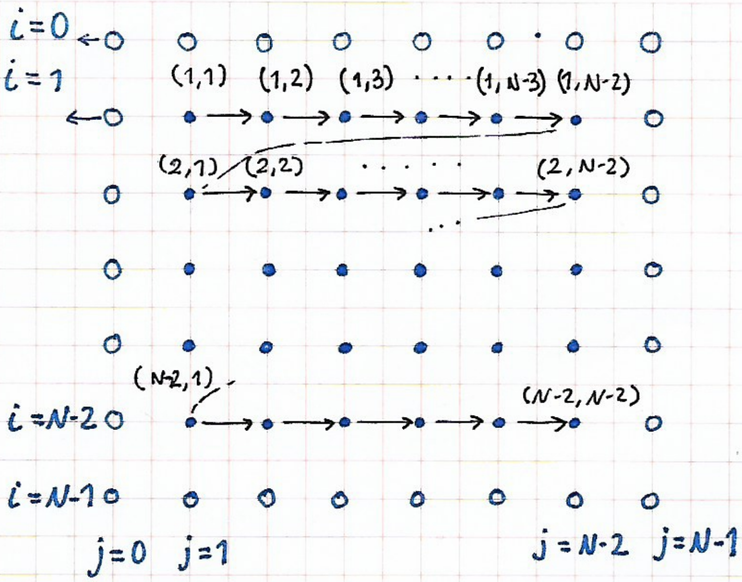


OBS: $N = nx = ny$

orden de elementos para vectores columna P, f:

$$
\begin{array}{c}
i,j \\
\left[\begin{array}{c}
1,1 \\ 1,2 \\ 1,3 \\ \vdots \\ 1,N-3 \\ 1,N-2 \\ 2,1 \\ 2,2 \\ 2,3 \\ \vdots \\ \vdots \\ N-2,N-3 \\ N-2,N-2
\end{array}\right]
\end{array}
$$

$\boxed{\text{CONDICIONES DE BORDE}}$

Sea i_paredes = numpy.where ($y \leq L/2$)
i_apertura = numpy.where ($y > L/2$)

① $P = 0$ en $(x = L, y > L/2)$ presión nula en la salida → i_apertura, $j = N-1$

$\Rightarrow P_{i,N-1} = 0$ para $i \in$ i_apertura

② $\dfrac{\partial P}{\partial x} = 0$ en $(x=0)$ pared izquierda y entrada

$$\left.\frac{P_{i,j+1} - P_{i,j}}{\Delta}\right|_{j=0} = 0 \quad \Rightarrow \quad P_{i,1} = P_{i,0}$$

③ $\dfrac{\partial P}{\partial x} = 0$ en $(x = L, y \leq L/2)$ pared derecha

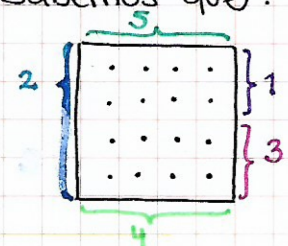
$$\left.\frac{P_{i,j} - P_{i,j-1}}{\Delta}\right|_{\substack{j=N-1 \\ i \in \text{i\_paredes}}} = 0 \quad \Rightarrow \quad P_{i,N-1} = P_{i,N-2}$$

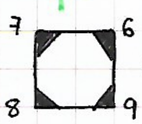
para $i \in$ i_paredes

④ $\dfrac{\partial P}{\partial y}$ en $(y=0)$ piso $\left.\dfrac{P_{i,j} - P_{i-1,j}}{\Delta}\right|_{i=N-1} \Rightarrow P_{N-1,j} = P_{N-2,j}$

⑤ $\dfrac{\partial P}{\partial y}$ en $(y=L)$ simetría $\left.\dfrac{P_{i+1,j} - P_{i,j}}{\Delta}\right|_{i=0} \Rightarrow P_{1,j} = P_{0,j}$

Sabemos que:





ESQUINAS

(1) $P_{i,N-1} = 0$ para ($i \in$ i_apertura)

(2) $P_{i,1} = P_{i,0}$

(3) $P_{i,N-1} = P_{i,N-2}$ para ($i \in$ i_paredes)

(4) $P_{N-1,j} = P_{N-2,j}$

(5) $P_{1,j} = P_{0,j}$

$\boxed{\text{CONDICIONES DE BORDE}}$ → dentro de la matriz.

$$P_{i,j+1} + P_{i+1,j} - 4P_{i,j} + P_{i,j-1} + P_{i-1,j} = f_{i,j} \cdot \Delta^2$$

(1) BORDE DERECHO SUPERIOR: ($j = N-2$)

$$\underbrace{\cancel{P_{i,N-1}}}_{0} + P_{i+1,N-2} - 4P_{i,N-2} + P_{i,N-3} + P_{i-1,N-2} = f_{i,N-2} \cdot \Delta^2 - 0$$

* solo para $i \in$ i_apertura

(2) BORDE IZQUIERDO: ($j = 1$)

$$P_{i,2} + P_{i+1,1} - 4P_{i,1} + \underbrace{\cancel{P_{i,0}}}_{P_{i,1}} + P_{i-1,1} = f_{i,1} \cdot \Delta^2$$

Reescribimos:

$$\overset{j+1}{P_{i,2}} + \overset{i+1}{P_{i+1,1}} - 3P_{i,1} + \overset{i-1}{P_{i-1,1}} = f_{i,1} \cdot \Delta^2$$

(3) BORDE DERECHO INFERIOR: ($j = N-2$)

$$\underbrace{\cancel{P_{i,N-1}}}_{P_{i,N-2}} + P_{i+1,N-2} - 4P_{i,N-2} + P_{i,N-3} + P_{i-1,N-2} = f_{i,N-2} \cdot \Delta^2$$

Reescribimos:

$$\overset{i+1}{P_{i+1,N-2}} - 3P_{i,N-2} + \overset{j-1}{P_{i,N-3}} + \overset{i-1}{P_{i-1,N-2}} = f_{i,N-2} \cdot \Delta^2$$

* solo para $i \in$ i_paredes.

(4) BORDE INFERIOR: ($i = N-2$)

$$P_{N-2,j+1} + \underbrace{\cancel{P_{N-1,j}}}_{P_{N-2,j}} - 4P_{N-2,j} + P_{N-2,j-1} + P_{N-3,j} = f_{N-2,j} \cdot \Delta^2$$

reescribimos

$$\overset{j+1}{P_{N-2,j+1}} - 3P_{N-2,j} + \overset{j-1}{P_{N-2,j-1}} + \overset{i-1}{P_{N-3,j}} = f_{N-2,j} \cdot \Delta^2$$

(5) BORDE SUPERIOR ($i = 1$)

$j+1$ $i+1$ $j-1$ ×

(6) Esquina superior derecha: $(i = 1,\ j = N-2)$

$$\underset{0}{\cancel{P_{1,N-1}}} + P_{2,N-2} - 4P_{1,N-2} + P_{1,N-3} + \underset{P_{1,N-2}}{\cancel{P_{0,N-2}}} = f_{1,N-2}\cdot\Delta^2$$

$$P_{2,N-2} - 3P_{1,N-2} + P_{1,N-3} = f_{i,j}\cdot\Delta^2$$

(7) Esquina superior izquierda: $(i = 1,\ j = 1)$

$$P_{1,2} + P_{2,1} - 4P_{1,1} + \underset{P_{1,1}}{\cancel{P_{1,0}}} + \underset{P_{1,1}}{\cancel{P_{0,1}}} = f_{1,1}\cdot\Delta^2$$

$$P_{1,2} + P_{2,1} - 2P_{1,1} = f_{1,1}\cdot\Delta^2$$

(8) Esquina inferior izquierda: $(i = N-2,\ j = 1)$

$$P_{N-2,2} + \underset{P_{N-2,1}}{\cancel{P_{N-1,1}}} - 4P_{N-2,1} + \underset{P_{N-2,1}}{\cancel{P_{N-2,0}}} + P_{N-3,1} = f_{N-2,1}\cdot\Delta^2$$

$$P_{N-2,2} - 2P_{N-2,1} + P_{N-3,1} = f_{N-2,1}\cdot\Delta^2$$

(9) Esquina inferior derecho: $(i = N-2,\ j = N-2)$

$$\underset{P_{N-2,N-2}}{\cancel{P_{N-2,N-1}}} + \underset{P_{N-2,N-2}}{\cancel{P_{N-1,N-2}}} - 4P_{N-2,N-2} + P_{N-2,N-3} + P_{N-3,N-2} = f_{N-2,N-2}\cdot\Delta^2$$

$$-2P_{N-2,N-2} + P_{N-2,N-3} + P_{N-3,N-2} = f_{N-2,N-2}\cdot\Delta^2$$

Matriz A: Diferencia entre i, $i+1$ es $(N-2)$ elementos
Diferencia entre j, $j+1$ es $+1$ elemento.