Motion Relative to Rotating Axes.

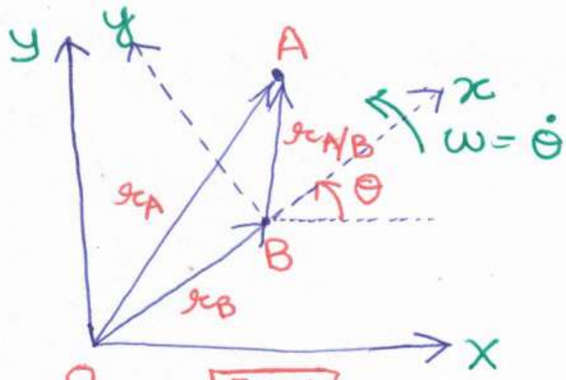


Fig 1

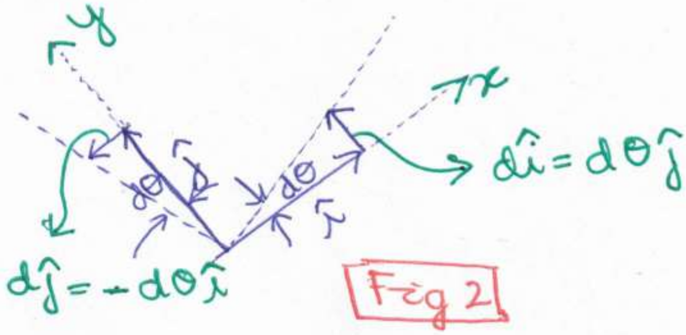


Fig 2

$(x-y)$: rotating frame

$(X-Y)$: fixed frame of reference

From Fig ①:

$$\vec{r}_A = \vec{r}_B + \vec{r}_{A/B}$$

$$\Rightarrow \vec{r}_A = \vec{r}_B + (x\hat{i} + y\hat{j})$$

Taking the time-derivative on both the sides of the eqn.

$$\dot{\vec{r}}_A = \dot{\vec{r}}_B + \frac{d}{dt}(x\hat{i} + y\hat{j}).$$

$$\Rightarrow \dot{\vec{r}}_A = \dot{\vec{r}}_B + (\dot{x}\hat{i} + \dot{y}\hat{j}) + (x\dot{\hat{i}} + y\dot{\hat{j}})$$

From Fig 2 :-

- $d\hat{i} = d\theta\,\hat{j}$

$$\Rightarrow \frac{d\hat{i}}{dt} = \frac{d\theta}{dt}\hat{j}$$

$$\Rightarrow \frac{d\hat{i}}{dt} = \omega\hat{j}$$

$$\Rightarrow \boxed{\dot{\hat{i}} = (\vec{\omega} \times \hat{i})}$$

- $d\hat{j} = -d\theta\,\hat{i}$

$$\Rightarrow \frac{d\hat{j}}{dt} = \frac{-d\theta}{dt}\hat{i}$$

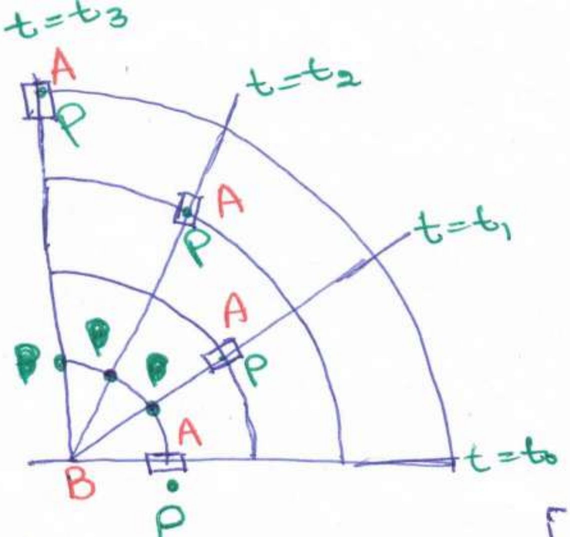
$$\Rightarrow \frac{d\hat{j}}{dt} = -\omega\hat{i}.$$

$$\Rightarrow \boxed{\dot{\hat{j}} = \vec{\omega} \times \hat{j}}$$

Here $\omega = \omega\hat{k}$

$$\therefore \dot{\vec{r}}_A = \dot{\vec{r}}_B + (\dot{x}\hat{i} + \dot{y}\hat{j}) + (\vec{\omega} \times x\hat{i} + \vec{\omega} \times y\hat{j})$$

$$\Rightarrow \boxed{\vec{v}_A = \vec{v}_B + \vec{\omega} \times \vec{r}_{A/B} + \vec{v}_{rel}}$$



$$\vec{v}_A = \vec{v}_B + \underbrace{(\omega \times r_{A/B}) + v_{rel}}_{v_{A/B}}$$

P' is a fixed point on the rotating frame and is coincident with point A' at the ~~point~~ time under consideration

$$\Rightarrow \vec{v}_A = \vec{v}_B + \vec{v}_{A/P} + \vec{v}_{P/B}$$

$\vec{v}_{A/P} \rightarrow \vec{v}_{rel}$ (which means A' moves radially outwards w.r.t P)

$\vec{v}_{P/B} \rightarrow (\omega \times r) \rightarrow$ (which means P' moves in a circular manner about B)

Differentiating the velocity expression w.r.t time, we can get the acceleration expression.

$$\therefore \dot{\vec{v}}_A = \dot{\vec{v}}_B + (\dot{\vec{\omega}} \times \vec{r}_{A/B}) + (\vec{\omega} \times \dot{\vec{r}}_{A/B}) + \dot{\vec{v}}_{rel}$$

$$\dot{\vec{r}}_{A/B} = \frac{d}{dt}(x\hat{i} + y\hat{j}) = (\dot{x}\hat{i} + \dot{y}\hat{j}) + (x\dot{\hat{i}} + y\dot{\hat{j}})$$

$$= \vec{v}_{rel} + \vec{\omega} \times \vec{r}_{A/B}$$

$$\therefore \vec{\omega} \times \dot{\vec{r}}_{A/B} = \vec{\omega} \times (\vec{\omega} \times \vec{r}_{A/B}) + \vec{\omega} \times \vec{v}_{rel}$$

Again, $\dot{\vec{\omega}} \times \vec{r}_{A/B} = \vec{\alpha} \times \vec{r}_{A/B}$

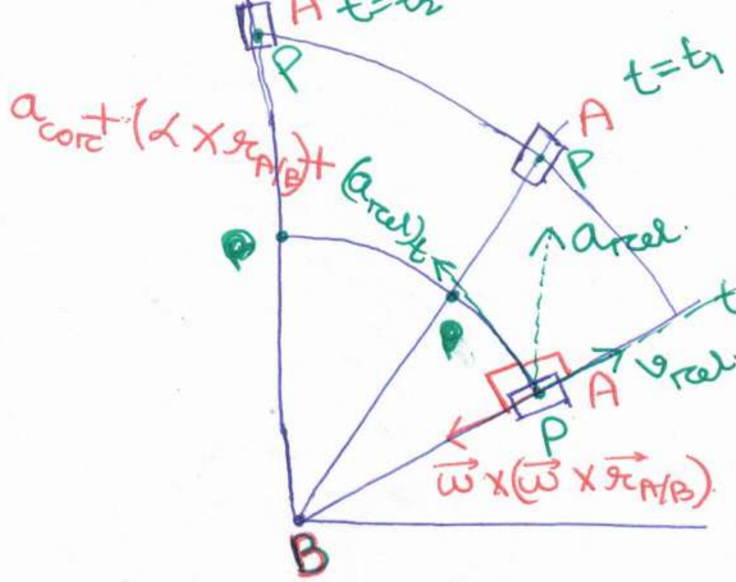
$$\dot{\vec{v}}_{rel} = \frac{d}{dt}(\dot{x}\hat{i} + \dot{y}\hat{j}) = (\ddot{x}\hat{i} + \ddot{y}\hat{j}) + (\dot{x}\dot{\hat{i}} + \dot{y}\dot{\hat{j}})$$

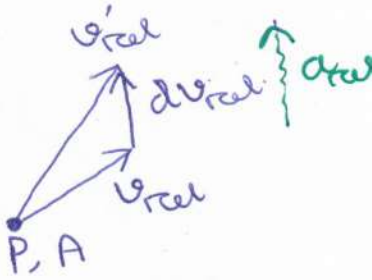
$$= \vec{a}_{rel} + (\vec{\omega} \times \dot{x}\hat{i} + \omega \times \dot{y}\hat{j})$$

$$= \vec{a}_{rel} + \vec{\omega} \times (\dot{x}\hat{i} + \dot{y}\hat{j})$$

$$\Rightarrow \dot{\vec{v}}_{rel} = (\vec{a}_{rel} + \vec{\omega} \times \vec{v}_{rel})$$

$$\therefore \boxed{\vec{a}_A = \vec{a}_B + \vec{\alpha} \times \vec{r}_{A/B} + \vec{\omega} \times (\vec{\omega} \times \vec{r}_{A/B}) + 2\vec{\omega} \times \vec{v}_{rel} + \vec{a}_{rel}}$$





[P' is a fixed point on the rotating frame and is co-incident with point A at the time under consideration]

$$\vec{a}_A = \vec{a}_B + \underbrace{(\vec{\alpha}\times\vec{r}_{A/B}) + \vec{\omega}\times(\vec{\omega}\times\vec{r}_{A/B}) + \vec{a}_{rel}}_{\vec{a}_{A/B}} + \underbrace{2\vec{\omega}\times\vec{v}_{rel}}_{\text{Coriolis acceleration}}$$

$$\Rightarrow \vec{a}_A = \vec{a}_B + \overset{a_{rel}}{\vec{a}_{A/P}} + \vec{a}_{P/B} + \overset{2\vec{\omega}\times\vec{v}_{rel}}{\vec{a}_{cor}}$$

$$\Rightarrow \vec{a}_A = \vec{a}_B + \left\{\underset{(a_{rel})_n}{(\vec{a}_{A/P})_n} + \underset{(a_{rel})_t}{(\vec{a}_{A/P})_t}\right\} + \left\{\underset{\vec{\omega}\times(\vec{\omega}\times\vec{r}_{A/B})}{(\vec{a}_{P/B})_n} + \underset{(\vec{\alpha}\times\vec{r}_{A/B})}{(\vec{a}_{P/B})_t}\right\} + \vec{a}_{cor}$$

- The coriolis acceleration is tangential to the path of the motion of the slider (or any component or object) as perceived from the rotating frame of reference.