Name (Roll Number):

No extra sheet will be provided or collected, **Time 20 mins., Max. Marks: 15**.

1. Suppose a finite Markov Chain with state space $\{1, 2, \ldots, M\}$ have the following transition probability matrix; $\boldsymbol{P} = ((p_{ij}))$, where $p_{ij} = \frac{1}{M}$, for $1 \leq i, j \leq M$. (a) Find the equivalent classes. (b) Find the period of each state. (c) Find $\sum_{n=2}^{\infty} f_{11}^n$. (d) Find $\lim_{n \to \infty} p_{11}^n$.
$(2+2+2+2 = 8)$

Solution: Observe $\boldsymbol{P}^n = \boldsymbol{P}$. Since $p_{ij} > 0$, for all $1 \leq i, j \leq M$, hence all the states communicate with each other, and each has period one. $\lim_{n \to \infty} p_{11}^n = 1/M$. All the states are recurrent, hence $\sum_{n=2}^{\infty} f_{11}^n = 1 - 1/M$

2. Suppose a finite Markov Chain with state space $\{1, 2, \ldots, M\}$, where $M > 2$, have the following transition probability matrix;

$$
P = \begin{bmatrix}
\frac{1}{M} & \frac{1}{M} & \cdots & \frac{1}{M} & \frac{1}{M} \\
0 & \frac{1}{M-1} & \cdots & \frac{1}{M-1} & \frac{1}{M-1} \\
0 & 0 & \frac{1}{M-2} & \cdots & \frac{1}{M-2} \\
\vdots & \vdots & \vdots & \vdots & \vdots \\
0 & 0 & \cdots & \frac{1}{2} & \frac{1}{2} \\
0 & 0 & \cdots & 0 & 1
\end{bmatrix}.
$$

Find the recurret and transient states. Find $\lim_{n\to\infty} p_{kk}^n$, for $k = 1, \ldots, M$.   (3+4=7)

Solution: Note that $p_{kk}^n = 1/(M - k + 1)^n$, for $k = 1, \ldots, M$. Hence, all the states are transient except the state 'M', as $\sum_{n=1}^{\infty} p_{kk}^n < \infty$, for $k = 1, \ldots, M - 1$, and $\sum_{n=1}^{\infty} p_{MM}^n = \infty$. $\lim_{n\to\infty} p_{kk}^n = 0$ if $k = 1, \ldots, M - 1$, and it is 1, for $k = M$.