol{\rho}_p$ that moves through a fluid with a viscosity of μ and a density of $\boldsymbol{\rho}$.

Various forces acting on the particle are:
- External force
- Buoyancy force
- Drag force

m = Mass of the spherical particle A_p

= Projected Area

u = Particle velocity

$$F = m\left(\frac{du}{dt}\right) = F_g - F_B - F_D$$

$$F_g = m\,g$$

$$F_B = m\left(\frac{\rho}{\rho_P}\right)g$$

$$F_D = \frac{C_D\, u^2 \rho\, A_P}{2}$$

$$\frac{du}{dt} = g(\rho_\rho - \rho)\rho_\rho - (C_D u^2 \rho A_P)/2m$$

At terminal velocity condition

$$\left(\frac{du}{dt} = 0, u = u_t\right)$$

For a spherical particle the eqn. reduces to:

$$u_t = \sqrt{\frac{4g(\rho_\rho - \rho)d_P}{3C_D\rho}} \qquad u_t^2 = \frac{4g(\rho_\rho - \rho)d_P}{3C_D\rho} \qquad C_D = \frac{4g(\rho_\rho - \rho)d_P}{3u_t^2\,\rho}$$

Particle Reynolds No. is

$$\mathrm{Re}_p = \frac{d_P\, u_t\, \rho}{\mu}$$

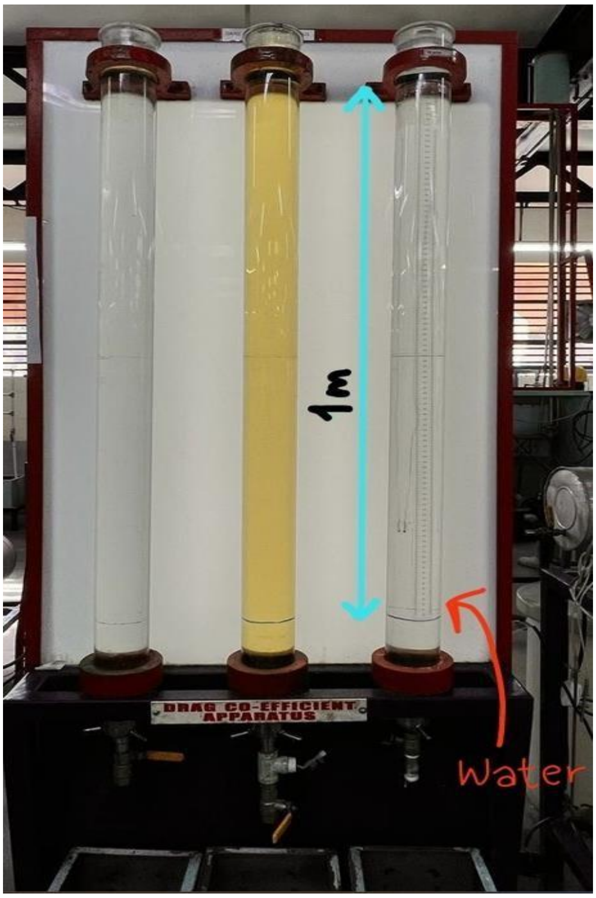
Plot of C_D vs R_{ep} (log log scale) is:

for $\mathrm{Re}_p \leq 1$ $\qquad C_D = \dfrac{24}{\mathrm{Re}_P}$ (Stokes Law region)

for $1000 < \mathrm{Re}_p < 200{,}000$ $\qquad C_D \approx 0.44$ i.e. Newton's Law Region

<u>Experiment Setup:</u>

We used the water column only because the lycopodium powder (Hydrophobic powder) cannot be used for other materials.



EXPERIMENTAL PROCEDURE:

(1) Select the set of balls with variety of materials and sizes.

(2) Measure the average diameter of each ball with screw gauge.

(3) Measure the mass of each ball with weighing balance.

(4) Calculate the density of ball materials from the measurements.

(5) Fill the tube with water.

(6) Measure the density of water.

(7) Now drop gently each object in column and note down the time taken (t) by the particle to cover a distance between two marked points on the column.

(8) Repeat the experiment by forming the lycopodium powder layer in water column.

MATERIALS REQUIRED:

- **Electricity Supply: Single phase, 220V AC, 50Hz, 5–15-amp Socket with earth connection**

- **Floor Area Required: 1.5m * 0.75m**
- **Objects-Aluminium Spherical balls, Brass Cylinder and brass cube.**
- **High Speed Camera with good lighting setup.**

Videos from high speed camera:

In the drive link provided below we have attached the videos recorded by the high speed camera show the streamlined (Air cavity) shape generated due to Lycopodium powder.

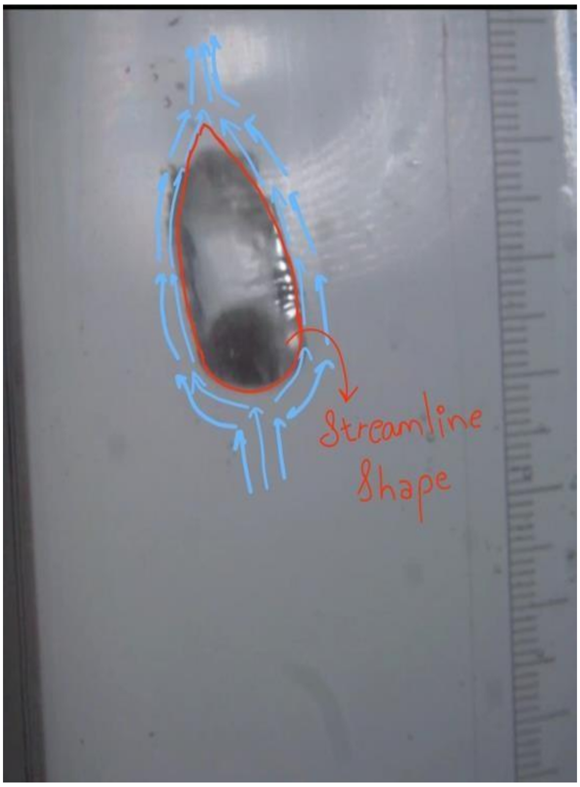
https://drive.google.com/drive/folders/1eeseUm9BkLHA4dFaBY5jOGfM2tKbfhZq?usp=sharing

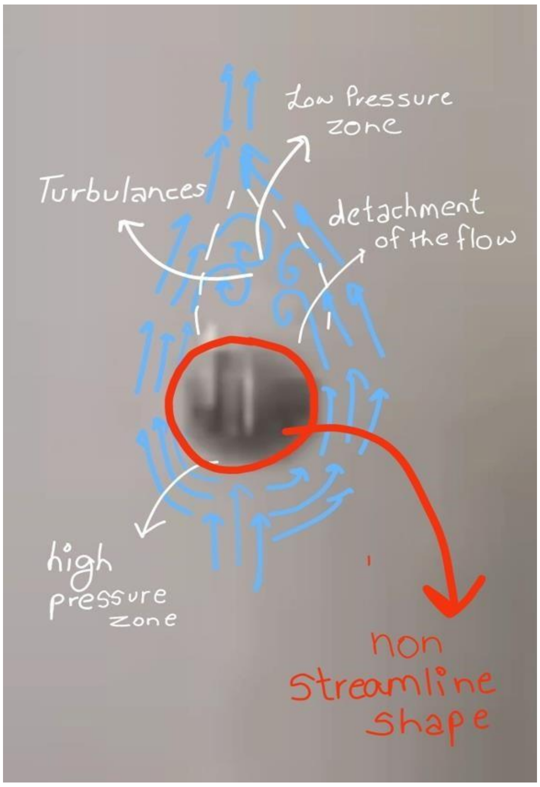
RESULTS & DISCUSSIONS:

Material	Dp, (mm)	Mass, g	X, (m)	T, (ms)	T_{lyco}, (ms)	U_t, (m/s)	$U_{t,lyco}$ (m/s)	% increase in U_t	ρ_{water}, (kg/m3)	η, (kg/m.s)	Re	Re_{lyco}	C_D	C_{Dlyco}	% change in C_D
Steel Ball	6.29	1	1	103	92	0.971	1.087	11.957	1053	0.001	6430.456	7199.315	0.592	0.473	20.219
	6.44	1.1	1	109	91	0.917	1.099	19.780	1053	0.001	6221.394	7452.000	0.679	0.473	30.300
	6.46	1.1	1	112	94	0.893	1.064	19.149	1053	0.001	6073.554	7236.574	0.719	0.507	29.560
	6.3	1.04	1	100	88	1.000	1.136	13.636	1053	0.001	6633.900	7538.523	0.559	0.433	22.560
	12.2	21.29	1	91	80	1.099	1.250	13.750	1053	0.001	14117.143	16058.250	2.289	1.769	22.715
Brass Cylender	12.23	21.65	1	94	82	1.064	1.220	14.634	1053	0.001	13700.202	15705.110	2.442	1.858	23.902
havg = 20mm	12.25	22	1	90	78	1.111	1.282	15.385	1053	0.001	14332.500	16537.500	2.239	1.682	24.889
	12.24	20.62	1	89	80	1.124	1.250	11.250	1053	0.001	14481.708	16110.900	2.189	1.769	19.202
	9.76	4	1	94	72	1.064	1.389	30.556	1053	0.001	10933.277	14274.000	0.766	0.449	31.331
Steel Ball	9.74	3.98	1	95	73	1.053	1.370	30.137	1053	0.001	10796.021	14049.616	0.780	0.461	30.953
	9.76	4	1	92	76	1.087	1.316	21.053	1053	0.001	11170.957	13522.737	0.733	0.500	31.758
	9.8	4.04	1	98	79	1.020	1.266	24.051	1053	0.001	10530.000	13062.532	0.835	0.543	35.017
		6.9	1	115	96	0.870	1.042	19.792	1053	0.001	8671.226	10387.406	1.731	1.206	30.314
Brass Cube		7	1	125	104	0.800	0.962	20.192	1053	0.001	7977.528	9588.375	2.045	1.416	30.778
side = 9.47mm		6.94	1	129	100	0.775	1.000	29.000	1053	0.001	7730.163	9971.910	2.178	1.309	29.907
		7.1	1	121	103	0.826	0.971	17.476	1053	0.001	8241.248	9681.466	1.916	1.388	27.539

CONCLUSION:

(1) The object with air cavity travels faster than the one without air cavity.





(2) Image from camera is attached above and we can conclude the following:

(a) When we added Lycopodium powder which generated Hydrophobic behaviour, we get streamlined shape (teardrop like shape) around the balls due to which they travels relatively faster.

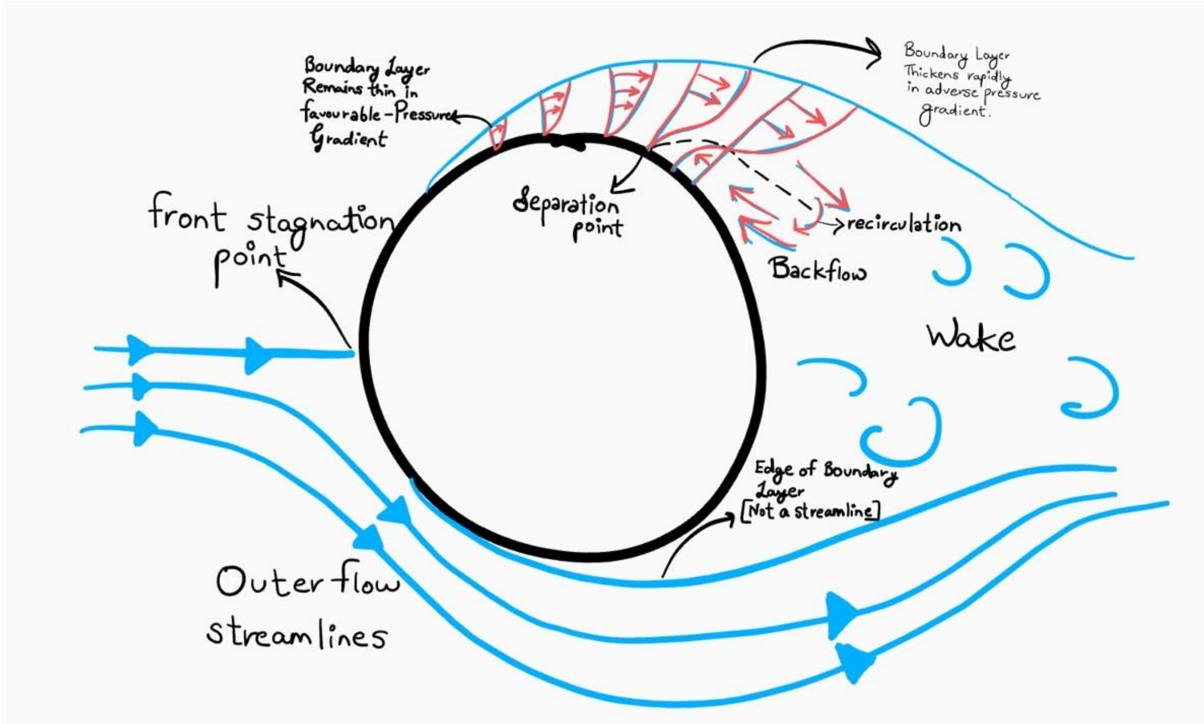
(b) When a normal sphere metal is dropped through water we get pressure differences and we can see from the image that the flow is detached at a point thereby creating a turbulent region. All this opposes the motion of the ball, resulting in greater drag force.

The following diagrams demonstrates how the streams pass thru each of the object and also take into consideration the wake region.

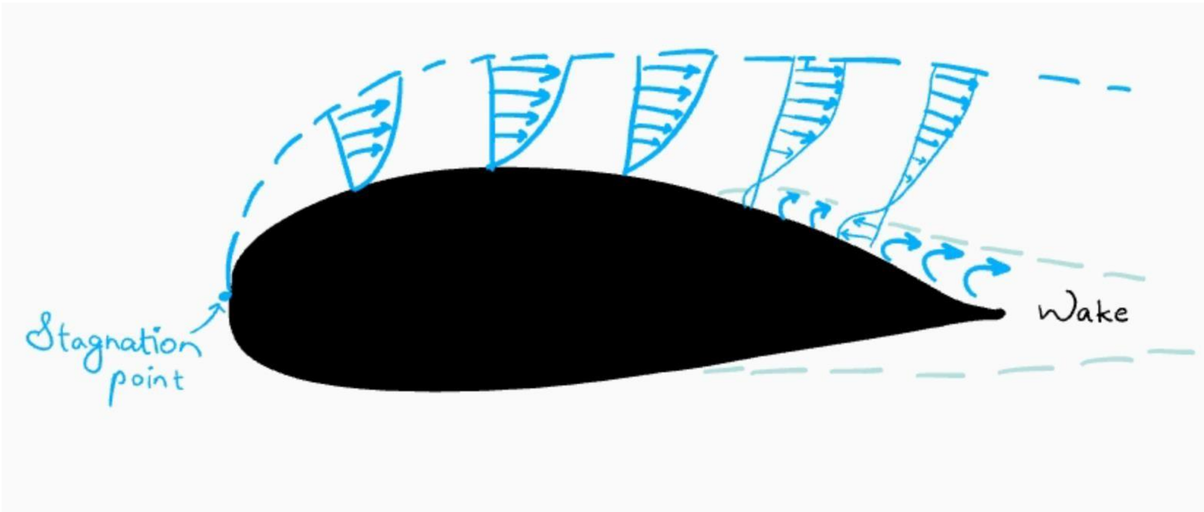
To define Wake-

In fluid mechanics, the term "wake" refers to the region of disturbed flow that forms behind an object moving through a fluid, such as air or water. When an object moves through a fluid, it creates a disturbance in the fluid flow, which results in the formation of eddies and vortices in the fluid behind the object. This region of disturbed flow is called the wake.

(i) Flow around a spherical object



(ii)Flow around a stream-line object



The factors such as backflow, turbulent or wake region, vortices contribute in increasing the drag coefficient.

In torpedo like shape the wake region is less thus we can come to conclusion that it take less time to flow through a fluid.

We saw significant decrement in the value of drag coefficient when we used Lycopodium powder in water.

(a) For steel balls of smaller size, the change in C_D lies between 20% to 30%

(b) For steel balls of larger size, the change in C_D lies between 30% to 35%

(c) For brass cylinder, the change in C_D lies between 20% to 25%

(d) For brass cube, the change in C_D lies between 25% to 30 %

PRECAUTIONS:

1. Stopwatch must be operated carefully while eliminating any time gap that might affect the stoppage time.
2. Make sure to measure the diameter of the balls at two different orientations.
3. Drop the balls in the fluid very gently.
4. The Angle of release should be maintained.

Nomenclature

F : Net force on body

F_D: Drag Force

F_B: Buoyant Force F_g:

Gravitational Force m:

mass of body/particle

C_D: Drag Coefficient ρ:

Density of liquid ρ_p:

Density of Particle

A_p: Projected Area of particle

U_t: Terminal Velocity

V_p: Volume of particle

Error Analysis:

Least Counts : Weighing machine : 0.01 gms Viscometer : 0.1 mPa-s

Timer - 0.01 seconds

Screw gauge : 0.01 mm

Scale used to measure dist X => 1 mm

C_D (drag coefficient) = $\dfrac{4g(\rho_p - \rho) D_p}{3 u_t^2 \rho} = \dfrac{4g(\rho_p - \rho) D_p t^2}{3 \rho X^2}$

Assuming gravity (g) is known accurately $\left(\frac{\Delta g}{g} = 0\right)$

$$\frac{\Delta C_D}{C_D} = \frac{\Delta(\rho_p - \rho)}{(\rho_p - \rho)} + \frac{\Delta D_p}{D_p} + 2\frac{\Delta t}{t} + \frac{\Delta \rho}{\rho} + \frac{\Delta X}{X}$$

$\frac{\Delta X}{X} \rightarrow \left(\frac{0.1\,cm}{100\,cm}\right) = 0.001$

{ Now $\rho_p = \dfrac{m_p}{V_p}$ → $\dfrac{\Delta \rho_p}{\rho_p} = \dfrac{\Delta m_p}{m_p} + \dfrac{3\Delta D_p}{D_p}$

$V_p = \frac{\pi}{6} D_p^3$

and $\rho = \dfrac{m_f}{V_f} = \left\{ \dfrac{\Delta m_f}{m_f} + \dfrac{\Delta V_f}{V_f} \right\}$

Assuming that the volume of fluid is known to an 0.1 mL accuracy,

$$\frac{\Delta V_f}{V_f} = \frac{0.1 (mL)}{10 (mL)} = \underline{0.01}$$

$$\Delta(\rho_p - \rho) = \Delta\rho_p + \Delta\rho = \frac{0.01}{m_p} + \frac{3 \times (0.01)}{D_p} + \frac{0.01}{m_f} + 0.01$$

$$\frac{\Delta C_D}{C_D} = \frac{1}{(\rho_p - \rho)} \times \left(\frac{0.01}{m_p} + \frac{0.03}{D_p} + \frac{0.01}{m_f} + 0.01 \right) + \frac{0.01}{D_p} + \frac{2 \times 0.01}{t} + 2 \times 0.001 \left(\frac{\Delta x}{x}\right)$$

* Use m_p & m_f in gms, ρ_p in gm/mm³, D_p in mm and t in seconds

Re_p (Reynolds Number of the Particle)

$$Re = \frac{\rho u_t D_p}{\eta}$$

$$\frac{\Delta Re}{Re} = \underbrace{\frac{\Delta \rho_t}{\rho_t}}_{\left(\frac{\Delta m_t}{m_t} + \frac{\Delta V_t}{V_t}\right)} + \underbrace{\frac{\Delta u_t}{u_t}}_{2\left(\frac{\Delta t}{t} + \frac{\Delta x}{x}\right)} + \frac{\Delta D_p}{D_p} + \frac{\Delta \eta}{\eta}$$

$$\frac{\Delta V_t}{V_t} = 0.01$$

$$\frac{\Delta Re}{Re} = \frac{0.01}{m_f} + 0.01 + 2 \times \left(\frac{0.01}{t}\right) + 2 \times (0.001) + \frac{0.01}{D_p} + \frac{0.1}{\eta}$$

where m_f is in gms

t is in seconds

D_p is in mm

and η is in m-Pa-s

Formula Derivation:

Deriving for the C_D formula for cylindrical and cubical blocks We know, net force equation $F_{net}= m(du/dt)= F_g - F_B – F_D$ Where $F_g= mg$; $F_D=C_Du^2\rho A_p/2$ $F_B=m(\rho/\rho_p)g$

Now, at terminal velocity condition, $du/dt=0$ and replace u with u_t So, $F_g -F_B – F_D$ ----(i) $Mg- m(\rho/\rho_p)g = C_D u_t^2 \rho A_p/2$ We can use $m=V_p \rho_p$ -----(ii) in equation (i) So, $V_p \rho_p g - V_p \rho g= C_D u_t^2 \rho A_p/2$ –-(iii)

Case 1: For cylinder

$V_p = \pi r^2 h$ **h = height of the cylinder**

$A_p = \pi r^2$ **r = radius of the cylinder**

$$C_D = 2gh(\rho_p - \rho)/u_t^2\rho$$

Case2: For Cube

$V_p = a^3$ **a = side of the cube**

$A_p = a^2$

$$C_D = 2ga(\rho_p - \rho)/u_t^2\rho$$

Case 3: For Sphere

$V_p = 4\pi r^3/3$ **r = radius of the sphere**

$A_p = \pi r^2$ **d = diameter of the sphere**

$$C_D = 4gd(\rho_p - \rho)/3u_t^2\rho$$

2. Density of water calculation:

Weight of water: 10.53gm ρ = mass/volume

$= 10.53 \times 10^{-3}/10^{-5}$

$= 1053\ Kg/m^3$

SAMPLE CALCULATIONS

Sample Calculations

Calculating C_D & Re

a) without powder

$$u_t = \frac{l}{t} = \frac{l}{1.03\,s} \simeq 0.971\ m/s$$

use $M = 1$ gm, $r = \frac{6.29}{2} \times 10^{-3}$

for steel ball

$$\rho_p = \frac{M}{V} = \frac{1 \times 10^{-3}}{\frac{4}{3} \times \pi \times \left(\frac{6.29}{2}\right)^3 \times 10^{-9}}$$

$$\rho_p = 7678.38\ kg/m^3$$

$$Re_p = \frac{\rho_f u_t D_p}{\mu} = \frac{1053 \times 0.971 \times 6.29/2 \times 10^{-3}}{0.001} = 6430.456$$

Now,

$$C_D = \frac{4 \times g \times (\rho_p - \rho) \times D_p}{3 u_t^2 \times \rho} = \frac{4 \times 9.81 \times (7678.38 - 1053) \times \frac{6.29}{2} \times 10^{-3}}{3 \times (0.971)^2 \times 1053}$$

$$\Rightarrow C_D = 0.592$$

% change in C_D

Now for same ball when passed with Lycopodium powder in it

we get, $u_t = 1.087$

& calculating values, we get

$$C_D)_{lyco} = 0.473$$

So % change in C_D is

$$\left(\frac{0.592 - 0.473}{0.592}\right) \times 100 \simeq 20.19\%$$