



ผลิตภัณฑ์	x	y
A	2	1
B	4	3
C	5	5
D	7	5

$$\text{Mean } \frac{2+4+5+7}{4} = 4.5 \quad \frac{1+3+5+5}{4} = 3.5$$

P'(x,y)	x	y
A'	$2 - 4.5 = -2.5$	$1 - 3.5 = -2.5$
B'	$4 - 4.5 = -0.5$	$3 - 3.5 = -0.5$
C'	$5 - 4.5 = 0.5$	$5 - 3.5 = 1.5$
D'	$7 - 4.5 = 2.5$	$5 - 3.5 = 1.5$

$$\begin{aligned} \text{cov}(x_1, x_1) &= \frac{\sum_{k=1}^4 (x_{1k} - \bar{x}_1)(x_{1k} - \bar{x}_1)}{4-1} \\ &= (x_{11} - \bar{x}_1)(x_{11} - \bar{x}_1) + (x_{12} - \bar{x}_1)(x_{12} - \bar{x}_1) + (x_{13} - \bar{x}_1)(x_{13} - \bar{x}_1) + (x_{14} - \bar{x}_1)(x_{14} - \bar{x}_1) \end{aligned}$$

3

$$= (2 - 4.5)(2 - 4.5) + (4 - 4.5)(4 - 4.5) + (5 - 4.5)(5 - 4.5) + (7 - 4.5)(7 - 4.5)$$

3

$$= \frac{13}{3} = 4.33 \times$$

$$\text{cov}(x_1, x_2) = \frac{(x_{11} - \bar{x}_1)(x_{21} - \bar{x}_2) + (x_{12} - \bar{x}_1)(x_{22} - \bar{x}_2) + (x_{13} - \bar{x}_1)(x_{23} - \bar{x}_2) + (x_{14} - \bar{x}_1)(x_{24} - \bar{x}_2)}{3}$$

3

$$= \frac{(2 - 4.5)(1 - 3.5) + (4 - 4.5)(3 - 3.5) + (5 - 4.5)(5 - 3.5) + (7 - 4.5)(5 - 3.5)}{3}$$

$$= \frac{14.85}{3} = 3.69 \times$$



$$\text{cov}(2,1) = \frac{(x_{21} - \bar{x}_2)(x_{11} - \bar{x}_1) + (x_{22} - \bar{x}_2)(x_{12} - \bar{x}_1) + (x_{23} - \bar{x}_2)(x_{13} - \bar{x}_1) + (x_{24} - \bar{x}_2)(x_{14} - \bar{x}_1)}{3}$$

4-1

$$= \frac{(-2.5)(-2.5) + (-0.5)(-0.5) + (0.5)(0.5) + (1.5)(2.5)}{3}$$

3

$$= \frac{11}{3} = 3.67 *$$

$$\text{cov}(2,2) = \frac{(x_{21} - \bar{x}_2)(x_{21} - \bar{x}_2) + (x_{22} - \bar{x}_2)(x_{22} - \bar{x}_2) + (x_{23} - \bar{x}_2)(x_{23} - \bar{x}_2) + (x_{24} - \bar{x}_2)(x_{24} - \bar{x}_2)}{3}$$

$$= \frac{(-2.5)(-2.5) + (-0.5)(-0.5) + (1.5)(1.5) + (1.5)(1.5)}{3}$$

$$= \frac{11}{3} = 3.67 *$$

$$\text{Covariance Matrix} = \begin{bmatrix} 4.33 & 3.67 \\ 3.67 & 3.67 \end{bmatrix}$$

Eigenvalues ( $\lambda$ )

$$\text{สมการ } \det(C - \lambda I) = 0$$

$$\det \left[ \begin{pmatrix} 4.33 & 3.67 \\ 3.67 & 3.67 \end{pmatrix} - \lambda \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \right] = 0$$

$$\det \left[ \begin{pmatrix} 4.33 & 3.67 \\ 3.67 & 3.67 \end{pmatrix} - \begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix} \right] = 0$$

$$\det \left[ \begin{pmatrix} 4.33 - \lambda & 3.67 \\ 3.67 & 3.67 - \lambda \end{pmatrix} \right] = 0$$

$$= (4.33 - \lambda)(3.67 - \lambda)$$

$$= (3.67)(3.67) = 13.49$$

$$(4.33 - \lambda)(3.67 - \lambda) - 13.49 = 0$$

$$= 19.8911 - 4.33\lambda - 3.67\lambda + \lambda^2 - 13.49 = 0$$

$$= \lambda^2 - 8\lambda - 2.42 = 0$$



$$\lambda = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(2.42)}}{2(1)}$$

$$= \frac{8 \pm \sqrt{64 - 9.68}}{2}$$

$$= \frac{8 \pm \sqrt{54.32}}{2} = \frac{8 \pm 7.37}{2} = 7.685 = \lambda_1$$

$$= \frac{8 - 7.37}{2} = 0.315 = \lambda_2$$

แก้สมการ  $(C - \lambda I)v_1 = 0$

$$\left[ \begin{pmatrix} 4.33 & 3.69 \\ 3.69 & 3.69 \end{pmatrix} - \lambda_1 \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \right] \begin{pmatrix} x \\ y \end{pmatrix} = 0$$

$$\left[ \begin{pmatrix} 4.33 & 3.69 \\ 3.69 & 3.69 \end{pmatrix} - \lambda_2 \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \right] \begin{pmatrix} x \\ y \end{pmatrix} = 0$$

$$\left[ \begin{pmatrix} 4.33 & 3.69 \\ 3.69 & 3.69 \end{pmatrix} - \begin{pmatrix} 7.685 & 0 \\ 0 & 7.685 \end{pmatrix} \right] = 0$$

$$\begin{pmatrix} 4.33 - 7.685 & 3.69 \\ 3.69 & 3.69 - 7.685 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = 0$$

$$\begin{pmatrix} -3.353 & 3.69 \\ 3.69 & -4.015 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = 0$$



$$\begin{pmatrix} -3.353x + 3.67y \\ 3.67x - 3.015y \end{pmatrix} = 0$$

$$\begin{array}{l|l} -3.353x + 3.67y = 0 & 3.67x - 4.015y \\ y = \frac{-3.353x}{3.67} & y = \frac{3.67}{-4.015}x \\ y = 0.91x & = 0.91x \end{array}$$

$$\text{so } y = 0.91x \text{ when } x = 1 \text{ so } v = [1, 0.91]$$

$$\|v\| = \sqrt{x^2 + y^2}$$

$$= \sqrt{1^2 + 0.91^2} = \sqrt{1 + 0.828} = \sqrt{1.828} \approx 1.35$$

$$g = v / \|v\|$$

$$x_{\text{unit}} = 1 / 1.35 \approx 0.94$$

$$y_{\text{unit}} = 0.91 / 1.35 \approx 0.69$$

Eigenvalue  $v_1 (\text{PC1}) \approx [0.94, 0.69]$

New Coordinate =  $A' \cdot \text{PC1}$

$$A'' = (-2.5)(0.94) + (-2.5)(0.69) \approx -3.595$$

$$B'' = (-0.5)(0.94) + (-0.5)(0.69) \approx -0.915$$

$$C'' = (0.5)(0.94) + (1.5)(0.69) \approx 1.405$$

$$D'' = (2.5)(0.94) + (1.5)(0.69) \approx 12.4995$$



จุด	x	y
E	1	3
F	2	3
G	4	2
H	5	1
Mean	$\frac{1+2+4+5}{4} = 3$	$\frac{3+3+2+1}{4} = \frac{11}{4} = 2.75$

$P'(x,y)$	x	y
E'	$1-3 = -2$	$3-2.75 = 0.25$
F'	$2-3 = -1$	$3-2.75 = 0.25$
G'	$4-3 = 1$	$2-2.75 = -0.75$
H'	$5-3 = 2$	$1-2.75 = -1.75$

$$\begin{aligned}
 \text{COV}(x_1, x_1) &= \frac{\sum_{k=1}^4 (x_{1k} - \bar{x}_1)(x_{1k} - \bar{x}_1)}{n-1} \\
 &= (1-3)(1-3) + (2-3)(2-3) + (4-3)(4-3) + (5-3)(5-3) \\
 &= (-2)(-2) + (-1)(-1) + (1)(1) + (2)(2) \\
 &= \frac{4+1+1+4}{3} = \frac{10}{3} = 3.33
 \end{aligned}$$

$$\begin{aligned}
 \text{COV}(x_1, x_2) &= \frac{(-2)(2.25) + (-1)(0.25) + (1)(-0.75) + (2)(-1.75)}{3} \\
 &= \frac{-4.5 + -0.25 - 0.75 - 3.5}{3} \\
 &= \frac{-9}{3} = -3
 \end{aligned}$$

$$\begin{aligned}
 \text{COV}(x_2, x_1) &= \frac{(2.25)(-2) + (0.25)(-1) + (-0.75)(1) + (-1.75)(2)}{3} \\
 &= \frac{-4.5 + 0.25 + 0.75 - 3.5}{3} \\
 &= \frac{-9}{3} = -3
 \end{aligned}$$



$$\begin{aligned} \text{COV}(x_2, x_2) &= (2.25)(2.25) + (0.25)(0.25) + (-0.75)(-0.75) + (-1.75)(-1.75) \\ &\quad - \frac{3}{3} \\ &= 5.06 + 0.06 + 0.56 + 3.06 = \frac{8.74}{3} = 2.91 \end{aligned}$$

$$\lambda = \text{Covariance Matrix} = \begin{bmatrix} 3.33 & -3 \\ -3 & 2.91 \end{bmatrix}$$

Eigenvalues ( $\lambda$ ) : ค้นหา  $\det(C - \lambda I) = 0$

$$\det \left[ \begin{pmatrix} 3.33 & -3 \\ -3 & 2.91 \end{pmatrix} - \lambda \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \right] = 0$$

$$\det \left[ \begin{pmatrix} 3.33 & -3 \\ -3 & 2.91 \end{pmatrix} - \begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix} \right] = 0$$

$$\det \left[ \begin{pmatrix} 3.33 - \lambda & -3 \\ -3 & 2.91 - \lambda \end{pmatrix} \right] = 0$$

$$= (3.33 - \lambda)(2.91 - \lambda) - (-3)(-3) = 9$$

$$= 9.69 - 3.33\lambda - 2.91\lambda + \lambda^2 - 9 = 0$$

$$= \lambda^2 - 6.24\lambda + .69 = 0$$

$$\lambda = -(-6.24) \pm \sqrt{(-6.24)^2 - 4(1)(0.69)}$$

$$2(1)$$

$$= 6.24 \pm \sqrt{38.94 - 2.76}$$

$$= \frac{6.24 \pm \sqrt{36.18}}{2} = \frac{6.24 \pm 6.02}{2} = \frac{12.26}{2} = 6.13 = \lambda_1$$

$$= \frac{6.24 - 6.02}{2} = \frac{0.22}{2} = 0.11 = \lambda_2$$

ห้ามมร  $(C - \lambda_1 I)v_1 = 0$

$$\left[ \begin{pmatrix} 3.33 & -3 \\ -3 & 2.91 \end{pmatrix} - \lambda_1 \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \right] \begin{pmatrix} x \\ y \end{pmatrix} = 0$$



$$\left[ \begin{pmatrix} 3.33 & -3 \\ -3 & 2.91 \end{pmatrix} - b_{13} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \right] \begin{pmatrix} x \\ y \end{pmatrix} = 0$$

$$\left[ \begin{pmatrix} 3.33 - b_{13} & -3 \\ -3 & 2.91 - b_{13} \end{pmatrix} \right] \begin{pmatrix} x \\ y \end{pmatrix} = 0$$

$$\begin{pmatrix} -2.8 & -3 \\ -3 & -3.22 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = 0$$

$$\begin{pmatrix} -2.8x & -3y \\ -3x & -3.22y \end{pmatrix} = 0 \quad \left| \begin{array}{l} -2.8x - 3y = 0 \\ y = -\frac{2.8}{-3}x \\ = 0.93x \end{array} \right| \quad \left| \begin{array}{l} -3x - 3.22y = 0 \\ y = -\frac{3}{-3.22}x \\ = 0.93x \end{array} \right|$$

จาก  $y = 0.93x$  และ  $x = 1$  จะได้  $v = [1, 0.93]$

$$\|v\| = \sqrt{x^2 + y^2}$$

$$= \sqrt{1^2 + 0.93^2} = \sqrt{1.8649} = 1.366$$

$$\hat{v} = v / \|v\|$$

$$x\_unit = \frac{1}{1.366} = 0.932$$

$$y\_unit = \frac{0.93}{1.366} = 0.681$$

Eigenvalue  $\lambda_1 (\rho C_1) \approx [0.93, 0.68]$

New Coordinate =  $A' \cdot \rho C_1$

$$A'' = (-2)(0.93) + (2.25)(0.68) \approx 0.07$$

$$F'' = (-1)(0.72) + (0.25)(0.68) \approx -0.56$$

$$G'' = (1)(0.93) + (-0.75)(0.68) \approx 0.92$$

$$H'' = (2)(0.93) + (-1.75)(0.68) \approx 0.27$$