Name: Kunal Ghosh

Course: M.Tech (Aerospace Engineering)

Subject: Machine Learning (E0-270)

SAP No.: 6000007645

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Importing the required Libraries

[1]:
```
import numpy as np
```

[2]:
```
import torch
```

[3]:
```
import torch.nn as nn
```

[4]:
```
import torch.optim as optim
```

[5]:
```
from torch.autograd import Variable
```

[6]:
```
from sklearn.preprocessing import MinMaxScaler
```

[7]:
```
import matplotlib.pyplot as plt
```

[8]:
```
import pandas as pd
```

Reading the data from the file

[9]:
```
Training_Set = pd.read_csv('Canada.csv')
```

Displaying the training set

[10]:
```
Training_Set
```

[10]:

	Date	T_idx	N	D	S	I	R
0	2020-01-22	0	37799407	0	37799407	0	0
1	2020-01-23	1	37799407	0	37799407	0	0
2	2020-01-24	2	37799407	0	37799407	0	0
3	2020-01-25	3	37799407	0	37799407	0	0
4	2020-01-26	4	37799407	0	37799406	1	0
..	...	...	...	...	...	...	...

```
183  2020-07-23    183  37799407  8918  37685022  105467  8918
184  2020-07-24    184  37799407  8922  37684305  106180  8922
185  2020-07-25    185  37799407  8928  37683950  106529  8928
186  2020-07-26    186  37799407  8933  37683631  106843  8933
187  2020-07-27    187  37799407  8944  37682949  107514  8944

[188 rows x 7 columns]
```

Factor N, Factor S, Factor I and Factor R will be used in PINNs

Factor N

[11]:
```python
Factor = Training_Set.to_numpy()
Factor_N = max(Factor[:,2])
Factor_N
```

[11]:
```
37799407
```

Dropping the columns of data which will NOT be used in training the Neural Network

[12]:
```python
Training_Set = Training_Set.drop("Date",axis=1)
Training_Set = Training_Set.drop("T_idx",axis=1)
Training_Set = Training_Set.drop("N",axis=1)
Training_Set = Training_Set.drop("D",axis=1)
Training_Set
```

[12]:
```
            S       I     R
0    37799407       0     0
1    37799407       0     0
2    37799407       0     0
3    37799407       0     0
4    37799406       1     0
..        ...     ...   ...
183  37685022  105467  8918
184  37684305  106180  8922
185  37683950  106529  8928
186  37683631  106843  8933
187  37682949  107514  8944

[188 rows x 3 columns]
```

[13]:
```python
Factor_S_Temp = Training_Set.to_numpy()
Factor_S = max(Factor_S_Temp[:,0])-min(Factor_S_Temp[:,0])
```

[14]:
```python
Factor_I_Temp = Training_Set.to_numpy()
Factor_I = max(Factor_I_Temp[:,1])-min(Factor_I_Temp[:,1])
```

[15]:
```
Factor_R_Temp = Training_Set.to_numpy()
Factor_R = max(Factor_R_Temp[:,2])-min(Factor_R_Temp[:,2])
```

Factor S

[16]:
```
Factor_S
```

[16]: 116458

Factor I

[17]:
```
Factor_I
```

[17]: 107514

Factor R

[18]:
```
Factor_R
```

[18]: 8944

Normalizing the data

[19]:
```
Normalizing = MinMaxScaler()
training_data = Normalizing.fit_transform(Training_Set)
```

Dataloading

We will be using a sliding time window approach. So, preparing the data for that.

The Sliding_Time_Windows(data, Sequence_Length) function will create each data points for training the neural networks (Both LSTM and PINNs)

Each data point consist of k days of data (stored in x) and the data of $(k+1)^{th}$ day as the target variable (stored in y). In the code, we have used k = Sequence_Length.

NOTE: We are having three variables S, I and R in the data

[20]:
```
def Sliding_Time_Windows(Data, Seq_Length):

    x = []
    y = []

    for i in range(len(Data)-Seq_Length-1):
        x_temp = Data[i:(i+Seq_Length),:]
        y_temp = Data[i+Seq_Length,:]
```

```
        x.append(x_temp)
        y.append(y_temp)

    return np.array(x),np.array(y)
```

Declaring the value of k (Sequence_Length)

[21]:
```
Sequence_Length = 4
x, y = Sliding_Time_Windows(training_data, Sequence_Length)
```

Shape of the data

[22]:
```
x.shape
```

[22]: (183, 4, 3)

Shape of the target variable

[23]:
```
y.shape
```

[23]: (183, 3)

Splitting the data into training set and test set

Amount of data used for training

[24]:
```
Training_Data_Amount = 0.67
```

[25]:
```
train_size = int(len(y) * Training_Data_Amount)
test_size = len(y) - train_size
```

Converting the data to the tensors and creating the training set as well as test set

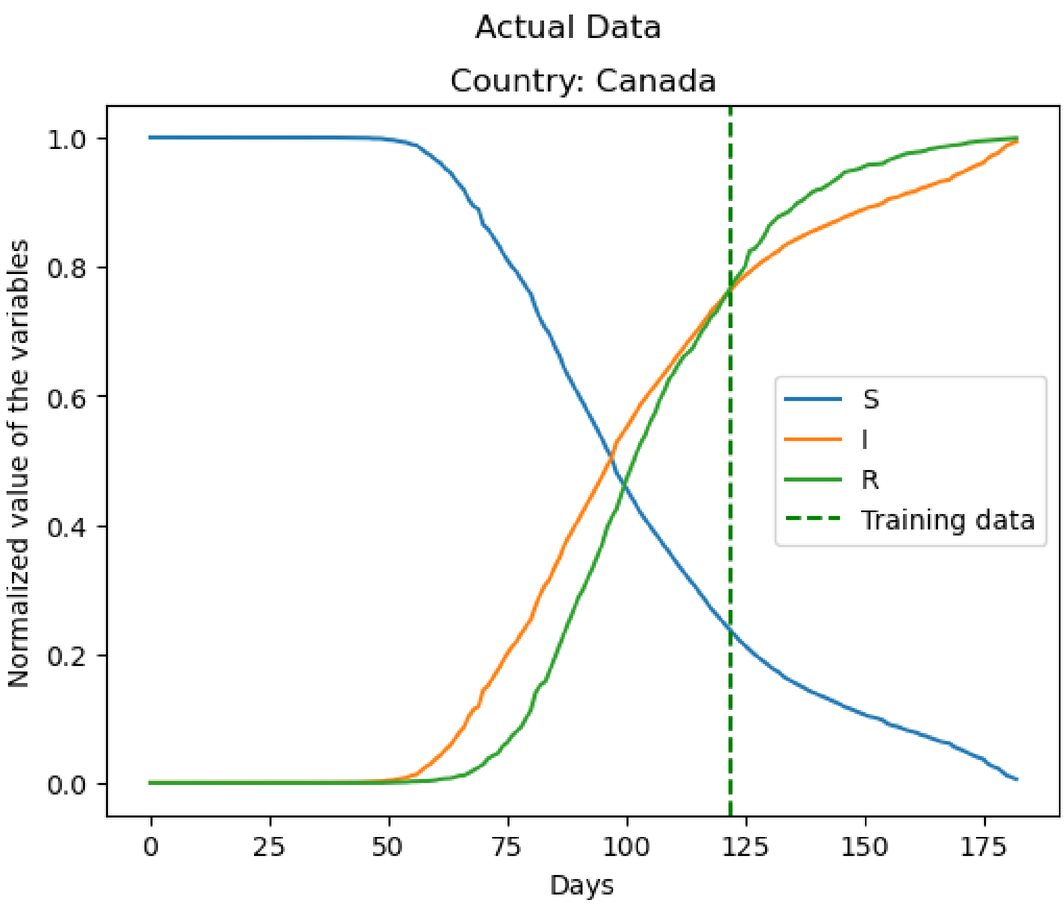
[26]:
```
Data_X = Variable(torch.Tensor(np.array(x)))
Data_Y = Variable(torch.Tensor(np.array(y)))
```

[27]:
```
Train_X = Variable(torch.Tensor(np.array(x[0:train_size])))
Train_Y = Variable(torch.Tensor(np.array(y[0:train_size])))
```

[28]:
```
Test_X = Variable(torch.Tensor(np.array(x[train_size:len(x)])))
Test_Y = Variable(torch.Tensor(np.array(y[train_size:len(y)])))
```

Plotting the actual data

[29]:
```
dataY_plot = Data_Y.data.numpy()
plt.plot(dataY_plot)
plt.axvline(x=train_size, c='g', linestyle='--')
plt.suptitle('Actual Data')
plt.title('Country: Canada')
plt.legend(["S","I","R","Training data"])
plt.xlabel("Days")
plt.ylabel("Normalized value of the variables")
plt.show()
```



Creating an LSTM class for the LSTM network

[30]:
```
class LSTM(nn.Module):
    def __init__(self, Size_Of_Inputs, Neurons_in_LSTM_Layer, Num_Layers,␣
 ↪Size_Of_Outputs):
        super(LSTM, self).__init__()
```

```
        self.Neurons_in_LSTM_Layer = Neurons_in_LSTM_Layer
        self.Num_Layers = Num_Layers
        self.lstm = nn.LSTM(Size_Of_Inputs, Neurons_in_LSTM_Layer, Num_Layers,␣
↪batch_first=True)
        self.FC = nn.Linear(Neurons_in_LSTM_Layer, Size_Of_Outputs)

    def forward(self, X):
        H0 = torch.zeros(self.Num_Layers, X.size(0), self.Neurons_in_LSTM_Layer).
↪to(X.device)
        C0 = torch.zeros(self.Num_Layers, X.size(0), self.Neurons_in_LSTM_Layer).
↪to(X.device)
        OUT, _ = self.lstm(X, (H0, C0))
        OUT = self.FC(OUT[:, -1, :])
        return OUT
```

Size of the input (It should be 3 because we have three variables S, I and R)

[31]:
```
input_size = 3
```

Size of the output (It should be 3 because we want to predict three variables S, I and R)

[32]:
```
output_size = 3
```

Number of the LSTM layers to be used

[33]:
```
Num_LSTM_Layers = 2
```

Number of neurons in LSTM layer

[34]:
```
Num_Neurons_LSTM_Layer = 20
```

Creating the LSTM network

[35]:
```
LSTM_model = LSTM(input_size,Num_Neurons_LSTM_Layer,Num_LSTM_Layers,output_size)
```

Loss function for the LSTM network

[36]:
```
Loss_Function = nn.MSELoss()
# Loss_Function = nn.L1Loss()
```

Learning Rate

[37]:
```
Learning_Rate = 0.0001
```

Optimizer for LSTM network

[38]:
```
optimizer = optim.Adam(LSTM_model.parameters(), lr=Learning_Rate)
```

Training the LSTM model

Number of the epochs

[39]:
```
Num_Epochs = 14000
```

[40]:
```
torch.manual_seed(0)
for epoch in range(Num_Epochs):
    Outputs = LSTM_model(Train_X)
    optimizer.zero_grad()
    loss = Loss_Function(Outputs, Train_Y)
    loss.backward()
    optimizer.step()
    if epoch % 250 == 0:
        print(f"Number of epochs: {epoch}\nLoss: {loss.item()}\n")
```

```
Number of epochs: 0
Loss: 0.33874133229255676

Number of epochs: 250
Loss: 0.13133107125759125

Number of epochs: 500
Loss: 0.05780916288495064

Number of epochs: 750
Loss: 0.040188811881542206

Number of epochs: 1000
Loss: 0.004352519288659096

Number of epochs: 1250
Loss: 0.0009548240923322737

Number of epochs: 1500
Loss: 0.0008291841368190944

Number of epochs: 1750
Loss: 0.0007727380725555122
```

```
Number of epochs: 2000
Loss: 0.0007208723109215498

Number of epochs: 2250
Loss: 0.0006698743673041463

Number of epochs: 2500
Loss: 0.0006189357955008745

Number of epochs: 2750
Loss: 0.0005670988466590643

Number of epochs: 3000
Loss: 0.0005131550715304911

Number of epochs: 3250
Loss: 0.0004558018990792334

Number of epochs: 3500
Loss: 0.00039410762838087976

Number of epochs: 3750
Loss: 0.0003283419064246118

Number of epochs: 4000
Loss: 0.0002610369410831481

Number of epochs: 4250
Loss: 0.00019779613649006933

Number of epochs: 4500
Loss: 0.00014631815429311246

Number of epochs: 4750
Loss: 0.00011162750888615847

Number of epochs: 5000
Loss: 9.131324623012915e-05

Number of epochs: 5250
Loss: 7.869250111980364e-05

Number of epochs: 5500
Loss: 6.908913201186806e-05

Number of epochs: 5750
Loss: 6.0796039178967476e-05
```

```
Number of epochs: 6000
Loss: 5.337798938853666e-05

Number of epochs: 6250
Loss: 4.671012356993742e-05

Number of epochs: 6500
Loss: 4.073369564139284e-05

Number of epochs: 6750
Loss: 3.541539263096638e-05

Number of epochs: 7000
Loss: 3.073656262131408e-05

Number of epochs: 7250
Loss: 2.6687668650993146e-05

Number of epochs: 7500
Loss: 2.3263901312020607e-05

Number of epochs: 7750
Loss: 2.0457788195926696e-05

Number of epochs: 8000
Loss: 1.8243948943563737e-05

Number of epochs: 8250
Loss: 1.6561258235014975e-05

Number of epochs: 8500
Loss: 1.5312400137190707e-05

Number of epochs: 8750
Loss: 1.4388502677320503e-05

Number of epochs: 9000
Loss: 1.3696830137632787e-05

Number of epochs: 9250
Loss: 1.3168874829716515e-05

Number of epochs: 9500
Loss: 1.2750168934871908e-05

Number of epochs: 9750
Loss: 1.2392791177262552e-05
```

```
Number of epochs: 10000
Loss: 1.2058607353537809e-05

Number of epochs: 10250
Loss: 1.17236204459914e-05

Number of epochs: 10500
Loss: 1.1382241609680932e-05

Number of epochs: 10750
Loss: 1.1035540410375688e-05

Number of epochs: 11000
Loss: 1.0703335647122003e-05

Number of epochs: 11250
Loss: 1.038765549310483e-05

Number of epochs: 11500
Loss: 1.0093542186950799e-05

Number of epochs: 11750
Loss: 9.824770131672267e-06

Number of epochs: 12000
Loss: 9.582920938555617e-06

Number of epochs: 12250
Loss: 9.360774129163474e-06

Number of epochs: 12500
Loss: 9.151702215604018e-06

Number of epochs: 12750
Loss: 8.951049494498875e-06

Number of epochs: 13000
Loss: 8.75541263667401e-06

Number of epochs: 13250
Loss: 8.566971700929571e-06

Number of epochs: 13500
Loss: 8.38869709696155e-06

Number of epochs: 13750
Loss: 8.216467904276215e-06
```

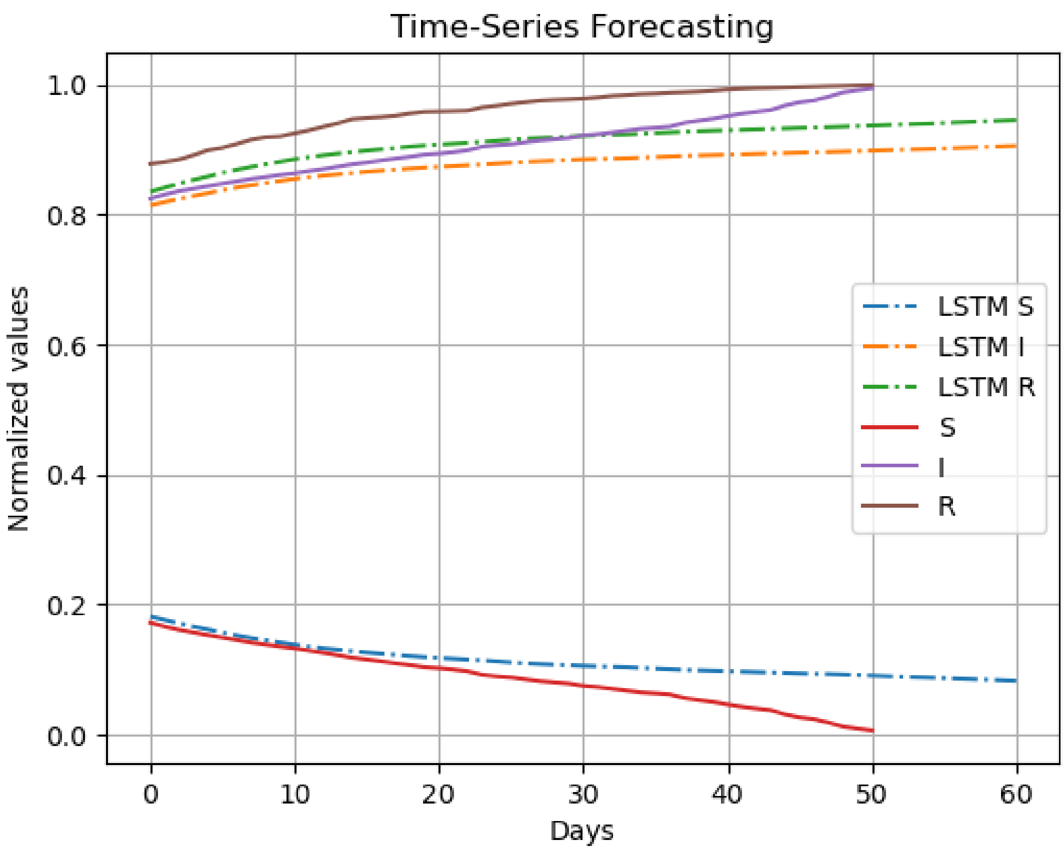
Comparing the actual data with the predicted data on the test dataset

[41]:
```
def Time_Step_Update(outputs,Train_X):
    outputs = outputs.unsqueeze(1)
    Train_X_clone = Train_X.clone().detach()
    Train_X_clone_1 = Train_X_clone[:,1:,:]
    temp = torch.cat((Train_X_clone_1,outputs),dim = 1)
    return temp
```

[42]:
```
LSTM_model.eval()
Data_LSTM = Test_X
Future_len = 10
for i in range(Future_len):
    train_predict_LSTM = LSTM_model(Data_LSTM)
    Data_LSTM = Time_Step_Update(train_predict_LSTM,Data_LSTM)
```

[43]:
```
data_predict_LSTM = train_predict_LSTM.data.numpy()
dataY_plot = Test_Y.data.numpy()
```

[44]:
```
plt.plot(data_predict_LSTM,linestyle= "-.")
plt.plot(dataY_plot[Future_len:,:])
plt.legend(["LSTM S","LSTM I","LSTM R","S","I","R"])
plt.title('Time-Series Forecasting')
plt.xlabel("Days")
plt.ylabel("Normalized values")
plt.grid()
plt.savefig("LSTM.png")
plt.show()
```



Saving the parameters of LSTM Model

[45]:
```
torch.save(LSTM_model,f"LSTM_MSE_LR_{Learning_Rate}_Epochs_{epoch}\
_seq_{Sequence_Length}_train_amount_{Training_Data_Amount}_Num_LSTM_Layers_\
{Num_LSTM_Layers}_Num_Neurons_LSTM_Layer_{Num_Neurons_LSTM_Layer}_.pth")
```

Physics Informed Neural Network (PINN)

Defining the Custom loss function for PINN (L1 Loss)

[46]:
```
def PINN_Loss_Function_SIR_L1(outputs, inputs, a, b, Factor_N, Factor_S,␣
 ↪Factor_I, Factor_R):

    # Define model inputs
    S = inputs[:, 0]
    I = inputs[:, 1]
    R = inputs[:, 2]
```

```
    # Define model outputs
    S_pred = outputs[:,0]
    I_pred = outputs[:,1]
    R_pred = outputs[:,2]

    # Compute differential equations using the original values
    dS_dt = (-a*S*I*Factor_I)/Factor_N
    dI_dt = (a*S*I*Factor_S)/Factor_N - b*I
    dR_dt = (b*I*Factor_I/Factor_R)

    # Compute differential equations using the predicted values
    dS_dt_pred = (-a*S_pred*I_pred*Factor_I)/Factor_N
    dI_dt_pred = (a*S_pred*I_pred*Factor_S)/Factor_N - b*I_pred
    dR_dt_pred = (b*I_pred*Factor_I/Factor_R)

    # Compute loss function
    loss = (torch.mean(torch.abs((dS_dt_pred - dS_dt))) + \
           torch.mean(torch.abs((dI_dt_pred - dI_dt)*(Factor_I/Factor_S))) + \
           torch.mean(torch.abs((dR_dt_pred - dR_dt)*(Factor_R/Factor_S)))) + \
           torch.mean(torch.abs(S_pred - S)) + \
           torch.mean(torch.abs((I_pred - I)*(Factor_I/Factor_S))) + \
           torch.mean(torch.abs((R_pred - R)*(Factor_R/Factor_S)))

    return loss
```

Defining the Custom loss function for PINN (MSE Loss)

[47]:
```
def PINN_Loss_Function_SIR_MSE(outputs,␣
↪inputs,a,b,Factor_N,Factor_S,Factor_I,Factor_R):

    # Define model inputs
    S = inputs[:, 0]
    I = inputs[:, 1]
    R = inputs[:, 2]

    # Define model outputs
    S_pred = outputs[:,0]
    I_pred = outputs[:,1]
    R_pred = outputs[:,2]

    # Compute differential equations using the original values
    dS_dt = (-a*S*I*Factor_I)/Factor_N
    dI_dt = (a*S*I*Factor_S)/Factor_N - b*I
    dR_dt = (b*I*Factor_I/Factor_R)
```

```
    # Compute differential equations using the predicted values
    dS_dt_pred = (-a*S_pred*I_pred*Factor_I)/Factor_N
    dI_dt_pred = (a*S_pred*I_pred*Factor_S)/Factor_N - b*I_pred
    dR_dt_pred = (b*I_pred*Factor_I/Factor_R)

    # Compute loss function
    loss = (torch.mean(torch.square((dS_dt_pred - dS_dt))) + \
           torch.mean(torch.square((dI_dt_pred - dI_dt)*(Factor_I/Factor_S)**2))␣
↪+ \
           torch.mean(torch.square((dR_dt_pred - dR_dt)*(Factor_R/
↪Factor_S)**2))) + \
           torch.mean(torch.square(S_pred - S)) + \
           torch.mean(torch.square((I_pred - I)*(Factor_I/Factor_S)**2)) + \
           torch.mean(torch.square((R_pred - R)*(Factor_R/Factor_S)**2))

    return loss
```

Creating the PINN

[48]:
```
Physics_Informed_model =␣
↪LSTM(input_size,Num_Neurons_LSTM_Layer,Num_LSTM_Layers,output_size)
```

Optimizer for PINN

[49]:
```
optimizer = optim.Adam(Physics_Informed_model.parameters(), lr=Learning_Rate)
```

Training the PINN model

Number of the epochs

[50]:
```
Num_Epochs = 10000
```

Infection rate

[51]:
```
a = 0.03
```

Recovery rate

[52]:
```
b = 0.00005
```

[53]:
```
torch.manual_seed(0)
for epoch in range(Num_Epochs):
    outputs = Physics_Informed_model(Train_X)
    optimizer.zero_grad()
    loss = PINN_Loss_Function_SIR_MSE(outputs,␣
↪Train_Y,a,b,Factor_N,Factor_S,Factor_I,Factor_R)
```

```
    # loss = PINN_Loss_Function_SIR_L1(outputs,␣
↪Train_Y,a,b,Factor_N,Factor_S,Factor_I,Factor_R)
    loss.backward()
    optimizer.step()
    if epoch % 250 == 0:
        print(f"Number of epochs: {epoch}\nLoss: {loss.item()}\n")
```

```
Number of epochs: 0
Loss: 0.496025949716568

Number of epochs: 250
Loss: 0.13331173360347748

Number of epochs: 500
Loss: 0.08797959238290787

Number of epochs: 750
Loss: 0.05752529203891754

Number of epochs: 1000
Loss: 0.007950923405587673

Number of epochs: 1250
Loss: 0.0008450091700069606

Number of epochs: 1500
Loss: 0.0007011437555775046

Number of epochs: 1750
Loss: 0.0006358582759276032

Number of epochs: 2000
Loss: 0.0005953919026069343

Number of epochs: 2250
Loss: 0.0005649508093483746

Number of epochs: 2500
Loss: 0.0005375672481022775

Number of epochs: 2750
Loss: 0.0005105331656523049

Number of epochs: 3000
Loss: 0.00048307463293895125

Number of epochs: 3250
```

```
Loss: 0.0004551461315713823

Number of epochs: 3500
Loss: 0.0004269155324436724

Number of epochs: 3750
Loss: 0.0003985800431109965

Number of epochs: 4000
Loss: 0.0003703096299432218

Number of epochs: 4250
Loss: 0.0003422235604375601

Number of epochs: 4500
Loss: 0.000314387958496809

Number of epochs: 4750
Loss: 0.00028681644471362233

Number of epochs: 5000
Loss: 0.00025947910035029054

Number of epochs: 5250
Loss: 0.00023231731029227376

Number of epochs: 5500
Loss: 0.0002052657655440271

Number of epochs: 5750
Loss: 0.00017828785348683596

Number of epochs: 6000
Loss: 0.00015143690689001232

Number of epochs: 6250
Loss: 0.00012496545969042927

Number of epochs: 6500
Loss: 9.949383093044162e-05

Number of epochs: 6750
Loss: 7.620069663971663e-05

Number of epochs: 7000
Loss: 5.681904076482169e-05

Number of epochs: 7250
```

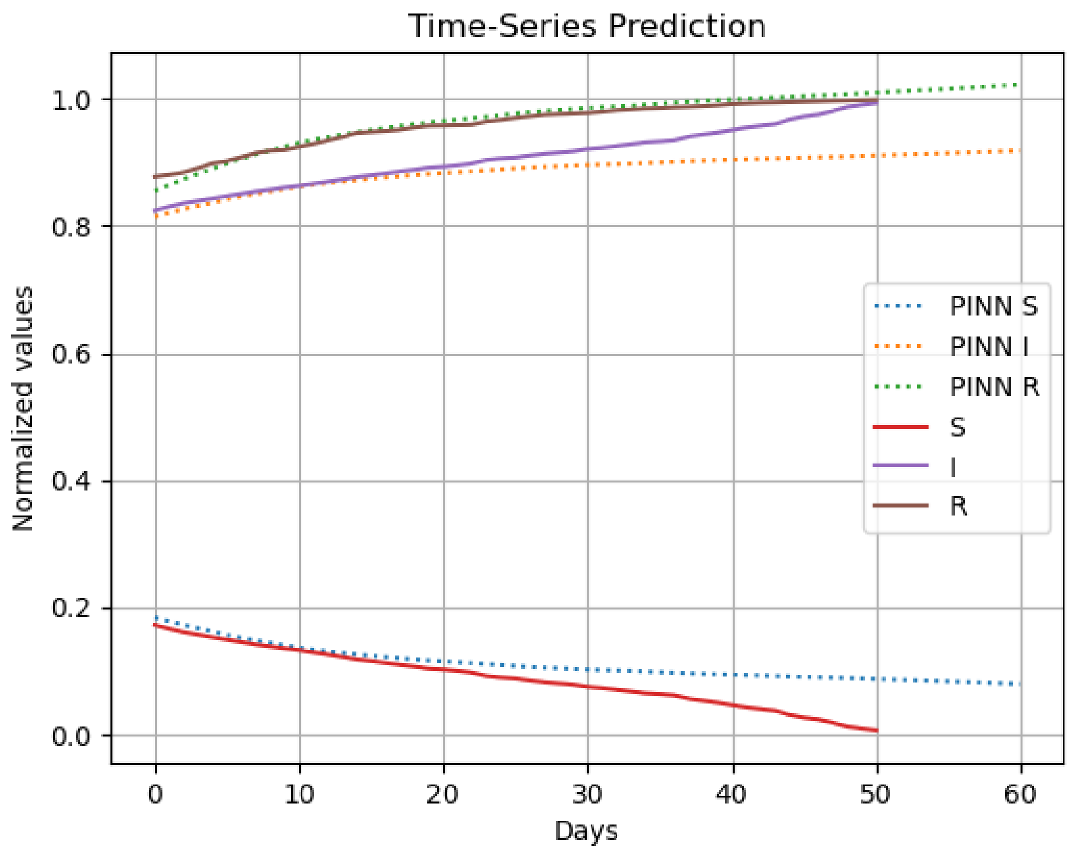
```
Loss: 4.302157321944833e-05

Number of epochs: 7500
Loss: 3.5158387618139386e-05

Number of epochs: 7750
Loss: 3.166797250742093e-05

Number of epochs: 8000
Loss: 3.031728192581795e-05

Number of epochs: 8250
Loss: 2.9667167837033048e-05

Number of epochs: 8500
Loss: 2.9186010578996502e-05

Number of epochs: 8750
Loss: 2.8748308977810666e-05

Number of epochs: 9000
Loss: 2.8331856810837053e-05

Number of epochs: 9250
Loss: 2.793049679894466e-05

Number of epochs: 9500
Loss: 2.7538722861208953e-05

Number of epochs: 9750
Loss: 2.7152456823387183e-05
```

Comparing the actual data with the predicted data on the test dataset

[54]:
```
Physics_Informed_model.eval()
LSTM_model.eval()
Data_PINN = Test_X
Data_LSTM = Test_X
Future_len = 10
for i in range(Future_len):
    train_predict_PINN = Physics_Informed_model(Data_PINN)
    train_predict_LSTM = LSTM_model(Data_LSTM)
    Data_PINN = Time_Step_Update(train_predict_PINN,Data_PINN)
    Data_LSTM = Time_Step_Update(train_predict_LSTM,Data_LSTM)
```

[55]:
```
data_predict_PINN = train_predict_PINN.data.numpy()
data_predict_LSTM = train_predict_LSTM.data.numpy()
dataY_plot = Test_Y.data.numpy()
```

[56]:
```
plt.title('Time-Series Prediction')
plt.plot(data_predict_PINN,linestyle= ":")
plt.plot(dataY_plot[Future_len:,:])
plt.legend(["PINN S","PINN I","PINN R","S","I","R"])
plt.xlabel("Days")
plt.ylabel("Normalized values")
plt.grid()
plt.savefig("PINN.jpg")
plt.show()
```



Saving the parameters of PINN Model

[57]:
```
torch.save(Physics_Informed_model,f"PINN_MSE_LR_{Learning_Rate}_Epochs_{epoch}\
_seq_{Sequence_Length}_train_amount_{Training_Data_Amount}_Num_LSTM_Layers_\
{Num_LSTM_Layers}_Num_Neurons_LSTM_Layer_{Num_Neurons_LSTM_Layer}_.pth")
```

Comparision of LSTM and PINN

[58]:
```
plt.title('Time-Series Prediction')
plt.plot(data_predict_PINN,linestyle= ":")
plt.plot(data_predict_LSTM,linestyle= "-.")
plt.plot(dataY_plot[Future_len:,:])
plt.legend(["PINN S","PINN I","PINN R","LSTM S","LSTM I","LSTM R","S","I","R"])
plt.xlabel("Days")
plt.ylabel("Normalized values")
plt.grid()
plt.savefig("LSTM_vs_PINN.jpg")
plt.show()
```