NCERT-Analog: 11.14.18

EE:1205 Signals and Systems
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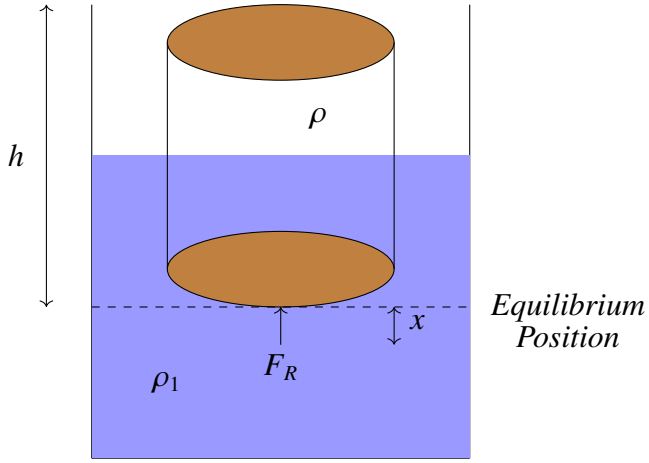
I. Question:

A cylindrical piece of cork of density of base area A and height h floats in a liquid of density ρ_1, The cork is depressed slightly and then released. Show that the cork oscillates up and down simple harmonically with a period $T = 2\pi\sqrt{\frac{h\rho}{\rho_1 g}}$

Solution:

Parameter	Description
ρ_1	Density of Liquid
ρ	Density of cork
h	Height of cylindrical cork
x	Displacement
T	Time period
A	Base area of cylindrical cork
F_R	Restoring Force
a	Acceleration
ω	Angular Frequency
$m = \rho Ah$	Mass of cylindrical cork

TABLE 1
Parameter Table



When we slightly displace cylindrical piece of cork by a depth x,

$$F_R = -(\rho_1 Ax)\, g \tag{1}$$

$$\implies a = -\frac{\rho_1 Ag}{m}x \tag{2}$$

$$a = -\omega^2 x \tag{3}$$

Comparing (2) and (3),

$$\omega^2 = \frac{\rho_1 Ag}{m} = \frac{\rho_1 Ag}{\rho Ah} \tag{4}$$

$$\implies \omega = \sqrt{\frac{\rho_1 g}{\rho h}} \tag{5}$$

$$T = \frac{2\pi}{\omega} \tag{6}$$

$$\therefore T = 2\pi\sqrt{\frac{h\rho}{\rho_1 g}} \tag{7}$$

Hence Proved.