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THAPAR INSTITUTE
OF ENGINEERING & TECHNOLOGY
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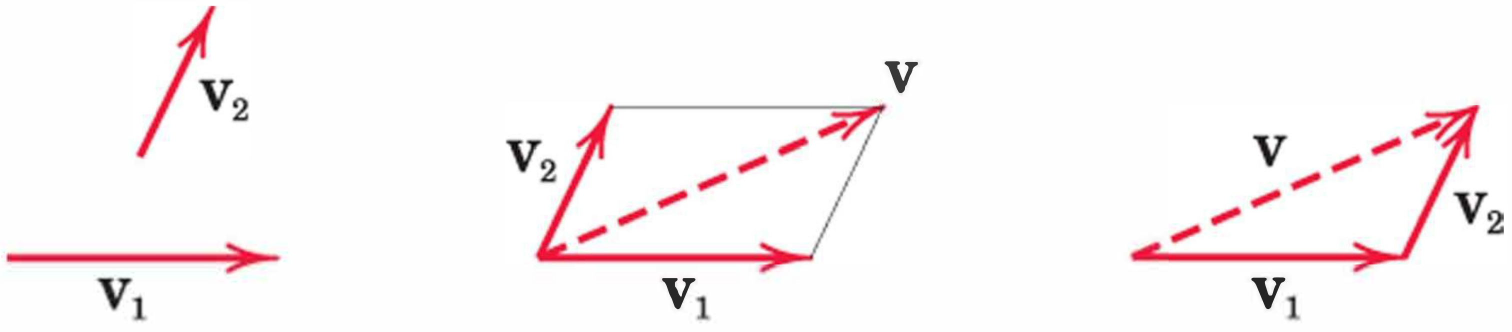
Scalars and Vectors

Scalars: only magnitude is associated.
Ex: time, volume, density, speed, energy, mass

Vectors: possess direction as well as magnitude, and must obey the parallelogram law of addition (and the triangle law).

Ex: displacement, velocity, acceleration,
force, moment, momentum

Equivalent Vector: $\mathbf{V} = \mathbf{V_1} + \mathbf{V_2}$ (Vector Sum)



Speed is the magnitude of velocity.

Vectors

A Vector **V** can be written as: **V** = *V***n**

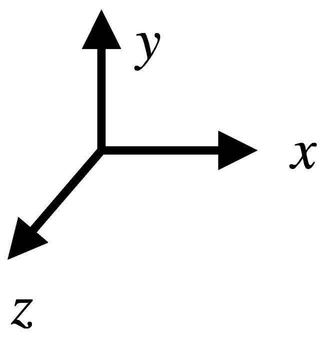
V = magnitude of **V**

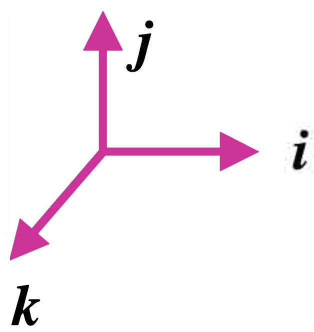
n = unit vector whose magnitude is one and whose direction coincides with that of **V**

Unit vector can be formed by dividing any vector, such as the geometric position vector, by its length or magnitude

Vectors represented by Bold and Non-Italic letters (**V**)

Magnitude of vectors represented by Non-Bold, Italic letters (*V*)

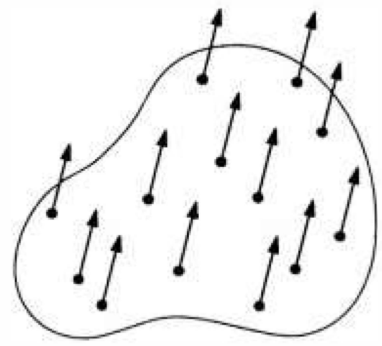




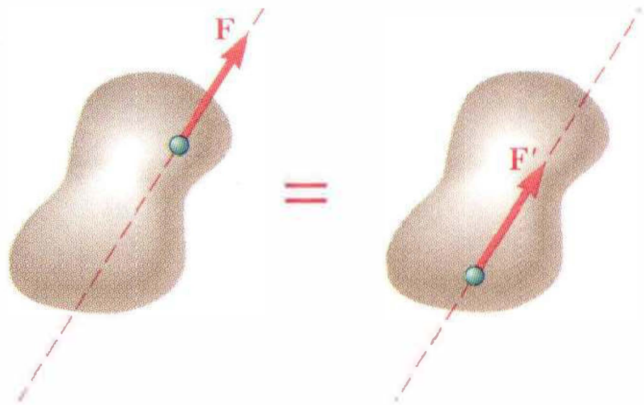
i, ***j***, ***k*** – unit vectors

Vectors

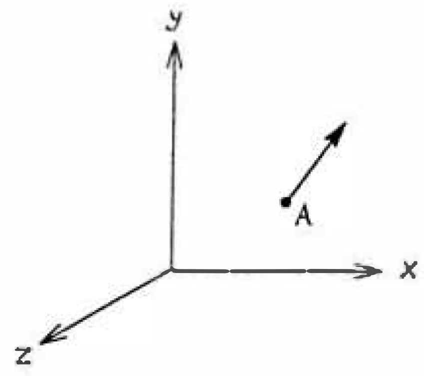
Free Vector: whose action is not confined to or associated with a unique line in space
Ex: Movement of a body without rotation.



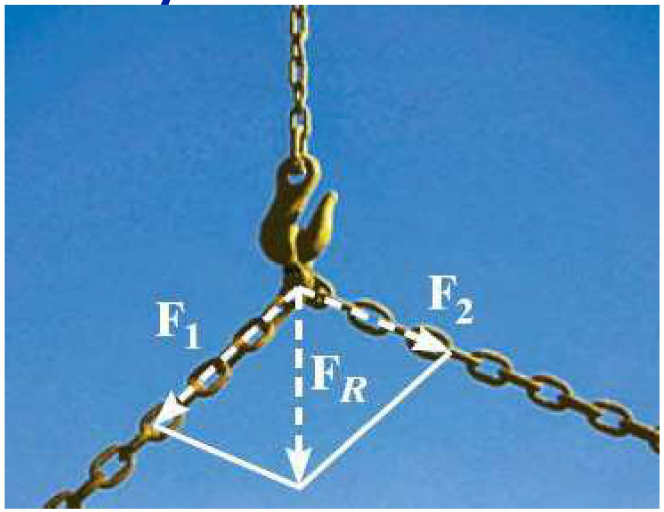
Sliding Vector: has a unique line of action in space but not a unique point of application
Ex: External force on a rigid body
→ Principle of Transmissibility
→ Imp in Rigid Body Mechanics



Fixed Vector: for which a unique point of application is specified
Ex: Action of a force on deformable body



Vector Addition: Procedure for Analysis



Parallelogram Law (Graphical)

Resultant Force (diagonal)

Components (sides of parallelogram)

Algebraic Solution

Using the coordinate system

Cosine law:

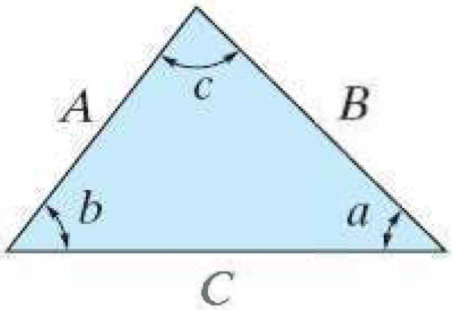
$$C = \sqrt{A^2 + B^2 - 2AB\cos c}$$

Sine law:

$$\frac{A}{\sin a} = \frac{B}{\sin b} = \frac{C}{\sin c}$$

Trigonometry (Geometry)

Resultant Force and Components from Law of Cosines and Law of Sines



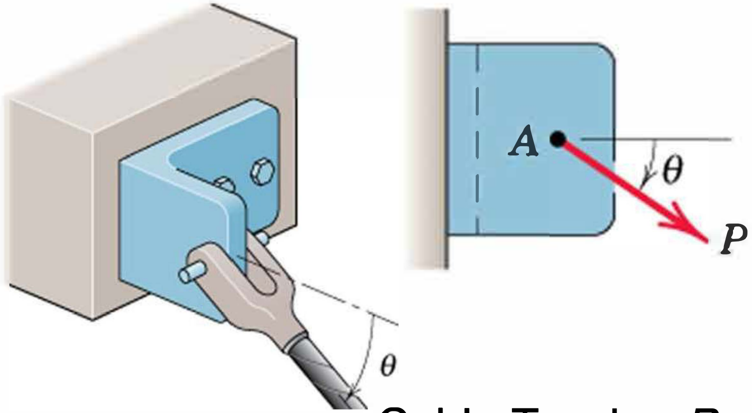
Force Systems

Force: Magnitude (P), direction (arrow) and point of application (point A) is important

Change in any of the three specifications will alter the effect on the bracket.

Force is a Fixed Vector

In case of rigid bodies, line of action of force is important (not its point of application if we are interested in only the resultant external effects of the force), we will treat most forces as



Cable Tension P

External effect: Forces applied (applied force); Forces exerted by bracket, bolts, Foundation (reactive force)

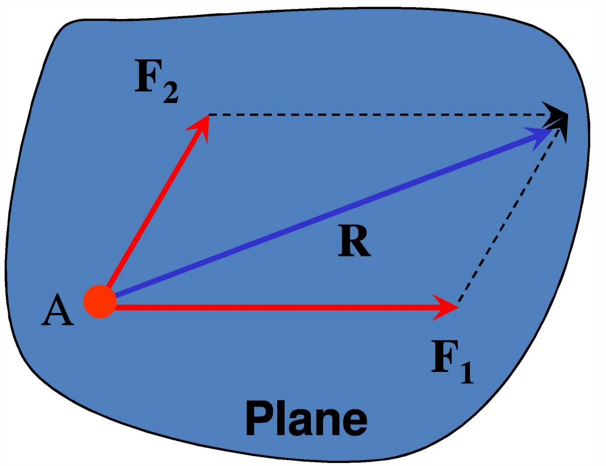
Internal effect: Deformation, strain pattern – permanent strain; depends on material properties of bracket, bolts, etc.

Force Systems

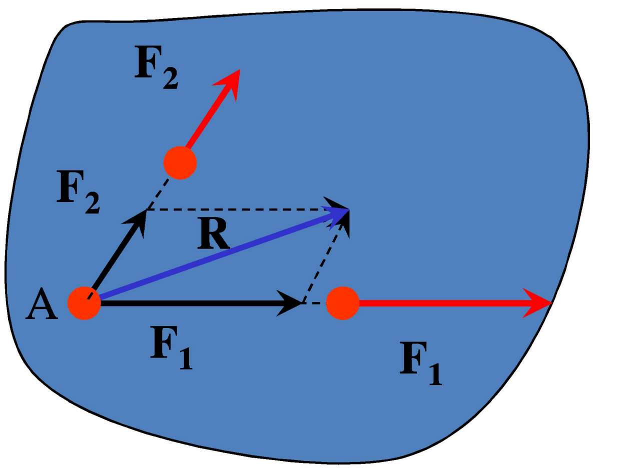
Concurrent force:

Forces are said to be concurrent at a point if their lines of action intersect at that point

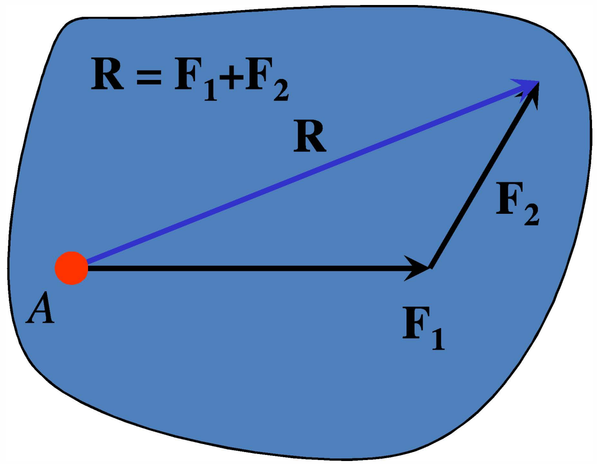
$\mathbf{F_1}$, $\mathbf{F_2}$ are concurrent forces; $\mathbf{R}$ will be on same plane; $\mathbf{R} = \mathbf{F_1} + \mathbf{F_2}$



Forces act at same point



Forces act at different point

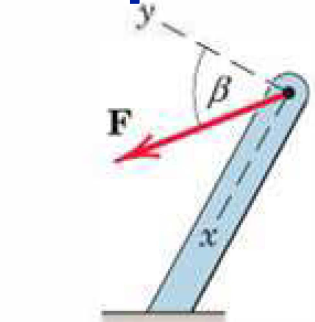


Triangle Law

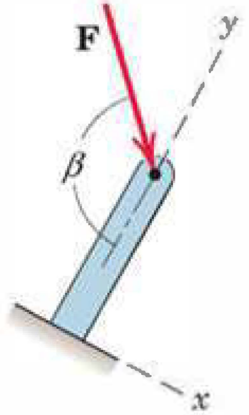
(Apply Principle of Transmissibility)

Components of Force

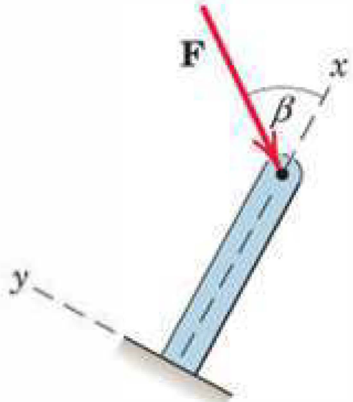
Examples



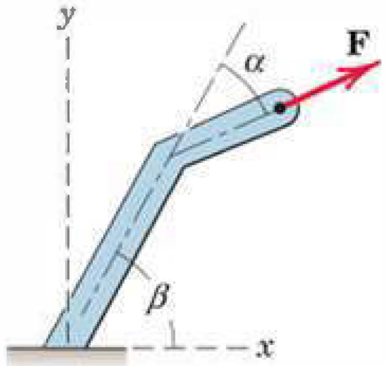
$F_x = F \sin \beta$
$F_y = F \cos \beta$



$F_x = F \sin(\pi - \beta)$
$F_y = -F \cos(\pi - \beta)$

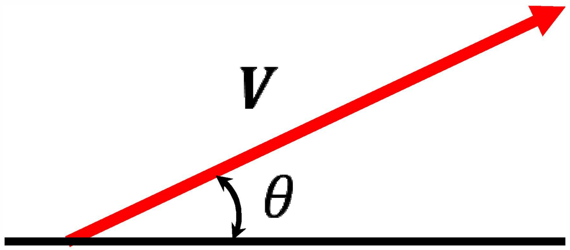


$F_x = -F \cos \beta$
$F_y = -F \sin \beta$

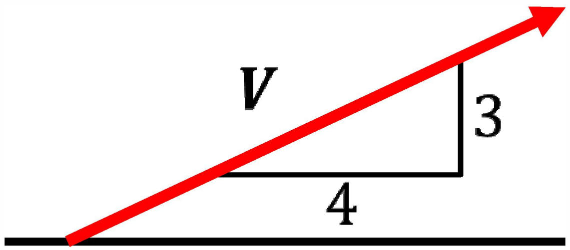


$F_x = F \cos(\beta - \alpha)$
$F_y = F \sin(\beta - \alpha)$

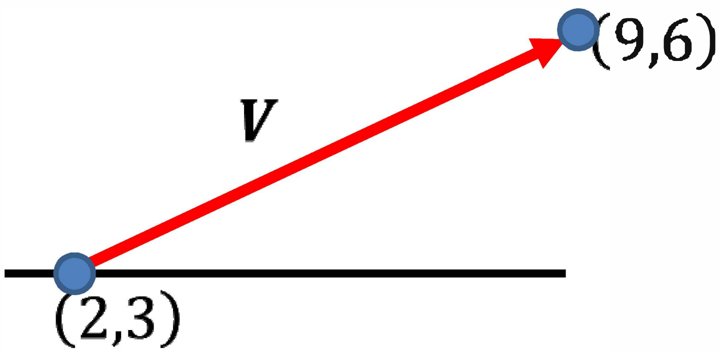
Vector



$$\boldsymbol{V} = V(cos\theta\boldsymbol{i} + sin\theta\boldsymbol{j})$$



$$\boldsymbol{V} = V\left(\frac{4\boldsymbol{i} + 3\boldsymbol{j}}{\sqrt{4^2 + 3^2}}\right)$$

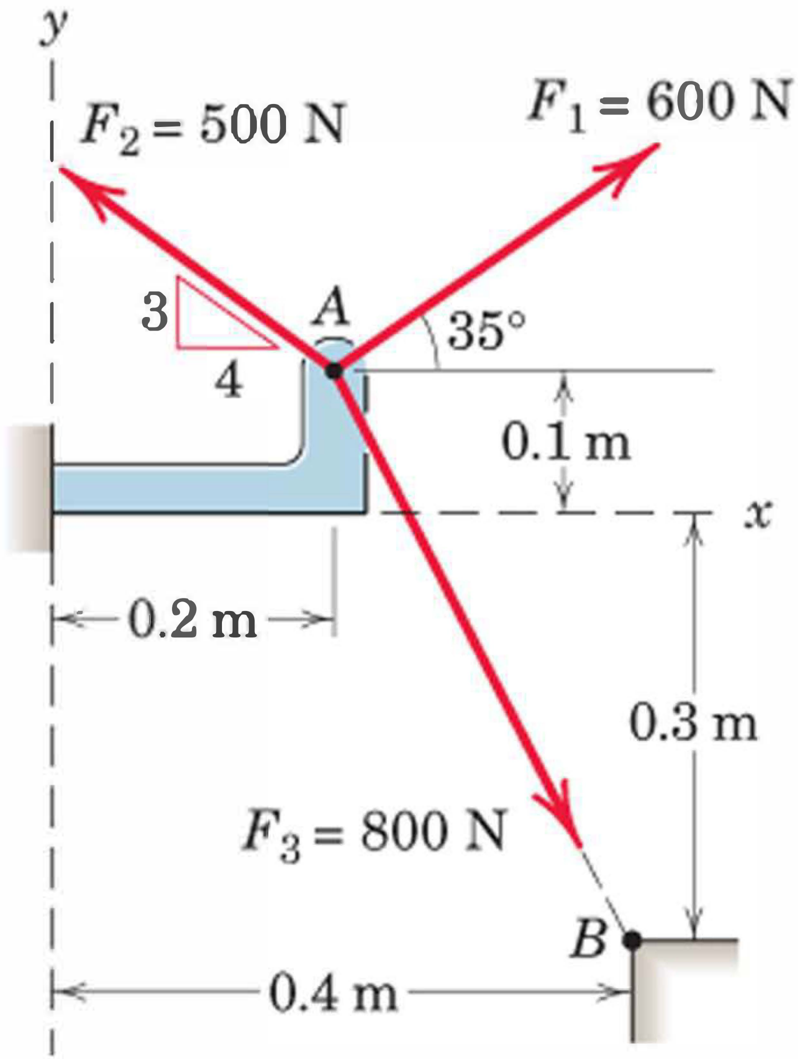


$$\boldsymbol{V} = V\left(\frac{(9-2)\boldsymbol{i} + (6-3)\boldsymbol{j}}{\sqrt{(9-2)^2 + (6-3)^2}}\right)$$

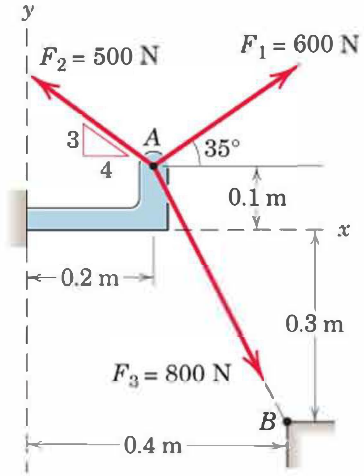
Components of Force

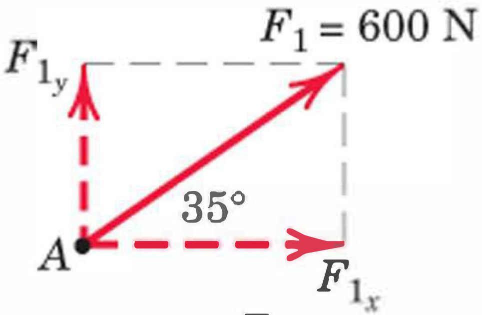
Example 1:

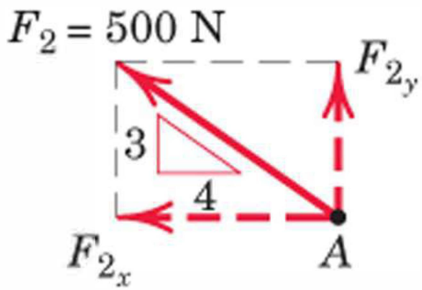
Determine the x and y scalar components of F_1, F_2, and F_3 acting at point A of the bracket

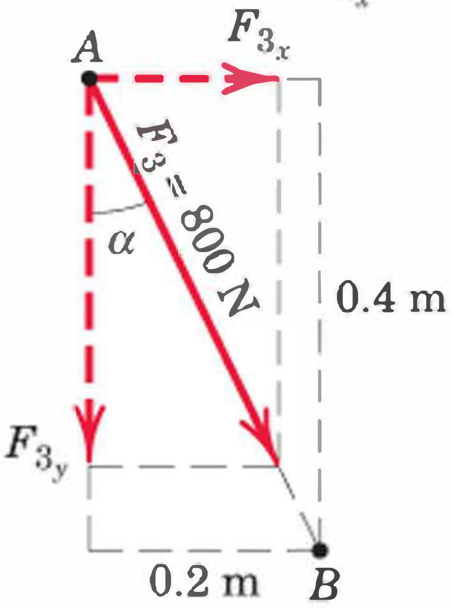


Components of Force









$$F_{1_x} = 600 \cos 35° = 491 \text{ N}$$

$$F_{1_y} = 600 \sin 35° = 344 \text{ N}$$

$$F_{2_x} = -500(\tfrac{4}{5}) = -400 \text{ N}$$

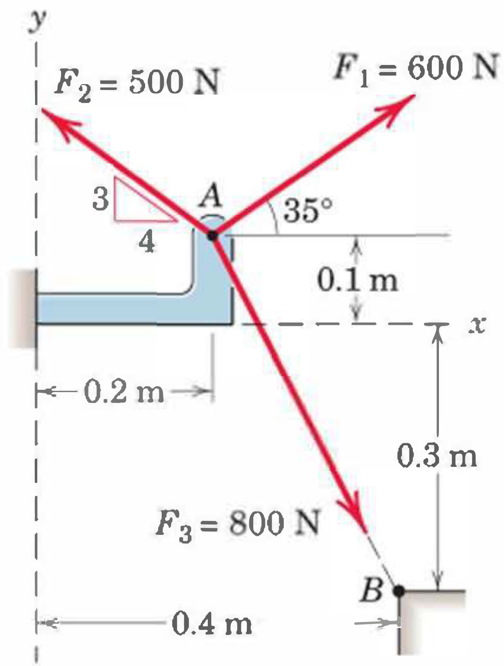
$$F_{2_y} = 500(\tfrac{3}{5}) = 300 \text{ N}$$

$$\alpha = \tan^{-1}\left[\frac{0.2}{0.4}\right] = 26.6°$$

$$F_{3_x} = F_3 \sin \alpha = 800 \sin 26.6° = 358 \text{ N}$$

$$F_{3_y} = -F_3 \cos \alpha = -800 \cos 26.6° = -716 \text{ N}$$

Components of Force



Alternative Solution

$$\boldsymbol{F}_1 = F_1 \boldsymbol{n}_1 = F_1 \frac{cos(35^o)\boldsymbol{i} + sin(35^o)\boldsymbol{j}}{\sqrt{\left(cos(35^o)\right)^2 + \left(sin(35^o)\right)^2}}$$

$$= 600[0.819\boldsymbol{i} + 0.5735\boldsymbol{j}]$$

$$= 491\boldsymbol{i} + 344\boldsymbol{j}$$

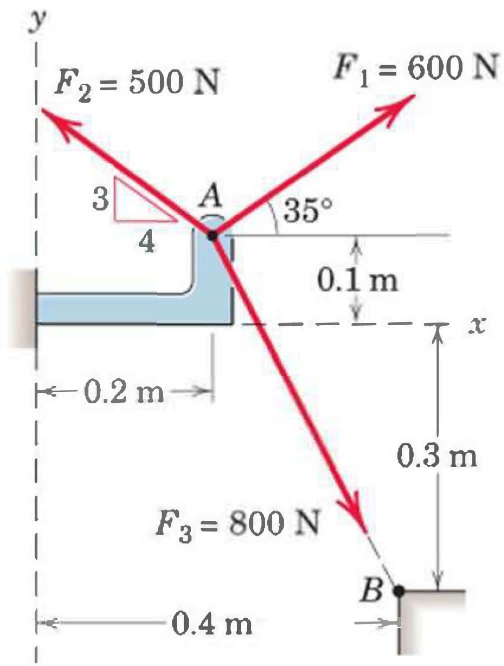
$F_{1x} = 491\ N$ $\qquad$ $F_{1y} = 344\ N$

$$\boldsymbol{F}_2 = F_2 \boldsymbol{n}_2 = F_2 \frac{-4\boldsymbol{i} + 3\boldsymbol{j}}{\sqrt{(-4)^2 + (3)^2}}$$

$$= 500[-0.8\boldsymbol{i} + 0.6\boldsymbol{j}] \quad = 400\boldsymbol{i} + 300\boldsymbol{j}$$

$F_{2x} = 400\ N$ $\qquad$ $F_{2y} = 300\ N$

Components of Force



Alternative Solution

$$\overrightarrow{AB} = 0.2\boldsymbol{i} - 0.4\boldsymbol{j}$$

$$\overline{AB} = \sqrt{(0.2)^2 + (-0.4)^2}$$

$$\boldsymbol{F_3} = F_3\boldsymbol{n_3} = F_3\frac{\overrightarrow{AB}}{\overline{AB}}$$

$$= 800\frac{0.2\boldsymbol{i} - 0.4\boldsymbol{j}}{\sqrt{(0.2)^2 + (-0.4)^2}}$$

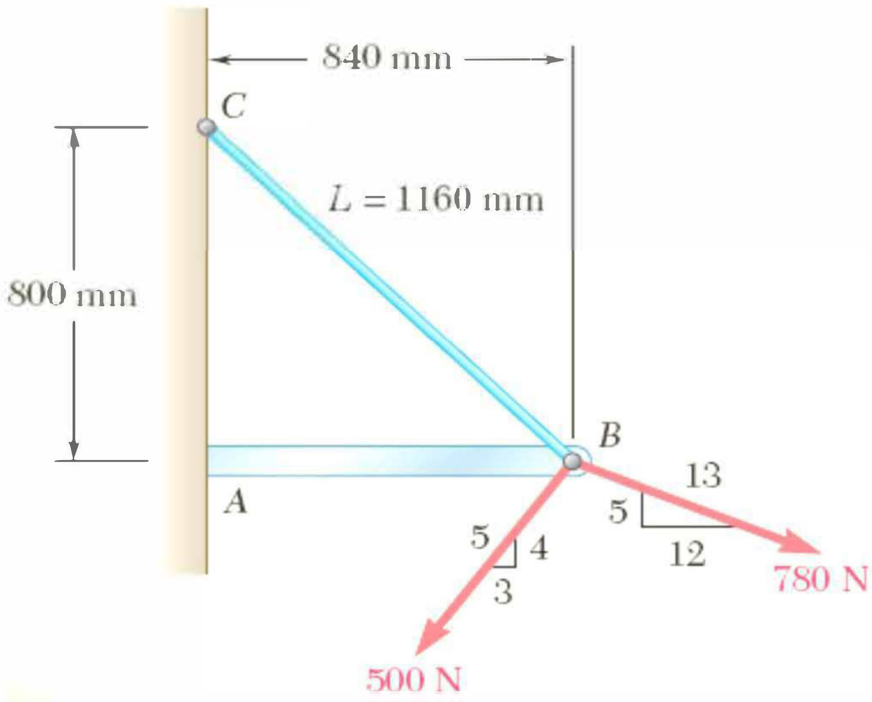
$$= 800[0.447\boldsymbol{i} - 0.894\boldsymbol{j}]$$

$$= 358\boldsymbol{i} - 716\boldsymbol{j}$$

$F_{3x} = 358\ N$ $\qquad$ $F_{3y} = 716\ N$

Components of Force

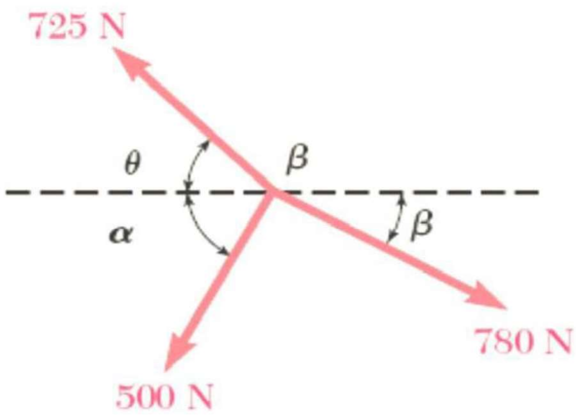
Example 3:Tension in cable *BC* is 725-N, determine the resultant of the three forces exerted at point *B* of beam *AB*.



Solution:

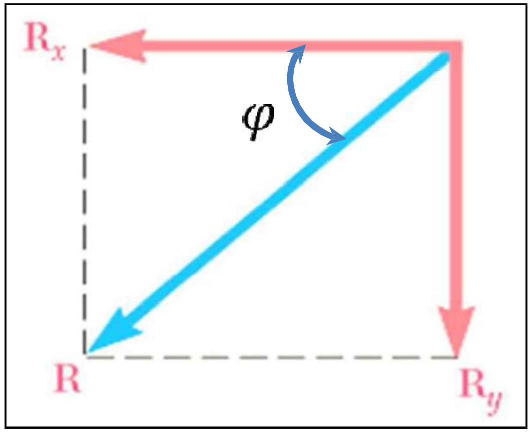
- Resolve each force into rectangular components.
- Determine the components of the resultant by adding the corresponding force components.
- Calculate the magnitude and direction of the resultant.

Components of Force



Resolve each force into rectangular components

Magnitude (N)	X-component (N)	Y-component (N)
725	-525	500
500	-300	-400
780	720	-300
	$R_x = -105$	$R_y = -200$



$$\mathbf{R} = R_x i + R_y j \qquad \mathbf{R} = (-105)i + (-200)j$$

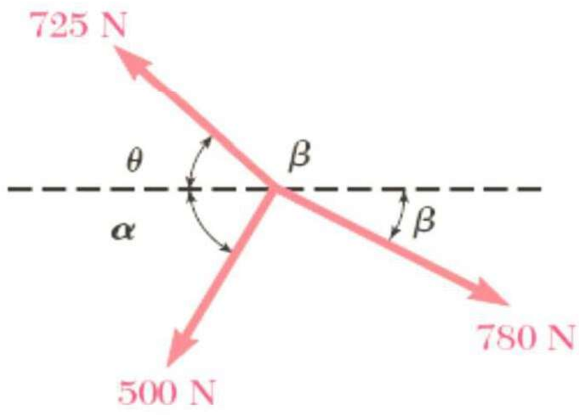
Calculate the magnitude and direction

$$tan\varphi = \frac{R_y}{R_x} = \frac{200}{105} \qquad \varphi = 62.3^o$$

$$\mathbf{R} = \sqrt{R_x^2 + R_y^2} = 225.9N$$

Components of Force

Alternate solution



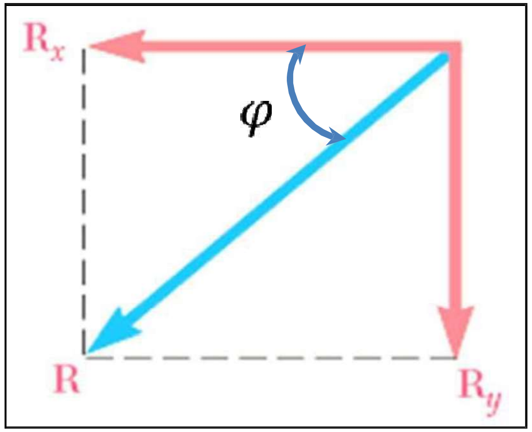
$$\boldsymbol{R} = \boldsymbol{F}_1 + \boldsymbol{F}_2 + \boldsymbol{F}_3$$

$$\boldsymbol{F}_1 = 725[-0.724\boldsymbol{i} + 0.689\boldsymbol{j}]$$

$$\boldsymbol{F}_2 = 500[-0.6\boldsymbol{i} - 0.8\boldsymbol{j}]$$

$$\boldsymbol{F}_2 = 780[0.923\boldsymbol{i} - 0.384\boldsymbol{j}]$$

$$\boldsymbol{R} = -105\boldsymbol{i} - 200\boldsymbol{j}$$



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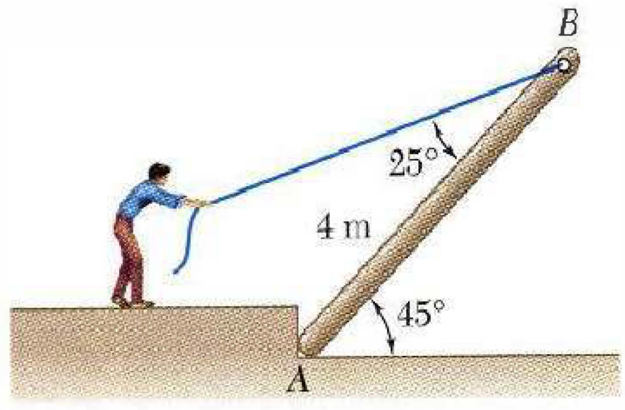
Rigid Body Equilibrium: Example



A man raises a 10 kg joist, of length 4 m, by pulling on a rope.

Find the tension in the rope and the reaction at A.

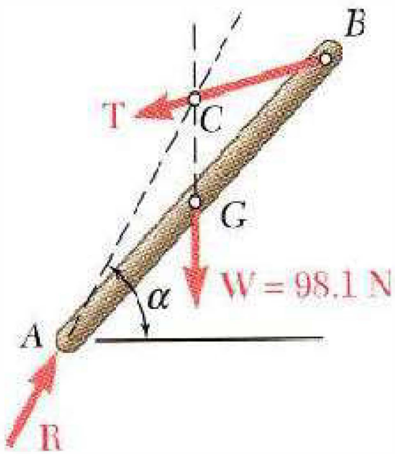
Solution:

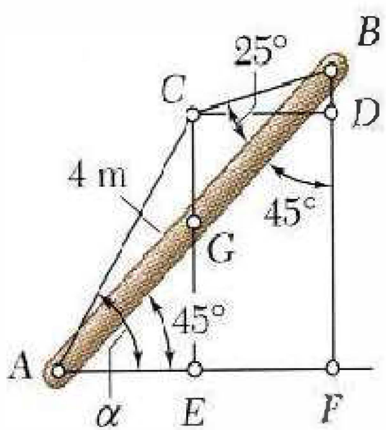
- Create a free-body diagram of the joist. Note that the joist is a 3 force body acted upon by the rope, its weight, and the reaction at A.

- The three forces must be concurrent for static equilibrium. Therefore, the reaction ***R*** must pass through the intersection of the lines of action of the weight and rope forces. Determine the direction of the reaction force ***R***.

- Utilize a force triangle to determine the magnitude of the reaction force ***R***.

Rigid Body Equilibrium: Example





- Create a free-body diagram of the joist.
- Determine the direction of the reaction force R.

$$AF = AB\cos 45 = (4\,\text{m})\cos 45 = 2.828\,\text{m}$$

$$CD = AE = \tfrac{1}{2}AF = 1.414\,\text{m}$$

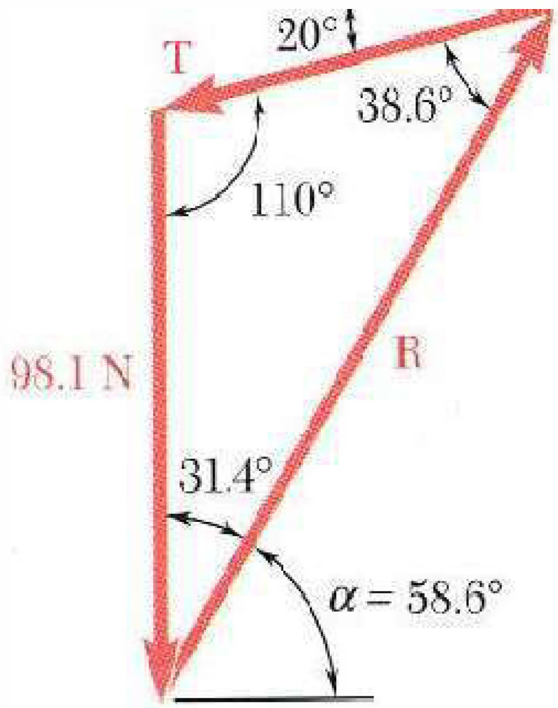
$$BD = CD\tan 20 = (1.414\,\text{m})\tan 20 = 0.515\,\text{m}$$

$$CE = BF - BD = (2.828 - 0.515)\,\text{m} = 2.313\,\text{m}$$

$$\tan\alpha = \frac{CE}{AE} = \frac{2.313}{1.414} = 1.636$$

$$\boxed{\alpha = 58.6^\circ}$$

Rigid Body Equilibrium: Example



- Determine the magnitude of the reaction force R.

$$\frac{T}{\sin 31.4^\circ} = \frac{R}{\sin 110^\circ} = \frac{98.1\,\text{N}}{\sin 38.6^\circ}$$

$$\boxed{\begin{aligned} T &= 81.9\,\text{N} \\ R &= 147.8\,\text{N} \end{aligned}}$$