Optimization Methods PS s7 Lab 3

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Short Introduction

By mistake I implemented the same objective function as in the previous lab (maximizing the volume of a cube with surface plots as input). In order not to redo the whole lab in the end I will derive the x, y and z from the plots.

2 - Function Values At X_0, X_1, X_m

	X_0	X_1	X_m
f	-0.0	-0.125	-0.0
g_1	-1	2	-0.8
h_1, h_2, h_3	$0,\ 0,\ 0$	$-1,\ -1,\ -1$	$0,\ -0.2,\ 0$

Table 1: Function evaluations at points X_0, X_1, and X_m.

3 - Code

```
def f(plot_1, plot_2, plot_3):
    return -(1/8) * plot_1 * plot_2 * plot_3


def g_1(plot_1, plot_2, plot_3):
    return plot_1 + plot_2 + plot_3 - 1


def h_1(plot_1, plot_2, plot_3): # -plot_1 <= 0
    return -plot_1


def h_2(plot_1, plot_2, plot_3): # -plot_2 <= 0
    return -plot_2


def h_3(plot_1, plot_2, plot_3): # -plot_3 <= 0
    return -plot_3


def f_with_restrictions(plot_1, plot_2, plot_3):
    r = 1
    return f(plot_1, plot_2, plot_3) + (1/r) * (g_1(plot_1, plot_2, plot_3)**2 +
                                                max(0, h_1(plot_1, plot_2, plot_3))**2 +
                                                max(0, h_2(plot_1, plot_2, plot_3))**2 +
                                                max(0, h_3(plot_1, plot_2, plot_3))**2)
```

Figure 1: Ojective and Restriction Functions

4 - Initial r Value Effect On Penalty Values

```
423, bauda = 4.9598233109928e-05

 bauda = 1.2225835627970804e-05

555, bauda = 3.0351408608103737e-06

, bauda = 7.56144810799971e-07

3, bauda = 1.8870765824678347e-07
```

Figure 2: r = 1

```
 bauda = 0.0065896362247266135

, bauda = 0.0013961827725584646

 bauda = 0.00032369167230884204

 bauda = 7.805036481437443e-05

9, bauda = 1.917010323419363e-05
```

Figure 3: r = 10

These pictures indicate that the lower the initial r value is the lower the penalty function's value will be during training. Which makes sense, because the total penalty is 1/r times the penalty function's value, which means that with higher r values the function parameters can stride away further from the restrictions without receiving much total penalty, leading to the increase of penalty function's value.

6 - Results

I got very similar results for all starting X's, each plot was 0.333... which is $\sim \frac{1}{3}$. However, because the task was not to find the optimal plot values, but rather the optimal x, y and z values. We will derive them using system of equations:

$$\begin{cases} 2xy = \frac{1}{3} \\ 2xz = \frac{1}{3} \\ 2yz = \frac{1}{3} \end{cases}$$

$$xy = xz$$

$$y = z$$

$$xy = yz$$

$$x = z$$

$$x = y = z$$

$$2x^2 = \frac{1}{3}$$

$$x^2 = \frac{1}{6}$$

$$x = \pm\frac{1}{\sqrt{6}}$$

$$\begin{cases} x = \frac{1}{\sqrt{6}} \\ y = \frac{1}{\sqrt{6}} \\ z = \frac{1}{\sqrt{6}} \end{cases}$$

$$\begin{cases} x = -\frac{1}{\sqrt{6}} \\ y = -\frac{1}{\sqrt{6}} \\ z = -\frac{1}{\sqrt{6}} \end{cases}$$

The negative solution does not satisfy the constraints, so the solution is that the rectangle shaped-box should be of shape: $x = y = z = \dfrac{1}{\sqrt{6}}$.