

# Introduction To Quantum Computing PS s7 Lab

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October 12, 2024

## Lab 1.1

### Exercise 1-5

Simplify:

$$\begin{aligned} ((\frac{1}{-i}) + (\frac{1}{-i})^2 + (\frac{1}{-i})^3 + \dots + (\frac{1}{-i})^{30})^{-3} &= (i + i^2 + i^3 + \dots + i^{30})^{-3} = ((i + i^2 + i^3 + i^4) + i^5 + \dots + i^{30})^{-3} = \\ (i^{29} + i^{30})^{-3} &= (i^1 + i^2)^{-3} = (i - 1)^{-3} = \frac{1}{(i-1)^3} = \frac{1}{(-1+i)*(-1+i)*(-1+i)} = \frac{1}{-2i*(-1+i)} = \frac{1}{2+2i} = \frac{1-i}{4} \end{aligned}$$

### Exercise 2-5

$$\begin{aligned} u + \overline{w}^2 + \frac{|z|}{|z+1|^2} &= (1+i) + (\overline{1-i})^2 + \frac{|1+2i|}{|(1+2i)+1|^2} = (1+i) + (1+i)^2 + \frac{\sqrt{1^2+2^2}}{|2+2i|^2} = (1+i) + [1+2i+i^2] + \\ \frac{\sqrt{5}}{(\sqrt{(2)^2+(2)^2})^2} &= (1+i) + [1+2i-1] + \frac{\sqrt{5}}{(\sqrt{8})^2} = (1+i) + [2i] + \frac{\sqrt{5}}{8} = 1+i+2i+\frac{\sqrt{5}}{8} = \left(1+\frac{\sqrt{5}}{8}\right) + 3i \end{aligned}$$

### Exercise 3-5

Given the complex number:

$$z = -30\sqrt{2} - 10\sqrt{6}i$$

Find the modulus  $r$ :

$$\begin{aligned} r &= \sqrt{(Re(z))^2 + (Im(z))^2} = \sqrt{(-30\sqrt{2})^2 + (-10\sqrt{6})^2} = \sqrt{(900 \times 2) + (100 \times 6)} = \sqrt{1800 + 600} = \\ \sqrt{2400} &= \sqrt{16 \times 150} = 4\sqrt{150} = 4\sqrt{25 \times 6} = 4 \times 5\sqrt{6} = 20\sqrt{6} \end{aligned}$$

Find the argument  $\theta$ :

$$\theta = \arctan\left(\frac{Im(z)}{Re(z)}\right) - \pi = \arctan\left(\frac{-10\sqrt{6}}{-30\sqrt{2}}\right) - \pi = \frac{\pi}{6} - \pi = -\frac{5\pi}{6}$$

So the exponent form is:

$$z = re^{\theta i} = 20\sqrt{6}e^{(-\frac{5\pi}{6})i}$$

### Exercise 4-15

Find x:  $3x^2 + 3x + 30 = 0$

$$D = b^2 - 4ac = 3^2 - 4 * 3 * 30 = -351$$

$$\sqrt{-351} = 3\sqrt{39} * i$$

$$x_{1,2} = \frac{-3 \pm 3\sqrt{39} * i}{2 * 3} = -0,5 \pm 0,5\sqrt{39} * i$$

### Exercise 5-15

Given:  $n = 9, z = -64i, \sqrt[9]{-64i} = ?$

$$64e^{-\frac{\pi}{2}i}$$

$$r^9 = 64$$

$$r = \sqrt[9]{64}$$

$$n\theta = \theta_0 + 2\pi k$$

$$\theta = \frac{-\frac{\pi}{2} + 2\pi k}{9}; k = 0, 1, 2, \dots, 8$$

$$z_0 = 64^{2/3} e^{-\frac{\pi}{18}i}$$

...

$$z_8 = \dots$$

### Exercise 6-5

$$2 \leq |z + 1 - i| \leq 10, \Im(z) < 1$$

$$2 \leq |x + y + 1 - i| \leq 10$$

$$2 \leq \sqrt{(x+1)^2 + (y-1)^2} \leq 10$$

$$4 \leq (x+1)^2 + (y-1)^2 \leq 100$$

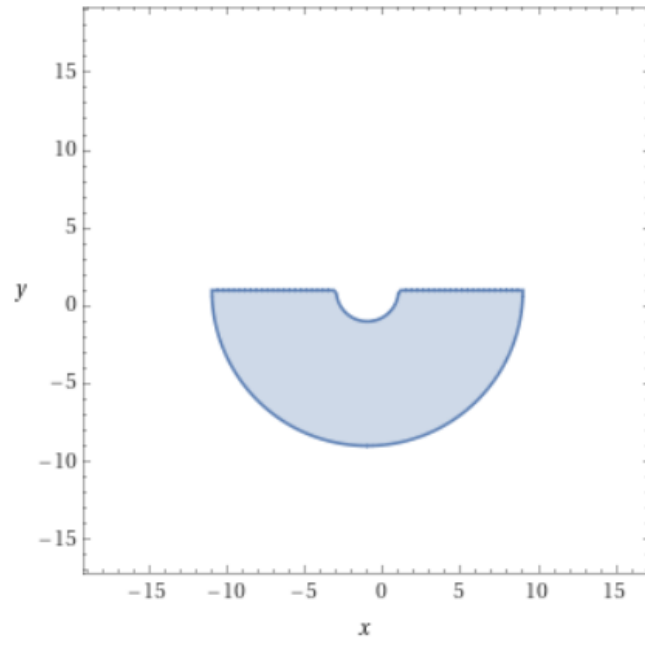


Figure 1: Solutions graph

## Lab 1.2

This part of the lab is very computations heavy. Therefore some of the calculations will be performed using python (numpy, scipy).

### Exercise 1-15

Are the given vectors:  $\vec{v}_1 = \begin{bmatrix} 1-i \\ -4-4i \\ -4-3i \\ 2-5i \end{bmatrix}$ ,  $\vec{v}_2 = \begin{bmatrix} -3-3i \\ -4+i \\ -2+2i \\ -3-4i \end{bmatrix}$ ,  $\vec{v}_3 = \begin{bmatrix} -3-i \\ -2+3i \\ -4-3i \\ -5-4i \end{bmatrix}$  linearly independent?

If the vectors are linearly independent, then the equation below is only true, when  $\alpha_1, \alpha_2, \alpha_3$  are all equal to 0:

$$\alpha_1 \begin{bmatrix} 1-i \\ -4-4i \\ -4-3i \\ 2-5i \end{bmatrix} + \alpha_2 \begin{bmatrix} -3-3i \\ -4+i \\ -2+2i \\ -3-4i \end{bmatrix} + \alpha_3 \begin{bmatrix} -3-i \\ -2+3i \\ -4-3i \\ -5-4i \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} \alpha_1 * (1-i) + \alpha_2 * (-3-3i) + \alpha_3 * (-3-i) \\ \alpha_1 * (-4-4i) + \alpha_2 * (-4+i) + \alpha_3 * (-2+3i) \\ \alpha_1 * (-4-3i) + \alpha_2 * (-2+2i) + \alpha_3 * (-4-3i) \\ \alpha_1 * (2-5i) + \alpha_2 * (-3-4i) + \alpha_3 * (-5-4i) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1-i & -3-3i & -3-i \\ -4-4i & -4+i & -2+3i \\ -4-3i & -2+2i & -4-3i \\ 2-5i & -3-4i & -5-4i \end{bmatrix} \cdot \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

We can rewrite this and solve using Gaussian elimination method

$$\begin{bmatrix} 1-i & -3-3i & -3-i & |0 \\ -4-4i & -4+i & -2+3i & |0 \\ -4-3i & -2+2i & -4-3i & |0 \\ 2-5i & -3-4i & -5-4i & |0 \end{bmatrix}$$

1st Gaussian elimination step

$$m_2 = \frac{-4-4i}{1-i} = \frac{(-4-4i) * (1+i)}{1^2 + 1^2} = \frac{-4-4i-4i+4}{2} = -4i$$

$$m_3 = \frac{-4-3i}{1-i} = \frac{(-4-3i) * (1+i)}{2} = \frac{-4-4i-3i+3}{2} = -\frac{1+7i}{2}$$

$$m_4 = \frac{2-5i}{1-i} = \frac{(2-5i) * (1+i)}{2} = \frac{2+2i-5i+5}{2} = \frac{7-3i}{2}$$

- Row 1 scaling by  $m_2$ :  $(3 - 3i) * -4i = -12i - 12$ ;  $(-3 - i) * -4i = 12i - 4$
- Row 2 subtraction:  $-4 + i - (-12i - 12) = 8 + 13i$ ;  $-2 + 3i - (12i - 4) = 2 - 9i$
- Row 1 scaling by  $m_3$ :  $(3 - 3i) * (-0, 5 - 3, 5i) = -1, 5 - 10, 5i + 1, 5i - 10, 5 = -12 - 9i$ ;  $(-3 - i) * (-0, 5 - 3, 5i) = -2 + 11i$
- Row 3 subtraction:  $-2 + 2i - (-12 - 9i) = 10 + 11i$ ;  $-4 - 3i - (-2 + 11i) = -2 - 14i$
- Row 1 scaling by  $m_4$ :  $(-3 - 3i) * \frac{7-3i}{2} = -15 - 6i$ ;  $(-3 - i) * \frac{7-3i}{2} = -12 + i$
- Row 4 subtraction:  $-3 - 4i - (-15 - 6i) = 12 + 2i$ ;  $-5 - 4i - (-12 + i) = 7 - 5i$

Resulting matrix:

$$\begin{bmatrix} 1+i & 3-3i & -3-i & |0 \\ 0 & 8+13i & 2-9i & |0 \\ 0 & 10+11i & -2-14i & |0 \\ 0 & 12+2i & 7-5i & |0 \end{bmatrix}$$

2nd Gaussian elimination step

$$m_3 = \frac{10+11i}{8+13i} = \frac{223}{233} - \frac{42}{233}i$$

$$m_4 = \frac{12+2i}{8+13i} = \frac{122}{233} - \frac{140}{233}i$$

- Row 2 scaling by  $m_3$ :  
 $(8 + 13i) * (\frac{223}{233} - \frac{42}{233}i) = 10 + 11i$ ;  
 $(2 - 9i) * (\frac{223}{233} - \frac{42}{233}i) = \frac{68}{233} - \frac{2091}{233}i$
- Row 3 subtraction:  
 $10 + 11i - (10 + 11i) = 0$ ;  
 $-2 - 14i - (\frac{68}{233} - \frac{2091}{233}i) = -\frac{534}{233} - \frac{1171}{233}i$

$$\begin{bmatrix} 1+i & 3-3i & -3-i & |0 \\ 0 & 8+13i & 2-9i & |0 \\ 0 & 0 & -\frac{534}{233} - \frac{1171}{233}i & |0 \\ 0 & 12+2i & 7-5i & |0 \end{bmatrix}$$

$$\begin{bmatrix} 1+i & 3-3i & -3-i & |0 \\ 0 & 8+13i & 2-9i & |0 \\ 0 & 0 & 1 & |0 \\ 0 & 12+2i & 7-5i & |0 \end{bmatrix}$$

$$\begin{bmatrix} 1+i & 3-3i & -3-i & | & 0 \\ 0 & 8+13i & 0 & | & 0 \\ 0 & 0 & 1 & | & 0 \\ 0 & 12+2i & 7-5i & | & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1+i & 0 & 0 & | & 0 \\ 0 & 8+13i & 0 & | & 0 \\ 0 & 0 & 1 & | & 0 \\ 0 & 12+2i & 7-5i & | & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & | & 0 \\ 0 & 1 & 0 & | & 0 \\ 0 & 0 & 1 & | & 0 \\ 0 & 0 & 0 & | & 0 \end{bmatrix}$$

$\alpha_1 = 0, \alpha_2 = 0, \alpha_3 = 0$ , this means that the vectors are linearly independent.

### Exercise 2-15

Find:  $A^2 + 3B^\dagger C^{-2} + (A^{-1})^\dagger$ .

$$\text{When: } A = \begin{bmatrix} -4+8i & 2+5i \\ -3+7i & 2+2i \end{bmatrix}, B = \begin{bmatrix} -3-6i & -7-i \\ 7-5i & -2+i \end{bmatrix}, C = \begin{bmatrix} 2-5i & -4-5i \\ -5+i & -9-4i \end{bmatrix}$$

We will first find  $A^{-1}$

$$\begin{bmatrix} -4+8i & 2+5i & | & 1 & 0 \\ -3+7i & 2+2i & | & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & \frac{(2+5i)}{(-4+8i)} & | & \frac{1}{(-4+8i)} & 0 \\ 0 & (2+2i) - \left(\frac{(2+5i)}{(-4+8i)} * (-3+7i)\right) & | & \frac{-3+7i}{(4-8i)} & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & \frac{(2+5i)}{(-4+8i)} & | & \frac{1}{(-4+8i)} & 0 \\ 0 & \frac{1}{20} - \frac{43}{20}i & | & \frac{-3+7i}{(4-8i)} & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & \frac{(2+5i)}{(-4+8i)} & | & \frac{1}{(-4+8i)} & 0 \\ 0 & 1 & | & \frac{3-7i}{17+9i} & \frac{1}{\frac{1}{20} - \frac{43}{20}i} \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & | & \frac{-2+14i}{15+95i} & -\left(\left(\frac{1}{\frac{1}{20} - \frac{43}{20}i}\right) * \left(\frac{(2+5i)}{(-4+8i)}\right)\right) \\ 0 & 1 & | & \frac{3-7i}{17+9i} & \frac{1}{\frac{1}{20} - \frac{43}{20}i} \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & | & \frac{26+8i}{185} & -\frac{79}{370} - \frac{67}{370}i \\ 0 & 1 & | & \frac{-6-73i}{185} & \frac{2}{185} + \frac{86}{185}i \end{bmatrix}$$

$$\begin{aligned}
A^{-1} &= \begin{bmatrix} \frac{26}{185} + \frac{8}{185}i & -\frac{79}{370} - \frac{67}{370}i \\ -\frac{6}{185} - \frac{73}{185}i & \frac{2}{185} + \frac{86}{185}i \end{bmatrix} \\
(A^{-1})^\dagger &= \begin{bmatrix} \frac{26}{185} + \frac{8}{185}i & -\frac{6}{185} + \frac{73}{185}i \\ -\frac{79}{370} + \frac{67}{370}i & \frac{2}{185} + \frac{86}{185}i \end{bmatrix} \\
A^2 &= \begin{bmatrix} -89 - 65i & -54 + 10i \\ -64 - 44i & -41 + 7i \end{bmatrix} \\
C^{-2} = C^{-1}C^{-1} &= \begin{bmatrix} \frac{503}{4425} + \frac{396}{4425}i & -\frac{172}{4425} - \frac{379}{4425}i \\ -\frac{331}{4425} - \frac{17}{4425}i & -\frac{206}{4425} + \frac{283}{4425}i \end{bmatrix} \begin{bmatrix} \frac{503}{4425} + \frac{396}{4425}i & -\frac{172}{4425} - \frac{379}{4425}i \\ -\frac{331}{4425} - \frac{17}{4425}i & -\frac{206}{4425} + \frac{283}{4425}i \end{bmatrix} \\
C^{-2} &= \begin{bmatrix} \frac{146682}{17850625} + \frac{526749}{17850625}i & \frac{206257}{17850625} - \frac{229351i}{17850625} \\ -\frac{86764}{17850625} - \frac{229798}{17850625}i & \frac{12836}{17850625} + \frac{11777}{17850625}i \end{bmatrix} \\
B^\dagger &= \begin{bmatrix} -3 - 6i & 7 + 5i \\ -7 + i & -2 + i \end{bmatrix} \\
3B^\dagger &= \begin{bmatrix} -9 - 18i & 21 + 15i \\ -21 + 3i & -6 + 3i \end{bmatrix}
\end{aligned}$$

So, putting  $A^2, 3B^\dagger, C^{-2}, (A^{-1})^\dagger$  into SciPy, we get this matrix:

$$\begin{bmatrix} -88.31 - 65.71i & -54.36 + 10.33i \\ -64.41 - 44.35i & -41.20 + 7.77i \end{bmatrix}$$

### Exercise 3-15

Rewrite  $\vec{v}_4$  using  $\vec{v}_1, \vec{v}_2, \vec{v}_3$ .

$$\begin{bmatrix} 2 - 4i & 2 - 5i & 2 - 3i \\ -1 - 3i & -2 - i & 4 - 4i \\ -4 - 4i & -1 + 3i & 1 - 5i \end{bmatrix} \cdot \begin{bmatrix} v_{41} \\ v_{42} \\ v_{43} \end{bmatrix} = \begin{bmatrix} -2 - 4i \\ 1 + i \\ 3 + 4i \end{bmatrix}$$

Multiplying the  $\vec{4}$  by the inverse matrix will give us the answer. The inverse, calculated using SciPy:

$$\begin{bmatrix} 0.06 + 0.095i & -0.083 - 0.247i & -0.089 + 0.201i \\ 0.084 + 0.07i & -0.078 + 0.095i & 0.094 - 0.109i \\ -0.078 + 0.046i & 0.224 + 0.128i & -0.026 - 0.073i \end{bmatrix}$$

These are the scalars that make  $\vec{v}_4$  by scaling:  $\vec{v}_1, \vec{v}_2, \vec{v}_3$ .

$$\begin{bmatrix} -0.6449302 - 0.51285819i \\ 0.65742101 - 0.40650257i \\ 0.65227774 + 0.25146951i \end{bmatrix}$$

### Exercise 4-15

Find an orthonormal basis for the given vectors:

$$\mathbf{v}_1 = \begin{bmatrix} 2 - 4i \\ -1 - 3i \\ -4 - 4i \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} 2 - 5i \\ -2 - i \\ -1 + 3i \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} 2 - 3i \\ 4 - 4i \\ 1 - 5i \end{bmatrix}$$

We can find an orthogonal basis for linearly independent vectors using Gram-Schmidt process and then normalize it.

$$\begin{aligned} \vec{u}_1 &= \vec{v}_1, \\ \vec{u}_2 &= \vec{v}_2 - \frac{\langle \vec{v}_2 | \vec{u}_1 \rangle}{\langle \vec{u}_1 | \vec{u}_1 \rangle} \vec{u}_1, \\ \vec{u}_3 &= \vec{v}_3 - \frac{\langle \vec{v}_3 | \vec{u}_1 \rangle}{\langle \vec{u}_1 | \vec{u}_1 \rangle} \vec{u}_1 - \frac{\langle \vec{v}_3 | \vec{u}_2 \rangle}{\langle \vec{u}_2 | \vec{u}_2 \rangle} \vec{u}_2. \end{aligned}$$

The calculations will be done using SciPy. End result is:

$$\vec{u}_1 = \begin{bmatrix} \frac{\sqrt{62} \cdot (1 - 2i)}{31} \\ \frac{\sqrt{62} \cdot (-1 - 3i)}{62} \\ \frac{2 \cdot \sqrt{62} \cdot (-1 - i)}{31} \end{bmatrix} \quad \vec{u}_2 = \begin{bmatrix} \frac{\sqrt{60047} \cdot (-5 - 136i)}{60047} \\ \frac{2 \cdot \sqrt{60047} \cdot (-43 + 6i)}{60047} \\ \frac{\sqrt{60047} \cdot (-35 + 181i)}{60047} \end{bmatrix} \quad \vec{u}_3 = \begin{bmatrix} \frac{\sqrt{1811821226} \cdot (139562 - 36868i)}{11776837969} \\ \frac{\sqrt{1811821226} \cdot (2 - 3i) \cdot (95123 + 41155i)}{23553675938} \\ \frac{\sqrt{1811821226} \cdot (120294 - 79562i)}{11776837969} \end{bmatrix}$$

### Exercise 5-15

Find the eigenvectors and eigenvalues of matrix A:

$$\mathbf{A} = \begin{pmatrix} 1 & 0 & -1 \\ 2i & i & 1 \\ 1 & 0 & i \end{pmatrix}$$

Eigenvectors are vectors that stay in the same span after the matrix transformation is applied:

$$\mathbf{A}\vec{v} = \lambda\vec{v}$$

$$\mathbf{A}\vec{v} = (\lambda\mathbf{I})\vec{v}$$

$$\mathbf{A}\vec{v} - (\lambda\mathbf{I})\vec{v} = \vec{0}$$

$$(\mathbf{A} - (\lambda\mathbf{I})) \cdot \vec{v} = \vec{0}$$



Because we are only looking for non-zero vectors, we will look for values of  $\lambda$  that can make a zero vector, which means that we are looking for matrix that squishes space:

$$\det \left( \begin{pmatrix} 1-\lambda & 0 & -1 \\ 2i & i-\lambda & 1 \\ 1 & 0 & i-\lambda \end{pmatrix} \right) = 0$$

We expand the determinant:

$$\begin{aligned} \det \left( \begin{pmatrix} 1-\lambda & 0 & -1 \\ 2i & i-\lambda & 1 \\ 1 & 0 & i-\lambda \end{pmatrix} \right) &= (1-\lambda) * \det \left( \begin{pmatrix} i-\lambda & 1 \\ 0 & i-\lambda \end{pmatrix} \right) - \det \left( \begin{pmatrix} 2i & i-\lambda \\ 1 & 0 \end{pmatrix} \right) = \\ &= (1-\lambda) * ((i-\lambda) * (i-\lambda)) - (-(i-\lambda)) = (1-\lambda) * (i-\lambda)^2 + (i-\lambda) \end{aligned}$$

So:

$$(1-\lambda) * (i-\lambda)^2 + (i-\lambda) = 0$$

$$(i-\lambda) * ((1-\lambda) * (i-\lambda) + 1) = 0$$

So,  $(i-\lambda) = 0$  or  $(1-\lambda) * (i-\lambda) + 1 = 0$ , if  $(i-\lambda) = 0$ , then  $\lambda_1 = i$  else:

$$(1-\lambda) * (i-\lambda) + 1 = 0$$

$$i - \lambda - \lambda i + \lambda^2 + 1 = 0$$

$$\lambda^2 - (1+i)\lambda + 1 + i = 0$$

$$D = (1+i)^2 - 4(1+i) = 1 + 2i - 1 - 4 - 4i = -4 - 2i$$

$$\lambda = \frac{1+i \pm \sqrt{-4-2i}}{2}$$

$$\sqrt{-4-2i} = 2\sqrt{5}e^{(\arctan(0,5)-\pi)i}$$

$$\lambda_2 = \frac{1+i + 2\sqrt{5}e^{(\arctan(0,5)-\pi)i}}{2} \approx 0,743 - 0,53i$$

$$\lambda_3 = \frac{1+i - 2\sqrt{5}e^{(\arctan(0,5)-\pi)i}}{2} \approx 0,257 + 1,53i$$

Previously calculated:  $\lambda_1 = i$

We can calculate an eigen vector for each of the eigen values:  $(\mathbf{A} - (\lambda\mathbf{I})) \cdot \vec{v} = \vec{0}$

$$\begin{pmatrix} 1-\lambda & 0 & -1 \\ 2i & i-\lambda & 1 \\ 1 & 0 & i-\lambda \end{pmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Calculating for  $\lambda_1$ :

$$\begin{pmatrix} 1-i & 0 & -1 \\ 2i & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} (1-i)x - z \\ 2ix + z \\ x \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$x = 0$$

$$-z = 0$$

$$z = 0$$

$y$  is a free variable, which means it can take on any value. So one set of eigen vectors is:  $\begin{bmatrix} 0 \\ y \\ 0 \end{bmatrix}$ , where

$y \in \mathbb{R}$ , this set of eigen vectors gets scaled by  $i$ . The other two eigen vectors were calculated using SciPy:

$$\lambda_2 = 0.743 - 0.529i$$

$$\mathbf{v}_2 = \begin{pmatrix} -0.449 - 0.278i \\ 0.790 + 0i \\ 0.031 - 0.309i \end{pmatrix}$$

$$\lambda_3 = 0.257 + 1.529i$$

$$\mathbf{v}_3 = \begin{pmatrix} 0.177 + 0.363i \\ 0.510 + 0.323i \\ 0.687 + 0i \end{pmatrix}$$

### Exercise 6-5

$$\text{Given: } \mathbf{A} = \begin{pmatrix} 1 & 0 & -1 \\ 2i & i & 1 \\ 1 & 0 & i \end{pmatrix} \text{ and } \mathbf{Z} = \begin{pmatrix} -2i & 2 \\ -1 & 2i \end{pmatrix}$$

Find:

- A)  $\mathbf{X}$ , so that  $\mathbf{AX}$  would be a non-diagonal Hermitian matrix.
- B)  $\mathbf{Y}$ , so that  $\mathbf{AY}$  would be a non-diagonal Unitary matrix.
- C)  $\mathbf{Z} \otimes \mathbf{A}$  and  $\mathbf{A} \otimes \mathbf{Z}$

A)

The Hermitian matrix condition is:  $\mathbf{AX} = (\mathbf{AX})^\dagger$ . We will pick a random matrix that meets this condition:

$$\mathbf{AX} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 1-i \\ 0 & 1+i & 0 \end{pmatrix}$$

Since this new random matrix is the product of applying transformation  $\mathbf{A}$  onto  $\mathbf{X}$ , that means that:

$$\mathbf{X} = \mathbf{A}^{-1}(\mathbf{AX})$$

$$\mathbf{A}^{-1} = \begin{pmatrix} 0.5 + 0.5i & 0 & 0.5 - 0.5i \\ -1.5 - 1.5i & -i & -0.5 + 1.5i \\ -0.5 + 0.5i & 0 & 0.5 - 0.5i \end{pmatrix}$$

$$\mathbf{X} = \mathbf{A}^{-1}(\mathbf{AX}) = \begin{pmatrix} 0.5 + 0.5i & 0 & 0.5 - 0.5i \\ -1.5 - 1.5i & -i & -0.5 + 1.5i \\ -0.5 + 0.5i & 0 & 0.5 - 0.5i \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 1-i \\ 0 & 1+i & 0 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & -2 & -1-i \\ 0 & 1 & 0 \end{pmatrix}$$

B)

The Unitarian matrix condition:  $\mathbf{UU}^\dagger = \mathbf{I}$ . If  $\mathbf{AY} = \mathbf{U}$ , then:  $\mathbf{Y} = \mathbf{A}^{-1}\mathbf{U}$

Here is a random Unitary matrix:

$$\mathbf{U} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\mathbf{Y} = \mathbf{A}^{-1}\mathbf{U} = \begin{pmatrix} 0 & 0.5 + 0.5i & 0.5 - 0.5i \\ -i & -1.5 - 1.5i & -0.5 + 1.5i \\ 0 & -0.5 + 0.5i & 0.5 - 0.5i \end{pmatrix}$$

C)

This exercise will be fully computed using SciPy:

$$\mathbf{Z} \otimes \mathbf{A} = \begin{bmatrix} -2i & 0 & 2i & 2 & 0 & -2 \\ 4 & 2 & -2i & 4i & 2i & 2 \\ -2i & 0 & 2 & 2 & 0 & 2i \\ -1 & 0 & 1 & 2i & 0 & -2i \\ -2i & -i & -1 & -4 & -2 & 2i \\ -1 & 0 & -i & 2i & 0 & -2 \end{bmatrix}$$

$$\mathbf{A} \otimes \mathbf{Z} = \begin{bmatrix} -2i & 2 & 0 & 0 & 2i & -2 \\ -1 & 2i & 0 & 0 & 1 & -2i \\ 4 & 4i & 2 & 2i & -2i & 2 \\ -2i & -4 & -i & -2 & -1 & 2i \\ -2i & 2 & 0 & 0 & 2 & 2i \\ -1 & 2i & 0 & 0 & -i & -2 \end{bmatrix}$$

## Lab 1.3

**Variant 15** - 1:15, 2:-, 3:-, 4:15, 5:15, 6:5.

### Exercise 1-15

Write a python function that applies the matrix  $\mathbf{M}$  to a vector  $k$  times. Also visualize system  $M$  as a graph. (Using chatGPT is allowed).

$$M = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$

This exercise was done in code.

### Exercise 2

Write python functions that:

- That generates a random doubly stochastic matrix with  $\mathbf{N}$  rows and columns.
- That checks if a given matrix is doubly stochastic.
- Applies a matrix  $\mathbf{k}$  times to an initial vector (note:  $\mathbf{k}$  can be negative as well).

### Exercise 3

Write python functions that:

- Generates a random quantum system matrix with  $\mathbf{N}$  rows and columns.
- Checks if a given matrix is a quantum system, dynamics matrix.
- Applies a matrix  $\mathbf{k}$  times to an initial vector (note:  $\mathbf{k}$  can be negative as well).
- Calculates the probability to go from state  $i$  to state  $j$  in time  $\mathbf{T}$  (at the start the system is at state  $i$  with a probability of 1).

### Exercise 4-15

Find the probability of each state.

$$|\psi\rangle = \begin{bmatrix} 4 + 3i \\ -5 + 3i \\ 6 - 4i \\ -4 - 3i \\ -1 - 5i \\ 7i \\ -7 - 5i \\ 2i \end{bmatrix}$$

### Exercise 5-15

Take the vector  $\psi$  from the previous exercise and vector  $\phi$ , normalize them and find the transition amplitude:  $\psi$  to  $\phi$ .

$$|\phi\rangle = \begin{bmatrix} -2 - 5i \\ 3i \\ 7 - 1i \\ -2 + 4i \\ 4i \\ -1 - 3i \\ -3 - 5i \\ 1 - 5i \end{bmatrix}$$

### Exercise 6

Write a python function which takes two 'sympy' matrices as parameters and returns their commutator.

## Lab 1.4

All exercises are done in code.

**Exercise 1**

**Exercise 2**

**Exercise 3**

**Exercise 4**

**Exercise 5**

**Exercise 6**