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Maths Assignment

Assignment 1: Vectors, Determinants and Matrices

Assignment 1

Vectors, Determinants & Matrices

1. Find the position vector of point $A(2, 4, -5)$

Ans $\bar{A} = A - O = (2, 4, -5) - (0, 0, 0)$
$= 2i, 4j, -5k$

2. Find the unit vector along $\bar{a} = 3i - 9j + \sqrt{10}k$

Soln

$$\hat{a} = \frac{\bar{a}}{|\bar{a}|} = \frac{3\hat{i} - 9j + \sqrt{10}k}{\sqrt{9 + 81 + 10}}$$

$$= \frac{3}{10}\hat{i} - \frac{9}{10}\hat{j} + \frac{\sqrt{10}}{10}\hat{k}$$

3. Show the points A, B, C are collinear.
$A(2, -1, 4)$ $B \equiv (3, 2, 5)$ $C(5, 8, 7)$

Ans

$\bar{AB} = \bar{B} - \bar{A} = (3, 2, 5) - (2, -1, 4)$
$= (1, 3, 1)$

$\bar{BC} = \bar{C} - \bar{B} = (5, 8, 7) - (3, 2, 5)$
$= (2, 6, 2)$

$\bar{AC} = \bar{C} - \bar{A} = (5, 8, 7) - (2, -1, 4)$
$= (3, 9, 3)$

For A, B, C to be collinear

$$\overline{AB} + \overline{BC} = \overline{AC}$$

LHS = RHS

$= \hat{i} + 3\hat{j} + \hat{k} + 2\hat{i} + 6\hat{j} + 2\hat{k}$

$= 3\hat{i} + 9\hat{j} + 3\hat{k}$

$= \overline{AC}$

4. Find scalar product $\bar{a} = 2i - 3j + k$
$\bar{b} = \hat{i} + 2\hat{j} - 3\hat{k}$

Ans

$\bar{a} . \bar{b} = (2\hat{i} - 3\hat{j} + \hat{k}) . (\hat{i} + 2\hat{j} - 3\hat{k})$

$= 2.(1) - 3(2) + 1(-3)$

$= 2 - 6 - 3$

$= -7$

5. Show the vectors are mutually $\perp$
$\bar{a} = -2\hat{i} - 3\hat{j} + \hat{k}$, $\bar{b} = \hat{i} + \hat{j} + 5\hat{k}$
$\bar{c} = -16\hat{i} + 11\hat{j} + \hat{k}$

Ans

If $\bar{a}, \bar{b}$ & $\bar{c}$ are mutually $\perp$
then $\bar{a}.\bar{b} = \bar{b}.\bar{c} = \bar{a}.\bar{c} = 0$

$\bar{a}.\bar{b} = (-2, -3, 1) . (1, 1, 5)$

$= -2 + -3(1) + 1(5)$

$= -5 + 5 = 0$

$\bar{a}.\bar{c} = (-2, -3, 1)(-16, 11, 1)$

$= -2(-16) + (-3)(11) + 1(1)$

$= +32 - 33 + 1 = 0$

$$\bar{b} \cdot \bar{c} = (\hat{i} + \hat{j} + 5\hat{k})(-16\hat{i} + 11\hat{j} + 1\hat{k})$$
$$= -16 + 11 + 5$$
$$= 0$$

6. Find cosine of the angle between the vectors $\bar{a}$ and $\bar{b}$ if
$\bar{a} = \hat{i} - 2j + k$, $\bar{b} = 2\hat{i} - 2j + 2\hat{k}$

Ans

$$\bar{a} \cdot \bar{b} = |\bar{a}||\bar{b}| \cos\theta \Rightarrow \theta = \cos^{-1}\left(\frac{\bar{a} \cdot \bar{b}}{|\bar{a}||\bar{b}|}\right)$$

$$\theta = \cos^{-1}\left(\frac{(i - 2j + k)(2i - 2j + 2\hat{k})}{\sqrt{1+4+1}\ \sqrt{4+4+4}}\right)$$

$$= \cos^{-1}\left(\frac{2 + 4 + 2}{\sqrt{6}\ \sqrt{12}}\right)$$

$$= \cos^{-1}\left(\frac{8}{\sqrt{2 \times 3 \times 2 \times 3 \times 2}}\right) = \cos^{-1}\left(\frac{\cancel{8}\,4}{3\,\cancel{6}\sqrt{2}}\right)$$

$$\theta = \cos^{-1}\left(\frac{4}{3\sqrt{2}}\right)$$

7. Find the projection of $\bar{b}$ on $\bar{a}$.

Ans $\bar{a} = 2i + 3j - 4\hat{k}$ and $\bar{b} = \hat{i} - \hat{j} - \hat{k}$

Projection of $\bar{b}$ on $\bar{a}$ is

$$\frac{\bar{b} \cdot \bar{a}}{|\bar{a}|} = \frac{(i - j - k)(2i + 3j - 4k)}{\sqrt{4 + 9 + 16}}$$

$$= \frac{2 - 3 + 4}{\sqrt{29}}$$

$$= \frac{3}{\sqrt{29}}$$

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8. Find $\bar{a} \times \bar{b}$
$\bar{a} = 3i - \hat{j} + 2\hat{k}$ $\quad \bar{b} = i + 5j - 2\hat{k}$

Ans

$$\bar{a} \times \bar{b} = \begin{vmatrix} i & j & k \\ 3 & -1 & 2 \\ 1 & 5 & -2 \end{vmatrix}$$

$$= i(-1 \times -2 - 5 \times 2) - j(3 \times -2 - 1 \times 2) + k(3 \times 5 - 1(-1))$$

$$= i(2 - 10) - j(-6 - 2) + k(15 + 1)$$

$$\bar{a} \times \bar{b} = -8\hat{i} - 8\hat{j} + 16\hat{k}$$

9. Find the sine of the angle b/w the vectors $\bar{a}$ and $\bar{b}$ if
$\bar{a} = i - 2\hat{k}$ $\quad \bar{b} = j - 4k$

Ans

$$\bar{a} \times \bar{b} = \begin{vmatrix} i & j & k \\ 1 & 0 & -2 \\ 0 & 1 & -4 \end{vmatrix} = i(0 - (-2)) - j(-4 - 0) + k(1 - 0)$$

$= ab\sin\theta\hat{n}$

$$\Rightarrow 2\hat{i} + 4\hat{j} + \hat{k} = |\bar{a}|\,|\bar{b}|\,.\sin\theta\,\hat{n}$$

$$= \sqrt{1+4}\,.\sqrt{1+16}\,.\sin\theta\,\hat{n}$$

$$\sin\theta = \frac{2\hat{i} + 4\hat{j} + \hat{k}}{\sqrt{5}\,.\sqrt{17}}$$

$$\theta = \sin^{-1}\left(\frac{2\hat{i} + 4\hat{j} + k}{\sqrt{85}}\right)$$

$5\hat{j} - 2\hat{k}$

$2 - 1 \times 2)$

b/w the

$- (-2))$

$4 - 0)$

$(1 - 0)$

$\theta \hat{n}$

$. \sin\theta \hat{n}$

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or

$$\bar{a} \cdot \bar{b} = ab\cos\theta$$

$$\cos\theta = \frac{\bar{a} \cdot \bar{b}}{ab} = \frac{(i - 2\hat{k})(j - 4k)}{\sqrt{1+4}\ \sqrt{1+16}}$$

$$= \frac{0 + 0 + 8}{\sqrt{85}} = \frac{8}{\sqrt{85}}$$

$$\cancel{\sin^2\theta = 1 - \cos^2\theta}$$

$$\sin(90 - \theta) = \cos\theta = \frac{8}{\sqrt{85}}$$

10. Find vector area of the Δ, the position vectors of whose vertices are $2\hat{i} + \hat{j} - 3\hat{k}$, $\hat{i} + 2\hat{j} + \hat{k}$, $3\hat{i} + j - 2\hat{k}$.

Ans

$$A \equiv (2, 1, -3)$$

$$B = (1, 2, 1)$$

$$C \equiv (3, 1, -2)$$

$$\bar{AB} = \bar{B} - \bar{A} = (1, 2, 1) - (2, 1, -3)$$

$$= (-1, 1, 4)$$

$$\bar{BC} = \bar{C} - \bar{B} = (3, 1, -2) - (1, 2, 1)$$

$$= (2, -1, -3)$$

$$\text{Area of } \Delta = \frac{1}{2} \times |\bar{AB} \times \bar{BC}|$$

$$\begin{vmatrix} i & j & k \\ -1 & 1 & 4 \\ 2 & -1 & -3 \end{vmatrix} = i(-3 - (-4)) - j(3 - 8) + k(+1 - 2)$$

$$= i + 5\hat{j} - \hat{k}$$

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Area of $\Delta = \frac{1}{2} \times \sqrt{1 + 25 + 1}$

$= \frac{1}{2} \times \sqrt{27} = \frac{3\sqrt{3}}{2}$

13.

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Find value of determinants where $i = \sqrt{-1}$

$$\begin{vmatrix} 1+3i & i-2 \\ -i-2 & 1-3i \end{vmatrix}$$

Ans

Ans

$D = (1+3i)(1-3i) - (i-2)(-i-2)$

$= 1^2 - (3i)^2 + (i-2)(i+2)$

$= 1 - 9i^2 + i^2 - 2^2$

$= 1 - 9(-1) + (-1) - 4$

$= 1 + 9 - 1 - 4$

$= 10 - 5$

$D = 5$

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12. Check if singular ~~o~~/or non-singular matrix.

Ans

(a) $\begin{bmatrix} 3 & 5 & 7 \\ -2 & 1 & 4 \\ 3 & 2 & 5 \end{bmatrix}$

(b) $\begin{bmatrix} 7 & 5 \\ -4 & 7 \end{bmatrix}$

$\Rightarrow$

$3(5-8) - 5(-10-12) + 7(-4-3)$

$= 3(-3) - 5(-22) + 7(-7)$

$= -9 + 110 - 49$

$= 52 \neq 0$

$\Rightarrow 49 - (-20)$

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$\neq 0$

Ans Both A & B are non-singular matrices.

13. If $A = \begin{bmatrix} 2 & 1 & 4 \\ 3 & 0 & 2 \\ 1 & 2 & 1 \end{bmatrix}$ $B = \begin{bmatrix} -1 & 1 & 1 \\ 2 & 3 & 0 \\ 1 & -3 & 1 \end{bmatrix}$

Find AB.

Ans

$$\begin{bmatrix} 2(-1)+1(2)+4(1) & 2(1)+1(3)+4(-3) & 2(1)+1(0)+4(1) \\ 3(-1)+0(2)+2(1) & 3(1)+0(3)+2(-3) & 3(1)+0\times0+2\times1 \\ 1\times-1+2\times2+1\times1 & 1\times1+2\times3+1(-3) & 1(1)+2(0)+1(1) \end{bmatrix}$$

$$= \begin{bmatrix} 4 & -7 & 6 \\ -1 & -3 & 5 \\ 4 & 4 & 2 \end{bmatrix}$$

14 Find A^T if $A = \begin{bmatrix} 2 & 4 & -1 \\ 3 & -1 & 2 \end{bmatrix}$

Ans $A^T = \begin{bmatrix} 2 & 3 \\ 4 & -1 \\ -1 & 2 \end{bmatrix}$

Assignment 2: Limits

Assignment 2

Limits

1. $$\lim_{x \to -3} \frac{\sqrt{2x+22} - 4}{x+3}$$

Sol^n

$$\lim_{x \to -3} \frac{(2x+22)^{1/2} - (4^2)^{1/2}}{x+3}$$

$$= \lim_{x \to -3} \frac{\sqrt{2x+22} - 4}{x+3} \times \frac{\sqrt{2x+22}+4}{\sqrt{2x+22}+4}$$

$$= \lim_{x \to -3} \frac{2x+22-16}{(x+3)(\sqrt{2x+22}+4)}$$

$$= \lim_{x \to -3} \frac{2x+6}{(x+3)(\sqrt{2x+22}+4)}$$

$$= \lim_{x \to -3} \frac{2\cancel{(x+3)}}{\cancel{(x+3)}(\sqrt{2x+22}+4)}$$

$$= \lim_{x \to -3} \frac{2}{\sqrt{2x+22}+4}$$

$$= \frac{2}{\sqrt{16}+4} = \frac{2}{4+4} = \frac{2}{8}$$

$$= \frac{1}{4}$$

2. $$\lim_{z \to 8} \frac{2z^2 - 17z + 8}{8 - z}$$

Sol^n

$$\lim_{z \to 8} \frac{(2z-1)\cancel{(z-8)}}{\cancel{(8-z)}\,-1}$$

$$\lim_{z \to 8} (1 - 2z) = 1 - 2 \times 8$$

$$\lim_{z \to 8} \frac{2z^2 - 17z + 8}{8 - z} = -15$$

3. $$\lim_{y \to 7} \frac{y^2 - 4y - 21}{3y^2 - 17y - 28}$$

Solⁿ

$$\lim_{y \to 7} \frac{y^2 - 7y + 3y - 21}{3y^2 - 21y + 4y - 28}$$

$$\lim_{y \to 7} \frac{y(y-7) + 3(y-7)}{3y(y-7) + 4(y-7)}$$

$$\lim_{y \to 7} \frac{\cancel{(y-7)}(y+3)}{\cancel{(y-7)}(3y+4)}$$

$$= \lim_{y \to 7} \frac{y+3}{3y+4}$$

$$= \frac{7+3}{3(7)+4} = \frac{10}{25} = \frac{2}{5}$$

4. $$\lim_{x \to 4} \frac{\sqrt{z} - 2}{z - 4}$$

Solⁿ

$$\lim_{x \to 4} \frac{\sqrt{z} - 2}{z - 4} \times \frac{\sqrt{z} + 2}{\sqrt{z} + 2}$$

$$= \lim_{x \to 4} \frac{\cancel{(z - 4)}}{\cancel{(z-4)}(\sqrt{z} + 2)}$$

$$= \frac{1}{\sqrt{4} + 2} = \frac{1}{2+2} = \frac{1}{4}$$

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5. $\lim_{t \to -3} \frac{6+4t}{t^2+1}$

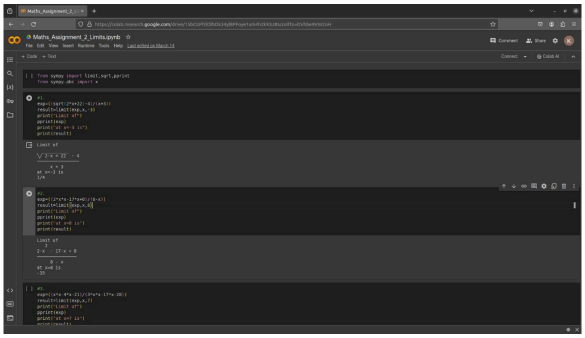
Soln

$\lim_{t \to -3} \frac{6+4t}{t^2+1}$

$= \frac{6+4(-3)}{(-3)^2+1}$

$= \frac{6-12}{9+1} = \frac{-6}{10}$

$= \frac{-3}{5}$



```
from sympy import limit,sqrt,pprint
from sympy.abc import x
```

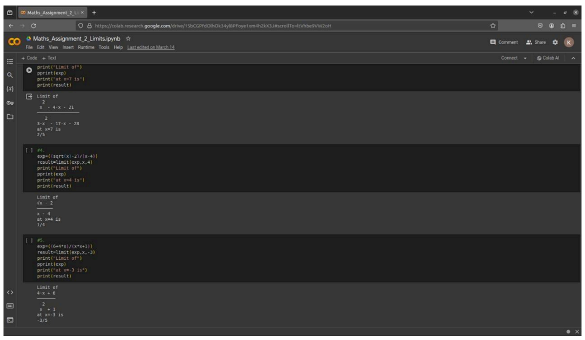
```
#1.
exp=((sqrt(2*x+22)-4)/(x+3))
result=limit(exp,x,-3)
print("Limit of")
pprint(exp)
print("at x=-3 is")
print(result)
```

```
Limit of
  __________
╲╱ 2·x + 22  - 4
────────────────
     x + 3
at x=-3 is
1/4
```

```
#2.
exp=((2*x*x-17*x+8)/(8-x))
result=limit(exp,x,8)
print("Limit of")
pprint(exp)
print("at x=8 is")
print(result)
```

```
Limit of
   2
2·x  - 17·x + 8
───────────────
     8 - x
at x=8 is
-15
```

```
#3.
exp=((x*x-4*x-21)/(3*x*x-17*x-28))
result=limit(exp,x,7)
print("Limit of")
pprint(exp)
print("at x=7 is")
print(result)
```



```
print("Limit of")
pprint(exp)
print("at x=7 is")
print(result)
```

```
Limit of
  2
 x  - 4·x - 21
────────────────
   2
3·x  - 17·x - 28
at x=7 is
2/5
```

```
#4.
exp=((sqrt(x)-2)/(x-4))
result=limit(exp,x,4)
print("Limit of")
pprint(exp)
print("at x=4 is")
print(result)
```

```
Limit of
√x - 2
──────
x - 4
at x=4 is
1/4
```

```
#5.
exp=((6+4*x)/(x*x+1))
result=limit(exp,x,-3)
print("Limit of")
pprint(exp)
print("at x=-3 is")
print(result)
```

```
Limit of
4·x + 6
───────
  2
 x  + 1
at x=-3 is
-3/5
```