Kinematics of Particles

Dynamics

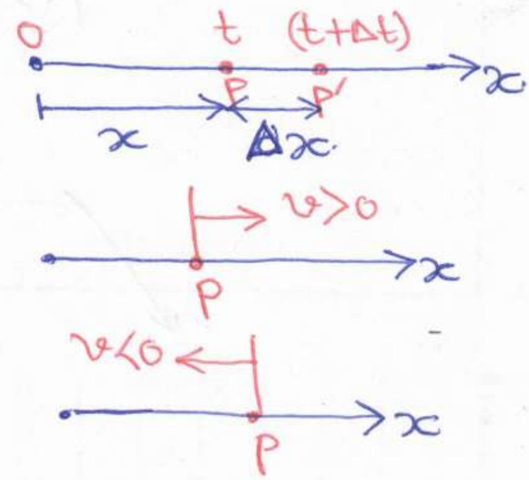
Kinematics | Kinetics

- **Kinematics**
  - study of geometry of motion of bodies/objects.
  - It is used to relate the displacement, velocity, acceleration, and time without reference to the cause of motion.
- **Kinetics**
  - study of the relations existing between the forces acting on a body, the mass of the body, and the motion of the body.
  - It is used to predict the motion caused by the given forces or to determine the forces required to produce a given motion.

Rectilinear Motion
↳ position, velocity, and acceleration of a particle as it moves along a straight line

Curvilinear Motion
↳ position, velocity, and acceleration of a particle as it moves a curved line.

Rectilinear Motion



- Consider a particle P which occupies a position P at time t and P' at $(t+\Delta t)$
- Average velocity $= \dfrac{\Delta x}{\Delta t}$
- Instantaneous velocity
  $$= v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$
  ↳ may be "+ve" or "−ve".
- Magnitude of velocity is referred to as particle speed

$$\begin{array}{c} \quad\quad v \quad\quad (v+\Delta v) \\ O \longrightarrow P' \longrightarrow P' \longrightarrow x \\ \quad\quad (t) \quad\quad (t+\Delta t) \end{array}$$

- Consider the particle with velocity v at time t and $(v+\Delta v)$ at $(t+\Delta t)$.

- Average acceleration $a = \dfrac{\Delta v}{\Delta t}$
- Instantaneous acceleration $a = \lim\limits_{\Delta t \to 0} \dfrac{\Delta v}{\Delta t} = \dfrac{dv}{dt}$

$$= \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2}$$

Determination of Motion of Particle

Three classes of motion may be defined
- acceleration given as a function of time, $a=f(t)$
- acceleration given as a function of position, $a=f(x)$
- acceleration given as a function of velocity, $a=f(v)$

① Acceleration given as a function of time, $a=f(t)$

$$\frac{dv}{dt} = a = f(t)$$

$$\Rightarrow dv = f(t)\,dt$$

$$\Rightarrow \int_{v_0}^{v(t)} dv = \int_0^t f(t)\,dt$$

$$\Rightarrow v(t) - v_0 = \int_0^t f(t)\,dt$$

$$\frac{dx}{dt} = v(t)$$

$$\Rightarrow dx = v(t)\,dt$$

$$\Rightarrow \int_{x_0}^{x(t)} dx = \int_0^t v(t)\,dt$$

$$\Rightarrow x(t) - x_0 = \int_0^t \left[v_0 + \int_0^t f(t)\,dt\right] dt$$

② Acceleration given as a function of position, $a=f(x)$

$$a = \frac{dv}{dt} = f(x)$$

$$\Rightarrow \frac{dv}{dx}\cdot\frac{dx}{dt} = f(x)$$

$$\Rightarrow v\,dv = f(x)\,dx$$

$$\Rightarrow \int_{v_0}^{v(x)} v\,dv = \int_{x_0}^{x} f(x)\,dx$$

$$\Rightarrow \frac{(v(x))^2}{2} - \frac{v_0^2}{2} = \int_{x_0}^{x} f(x)\,dx$$

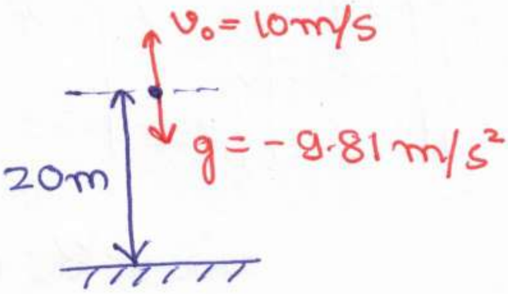
Eg. A ball is tossed with a vertical velocity of 10 m/s from a window situated 20 m above the ground. Determine

(a) velocity and elevation of the ball above the ground at time 't'

(b) highest elevation reached by the ball and the corresponding time

(c) time when ball hits the ground and the corresponding velocity.

Solution.



(a) $a = \frac{dv}{dt}$

$\Rightarrow \int_{v_0}^{v} dv = \int_0^t a\,dt$

$\Rightarrow v - v_0 = -gt$

$\Rightarrow v = v_0 - gt$

$\Rightarrow \boxed{v = 10 - 9.81t}$

• $v = \frac{dy}{dt}$

$\Rightarrow \int_{20}^{y} dy = \int_{10}^{t} v\,dt$

$\Rightarrow y - 20 = \int_0^t (10 - 9.81)\,dt$

$\Rightarrow \boxed{y = 20 + 10t - \frac{9.81t^2}{2}}$

(b) At the highest elevation, $v = 0$

$\Rightarrow 10 - 9.81t = 0$

$\Rightarrow t = \frac{10}{9.81} = \underline{1.019\,s}$

Substituting $t = 1.019$ s in the expression of y:

$y = 20 + 10(1.019) - \frac{9.81}{2}(1.019)^2$

$\Rightarrow \boxed{y = 25.1\,m}$

(c) When the ball hits the ground $y = 0$

$\Rightarrow 20 + 10t - 4.905t^2 = 0$

$\Rightarrow t = -1.243\,s$ (meaning less)

$= \underline{3.28\,s}$ ✓

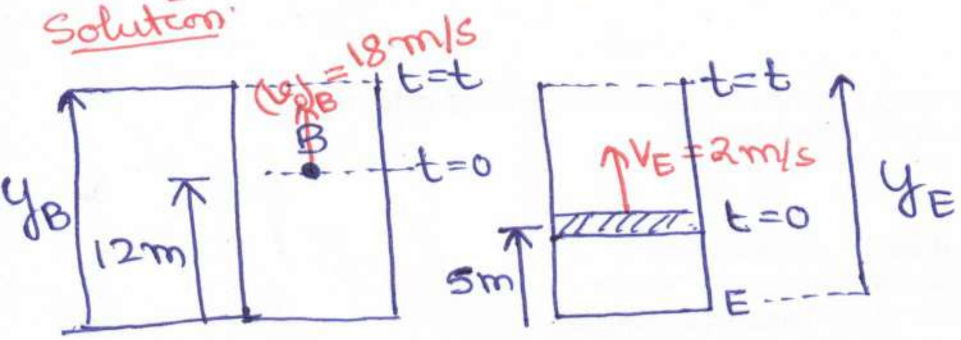
• $v = 10 - 9.81(t)$

$= 10 - 9.81(3.28)$

$= -22.2\,m/s^2$.

Eg: A ball is thrown vertically from 12m level in an elevator shaft with an initial velocity of 18 m/s. At the same instant, an open-elevator platform passes 5 m level upwards at an uniform velocity of 2 m/s.
Determine :-
(a) when and where the ball hits the elevator
(b) relative velocity of ball and elevator at contact.

Solution:



- $v_B = (v_0)_B - gt$

$\Rightarrow v_B = 18 - 9.81t$

- $y_B = (v_0 t) - \frac{1}{2}gt^2 + y_0$

$\Rightarrow y_B = 12 + 18t - \frac{1}{2}(9.81)t^2$

$v_E = 2$ m/s (uniform velocity)

$y_E = (y_0)_E + v_E t$

$\Rightarrow y_E = 5 + 2t$

(a) For the ball to hit the elevator, $y_E = y_B$

$$\Rightarrow 12 + 18t - \frac{9.81}{2}t^2 = 5 + 2t$$

$$\Rightarrow 4.905t^2 - 16t - 12 = 0$$

$$\Rightarrow t = -0.63\,s \text{ (meaningless)}$$

$$= \underline{3.89\,s}$$

$y_E = 5 + 2(3.89) = \underline{12.78\,m}$ (position of elevator at the instance when ball hits it)

(b) $v_{B/E} = v_B - v_E$

$$= 18 - 9.81(3.65) - 2.$$

$$\Rightarrow v_{B/E} = \underline{-19.81\,m/s}$$

③ Acceleration as a function of velocity, $a = f(v)$

$$\frac{dv}{dt} = a = f(v)$$

$$\Rightarrow \frac{dv}{f(v)} = dt$$

$$\Rightarrow \int_{v_0}^{v(t)} \frac{dv}{f(v)} = \int_0^t dt$$

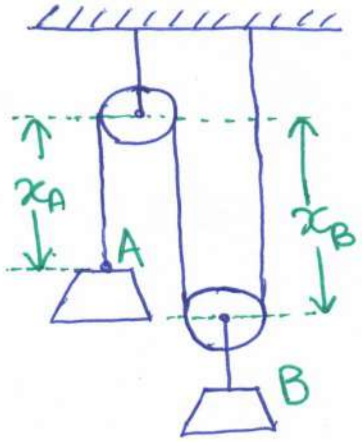
$$\Rightarrow \int_{v_0}^{v(t)} \frac{dv}{f(v)} = t$$

$$v\frac{dv}{dx} = f(v)$$

$$\Rightarrow \int_{x_0}^{x(t)} dx = \int_{v_0}^{v(t)} \frac{v\,dv}{f(v)}$$

$$\Rightarrow x(t) - x_0 = \int_{v_0}^{v(t)} \frac{v\,dv}{f(v)}$$

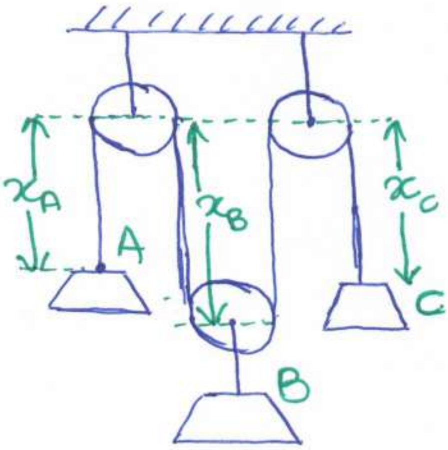
Motion of Several Particles: Dependent Motion.



- Position of block B depends on the position of block A.
- As the length of the rope is constant, it follows that the sum of lengths of segments must be constant.

$$x_A + 2x_B = \text{constant}$$

(one ~~diagram~~ degree of freedom).

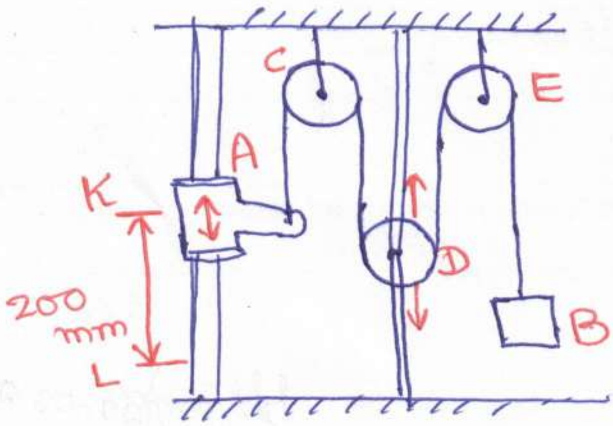


- Position of three blocks are dependent.

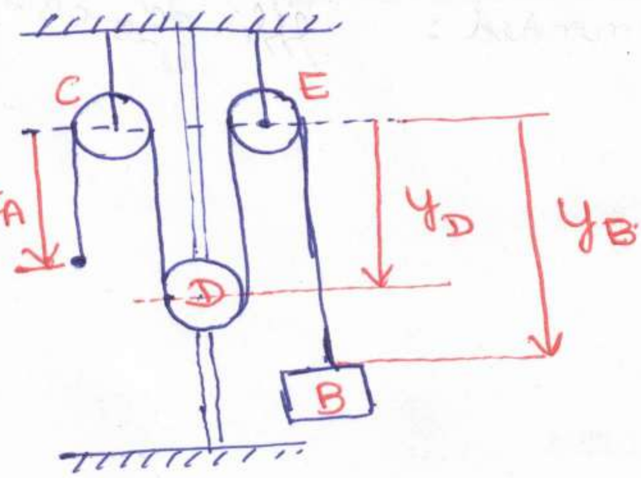
$$x_A + 2x_B + x_C = \text{constant}$$

(two degrees of freedom)

Eg. Pulley 'D' moves down at a velocity of 75 mm/s, which is also attached to the collar A. At time $t=0$, collar 'A' starts moving down from K with constant acceleration and zero initial velocity. Knowing the velocity of collar 'A' is 300 mm/s as it passes L, determine the change in elevation, velocity, and acceleration of block B, when collar 'A' is at 'L'.



Solution



$$y_A + 2y_D + y_B = \text{constant}$$

$$\Rightarrow \dot{y}_A + 2\dot{y}_D + \dot{y}_B = 0$$

$$\Rightarrow v_A + 2v_D + v_B = 0$$

Differentiating the velocities w.r.t the time, we get

$$a_A + 2a_D + a_B = 0$$

(*) For collar A:-

$v_0 = 0$

$(v_A)_L = 300 \, m/s$.

$y_A = 200 \, mm$.

- Using the relation -

$v_A^2 = v_0^2 + 2a_A y$

$\Rightarrow (300)^2 = 2a_A(200)$.

$\Rightarrow a_A = 225 \, mm/s$.

- Also, $v_A = v_0 + a_A t$

$\Rightarrow t = \frac{300}{225} = 1.33 s$

(*) For pulley D, it moves in a rectilinear motion with uniform velocity of 75 mm/s.

$\therefore y_D = (75)(1.33) = 100 \, mm$.

$\therefore y_A + 2y_D + y_B = \text{constant} = (y_A)_0 + 2(y_D)_0 + (y_B)_0$

$\Rightarrow [y_A - (y_A)_0] + 2[y_D - (y_D)_0] + [y_B - (y_B)_0] = 0$

$\Rightarrow 200 + 2(100) + [y_B - (y_B)_0] = 0$

$\Rightarrow \boxed{[y_B - (y_B)_0] = -400 \, mm}$

Again, $\dot{y}_A + 2\dot{y}_D + \dot{y}_B = 0$

$\Rightarrow v_A + 2v_D + v_B = 0$

$\Rightarrow 300 + 2(75) + v_B = 0$

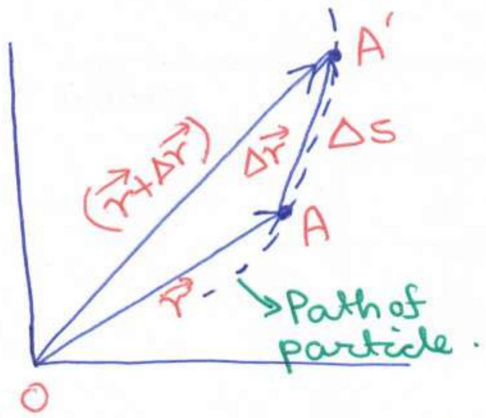
$\Rightarrow \boxed{v_B = -450 \, mm/s}$

Similarly, $a_A + 2a_D + a_B = 0$

$\Rightarrow 225 + 0 + a_B = 0$

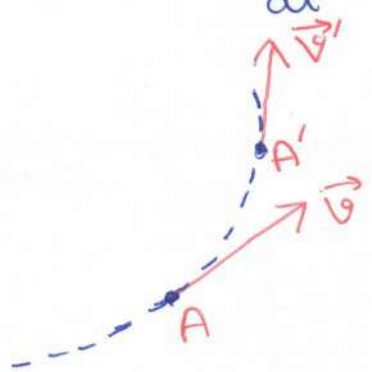
$\Rightarrow \boxed{a_B = -225 \, m/s^2}$

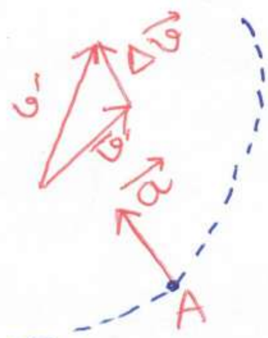
Plane Curvilinear Motion



- At time t, particle is at point A whose position vector is given as $\vec{r}$.
- At time $(t+\Delta t)$, the particle moves to A' whose position vector is given as $(\vec{r}+\Delta\vec{r})$

- The complete information of the particle is known if the position vector (magnitude and direction) at a given instance is specified.
- Average velocity $(\vec{V}_{av}) = \dfrac{\Delta\vec{r}}{\Delta t}$
- Average speed $= \dfrac{\Delta s}{\Delta t}$
- Instantaneous velocity $(\vec{V}) = \lim_{\Delta t\to 0} \dfrac{\Delta\vec{r}}{\Delta t} = \dfrac{d\vec{r}}{dt} = (\dot{\vec{r}})$
- Speed $|V| = \dfrac{ds}{dt} = \dot{s}$



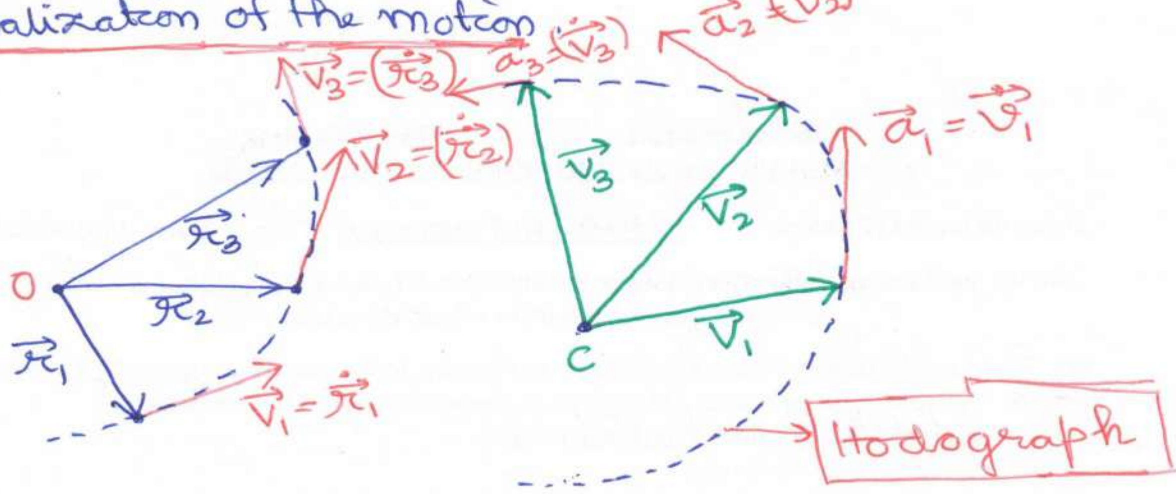


- As $\Delta\vec{r}\to 0$, the velocity vector becomes tangent to the curved path at the point of consideration.

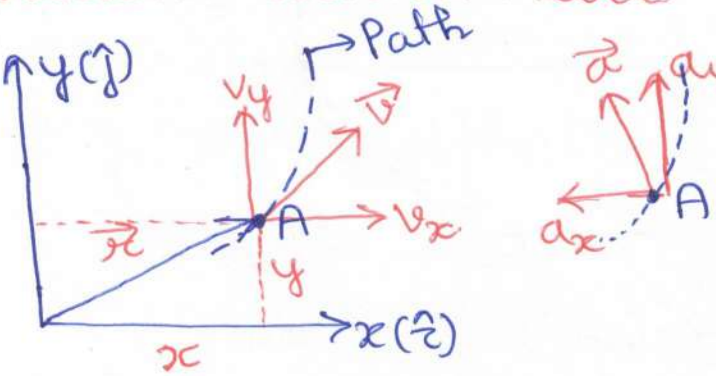
- In general, the acceleration of the particle is neither tangent nor normal to the curved path at the point of consideration.
- The direction of acceleration $(\vec{a})$ is along the direction $(\Delta\vec{v})$.

$$\vec{a} = \lim_{\Delta t\to 0} \frac{\Delta\vec{v}}{\Delta t} = \frac{d\vec{v}}{dt} = (\dot{\vec{v}})$$

Visualization of the motion



Rectangular Coordinates (x-y).

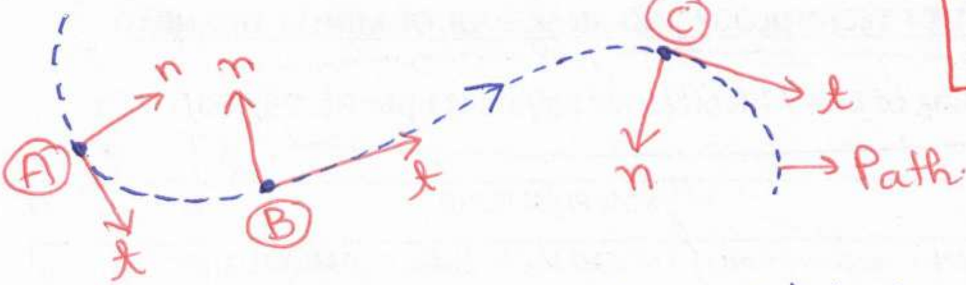


- $\vec{r} = x\hat{i} + y\hat{j}$.
- $\vec{V} = \dot{\vec{r}} = \frac{d}{dt}(x\hat{i}+y\hat{j}) = \hat{i}\frac{dx}{dt} + x\cancelto{0}{\frac{d\hat{i}}{dt}} + \hat{j}\frac{dy}{dt} + y\cancelto{0}{\frac{d\hat{j}}{dt}}$

$$\Rightarrow \boxed{\vec{V} = \dot{x}\hat{i} + \dot{y}\hat{j}}$$

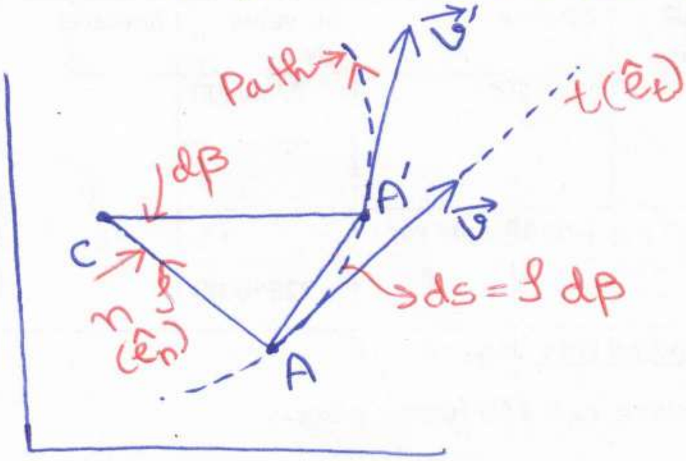
- $\boxed{\vec{a} = \dot{\vec{V}} = \ddot{x}\hat{i} + \ddot{y}\hat{j}}$
- $|\vec{V}| = \sqrt{V_x^2 + V_y^2}$, $\tan\theta = \frac{V_y}{V_x}$.
- $|\vec{a}| = \sqrt{a_x^2 + a_y^2}$

Normal and Tangent Coordinates (n-t)



$$\boxed{\text{Radius of curvature } \rho = \frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|}}$$

- The positive direction of 'n' at any position is always taken towards the center of curvature of the path.
- The +ve direction of 'n' changes from one side to the other, if the curvature changes direction.



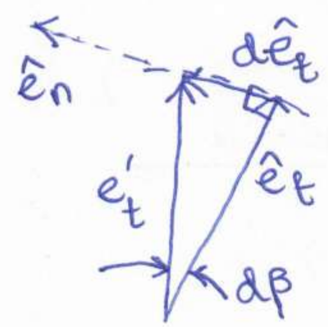
- $\vec{V} = |\vec{V}|\hat{e}_t$

$$= \frac{ds}{dt}\hat{e}_t$$

$$= \frac{d}{dt}(\rho\, d\beta)\hat{e}_t$$

$$= \rho\frac{d\beta}{dt}\hat{e}_t = \rho\dot{\beta}\hat{e}_t$$

$$|\vec{V}| = \rho\frac{d\beta}{dt}$$



- $\vec{a} = \dfrac{d\vec{v}}{dt}$

$$= \frac{d}{dt}\left(\rho\frac{d\beta}{dt}\hat{e}_t\right)$$

$$= \rho\frac{d^2\beta}{dt^2}\hat{e}_t + \rho\frac{d\beta}{dt}\frac{d}{dt}(\hat{e}_t)$$

$$= \underbrace{\rho\frac{d^2\beta}{dt^2}}_{a_t}\hat{e}_t + \underbrace{\rho\left(\frac{d\beta}{dt}\right)^2}_{a_n}\hat{e}_n$$

$$\Rightarrow \boxed{\vec{a} = |\dot{\vec{v}}|\hat{e}_t + \frac{|\vec{V}^2|}{\rho}\hat{e}_n}$$

where $\boxed{a_n = \dfrac{|\vec{V}|^2}{\rho}}$

$\boxed{a_t = |\dot{\vec{V}}|}$

$d\hat{e}_t = \{|\hat{e}_t|\, d\beta\}\hat{e}_n$

$\Rightarrow d\hat{e}_t = d\beta\,\hat{e}_n$

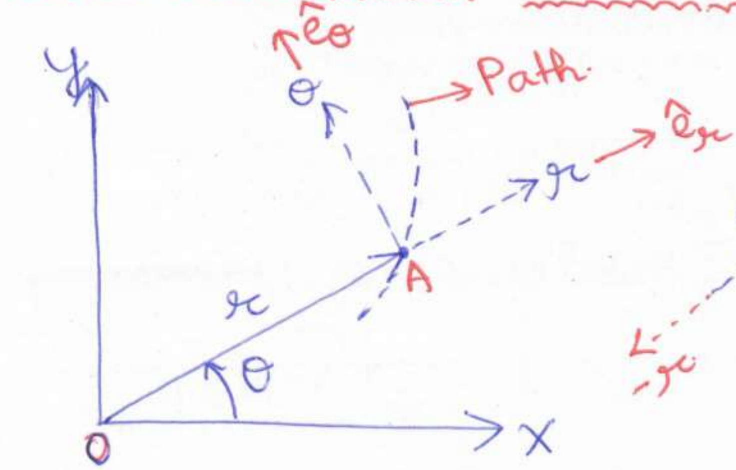
$\Rightarrow \cancel{\frac{d\hat{e}_t}{dt}} = \cancel{\frac{d\beta}{dt}}\hat{e}_n$

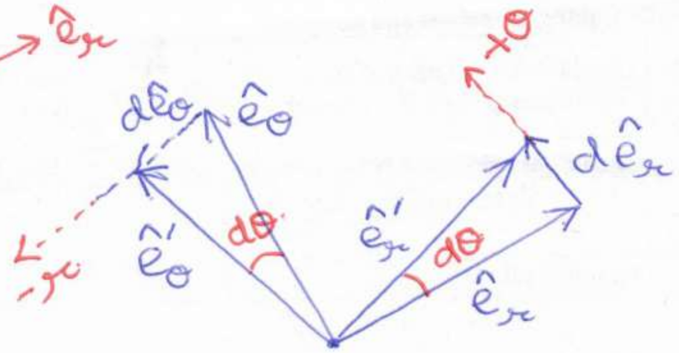
$\Rightarrow \dfrac{d\hat{e}_t}{d\beta} = \hat{e}_n$

- $\dfrac{d\hat{e}_t}{dt} = \dfrac{d\hat{e}_t}{d\beta}\cdot\dfrac{d\beta}{ds}\cdot\dfrac{ds}{dt}$

$$= \left(\hat{e}_n \frac{1}{\rho}|\vec{v}|\right) = \frac{1}{\rho}\left(\rho\frac{d\beta}{dt}\right)\hat{e}_n$$

Polar Co-ordinates: Radial & Transverse Components





$\vec{r} = |\vec{r}|\hat{e}_r$

• $\vec{v} = \frac{d\vec{r}}{dt} = \frac{d}{dt}(|\vec{r}|\hat{e}_r)$

$\Rightarrow \vec{v} = \frac{d|\vec{r}|}{dt}\hat{e}_r + |\vec{r}|\frac{d\hat{e}_r}{dt}$

$\Rightarrow \vec{v} = |\dot{\vec{r}}|\hat{e}_r + |\vec{r}|\frac{d\hat{e}_r}{d\theta}\frac{d\theta}{dt}$

$\Rightarrow \boxed{\vec{v} = \dot{r}\hat{e}_r + (r\dot{\theta})\hat{e}_\theta}$

• $d\hat{e}_r = |\hat{e}_r| d\theta\, \hat{e}_\theta$

$\Rightarrow d\hat{e}_r = d\theta\, \hat{e}_\theta$

• $d\hat{e}_\theta = |\hat{e}_\theta d\theta| (-\hat{e}_r)$

$\Rightarrow d\hat{e}_\theta = -d\theta\, \hat{e}_r$

$\therefore v_r = \dot{r}$

$v_\theta = r\dot{\theta}$

• $\vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}(\dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta)$

$\Rightarrow \vec{a} = \ddot{r}\hat{e}_r + \dot{r}\frac{d}{dt}(\hat{e}_r) + \dot{r}\dot{\theta}\hat{e}_\theta + r\ddot{\theta}\hat{e}_\theta + r\dot{\theta}\frac{d}{dt}\hat{e}_\theta$

$\Rightarrow \vec{a} = \ddot{r}\hat{e}_r + \dot{r}\frac{d\hat{e}_r}{d\theta}\cdot\frac{d\theta}{dt} + \dot{r}\dot{\theta}\hat{e}_\theta + r\ddot{\theta}\hat{e}_\theta + r\dot{\theta}\frac{d\hat{e}_\theta}{d\theta}\frac{d\theta}{dt}$

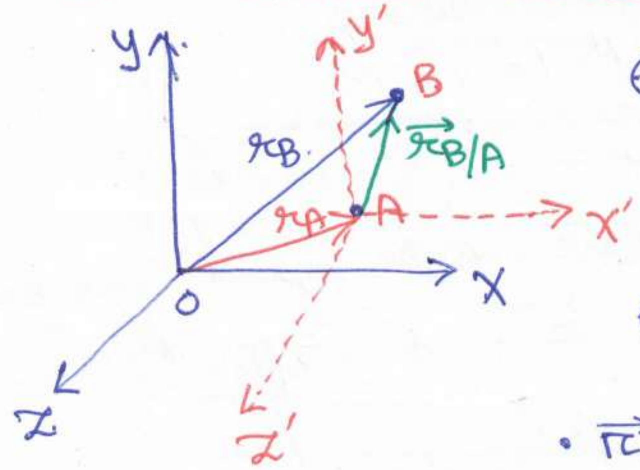
$\Rightarrow \vec{a} = \ddot{r}\hat{e}_r + \dot{r}\dot{\theta}\hat{e}_\theta + \dot{r}\dot{\theta}\hat{e}_\theta + r\ddot{\theta}\hat{e}_\theta - r(\dot{\theta})^2\hat{e}_r$

$\Rightarrow \boxed{\vec{a} = (\ddot{r} - r\dot{\theta}^2)\hat{e}_r + (2\dot{r}\dot{\theta} + r\ddot{\theta})\hat{e}_\theta}$

$a_r = \ddot{r} - r\dot{\theta}^2$

$a_\theta = 2\dot{r}\dot{\theta} + r\ddot{\theta}$

Motion Relative to a Frame in Translation



* $OXYZ$: Fixed frame of reference
  $\vec{r}_A \rightarrow$ position vector of A
  $\vec{r}_B \rightarrow$ position vector of B.

* $AX'Y'Z'$: Moving frame of reference.

- $\vec{r}_{B/A}$ defines the position of B w.r.t the moving frame $AX'Y'Z'$.

$$\boxed{\vec{r}_{B/A} = \vec{r}_B - \vec{r}_A} \quad \text{-----} (1)$$

Differentiating eqn. (1) w.r.t time, we get:

$$\boxed{\vec{v}_{B/A} = \vec{v}_B - \vec{v}_A} \quad \text{------} (2)$$

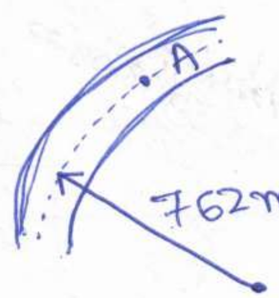
Differentiating eqn. (2) w.r.t time, we get:

$$\boxed{\vec{a}_{B/A} = \vec{a}_B - \vec{a}_A} \quad \text{----} (3)$$

Eg. A motorist is travelling on the curved section of highway at 96 km/hr. The motorist applies brake causing a constant deceleration rate. Knowing that after 8 s, the speed has been reduced to 72 km/hr, determine the acceleration of the automobile immediately after the brakes are applied.

Solution

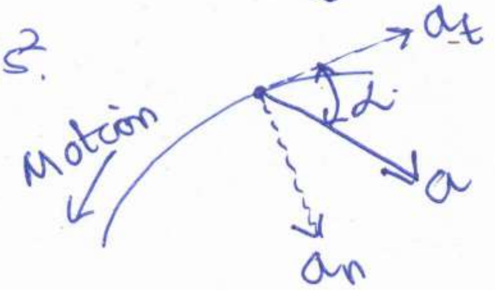
$$u_0 = 96 \times \frac{\cancel{1000}^{5}}{\cancel{3600}_{18}} = 26.7\,m/s$$



$$v = \cancel{72}^{4} \times \frac{5}{\cancel{18}} = 20\,m/s$$

$$v = u_0 + at$$

$$\Rightarrow a_t = \frac{-26.7 + 20}{8} = (-)0.8375\,m/s^2$$

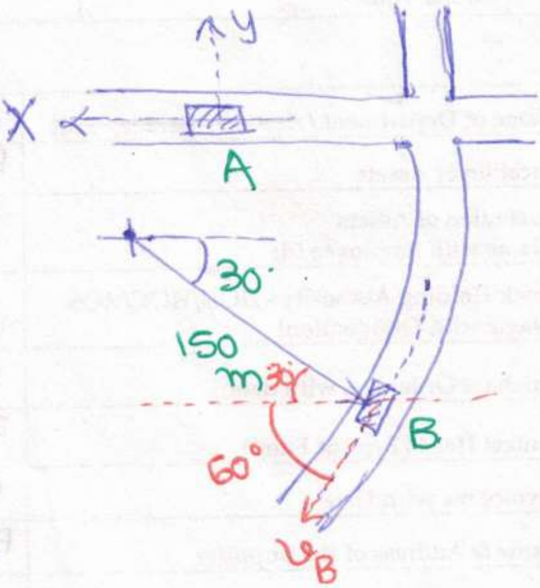


- $a_n = \frac{v^2}{\rho} = \frac{(26.7)^2}{762} = 0.94\,m/s^2$

$$a = \sqrt{a_n^2 + a_t^2} = 1.26\,m/s$$

$$\alpha = \tan^{-1}\left(\frac{a_n}{a_t}\right) = 48.4^\circ$$

Eg. Car 'A' is accelerating in the direction of its motion at a rate of $1.2 m/s^2$. Car 'B' is rounding a curve of 150-m radius at a constant speed of 54 km/hr. Determine the velocity and acceleration of car 'B' w.r.t the driver of 'A', if the car 'A' has reached a speed of 72 km/hr for the positions represented.



Solution.

$\boxed{\text{Velocity}}$ $\vec{v}_{B/A} = \vec{v}_B - \vec{v}_A$

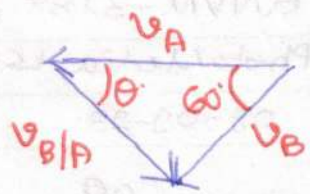
$v_B = 54 km/hr$

$= 54 \times \frac{5}{18} = 15 m/s$

$v_A = 72 km/hr$

$= 72 \times \frac{5}{18} = 20 m/s$

$\frac{v_{B/A}}{\sin 60} = \frac{v_B}{\sin\theta}$



Using cosine law -

$v_{B/A} = \sqrt{v_A^2 + v_B^2 - 2v_A v_B \cos 60°} = \underline{18.03 m/s}$

$\therefore$ $\cancel{\theta = \sin^{-1}\left(\frac{v_{B/A}}{v_B \sin 60}\right)}$ $\theta = \sin^{-1}\left(\frac{v_B \sin 60}{v_{B/A}}\right) = \underline{46.09°}$

$\boxed{\text{Acceleration}}$

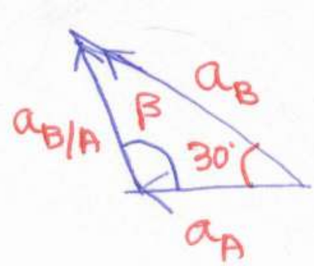
Since, car B moves at constant speed, $a_T = 0$

- $a_n = \frac{v_B^2}{\rho} = \frac{(15)^2}{150} = 1.5 m/s^2 = a_B$
- $a_A = 1.2 m/s^2$.

Using cosine law -

$a_{B/A} = \sqrt{a_A^2 + a_B^2 - 2a_A a_B \cos 30°}$

$\Rightarrow a_{B/A} = \underline{0.756 m/s^2}$

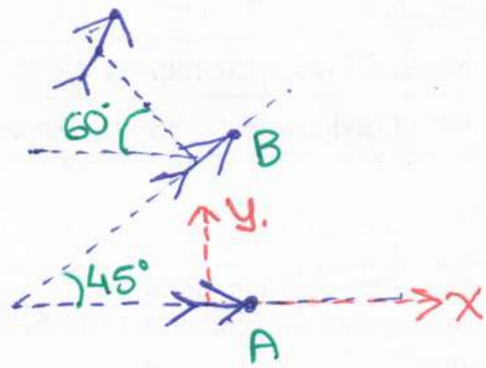


$\frac{a_B}{\sin\beta} = \frac{a_{B/A}}{\sin 30°} \Rightarrow \underline{\beta = 82.2°}$

(c) Relative acceleration of the collar w.r.t the arm is like the motion of collar w.r.t the arm is rectilinear and defined by r- coordinate.

$$a_{B/OA} = \ddot{r} = -0.24 \, m/s^2$$

Eg: Passengers in the jet 'A' flying east at a speed 800 km/h observe a second jet 'B'. Although the nose of jet 'B' is pointing toward 45° northeast direction, plane 'B' appears to the passengers in 'A' to be moving away at 60° as shown. Determine the true velocity of jet 'B'.



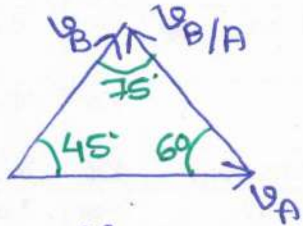
Solution

$$\vec{v}_{B/A} = \vec{v}_B - \vec{v}_A$$

$$\Rightarrow \vec{v}_B = \vec{v}_{B/A} + \vec{v}_A$$

$$v_A = 800 \times \frac{5}{18} \, m/s$$

$$= 222.22 \, m/s$$



Using sine law of triangles:-

$$\frac{v_{B/A}}{\sin 45^\circ} = \frac{v_A}{\sin 75^\circ} = \frac{v_B}{\sin 60^\circ}$$

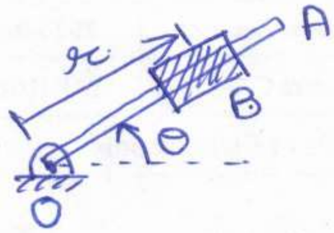
$$\Rightarrow v_B = \left(\frac{\sin 60^\circ}{\sin 75}\right) v_A = 199.23 \, m/s.$$

$$v_{B/A} = \left(\frac{\sin 45^\circ}{\sin 75}\right) v_A = 162.67 \, m/s$$

* This problem can also be solved using vector algebra also!!!

Eg: Rotation of the arm about 'O' is defined by $\theta = 0.15t^2$, where 'θ' and 't' are in radians and seconds, respectively. Collar B slides along the arm such that $r = 0.9 - 0.12t^2$, where 'r' is in meters. After the arm has rotated through 30°, determine :-

(a) total velocity of the collar.
(b) total acceleration of the collar.
(c) relative acceleration of collar w.r.t the arm.



Solution

Evaluation of 't' for $\theta = 30°$

$$\theta = 0.15t^2$$

$$\Rightarrow 30 \times \frac{\pi}{180} = 0.15t^2$$

$$\Rightarrow \underline{t = 1.87s}$$

$r = 0.9 - 0.12t^2$

$\Rightarrow r = 0.9 - 0.12(1.87)^2$

$\Rightarrow r = 0.481m$

• $\dot{r} = -0.24t = (-0.24)(1.87$

$\Rightarrow \dot{r} = -0.449 m/s$

• $\ddot{r} = -0.24 m/s^2$

* $\dot{\theta} = 0.3t = 0.561 rad/s$
* $\ddot{\theta} = 0.3 rad/s^2$
* $\theta = 0.15t^2 = 0.524 rad$

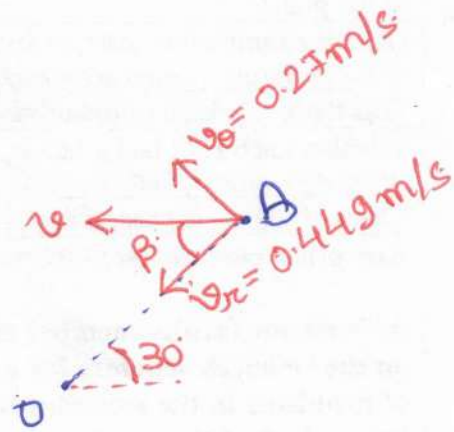
(a) $v = \sqrt{v_\theta^2 + v_r^2}$

$v_\theta = r\dot{\theta} = (0.481)(0.561) = 0.27 m/s$

$v_r = \dot{r} = -0.449 m/s$

$v = \sqrt{(0.27)^2 + (-0.449)^2} = 0.524 m/s$

$\beta = \tan^{-1}\frac{v_\theta}{v_r} = 31.0°$



(b) $a = \sqrt{a_r^2 + a_\theta^2}$

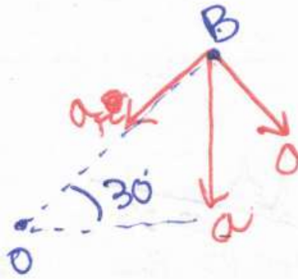
* $a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$

$= (0.481)(0.3) + 2(-0.449)(0.561)$

$= -0.359 m/s^2$

* $a_r = \ddot{r} - r\dot{\theta}^2 = -0.24 - (0.481)(0.561)^2$

$= -0.391 m/s^2$



$a = \sqrt{(-0.359)^2 + (-0.391)^2} = 0.531 m/s^2$