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- CST SPL 1
- 18

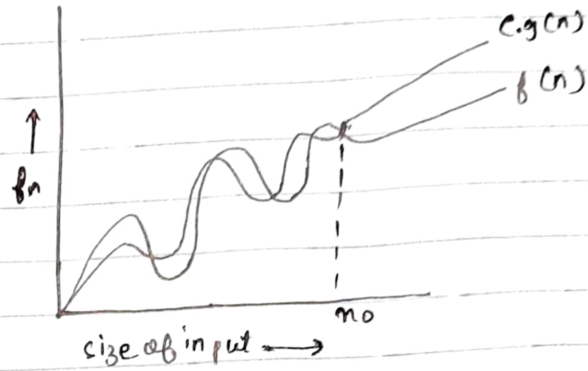
# Design and Analysis of Algorithms

## TUTORIAL 1

### Asymptotic Notations

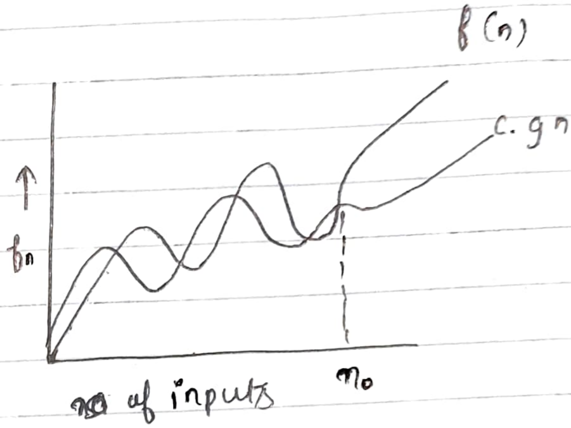
They help you find the complexity of an algorithm when input is very large.

(I) Big O (O)




$f(n) = O(g(n))$
iff $f(n) \le c.g(n)$
$\forall\ n \ge n_0$
for some constant $c>0$
$\Rightarrow$ g(n) is tight upper bound of f(n)

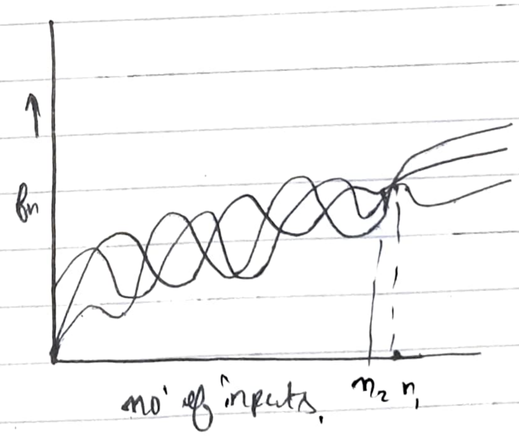
(II) Big Omega ($\Omega$)




$f(n) = \Omega(g(n))$
g(n) is 'tight' lowerbound of f(n)
$f(n) = \Omega(g(n))$
iff $f(n) \ge c.g(n)$
$\forall\ n \ge n_0$ for some constant $c>0$

(III) Theta ($\Theta$)

$f(n) = \Theta(g(n))$
g(n) is both 'tight' upper and lower bound of f(n)




$f(n) = \Theta(g(n))$
iff
$c_1\,g(n) \le f(n) \le c_2.g(n)$
$\forall\ n \ge \max(n_1, n_2)$
for some constant $c_1>0$ & $c_2>0$

4) Small o (o)

$f(n) = o(g(n))$

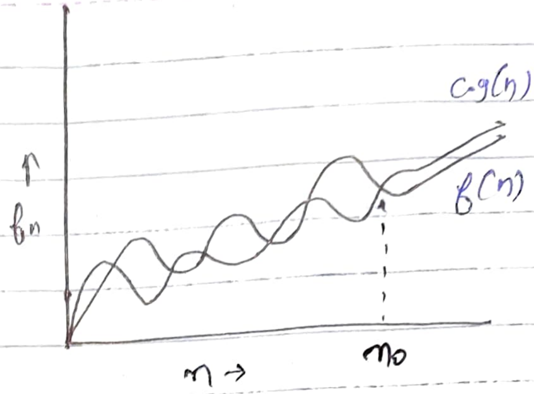
g(n) is upper bound of fn (f(n))

$f(n) = o(g(n))$

when $f(n) < c.g(n)$

$\forall\ n > n_0$

& $\forall\ c > 0$




(5) Small omega (ω)

$f(n) = \omega(g(n))$

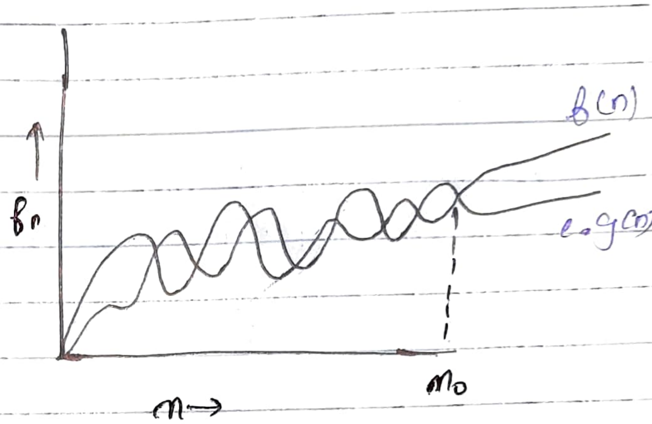
g(n) is lower bound of f(n)

$f(n) = \omega(g(n))$

when $f(n) > c.g(n)$

$\forall\ n > n_0$

& $\forall\ c > 0$



Q2 for (i 1 & n) // i = 1, 2, 4, 8 ... n
{ i i*2 } // O(1)

$\sum_{i=1}^{n}$ 1 + 2 + 4 + 8 + ... n

GP Kth value → $T_K = ar^{k-1}$
$= 1 \times 2^{k-1}$
$n = 2^k$

$2n = 2^K$
$\log 2n = K \log 2$
$\log 2 + \log n = K \log 2$
$\log n + 1 = K$

$O(K) = O(1 + \log n)$
$= O(\log n)$

Q3 $T(n) = 3T(n-1)$ — ①

put n = n-1
$T(n-1) = 3T(n-2)$ — ②
from ① & ②

$T(n) = 3(3T(n-2))$
$= 9T(n-2)$ — ③

putting n = n-2 in ①
$T(n) = 3(T(n-3))$
$T(n) = 27(T(n-3))$
$T(n) = 3^K(T(n-K))$

putting, $n - K = 0$

$\Rightarrow n = K$

$T(n) = 3^n [T(n - n)]$

$T(n) = 3^n T(0)$

$T(n) = 3^n \times 1$ $\quad [T(0) = 1]$

$T(n) = O(3^n)$

Q4 $\quad T(n) = 2T(n-1) - 1$ — ①

Let $n = n-1$

$T(n-1) = 2T(n-2) - 1$ — ②

from ① & ②

$T(n) = 2[2T(n-2) - 1] - 1$

$T(n) = 4T(n-2) - 2 - 1$ — ③

Let $n = n-2$

$T(n-2) = 2T(n-3) - 1$ — ④

from ③ & ④

$T(n) = 4[2T(n-3) - 1] - 2 - 1$

$T(n) = 8T(n-3) - 4 - 2 - 1$

$T(n) = 2^K T(n-K) - 2^{K-1} - 2^{K-2}$

$G.P = 2^{K-1} + 2^{K-2} + 2^{K-3} + \cdots 1$

$a = 2^{K-1}$

$r = 1/2$

$= \dfrac{a(1-r^n)}{1-r}$

$= \dfrac{2^{K-1}(1-(1/2)^n)}{1/2}$

$= 2^k (1 - (1/2)^k)$

$= 2^k - 1$

Let $n - k = 0$

$n = k$

$T(n) = 2^n T(n-n) - (2^n - 1)$

$T(n) = 2^n \cdot 1 - (2^n - 1)$

$T(n) = 2^n - (2^n - 1)$

$T(n) = O(1)$

Q5 $P = 1, 2, 3, 4, 5, 6 \ldots$

$S = 1 + 3 + 6 + 10 + 15 + 21 \ldots n$

Sum of $S = 1 + 3 + 6 + 10 + \ldots + T_n$ —— (1)

also $S = 1 + 3 + 6 + 10 + \ldots T_{n-1} + T_n$ —— (2)

from (1) — (2)

$0 = 1 + 2 + 3 + 4 + \ldots n - T_n$

$T_k = 1 + 2 + 3 + 4 + \ldots k$

$T_k = \frac{1}{2} (k+1)$

for k iterations

$1 + 2 + 3 + \ldots + k <= n$

$\frac{k(k+1)}{2} <= n$

$\frac{k^2 + k}{2} <= n$

$O(k^2) <= n$

$k = O(\sqrt{n})$

$T(n) = O(\sqrt{n})$

Q6

$i^2 <= n$

$i <= \sqrt{n}$

$i = 1, 2, 3, 4, \ldots, \sqrt{n}$

$\sum_{i=1}^{n} 1 + 2 + 3 + 4 + \ldots + \sqrt{n}$

$T(n) = \frac{\sqrt{n} \times (\sqrt{n} + 1)}{2}$

$T(n) = \frac{n \times \sqrt{n}}{2}$

$T(n) = O(n)$

Q7

for ~~k = k/2~~ $k = k * 2$

$k = 1, 2, 4, 8, \ldots n$

GP → $a = 1$, $r = 2$

$= \frac{a(r^n - 1)}{r - 1}$

$= \frac{1(2^k - 1)}{1}$

$n \to 2^k$

$\log n = k$

| i | j | k |
|---|---|---|
| 1 | log n | log n × log n |
| 2 | log n | log n × log n |
| 3 | ⋮ | ⋮ |
| ⋮ | ⋮ | ⋮ |
| n | log n | log n . log n |

$\therefore O(n \log^2 n)$

(8)

$T(n) = T(n/3) + n^2$

$a=1$ , $b=3$ , $f(n) = n^2$

$c = \log_3 1 = 0$

$n^0 = 1 \quad < \quad (f(n) = n^2)$

$T(n) = \Theta(n^2)$

(9)

for $i = 1 \Rightarrow j = 1, 2, 3, 4 \ldots\ldots n = n$

for $i = 2 \Rightarrow j = 1, 3, 5, \ldots\ldots n = n/2$

for $i = 3 \Rightarrow j = 1, 4, 7, \ldots\ldots n = n/3$

for $i = n \Rightarrow j = 1 \ldots\ldots$

$\Rightarrow \sum_{j=n}^{1} n + \frac{n}{2} + \frac{n}{3} + \frac{n}{4} + \ldots + 1$

$\Rightarrow \sum_{j=n}^{1} n\left[1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \ldots \frac{1}{n}\right]$

$\sum_{j=n} n[\log n]$

$T(n) = [n \log n]$

$T(n) = O(n \log n)$

(10) given $n^k$ & $e^n$

relation b/w $n^k$ & $e^n$ is

$n^k = O(e^n)$

as $n^k \leq a e^n$

$\forall\ n \geq n_0$ & some constant $a > 0$

for $n_0 = 1$

$e = 2$

$1^k \leq a.$

$n_0 = 1$ &