Problem 1

In this problem, we need to write a matlab function to compute 2D Fast Fourier Transform (Recursive Formulation).

Approach

- To compute 2D FFT we need to
  - do 1D FFT on each row (real to complex) . . . . (1)
  - do 1D FFT on each column resulting from (1) (complex to complex)
- So, we need to write code 1D FFT. I used this algorithm to write code for 1D FFT.

RECURSIVE-FFT(a)

```
1  n ← length[a]            ▷ n is a power of 2.
2  if n = 1
3     then return a
4  ω_n ← e^(2πi/n)
5  ω ← 1
6  a^[0] ← (a_0, a_2, . . . , a_(n−2))
7  a^[1] ← (a_1, a_3, . . . , a_(n−1))
8  y^[0] ← RECURSIVE-FFT(a^[0])
9  y^[1] ← RECURSIVE-FFT(a^[1])
10 for k ← 0 to n/2 − 1
11       do y_k ← y_k^[0] + ω y_k^[1]
12          y_(k+(n/2)) ← y_k^[0] − ω y_k^[1]
13          ω ← ω ω_n
14 return y                 ▷ y is assumed to be column vector.
```

Code

1. 2D FFT code which uses 1D FFT.

```
FFT_row = zeros(size(A));
FFT_col = zeros(size(A));

%Perform FFT on each row
for i=1:size(A,1)
    FFT_row(i,:) = recursive_FFT(A(i,:));
end

%Perform FFT on each column
for i=1:size(A,2)
    FFT_col(:,i) = recursive_FFT(FFT_row(:,i).');
end

% Plot the 2D FFT
C = fftshift(FFT_col);
```

```
figure, imagesc(mat2gray(log(abs(C)+1))), colormap gray;
title('Using Recursive FFT function');
```

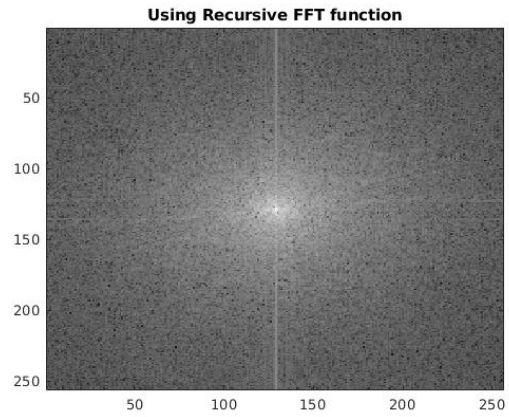
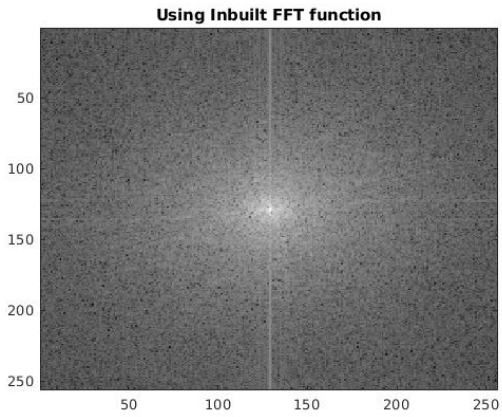
2. Recursive 1D FFT.

```
function FFT = recursive_FFT(x)
% Only works if N = 2^k
N = length(x);
even_part = zeros(1,floor(N/2));
odd_part = zeros(1,floor((N+1)/2));
for i=1:2:N
    odd_part(1,floor((i+1)/2))=x(1,i);
end
for i=2:2:N
    even_part(1,floor(i/2))=x(1,i);
end
if N == 1
    FFT = x;
else
    ODD_FFT = recursive_FFT(odd_part);
    EVEN_FFT = recursive_FFT(even_part);
    FFT = zeros(N,1);
    Exp_vec = exp(-1i*2*pi*((0:N/2-1)')/N);
    tmp = Exp_vec .* EVEN_FFT;
    FFT = [(ODD_FFT + tmp);(ODD_FFT -tmp)];
end
```

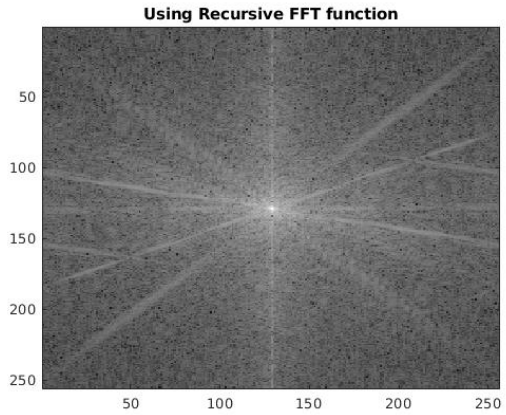
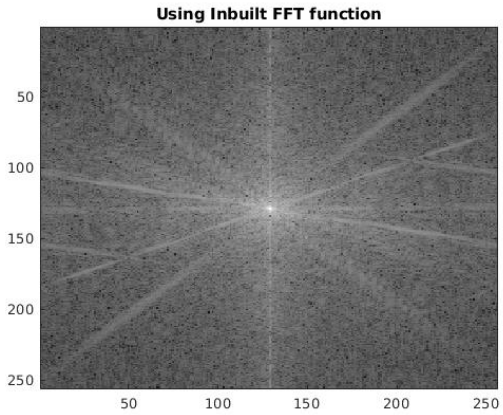
3. 2D FFT using inbuilt function `fft2`.

```
B = fftshift(fft2(A));
figure, imagesc(mat2gray(log(abs(B)+1))), colormap gray;
title('Using Inbuilt FFT function');
```

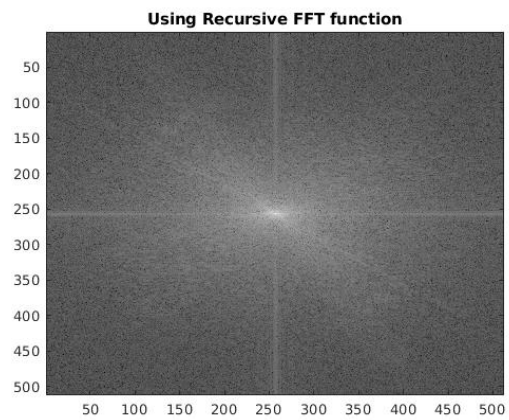
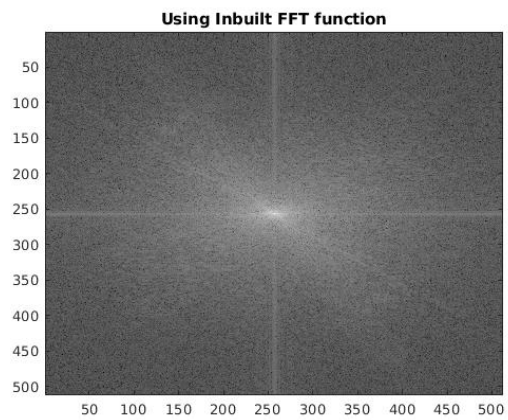
Img1.jpg

My Recursive FFT function	Inbuilt FFT2 function
	

Img2.jpg

My Recursive FFT function	Inbuilt FFT2 function
	

Img3.jpg

My Recursive FFT function	Inbuilt FFT2 function
	

Observation

As we can see the output results of both inbuilt 2D FFT function (`fft2`) and my recursive 2D FFT function are almost identical. My spectrogram works only if the input image is some power of 2.