$y = \sqrt{(x+6)^2 + 25} + \sqrt{(x-6)^2 + 121}$, say $y = y_1 + y_2$

Let $y_1 = \sqrt{(x+6)^2 + 25}$, $y_2 = \sqrt{(x-6)^2 + 121}$

for y_1

$$\frac{dy_1}{dx} = \frac{1}{2}\left[(x+6)^2 + 25\right]^{-1/2} \cdot 2(x+6)$$

$$\frac{dy_1}{dx} = \frac{x+6}{\sqrt{(x+6)^2 + 25}}$$

At stationary point $\frac{dy_1}{dx} = 0$

$$x + 6 = 0 \cdot \sqrt{(x+6)^2 + 25}$$

$$x + 6 = 0$$

$$x = -6$$

$$\therefore \frac{d^2y_1}{dx^2} = \frac{1}{\sqrt{(x+6)^2 + 25}} + (x+6)\left(-\frac{1}{2}\right)\left[(x+6)^2 + 25\right]^{-3/2} \cdot 2(x+6)$$

$$= \frac{1}{\sqrt{(x+6)^2 + 25}} - \frac{(x+6)^2}{\left[\sqrt{(x+6)^2 + 25}\right]^3}$$

$$\left.\frac{d^2y_1}{dx^2}\right|_{x=-6} = \frac{1}{5} > 0.$$

$$\Rightarrow x_{min} = -6$$

Then $y_{min} = \sqrt{(-6+6)^2 + 25}$

$$y_{min} = \sqrt{25} = 5$$

Min $(x_1, y_1) = \sqrt{6,15}$ ~~$[+6,15]$~~ $[-6, 5]$

For $y_2 = \sqrt{(x-6)^2 + 121}$

$$\frac{dy_2}{dx_2} = \frac{1}{2}\left[(x-6)^2 + 121\right]^{-1/2} \cdot 2(x-6)$$

$$\frac{dy_2}{dx_2} = \frac{x-6}{\sqrt{(x-6)^2 + 121}}$$

At stationary point $\frac{dy_2}{dx_2} = 0$

$$x - 6 = 0 \; ; \; x = 6$$

$$\frac{d^2y_2}{dx_2^2} = \frac{1}{\sqrt{(x-6)^2 + 121}} + (x-6) \cdot -\frac{1}{2}\left[(x-6)^2 + 121\right]^{-3/2} \cdot 2(x-6)$$

$$\frac{d^2y_2}{dx_2^2} = \frac{1}{\sqrt{(x-6)^2 + 121}} - \frac{(x-6)^2}{\left[\sqrt{(x-6)^2 + 121}\right]^3}$$

$$\left.\frac{d^2y_2}{dx_2^2}\right|_{x=6} = \frac{1}{\sqrt{121}} \Rightarrow \frac{1}{11} > 0$$

$$\Rightarrow x_{min} = 6$$

$$y_{min} = \sqrt{(6-6)^2 + 121}$$

$$y_{min} = \sqrt{121} \Rightarrow 11$$

$$Min\,(x_2, y_2) = (6, 11)$$

For $y = \sqrt{(x+6)^2 + 25} + \sqrt{(x-6)^2 + 121}$ for $y = y_1 + y_2$

For minimum value of y is

$$y = 5 + 11$$

$$y = 16$$

OGUNKOYA . BUKUNMI T.