Post_processing

April 30, 2023

1 Question (1)

1.1 Consider the viscous Burgers' equation $u_t + uu_x = \alpha u_{xx}$, where u(x, t) is the velocity component along the x−direction, and is the kinematic viscosity. Solve this 1D equation in a periodic domain of size 1.0 for the following cases. Use $\Delta t = 0.0004$, $t_{end} = 0.075$, and $u(x, 0) = sin(4\pi x) + sin(6\pi x) + sin(10\pi x)$.

1.2 (b) Use Euler and second order central difference schemes to solve the equation with $\alpha = 0.001$ for a grid resolution of 1024. Compare with part (a) solution and comment on the nature of solution.

2 Answer (1)(b):

3 Import the necessary libraries

[1]:
```
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
```

4 Reading the name of the files generated

[2]:
```
Output_files = pd.read_csv("Output_file_names.csv",delimiter=",",header=None).
 ↪to_numpy()
Output_files = np.squeeze(Output_files)
Output_files = Output_files.tolist()
```

4.1 Output files generated

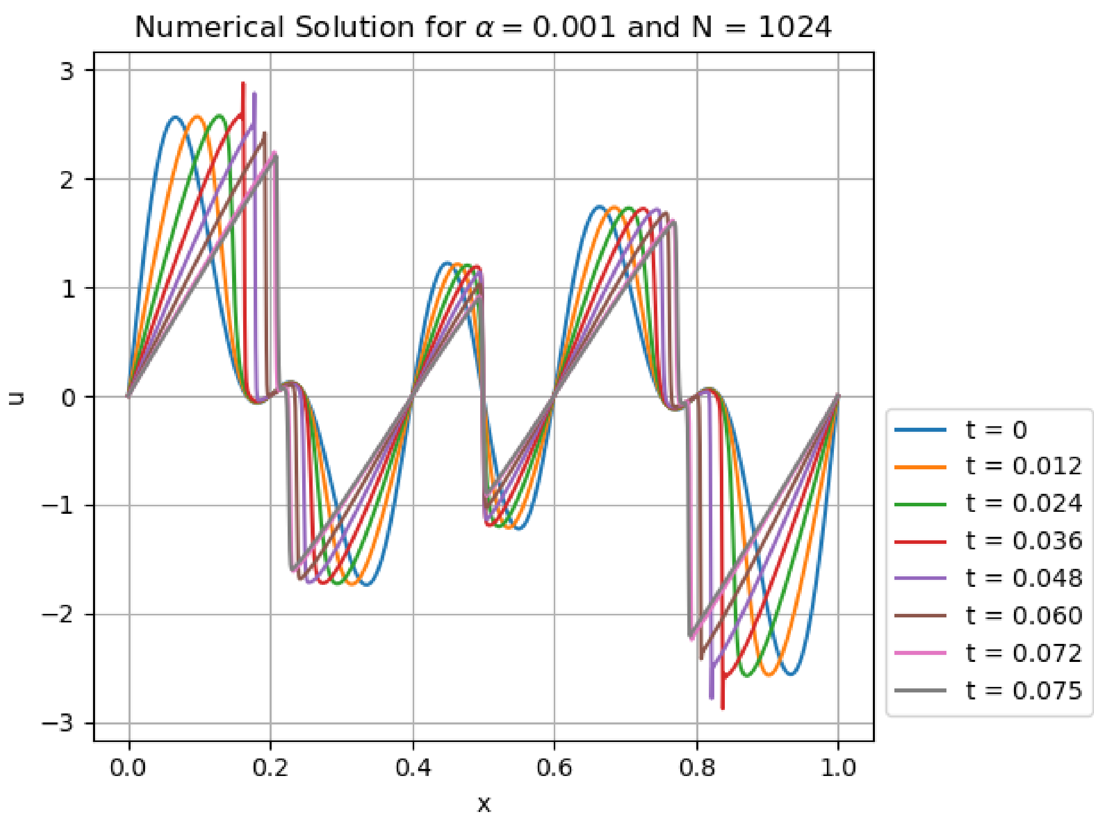
[3]:
```
Output_files
```

[3]:
```
['Question_1_b_u_n_t_0.000000_N_1024_.csv',
 'Question_1_b_u_n_t_0.012000_N_1024_.csv',
 'Question_1_b_u_n_t_0.024000_N_1024_.csv',
 'Question_1_b_u_n_t_0.036000_N_1024_.csv',
 'Question_1_b_u_n_t_0.048000_N_1024_.csv',
```

```
'Question_1_b_u_n_t_0.060000_N_1024_.csv',
'Question_1_b_u_n_t_0.072000_N_1024_.csv',
'Question_1_b_u_n_t_0.075000_N_1024_.csv']
```

4.2 Numerical solution for $\alpha = 0.001$ and grid resolution of 1024

[4]:
```
for output_file in Output_files:
    Numerical_Solution = pd.read_csv(output_file,delimiter=",",header=None).
 ↪to_numpy()
    x = np.linspace(0, 1,Numerical_Solution.shape[0])
    plt.plot(x,Numerical_Solution)
plt.xlabel("x")
plt.ylabel("u")
plt.legend(["t = 0","t = 0.012","t = 0.024","t = 0.036","t = 0.048","t = 0.
 ↪060","t = 0.072","t = 0.075"],bbox_to_anchor=(1,0.5))
plt.title(r"Numerical Solution for $\alpha = 0.001$ and N = 1024")
plt.grid()
plt.tight_layout()
plt.savefig("Question_1_b.png",dpi = 500)
plt.show()
```



4.3 Comment on the errors:

4.3.1 1. From the above graph it is apparent that the error in this case is both dissipative and dispersive in nature. In part (a) solution the nature of the error was dissipative and NOT dispersive.

4.3.2 2. We are also getting the same result about the nature of the error using the modified equation.

4.3.3 3. We can observe small kinks in the solution (associated with t = 0.036, 0.048 and 0.060), which may be because the absolute value of negative viscosity term becoming higher, resulting in negative damping. Interestingly, the solution does not blow up, indicating that the negative viscosity is not very high and the overall viscosity remains mostly positive.