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MODELS FOR IAU'S AUTONOMOUS GUIDED VEHICLE

Magnus Nørgaard

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TECHNICAL REPORT

IMM-REP-1997-15

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1 Introduction

Models of dynamic systems are required for many different reasons. In this project models are of course needed for being able to design observers (Kalman filters). However, to test the observers an inherent part of the design problem is control system design and simulation of the system. These problems too require models of the system under consideration. Depending the application different levels of model complexity are possible, but often it is difficult to know in advance which features are relevant in the model and which are not. It is common to use quite simple models, preferably linear, for observer and control system design while for simulation it is often desirable to use more a complex model.

I this report models of different complexity are derived for the Autonomous Guided Vehicle (AGV) at the Department of Automation. Initially a relatively accurate model is derived and subsequently different simplifications of the model is proposed. The report has been organized in three sections, where the first section by far is the largest: modeling the dynamics, the kinematic transformation and implementation of a simulation model. The models are derived symbolically. The physical vehicle constants can be found in appendix A.

Naturally a compound model describing the vehicle dynamics and kinematics is not enough to design good observers and control systems. To do this it is also necessary to model the all the sensors on the system. However, since the sensors in this case are quite complicated devices it has been found convenient to devote an entire report for this purpose (Norgaard et al. 1997b).

2 Dynamic Modeling of the AGV

A dynamic model of the AGV was derived in [And95] and later extended slightly in [Lar96]. This note is essentially a recapitulation of these derivations. It follows quite closely the approach taken in [And95], but incorporates the extensions considered in [Lar96]. The extensions comprise the centrifugal and coriolis forces, the coloumb friction in the DC-motors and that the two driving wheels on the AGV can have different radius.

2.1 The actuator dynamics

The vehicle has two driving wheels in the front and a castor wheel in the rear end. Each of the two driving wheels are actuated by an armature controlled DC-motor to enable differential drive. The motors give rise to a force on each wheel in the driving direction of the vehicle. Or, when seen from the DC-motors, a corresponding torque disturbance acting on the motor shafts. A diagram of an often used DC-motor model (see f. ex. [Let78]) is shown in Fig. 1.

While most motor parameters can be found in the data sheet for the motor, Coulomb friction and stiction are typically found experimentally as discussed in [Let78].

The model of the friction can take many different levels of sophistication. By neglecting Coulomb friction and stiction a linear model suitable for most observer and control system

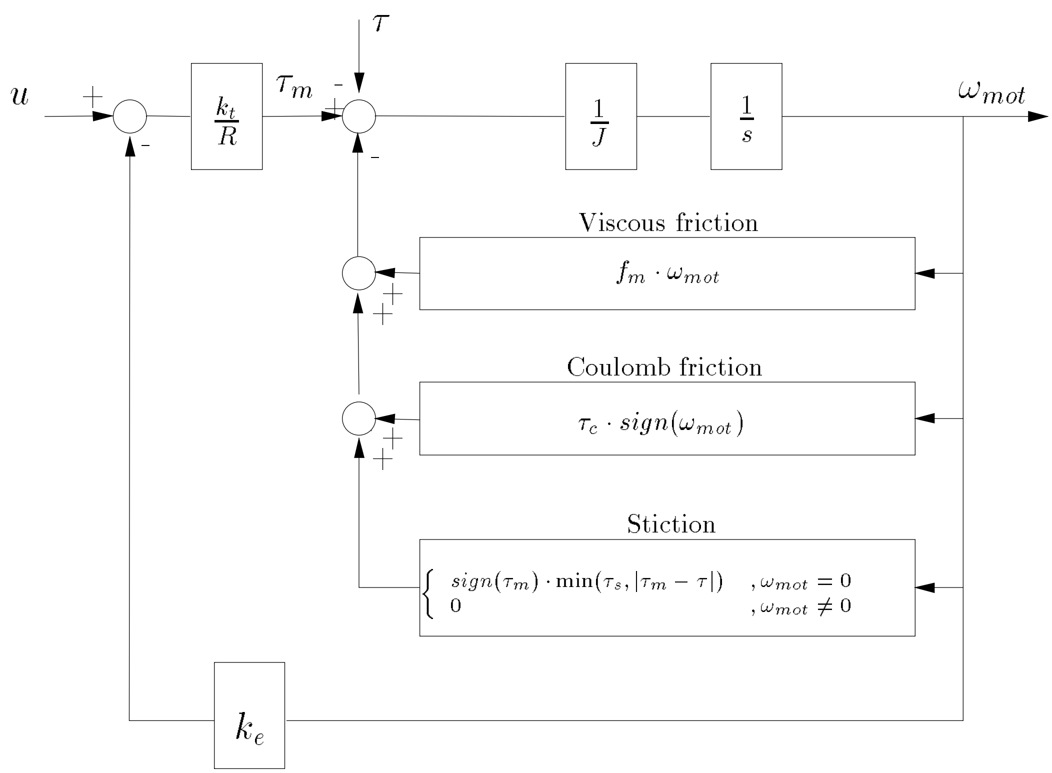

Figure 1. *Diagram of an armature controlled DC-motor. J is the moment of inertia, R is the resistance, k_e the voltage constant, f_m the viscous friction coefficient, τ_c the (magnitude of the) torque due to Coulomb friction, τ_s the maximum torque due to stiction, and τ the torque disturbance caused by the forces acting on the wheels.*

designs is obtained.

$$J\dot{\omega}_{mot} = \frac{k_t}{R}(u - k_e\omega_{mot}) - \tau - f_m\omega_{mot} \tag{1}$$

The simplified model is shown in Fig. 2

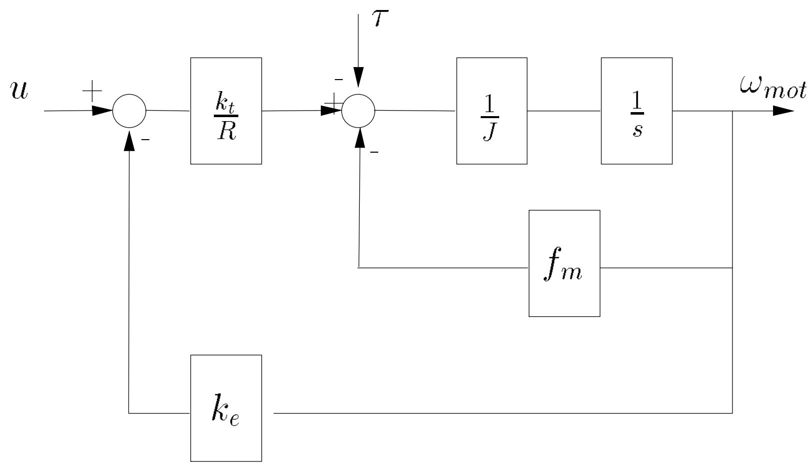

Figure 2. *Model of DC-motor obtained by neglecting nonlinear friction terms.*

In order to perform realistic simulations it is be desirable to use the accurate model depicted in Fig. 1. Unfortunately this model is not suitable for simulation. Its demand for infinite acceleration when the rotation changes direction is difficult to handle efficiently in most simulation packages. In the MSL-library for Simulink [PRS93] a friction model that provides a reasonable trade-off between accuracy and simulation speed has been proposed as an alternative. The principle of this model is depicted in Fig. 3.

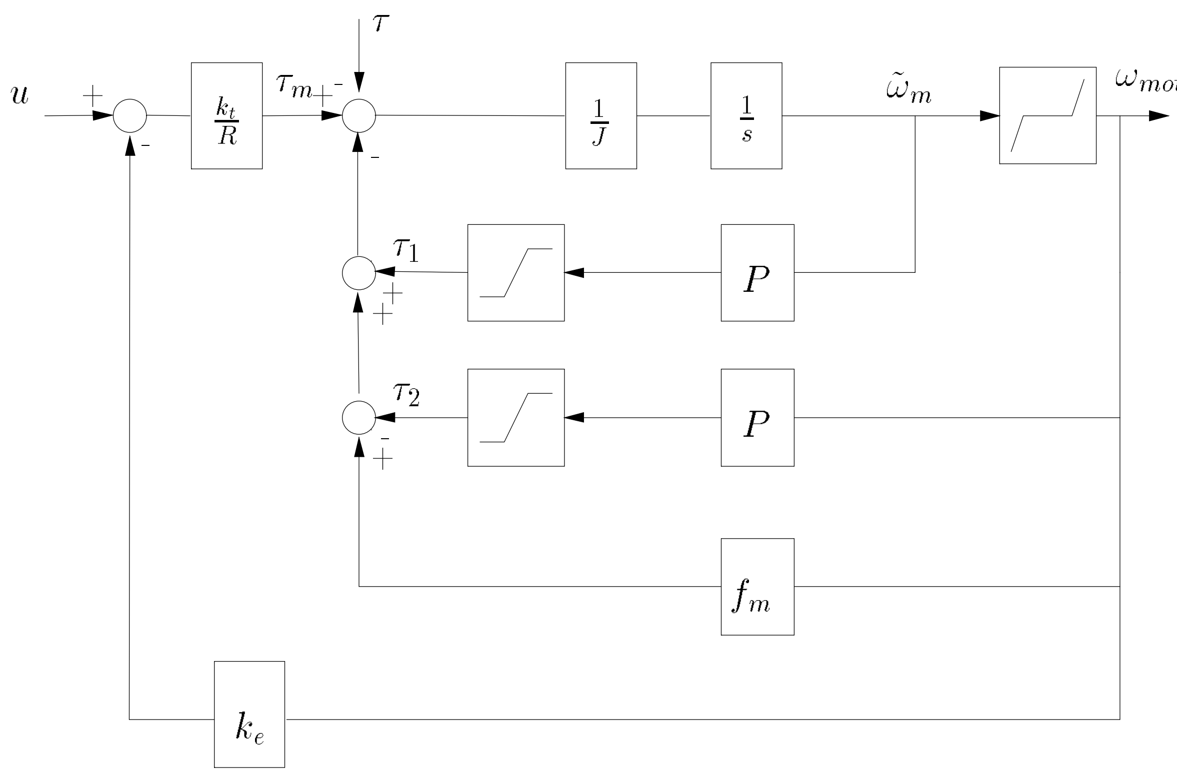

Figure 3. *A principle diagram for an accurate DC-motor model suitable for simulation purposes.*

The variables ω_{mot}, τ_1 and τ_2 in Fig. 3 are determined as follows

$$\omega_{mot} = \begin{cases} \tilde{\omega}_m - \frac{\tau_s}{P} & , \quad \tilde{\omega}_m < -\frac{\tau_s}{P} \\ \tilde{\omega}_m + \frac{\tau_s}{P} & , \quad \tilde{\omega}_m > \frac{\tau_s}{P} \\ 0 & , \quad otherwise \end{cases} \tag{2}$$

$$\tau_1 = \begin{cases} -\tau_s & , \quad \omega_{mot} < 0 \\ \tau_s & , \quad \omega_{mot} > 0 \\ P\tilde{\omega}_m & , \quad otherwise \end{cases} \tag{3}$$

$$\tau_2 = \begin{cases} -(\tau_s - \tau_c) & , \quad \omega_{mot} < -\frac{\tau_s - \tau_c}{P} \\ \tau_s - \tau_c & , \quad \omega_{mot} > \frac{\tau_s - \tau_c}{P} \\ P\omega_{mot} & , \quad otherwise \end{cases} \tag{4}$$

The gain P is a design parameter. As it is increased the model will approach the model in Fig. 1 at the expense of an increased simulation speed and accuracy. It is recommended to select P such that the dynamics of the friction loops are a factor $10 - 100$ faster than the dynamics of the complete system. In section 4 of this report it is explained how to derive a reasonable value for P.

2.2 Amplifier and DA-converter

The amplifier dynamics can in general be neglected as it is much faster than the dynamics of the vehicle. It is however not recommended to neglect the limitations in voltage and current. The limitations are introduced in the model as shown in Fig. 4. Hard limita-

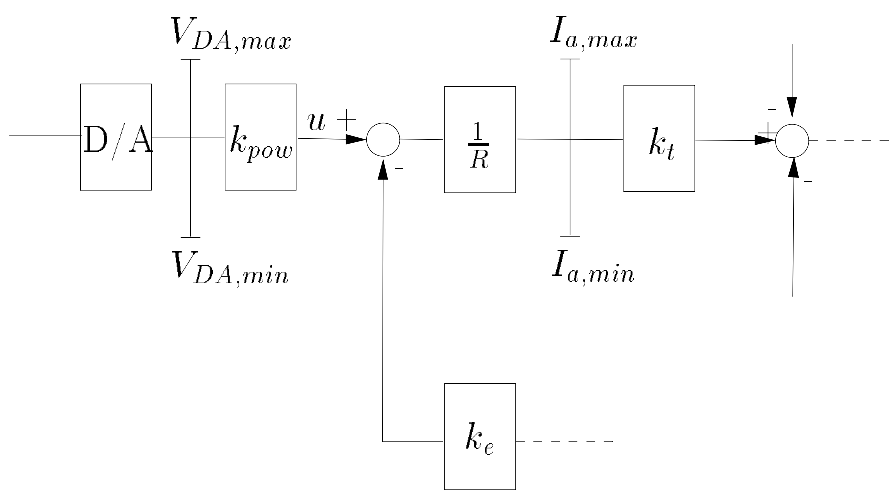

Figure 4. *Voltage and current limitation in the amplifier.*

tions like the above mentioned are rather difficult to handle in the design of controller and observer. What should be done is to include the limitations in the simulation model. In this way one can explore whether the limitations will excert an unfortunate influence on the control performance. The limitations should also be kept in mind when reference trajectories for the controller are generated. In particular it should be noted that the current limitation imposes a limitation in acceleration. Assuming that the identical motors, gears, and wheels are used on both sides of the vehicle, the upper acceleration limit is

$$a_{lim} = \frac{I_{max}k_t - \tau_{friction}}{J_{tot}} \frac{r}{N}, \qquad J_{tot} = J + \frac{1}{2}\frac{Mr^2}{N^2} \tag{5}$$

M is the mass of the vehicle, r is the wheel radius, and N is the gear ratio.

2.3 The torque disturbances

In this section expressions for the torques acting on each of the two motor shafts will be derived. The determination of the torques involves several mechanical considerations and it is convenient to separate the process into a few small and manageable stages. The same approach as in [And95] has been taken, although the final expressions obtained in this note are more detailed. The reader is referred to [And95] for a more thorough treatment of the actual modeling process.

The first stage is to consider the forces acting on the vehicle in the forward and sideway direction, respectively. To facilitate the derivation a frame of reference fixed to the body of the vehicle is introduced. This is shown in Fig. 5. Since the vehicle is moving this is a non-inertial system and it becomes necessary to consider fictive forces. ω is the angular

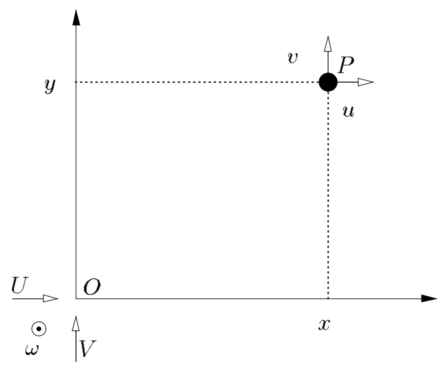

Figure 5. *The coordinate system with origin in the vehicle's point of rotation. The origin is denoted O while P is another point on the vehicle. U, V is the velocity of the origin in the x and y directions and u, v is the velocity of the point P in these directions.*

velocity of P around the point of rotation and it is thus perpendicular to the (x, y)-plane. The relation between u, v and U, V is described by:

$$\begin{cases} u = U - \omega y \\ v = V + \omega x \end{cases} \tag{6}$$

The acceleration of the point P in the x and y directions is obtained as the time derivative of (6):

$$\begin{cases} \dot{u} = \dot{U} - \dot{\omega} y - \omega \dot{y} = \dot{U} - \dot{\omega} y - \omega(V + \omega x) = \dot{U} - \dot{\omega} y - \omega V - \omega^2 x \\ \dot{v} = \dot{V} + \dot{\omega} x + \omega \dot{x} = \dot{V} + \dot{\omega} x + \omega(U - \omega y) = \dot{V} + \dot{\omega} x + \omega U - \omega^2 y \end{cases} \tag{7}$$

Application of Newton's second law in the x and y directions thus give:

$$F_x = M(\dot{U} - \dot{\omega} y - \omega V - \omega^2 x) \tag{8}$$

$$F_y = M(\dot{V} + \dot{\omega} x + \omega U - \omega^2 y) \tag{9}$$

where M is the mass of the vehicle. The second term in the parentheses corresponds to the *angle acceleration force*, the third term to the *coriolis force*, and the fourth term to the *centrifugal force*.

The forces F_x and F_y can each be assumed to be composed of two contributions: a force in the x (y) direction acting on each of the two driving wheels. The four force components are depicted in Fig. 6.

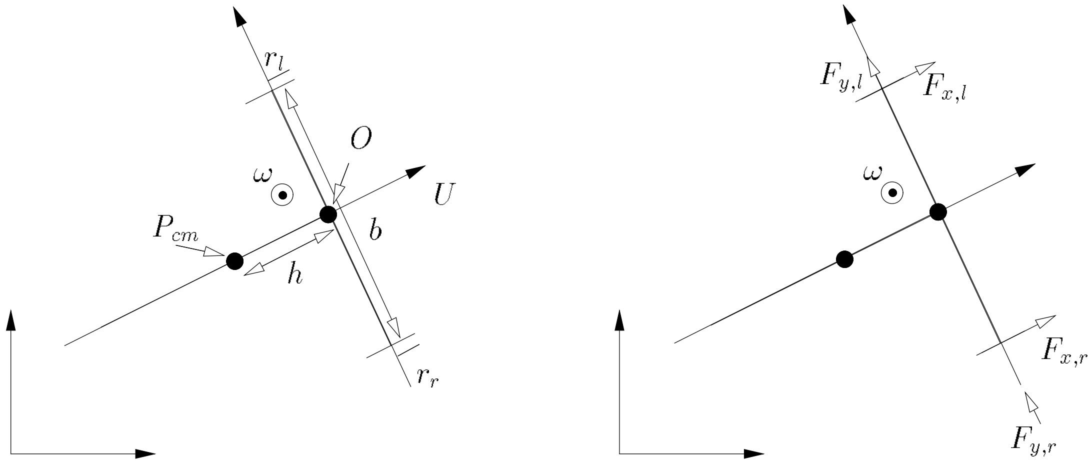

Figure 6. *The geometry of the vehicle. Origin of the body fixed frame of reference is the point of rotation (O) and reference is the center of mass (P_{cm}). Left panel: h is the distance between O and P_{cm}, b is the distance between the wheels and r_l, r_r are the radii of the two driving wheels. Right panel: the forces acting on the driving wheels in the points of floor contact.*

If P is chosen as the vehicle's center of mass and is assumed to be located on the x-axis the following will hold: $y = 0, V = 0, x = -h$. This leads to

$$F_{x,r} + F_{x,l} = M(\dot{U} + \omega^2 h) \tag{10}$$

$$F_{y,r} + F_{y,l} = M(-\dot{\omega} h + \omega U) \tag{11}$$

Introduce I as the moment of inertia measured w.r.t. the center of mass (P_{cm}) and let $\tau_{p,ext}$ be the sum of all torques referred to this point. The angular momentum theorem states:

$$I\dot{\omega} = \tau_{p,ext} \tag{12}$$

The total torque in the point of rotation, O, is

$$\tau_{o,ext} = \frac{b}{2}F_{x,r} - \frac{b}{2}F_{x,l} \tag{13}$$

and since

$$\tau_{o,ext} = \tau_{p,ext} + r_c \times F_{ext} \tag{14}$$

where r_c is the vector from O to P, the angular momentum theorem can also be written:

$$I\dot{\omega} = \frac{b}{2}F_{x,r} - \frac{b}{2}F_{x,l} + h(F_{y,r} + F_{y,l}) \tag{15}$$

Inserting (11) in (15) gives

$$\frac{1}{b}(I + Mh^2)\dot{\omega} = \frac{1}{2}F_{x,r} - \frac{1}{2}F_{x,l} + \frac{1}{b}Mh\omega U \tag{16}$$

and from (10) it follows that

$$\begin{gathered} F_{x,r} = M(\dot{U} + \omega^2 h) - F_{x,l} \\ \Updownarrow \\ F_{x,l} = M(\dot{U} + \omega^2 h) - F_{x,r} \end{gathered} \tag{17}$$

If these relations are merged it leads to

$$\frac{1}{b}(I+Mh^2)\dot{\omega} = \frac{M}{2}(\dot{U}+\omega^2 h) + \frac{h}{b}M\omega U - F_{x,l} \tag{18}$$

$$\frac{1}{b}(I+Mh^2)\dot{\omega} = -\frac{M}{2}(\dot{U}+\omega^2 h) + \frac{h}{b}M\omega U + F_{x,r} \tag{19}$$

or, by separating the forces on the left hand side,

$$F_{x,l} = -\frac{I+Mh^2}{b}\dot{\omega} + \frac{Mh}{2}\omega^2 + \frac{Mh}{b}\omega U + \frac{M}{2}\dot{U} \tag{20}$$

$$F_{x,r} = \frac{I+Mh^2}{b}\dot{\omega} + \frac{Mh}{2}\omega^2 - \frac{Mh}{b}\omega U + \frac{M}{2}\dot{U} \tag{21}$$

Before these expressions are inserted in the actuator models it is necessary to express the forces as functions of the angular acceleration and velocity of the two motor shafts instead of as functions of the vehicle's forward and angular acceleration and velocity. First ω and U is substituted by utilizing their relation to the wheel velocities:

$$U = \frac{v_r + v_l}{2} \Rightarrow \dot{U} = \frac{\dot{v}_r + \dot{v}_l}{2} \tag{22}$$

$$\omega = \frac{v_r - v_l}{2} \Rightarrow \dot{\omega} = \frac{\dot{v}_r - \dot{v}_l}{2} \tag{23}$$

which, when inserted in (20) and (21), leads to

$$F_{x,l} = \left(\frac{M}{4} - \frac{I+Mh^2}{b^2}\right)\dot{v}_r + \left(\frac{M}{4} + \frac{I+Mh^2}{b^2}\right)\dot{v}_l + \frac{Mh}{b^2}(v_r^2 - v_r v_l) \tag{24}$$

$$F_{x,r} = \left(\frac{M}{4} + \frac{I+Mh^2}{b^2}\right)\dot{v}_r + \left(\frac{M}{4} - \frac{I+Mh^2}{b^2}\right)\dot{v}_l + \frac{Mh}{b^2}(v_l^2 - v_r v_l) \tag{25}$$

subsequently it is used that

$$\begin{aligned} v_l &= r_l \omega_l \\ v_r &= r_r \omega_r \end{aligned} \tag{26}$$

(it is assumed that the radii of the two wheels might be different)

The forces can then be expressed as functions of the angular acceleration and velocity of the two wheels

$$F_{x,l} = \left(\frac{M}{4} - \frac{I+Mh^2}{b^2}\right) r_r\dot{\omega}_r + \left(\frac{M}{4} + \frac{I+Mh^2}{b^2}\right) r_l\dot{\omega}_l + \frac{Mh}{b^2}(r_r^2\omega_r^2 - r_r r_l \omega_r \omega_l) \tag{27}$$

$$F_{x,r} = \left(\frac{M}{4} + \frac{I+Mh^2}{b^2}\right) r_r\dot{\omega}_r + \left(\frac{M}{4} - \frac{I+Mh^2}{b^2}\right) r_l\dot{\omega}_l + \frac{Mh}{b^2}(r_l^2\omega_l^2 - r_r r_l \omega_r \omega_l) \tag{28}$$

The complete dynamic model

If the transmission from motor to wheel includes the gear $1/N$ the following relations connect the actuator model with (27) and (28):

$$\omega_{mot} = N\omega_{wheel} \tag{29}$$

$$F = \frac{N}{r}\tau \tag{30}$$

The complete model is thus given by

$$\begin{aligned}
\tau_l &= \frac{r_r r_l}{N^2}\left(\frac{M}{4} - \frac{I + Mh^2}{b^2}\right)\dot{\omega}_{mot,r} + \frac{r_l^2}{N^2}\left(\frac{M}{4} + \frac{I + Mh^2}{b^2}\right)\dot{\omega}_{mot,l} \\
&\quad + \frac{Mhr_l}{b^2N^3}(r_r^2\omega_{mot,r}^2 - r_r r_l \omega_{mot,r}\omega_{mot,l}) &(31)\\
\tau_r &= \frac{r_r^2}{N^2}\left(\frac{M}{4} + \frac{I + Mh^2}{b^2}\right)\dot{\omega}_{mot,r} + \frac{r_r r_l}{N^2}\left(\frac{M}{4} - \frac{I + Mh^2}{b^2}\right)\dot{\omega}_{mot,l} \\
&\quad + \frac{Mhr_r}{b^2N^3}(r_l^2\omega_{mot,l}^2 - r_r r_l \omega_{mot,r}\omega_{mot,l}) &(32)\\
J_l\dot{\omega}_{mot,l} &= \frac{k_{t,l}}{R_l}u_l - \frac{k_{e,l}k_{t,l}}{R_l}\omega_{mot,l} - \tau_l - f_{m,l}\omega_{mot,l} - \tau_{c,l}sign(\omega_{mot,l}) - \tau_{stic,l} &(33)\\
J_r\dot{\omega}_{mot,r} &= \frac{k_{t,r}}{R_r}u_r - \frac{k_{e,r}k_{t,r}}{R_r}\omega_{mot,r} - \tau_r - f_{m,r}\omega_{mot,r} - \tau_{c,r}sign(\omega_{mot,r}) - \tau_{stic,r} &(34)
\end{aligned}$$

In addition to these expressions one should in principle include the voltage and current limitations in DA-converter and amplifier.

For control design purposes it is most oftyen desirable to work with a simplified linear model. By neglecting the coriolis and centrifugal forces as well as the Coloumb friction the following simplified expressions for the torque disturbances appears (this is similar to the model derived in [And95]):

$$\begin{aligned}
\tau_l &= \frac{r_r r_l}{N^2}\left(\frac{M}{4} - \frac{I + Mh^2}{b^2}\right)\dot{\omega}_{mot,r} + \frac{r_l^2}{N^2}\left(\frac{M}{4} + \frac{I + Mh^2}{b^2}\right)\dot{\omega}_{mot,l} &(35)\\
\tau_r &= \frac{r_r^2}{N^2}\left(\frac{M}{4} + \frac{I + Mh^2}{b^2}\right)\dot{\omega}_{mot,t} + \frac{r_r r_l}{N^2}\left(\frac{M}{4} - \frac{I + Mh^2}{b^2}\right)\dot{\omega}_{mot,l} &(36)\\
J_l\dot{\omega}_{mot,l} &= \frac{k_{t,l}}{R_l}u_l - \frac{k_{e,l}k_{t,l}}{R_l}\omega_{mot,l} - \tau_l - f_{m,l}\omega_{mot,l} &(37)\\
J_r\dot{\omega}_{mot,r} &= \frac{k_{t,r}}{R_r}u_r - \frac{k_{e,r}k_{t,r}}{R_r}\omega_{mot,r} - \tau_r - f_{m,r}\omega_{mot,r} &(38)
\end{aligned}$$

3 The Kinematic Model

The kinematic equations express the relationship between the vehicle variables plus their derivatives and the position, velocity and acceleration in a world fixed coordinate system. The dynamic model derived above provides the angular velocity of the driving wheels for a given control input to the motors. From these velocities it is straightforward to derive the translational and angular velocity of the vehicle:

$$\begin{aligned}
U &= \frac{r_r\omega_r + r_l\omega_l}{2} &(39)\\
\omega &= \frac{r_r\omega_r - r_l\omega_l}{b} &(40)
\end{aligned}$$

Based on these velocities the kinematic equations can be expressed as follows

$$\dot{x} = Ucos(\theta) \tag{41}$$
$$\dot{y} = Usin(\theta) \tag{42}$$
$$\dot{\theta} = \omega \tag{43}$$

and

$$\ddot{x} = \dot{U}cos(\theta) - U\omega sin(\theta) = \dot{U}cos(\theta) - \omega\dot{y} \tag{44}$$
$$\ddot{y} = \dot{U}sin(\theta) + U\omega cos(\theta) = \dot{U}sin(\theta) + \omega\dot{x} \tag{45}$$
$$\ddot{\theta} = \frac{r_r\dot{\omega}_r - r_l\dot{\omega}_l}{b} \tag{46}$$

4 Implementation of a Simulation Model

To accommodate a simulation of the vehicle it is convenient to introduce the following constants to simplify the expressions used for determining the torques (31),(32) (or (35),(36)):

$$k_{ll} = \frac{r_l^2}{N^2}\left(\frac{M}{4} + \frac{I + Mh^2}{b^2}\right) \tag{47}$$
$$k_{rr} = \frac{r_r^2}{N^2}\left(\frac{M}{4} + \frac{I + Mh^2}{b^2}\right) \tag{48}$$
$$k_{lr} = \frac{r_l r_r}{N^2}\left(\frac{M}{4} - \frac{I + Mh^2}{b^2}\right) \tag{49}$$
$$k_{llr} = \frac{Mhr_l^2 r_r}{b^2N^3} \tag{50}$$
$$k_{lrr} = \frac{Mhr_l r_r^2}{b^2N^3} \tag{51}$$

Leading to

$$\tau_l = k_{lr}\dot{\omega}_{mot,r} + k_{ll}\dot{\omega}_{mot,l} + k_{lrr}\omega_{mot,r}^2 - k_{llr}\omega_{mot,r}\omega_{mot,l} \tag{52}$$
$$\tau_r = k_{rr}\dot{\omega}_{mot,r} + k_{lr}\dot{\omega}_{mot,l} + k_{llr}\omega_{mot,l}^2 - k_{lrr}\omega_{mot,r}\omega_{mot,l} \tag{53}$$

Algebraic loops are difficult to handle in simulations, but by a little elaboration on the previous expressions these can effectively be eliminated.

$$\tilde{\tau}_l = \frac{k_{t,l}}{R_l}u_l - \frac{k_{e,l}k_{t,l}}{R_l}\omega_{mot,l} - f_{m,l}\omega_{mot,l} - \tau_{Coul+stic,l} - k_{lrr}\omega_{mot,r}^2 + k_{llr}\omega_{mot,r}\omega_{mot,l} \tag{54}$$
$$\tilde{\tau}_r = \frac{k_{t,r}}{R_r}u_r - \frac{k_{e,r}k_{t,r}}{R_r}\omega_{mot,r} - f_{m,r}\omega_{mot,r} - \tau_{Coul+stic,r} - k_{llr}\omega_{mot,l}^2 + k_{lrr}\omega_{mot,r}\omega_{mot,l} \tag{55}$$

in which case the dynamical model can be written as

$$(J_l + k_{ll})\,\dot{\omega}_{mot,l} + k_{lr}\dot{\omega}_{mot,r} = \tilde{\tau}_l \tag{56}$$
$$(J_r + k_{rr})\,\dot{\omega}_{mot,r} + k_{lr}\dot{\omega}_{mot,l} = \tilde{\tau}_r \tag{57}$$

$\Updownarrow$

$$\left[J_l + k_{ll} - \frac{k_{lr}^2}{J_r + k_{rr}}\right]\dot{\omega}_{mot,l} = \tilde{\tau}_l - \frac{k_{lr}}{J_r + k_{rr}}\tilde{\tau}_r \tag{58}$$

$$\left[J_r + k_{rr} - \frac{k_{lr}^2}{J_l + k_{ll}}\right]\dot{\omega}_{mot,r} = \tilde{\tau}_r - \frac{k_{lr}}{J_l + k_{ll}}\tilde{\tau}_l \tag{59}$$

When using the friction model shown in Fig. 3 it is necessary to select the gain P. As was mentioned it is recommended to choose this such that the dynamics due to Coloumb friction and stiction are a factor $10 - 100$ faster than dynamics of the vehicle. A slightly simplified model can be obtained by neglecting all nonlinear terms and the cross-couplings between the two motors and by regarding the coefficients to the angular velocities on the left hand side of (58) and (59) as equivalent moments of inertia

$$\tilde{J}_l = J_l + k_{ll} - \frac{k_{lr}^2}{J_r + k_{rr}} \tag{60}$$

$$\tilde{J}_r = J_r + k_{rr} - \frac{k_{lr}^2}{J_l + k_{ll}} \tag{61}$$

This corresponds to an model for each wheel as shown in Figure 3, but where the external torque (τ) is eliminated and the motor inertia is substituted for the equivalent moment of inertia given above. The gain, P can thus be selected according to (10 is preferred for rapid simulations)

$$P_l = [10 - 100] \times \frac{k_{e,l}k_{t,l} + f_{m,l}R_l}{R_l} \tag{62}$$

(the gain for the right motor is chosen in a similar fashion).

References

[And95] Gert L. Andersen. *Modelling and Control of a Mobile Robot.* PhD thesis, IAU, DTU, 1995.

[Bak97] M. Bak. Sensor fusion using kalman filering. Master's thesis, IMM/IAU, DTU, 1997. (In Danish).

[BF96] J. Borenstein and L. Feng. Measurement and correction of systematic odometry errors in mobile robots. *IEEE Trans. on Robotics and Automation*, XX(XX):xx–xx, 1996.

[Lar96] Mikael B. Larsen. Sensor fusion. Master's thesis, IMM/IAU, DTU, 1996. (In Danish).

[Let78] Nils Leth. *Servomechanisms, Vol. 1.* IAU, DTU, 1978. (In Danish).

[PRS93] Anders Pjeturson, Ole Ravn, and Maciej Szymkat. *Guidelines for generating MSL elements.* IAU, DTU, 1993.

A Vehicle Constants

Frame dimensions:

	Length, l	$0.65\,m$
	Width, w	$0.47\,m$
	Height	$0.55\,m$
	Weight	$51.0\,kg$
	Front to point of rotation, d_{of}	$0.11\,m$
	Wheel base to castor, d_{wc}	$0.50\,m$

Wheels:

	Distance between wheels, b	$0.54\,m$
	Radius, r	$0.10\,m$

Battery dimensions:

	Length, l_b	$0.33\,m$
	Width, w_b	$0.15\,m$
	Height	$0.27\,m$
	Weight	$34.0\,kg$
Total weight, M		$85.0\,kg$
Moment of inertia, I		$5.46\,kgm^2$
Center of mass relative to point of rotation, h		$0.12\,m$

Actuator characteristics:

	Moment of inertia, J	$2.50 \times 10^{-5}\,kgm^2$
	Resistance, R	$0.38\,\Omega$
	Voltage constant, k_e	$2.70 \times 10^{-2}\,V sec/rad$
	Viscous friction, f_m	$0\,Nmsec/rad$
	Gear ratio, N	36
	Stiction (attacking on motor shaft), τ_c	$0.065\,Nm$
	Coulomb friction (on motor shaft), τ_s	$0.065\,Nm$

Power amplifier:

Amplification, k_{pow}	2.0
Current limitation	$\left[\ -5.5A, \quad +5.5A\ \right]$
Accelation limitation	$\left[\ -0.657m/s^2, \quad +0.657m/s^2\ \right]$

Power system:

Total capacity	110 Ah
Stabilized voltage supplies	$5V/25A$, $12V/2A$, $-12V/0.5A$
Average power consumption	$20A$

DA conversion:

Range	$[-5V, +5V]$
Resolution	12 bit
Converter gain α_{DA}	$2.44 \times 10^{-3}\, V/bit$

Sampling period:

T_s	40 msec

A.1 Calculation of vehicle constants

Center of mass

The total mass of the vehicle can be determined by placing an ordinary weight under each of the three wheels and then add the three measured masses. The center of mass can subsequently be determined from simple force considerations: the gravitation in the center of mass should correspond to the normal forces from the floor below each wheel (see Fig. 7).

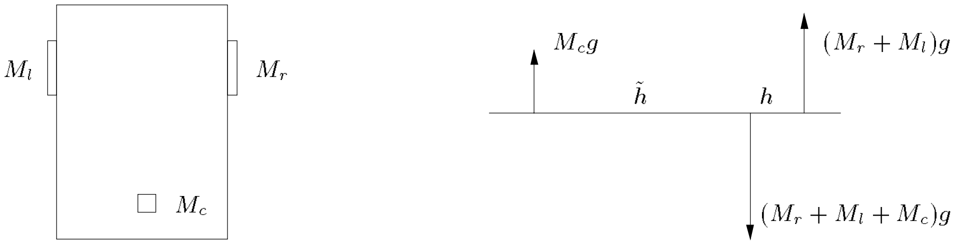

Figure 7. *Dertermination of the center of mass. It is assumed that the mass is distributed symmetrically so that the same mass is measured under each of the two driving wheels.* $h + \tilde{h} = d_{wc}$.

$$h = \frac{M_c}{M_l + M_r + M_c} d_{wc} \tag{63}$$

Moment of inertia

It is assumed that the mass of the battery is distributed homogeneously over its bottom sheet and that the mass of the vehicle frame is distributed homogenously over the floor of the frame. It is further assumed that the center of the battery is right above the point of rotation. The moment of inertia around the point of rotation is then calculated as

$$\begin{aligned} I &= I_f + I_b \\ &= \tfrac{1}{12} M_f (l^2 + w^2) + M_f (\tfrac{l}{2} - l_{of})^2 + \tfrac{1}{12} M_b (l_b^2 + w_b^2) \end{aligned} \tag{64}$$

Viscous friction, Coulomb friction and stiction

It is assumed that the friction is the same on both wheels. The sum of Coulomb friction and stiction can be measured by slowly increasing the control signal to both motors until the vehicle starts moving. Most of the friction is in gear and wheel bearings, but for convenience it is assumed to attack directly on the motor shafts. If the control input required to drive the vehicle is u, the stiction on each motor shaft is

$$\tau_c + \tau_s = \frac{k_t}{K_{amplifier} R} u \tag{65}$$

The coloumb friction alone can be found by gradually decreasing the control input while the vehicle is moving. When the control signal that just keeps the vehicle running has been found, the coloumb friction, τ_c is determined from the expression above.

Often the variations in Coulomb friction superseeds the stiction and for this reason it is common to neglect the stiction from the model.

If a velocity controller has been designed the Coulomb friction and viscous friction are quite easiliy estimated by driving at two different velocities. Assuming that the two velocities are $\omega_{mot,1}$ and $\omega_{mot,r}$ and that the velocities required to keep these velocities are $(u_{1,l}, u_{1,r})$ and $(u_{2,l}, u_{2,r})$. The friction coefficients are then calculated from

$$
\begin{aligned}
f_{m,l} &= \frac{\frac{k_{pow}k_{t,l}}{R_l}(u_{2,l} - u_{1,l}) - \frac{k_{e,l}k_{t,l}}{R_l}(\omega_{mot,2} - \omega_{mot,1})}{\omega_{mot,2} - \omega_{mot,1}} && (66) \\
\tau_{c,l} &= \frac{k_{pow}k_{t,l}}{R_l}u_{2,l} - \left(\frac{k_{e,l}k_{t,l}}{R_l} - f_{m,l}\right)\omega_{mot,2} && (67) \\
f_{m,r} &= \frac{\frac{k_{pow}k_{t,r}}{R_r}(u_{2,r} - u_{1,r}) - \frac{k_{e,r}k_{t,r}}{R_r}(\omega_{mot,2} - \omega_{mot,1})}{\omega_{mot,2} - \omega_{mot,1}} && (68) \\
\tau_{c,r} &= \frac{k_{pow}k_{t,r}}{R_r}u_{2,r} - \left(\frac{k_{e,r}k_{t,r}}{R_r} - f_{m,r}\right)\omega_{mot,2} && (69) \\
& && (70)
\end{aligned}
$$

More accurate estimates can be obtained by collecting the data from experiments at several different velocities and then determine the friction coefficients by least squares estimation.

Wheel base and wheel radius

These quantities are particularly important to know precisely since they have impact on the accuracy at which the pose can be determined by odometry. An experimental method for calibration of wheel base and wheel radii is described in [BF96] and [BF96]. In [Bak97] the method was successfully applied to IAU's AGV.