

Department of Mathematics and Computing

Engineering Mathematics-I

Tutorial Sheet-3

- Evaluate the following integrals.
(i) $\int_0^a \int_0^b (x^2 + y^2) dy dx$ (ii) $\int_1^2 \int_1^x xy^2 dy dx$ (iii) $\int_0^1 \int_{\sqrt{y}}^{2-y} x^2 dx dy$
- Evaluate $\iint_R (x + 2y) dx dy$, where R is the region bounded by the parabolas $y = 2x^2$ and $y = 1 + x^2$.
Ans: $\frac{32}{15}$
- Evaluate $\iint_R x^3 dx dy$, where $R = \{(x, y) : 1 \leq x \leq e, 0 \leq y \leq \ln x\}$.
Ans: $\frac{3}{16}e^4 + \frac{1}{16}$
- Evaluate $\iint_R xy^2 dx dy$, where R is the triangle with vertices $(0, 0)$, $(1, 0)$ and $(1, 1)$.
Ans: $\frac{1}{15}$
- Evaluate the following by change of order of integration.
(i) $\int_0^a \int_{\frac{x^2}{a}}^{2a-x} xy dy dx$ (ii) $\int_0^1 \int_y^{2-y} xy dx dy$ (iii) $\int_0^1 \int_x^1 \frac{y}{x^2+y^2} dy dx$ (iv) $\int_0^1 \int_x^1 \sin y^2 dy dx$
- Changing into polar co-ordinates, evaluate the following integrals.
(i) $\int_0^a \int_y^a \frac{x^2}{(x^2+y^2)^{3/2}} dy dx$ (ii) $\int_0^2 \int_0^{\sqrt{2x-x^2}} \frac{x}{(x^2+y^2)} dy dx$ (iii) $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)} dy dx$
- Evaluate $\iint_R (x-y)^2 \cos^2(x+y) dx dy$, where R is the rhombus with successive vertices at $(\pi, 0)$, $(2\pi, \pi)$, $(\pi, 2\pi)$ and $(0, \pi)$.
- Using double integration, evaluate the area of (i) cardioid $r = a(1 - \cos\theta)$, and (ii) lemniscate $r^2 = a^2 \cos 2\theta$.
- Using double integration, evaluate the area lying between the parabola $y = 4x - x^2$ and the line $y = x$.
- Using double integration, evaluate the area lying between the curves $xy = 2$, $4y = x^2$ and $y = 4$.
- Using double integration, find the volume of the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$.
- Find the volume of cylinder $x^2 + y^2 = a^2$ above the xy -plane cut by the plane $x + y + z = 2a$.
- A circular hole of radius b is made centrally through a sphere of radius a . Find the volume of remaining part.
- Find (i) the mass, (ii) center of mass, and (iii) moment of inertia about axes of a lamina with density function $f(x, y) = 6x$ of triangular shape bounded by the x -axis, the line $y = x$, and the line $y = 2 - x$.
- Let R be the unit square, i.e., $R = \{(x, y) : 0 \leq x \leq 1, 0 \leq y \leq 1\}$. Suppose the density at a point (x, y) of R is given by the function $f(x, y) = \frac{1}{y+1}$, i.e., R is denser near the x -axis. Then find (i) the mass, (ii) center of mass, and (iii) moment of inertia about axes