Engineering Mathematics-I (NMCI101)
IIT (ISM) Dhanbad
Tutorial Sheet 3(b)

1. Find the volume inside the unit sphere $x^2 + y^2 + z^2 = 1$.

2. Find the volume inside the ellipsoid

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1.$$

3. Find the volume of tetrahedron T bounded by $x \geq 0, y \geq 0, z \geq 0$ and $2x + 3y + z \leq 6$.

4. Evaluate the triple integral

$$\iiint_G \sqrt{x^2 + z^2}\, dV,$$

where G is the region bounded by the paraboloid $y = x^2 + z^2$ and the plane $y = 4$.

5. Set up the limits of integration for evaluating the triple integral of a function $F(x, y, z)$ over the tetrahedron D with vertices $(0,0,0)$, $(2,0,0)$, $(0,2,0)$, and $(0,0,2)$.

6. Evaluate the following integrals.

   (i) $\int_0^a \int_0^a \int_0^a (xy + yz + zx)\, dx\, dy\, dz$ **(ii)** $\int_0^4 \int_0^{2\sqrt{z}} \int_0^{\sqrt{4z - x^2}} dy\, dx\, dz$

   (iii) $\int_0^2 \int_0^2 \int_0^z (4 - x^2)(2x + y)\, dx\, dy\, dz$

7. A solid "trough" of constant density ρ bounded below by the surface $z = 4y$, above by the plane $z = 4$, and on the ends by the planes $x = 1$ and $x = -.1$. Find the center of mass and the moments of inertia with respect to the three axes.

8. Find the moment of inertia of a solid sphere W of uniform density and radius a about the z-axis.

9. Find the center of gravity (centroid) of a solid object bounded by the paraboloid $z = x^2 + y^2$ and the plane $z = 4$ using triple integration.

10. Find the mass of a solid hemisphere of radius R with a density function $\rho(x, y, z) = kz$, where k is a constant. The hemisphere is located above the xy-plane (i.e., $z \geq 0$).

11. Evaluate the triple integral

$$\iiint\limits_{T} x\,y\,z\,dx\,dy\,dz$$

where T is the region in the xyz-space bounded by the planes $x = 0$, $y = 0$, $z = 0$, $x + y = 1$, and $z = x + y$. Use the transformation $u = x + y$, $v = x - y$, and $w = z$.