

# Department of Mathematics and Computing

## Engineering Mathematics-I

### Tutorial Sheet-3

1. Evaluate the following integrals.  
 (i)  $\int_0^a \int_0^b (x^2 + y^2) dy dx$     (ii)  $\int_1^2 \int_1^x xy^2 dy dx$     (iii)  $\int_0^1 \int_{\sqrt{y}}^{2-y} x^2 dx dy$
2. Evaluate  $\iint_R (x + 2y) dx dy$ , where  $R$  is the region bounded by the parabolas  $y = 2x^2$  and  $y = 1 + x^2$ .  
 Ans:  $\frac{32}{15}$
3. Evaluate  $\iint_R x^3 dx dy$ , where  $R = \{(x, y) : 1 \leq x \leq e, 0 \leq y \leq \ln x\}$ .  
 Ans:  $\frac{3}{16}e^4 + \frac{1}{16}$
4. Evaluate  $\iint_R xy^2 dx dy$ , where  $R$  is the triangle with vertices  $(0, 0)$ ,  $(1, 0)$  and  $(1, 1)$ .  
 Ans:  $\frac{1}{15}$
5. Evaluate the following by change of order of integration.  
 (i)  $\int_0^a \int_{\frac{x^2}{a}}^{2a-x} xy dy dx$     (ii)  $\int_0^1 \int_y^{2-y} xy dx dy$     (iii)  $\int_0^1 \int_x^1 \frac{y}{x^2+y^2} dy dx$     (iv)  $\int_0^1 \int_x^1 \sin y^2 dy dx$
6. Changing into polar co-ordinates, evaluate the following integrals.  
 (i)  $\int_0^a \int_y^a \frac{x^2}{(x^2+y^2)^{3/2}} dy dx$     (ii)  $\int_0^2 \int_0^{\sqrt{2x-x^2}} \frac{x}{(x^2+y^2)} dy dx$     (iii)  $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)} dy dx$
7. Evaluate  $\iint_R (x-y)^2 \cos^2(x+y) dx dy$ , where  $R$  is the rhombus with successive vertices at  $(\pi, 0)$ ,  $(2\pi, \pi)$ ,  $(\pi, 2\pi)$  and  $(0, \pi)$ .
8. Using double integration, evaluate the area of (i) cardioid  $r = a(1 - \cos\theta)$ , and (ii) lemniscate  $r^2 = a^2 \cos 2\theta$ .
9. Using double integration, evaluate the area lying between the parabola  $y = 4x - x^2$  and the line  $y = x$ .
10. Using double integration, evaluate the area lying between the curves  $xy = 2$ ,  $4y = x^2$  and  $y = 4$ .
11. Using double integration, find the volume of the ellipsoid  $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ .
12. Find the volume of cylinder  $x^2 + y^2 = a^2$  above the  $xy$ -plane cut by the plane  $x + y + z = 2a$ .
13. A circular hole of radius  $b$  is made centrally through a sphere of radius  $a$ . Find the volume of remaining part.
14. Find (i) the mass, (ii) center of mass, and (iii) moment of inertia about axes of a lamina with density function  $f(x, y) = 6x$  of triangular shape bounded by the  $x$ -axis, the line  $y = x$ , and the line  $y = 2 - x$ .
15. Let  $R$  be the unit square, i.e.,  $R = \{(x, y) : 0 \leq x \leq 1, 0 \leq y \leq 1\}$ . Suppose the density at a point  $(x, y)$  of  $R$  is given by the function  $f(x, y) = \frac{1}{y+1}$ , i.e.,  $R$  is denser near the  $x$ -axis. Then find (i) the mass, (ii) center of mass, and (iii) moment of inertia about axes