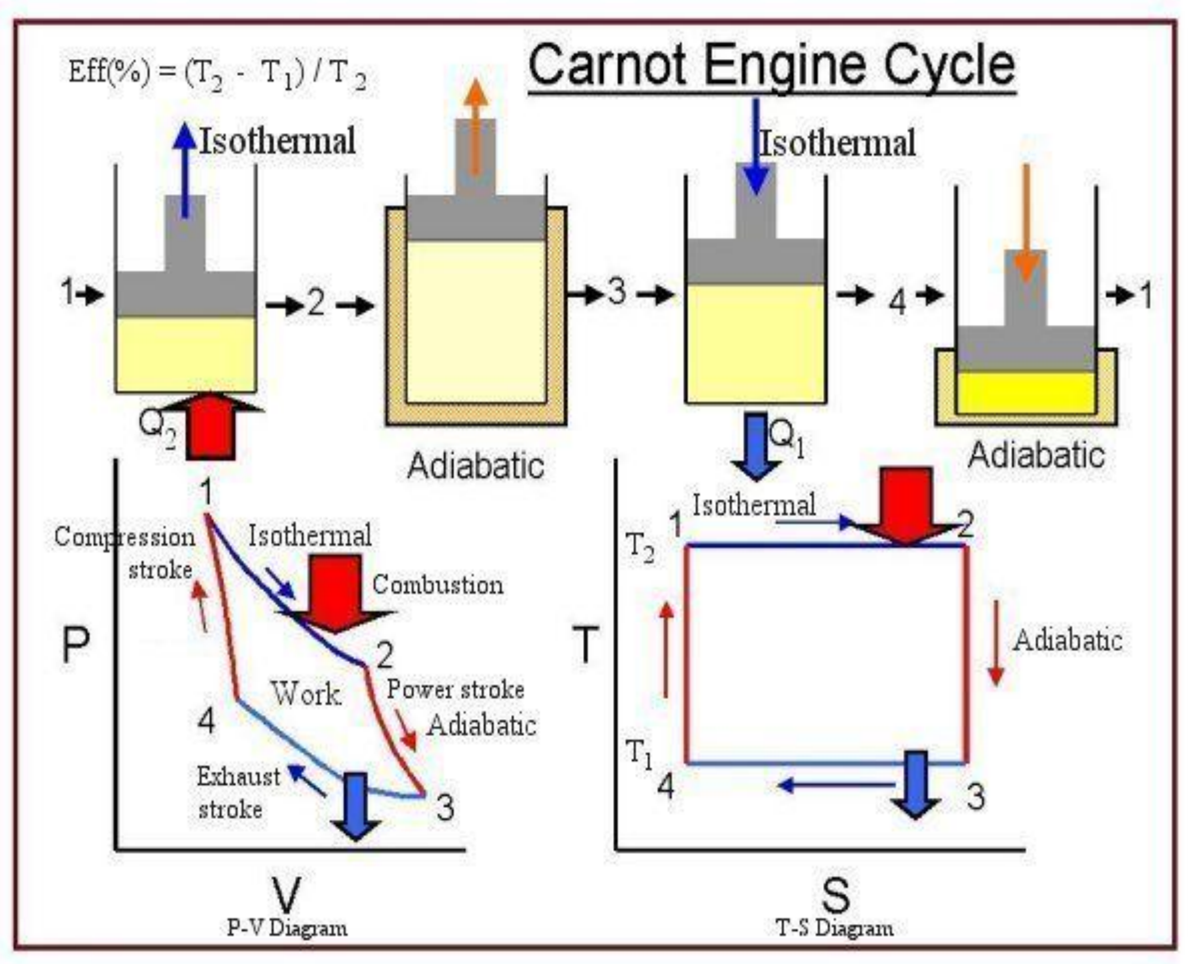
CARNOT'S REVERSIBLE ENGINE

A heat engine is a device that is used to convert **heat energy** into **mechanical work.** A **Carnot heat engine** is a heat engine that operates on the Carnot cycle. The basic model for this engine was developed by Nicolas Léonard Sadi Carnot in 1824. The Carnot engine model was graphically expanded by Benoît Paul Émile Clapeyron in 1834 and mathematically explored by Rudolf Clausius in 1857, a work that led to the fundamental thermodynamic concept of entropy.

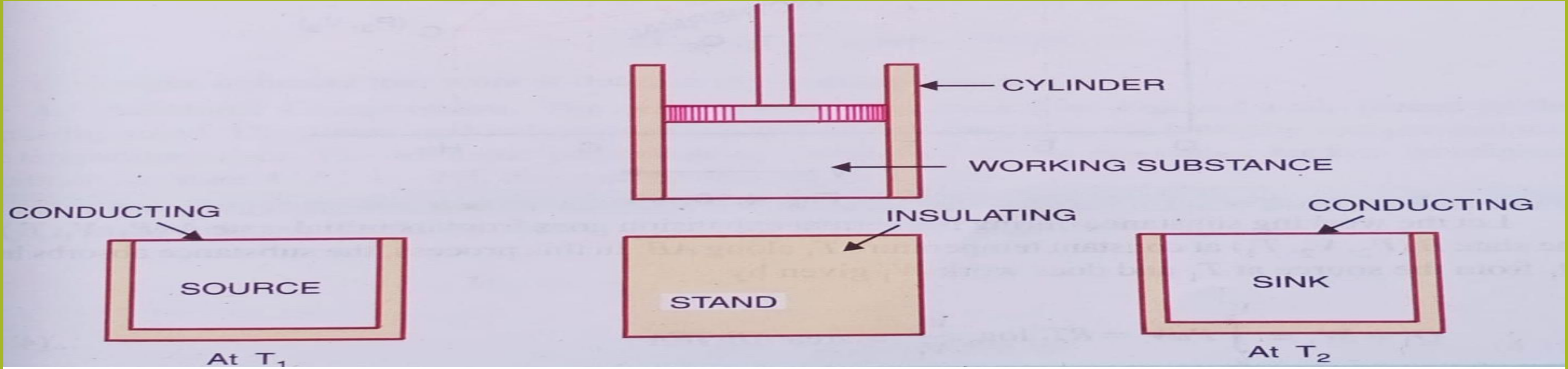


The Carnot engine is the most efficient engine which is theoretically possible. The efficiency depends only upon the absolute temperatures of the hot and cold heat reservoirs between which it operates.

A heat engine acts by transferring energy from a warm region to a cool region of space and, in the process, converting some of that energy to mechanical work.

The cycle may also be reversed. The system may be worked upon by an external force, and in the process, it can transfer thermal energy from a cooler system to a warmer one, thereby acting as a refrigerator or heat pump rather than a heat engine.

Every thermodynamic system exists in a particular state. A thermodynamic cycle occurs when a system is taken through a series of different states and finally returned to its initial state. In the process of going through this cycle, the system may perform work on its surroundings, thereby acting as a heat engine.



For any engine, there are three essential requisites.

SOURCE: The source should be at a fixed high-temperature T_1 from which the heat engine can draw heat. It has infinite thermal capacity and any amount of heat can be drawn from it at constant temperature T_1.

SINK: The sink should be at a fixed lower temperature T_2 to which any amount of energy can be rejected. It also has infinite thermal capacity and its temperature remains at a constant temperature T_2.

WORKING SUBSTANCE: A cylinder with non-conducting sides and a conducting bottom contains the perfect gas as a working substance. A perfect non-conducting and frictionless piston are fitted into the cylinder. The working substance undergoes a complete cyclic operation. A perfect non-conducting stand is also provided so that the working substance can undergo an adiabatic operation.

CARNOT'S CYCLE

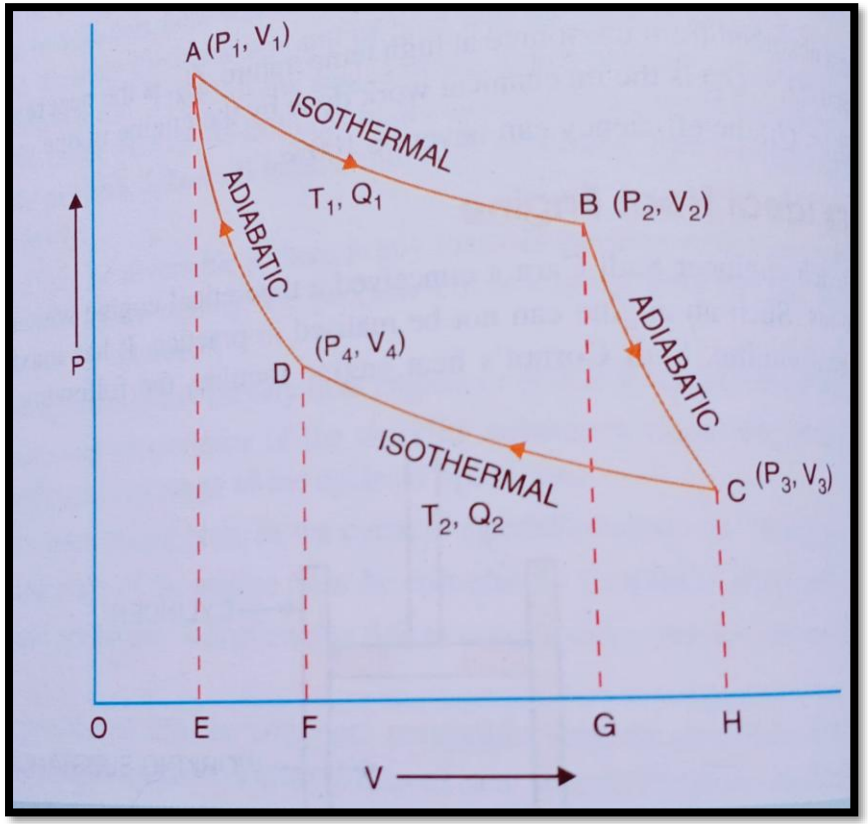
The most efficient heat engine cycle is the Carnot cycle, consisting of two isothermal processes and two adiabatic processes. The Carnot cycle can be thought of as the most efficient heat engine cycle allowed by physical laws. When the second law of thermodynamics states that not all the supplied heat in a heat engine can be used to do work, the Carnot efficiency sets the limiting value on the fraction of the heat which can be so used.

In order to approach the Carnot efficiency, the processes involved in the heat engine cycle must be reversible and involve no change in entropy. This means that the Carnot cycle is an idealization, since no real engine processes are reversible and all real physical processes involve some increase in entropy.

The working substance is subjected to the following cycle or quasi-static operations known as Carnot's cycle to obtain a continuous supply of work.

- **ISOTHERMAL EXPANSION**: The cylinder is first placed on the source so that the gas acquires the temperature T1 of the source. It is then allowed to undergo quasi-static expansion. As the gas expands, its temperature tends to fall. Heat passes into the cylinder through the perfectly conducting base which is in contact with the source. The gas, therefore, undergoes slow isothermal expansion at the constant temperature T1. **Let the working substance during isothermal expansion goes from its initial state A(P_1, V_1, T_1) to the state B(P_2, V_2, T_1) at constant temperature T1 and does work W1 given** by

$$Q_1 = W_1 = \int_{V_1}^{V_2} P\, dV = RT_1 \log_e \frac{V_2}{V_1} = \text{area ABGEA}$$



Graph between P and V

ADIABATIC EXPANSION: The cylinder is now removed from the source and placed on the insulating stand. The gas is allowed to undergo slow adiabatic expansion, performing external work at the expense of its internal energy, until its temperature falls to T_2, the same as that of the sink.

This process is represented by the adiabatic BC, starting from state B $(\boldsymbol{P_2, V_2, T_1})$ to the state C$(\boldsymbol{P_3, V_3, T_2})$. In this process, there is no transfer of heat, the temperature of the substance falls to T_2 and it does some external work $\boldsymbol{W_2}$ given by

$$W_2 = \int_{V_2}^{V_3} P.dV = K\int_{V_2}^{V_3} \frac{dV}{V^{\gamma}} \qquad \textbf{(During adiabatic process, } PV^{\gamma} \textbf{ = constant = K)}$$

$$= \frac{KV_3^{1-\gamma} - KV_2^{1-\gamma}}{1-\gamma}$$

$$= \frac{P_3V_3 - P_2V_2}{1-\gamma} \qquad \textbf{(Since, } P_2V_2^{\gamma} = P_3V_3^{\gamma} = K)$$

$$= \frac{RT_2 - RT_1}{1-\gamma}$$

$$= \frac{R(T_1 - T_2)}{\gamma - 1} = \textbf{Area BCHGB}$$

ISOTHERMAL COMPRESSION: The cylinder is now removed from the insulating stand and is placed on the sink which is at a temperature T_2 . The piston is now slowly moved inwards so that the work is done on the gas. The temperature tends to increase due to heat produced by compression since the conducting base of the cylinder is in contact with the sink, the heat developed passes to the sink and the temperature of the gas remains constant at T_2. Thus the gas undergoes isothermal compression at the constant temperature T_2 and gives up some heat to the sink.

This operation is represented by the isothermal CD, starting from the state **C**$(\boldsymbol{P_3, V_3, T_2})$ to the state **D**$(\boldsymbol{P_4, V_4, T_2})$**.** In this process, the substance rejects heat Q_2 to the sink at T_2 and the work $\boldsymbol{W_3}$ is done on the substance is given by

$$\boldsymbol{Q_2 = W_3 = \int_{V_3}^{V_4} P\, dV = RT_2 \log \frac{V_4}{V_3}}$$

$$\boldsymbol{= -RT_2 \log \frac{V_3}{V_4}} \quad \textbf{= area CHFDC}$$

(− ve sign indicates that work is done on the working substance)

ADIABATIC COMPRESSION: The cylinder is now removed from the sink and again placed on the insulating stand. The piston is slowly moved inwards so that the gas is adiabatically compressed and the temperature rises. The adiabatic compression is continued till the gas comes back to its original condition i.e. state **A**$(\boldsymbol{P_1}, \boldsymbol{V_1}, \boldsymbol{T_1})$, thus completing one full circle.

This operation is represented by adiabatic DA, starting from **D**$(\boldsymbol{P_4}, \boldsymbol{V_4}, \boldsymbol{T_2})$ to the final state **A**$(\boldsymbol{P_1}, \boldsymbol{V_1}, \boldsymbol{T_1})$. In this process, the work $\boldsymbol{W_4}$ is done on the substance and is given by

$$\boldsymbol{W_4 = \int_{V_4}^{V_1} P \cdot dV}$$

$$= -\frac{R(T_1 - T_2)}{\gamma - 1} = \textbf{area DFEAD}$$

(−ve sign indicates that the work done on the working substance. Since $\boldsymbol{W_2}$ and $\boldsymbol{W_4}$ are equal opposite, they cancel each other.)

WORK DONE BY THE ENGINE PER CYCLE

During the cycle, the working substance absorbs an amount of heat $\boldsymbol{Q_1}$ from the source and rejects $\boldsymbol{Q_2}$ to the sink.

Hence, the net amount of heat absorbed by the gas per cycle

$$= \boldsymbol{Q_1 - Q_2}$$

The net work done by the engine per cycle

$$= \boldsymbol{w_1 + w_2 + w_3 + w_4} \qquad (\boldsymbol{Since, w_2 = -w_4})$$

$$= \boldsymbol{w_1 + w_3}$$

From the graph the net work done by the cycle

= area ABGEA + area BCHGB area – CHFDC – area DFEAD

= area ABCDA

Thus, the area enclosed by the Carnot's cycle consisting of two isothermals and two adiabatics gives the net amount of work done per cycle.

In the cyclic process,

Net heat absorbed = Net work done per cycle

$$\boldsymbol{Q_1 - Q_2 = w_1 + w_3}$$

$$\boldsymbol{Q_1 - Q_2 = RT_1 \log_e \frac{V_2}{V_1} - RT_2 \log_e \frac{V_3}{V_4}} \qquad \textbf{............(1)}$$

Since the points, A and D lie on the same adiabatic DA

$$T_1 V_1^{\gamma-1} = T_2 V_4^{\gamma-1}$$

$$\frac{T_2}{T_1} = \left[\frac{V_1}{V_4}\right]^{\gamma-1} \qquad \text{...............(2)}$$

Similarly, points B and C lie on the same adiabatic BC

$$T_1 V_2^{\gamma-1} = T_2 V_3^{\gamma-1}$$

$$\frac{T_2}{T_1} = \left[\frac{V_2}{V_3}\right]^{\gamma-1} \qquad \text{.................(3)}$$

From equations (2) and (3),

$$\left[\frac{V_1}{V_4}\right]^{\gamma-1} = \left[\frac{V_2}{V_3}\right]^{\gamma-1}$$

or $$\frac{V_1}{V_4} = \frac{V_2}{V_3}$$

or $$\frac{V_2}{V_1} = \frac{V_3}{V_4}$$

Substituting in equation (1), we get

Net work done = $\boldsymbol{Q_1 - Q_2 = RT_1 \, log_e \frac{V_2}{V_1} - RT_2 \, log_e \frac{V_2}{V_1}}$

$$\boldsymbol{Q_1 - Q_2 = R(T_1 - T_2) \log_e \frac{V_2}{V_1}}$$

EFFICIENCY

The efficiency of the heat engine is the **rate of the quantity of heat converted into work per cycle to the total amount of heat absorbed per cycle.**

Efficiency, $$\eta = \frac{Useful\ output}{Input} = \frac{w}{Q_1}$$

$$= \frac{(Q_1 - Q_2)}{Q_1}$$

$$= \frac{R(T_1 - T_2)\log_e \frac{V_2}{V_1}}{RT_1 \log_e \frac{V_2}{V_1}}$$

$$\eta = \frac{T_1 - T_2}{T_1}$$

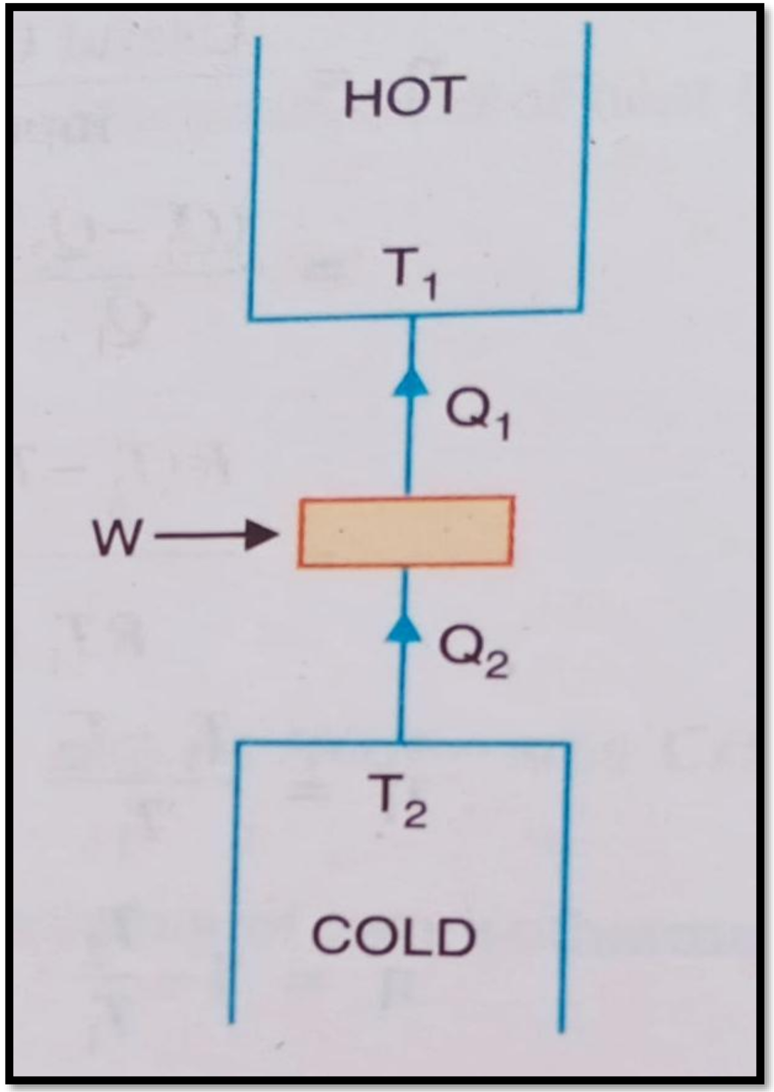
$$\eta = 1 - \frac{T_2}{T_1}$$

From this equation we conclude that the efficiency depends upon the temperature of the source and sink and is always less than unity. The efficiency is independent of working substance.

CARNOT'S CYCLE (REFRIGERATOR)

Carnot's cycle is perfectly reversible. It can work as a **Heat engine** and also as a **refrigerator.**
When it is operated as a refrigerator, it absorbs heat Q_2 from the sink at the temperature T_2. W amount of work is done on it by the external means and rejects heat Q_1 to the source at temperature T_1, $(T_1 > T_2)$ as shown in the figure.

In this case, heat flows from a body at a temperature T_2 to a body at a higher temperature T_1 with the help of external work done on the working substance. This action is that of a refrigerator. In every cycle, heat Q_2 is extracted from the cold body. This will not be possible if the cycle is not completely reversible.



COEFFICIENT OF PERFORMANCE

The amount of heat absorbed at the lower temperature is Q_2. The amount of work done on the working substance by the external agency = W and the amount of heat rejected = Q_1 . Here Q_2 is the desired refrigerating effect in each cycle.

Therefore, Coefficient of performance, $\mathbf{P} = \frac{Q_2}{w} = \frac{Q_2}{Q_1 - Q_2}$

Also, $\mathbf{P} = \frac{T_2}{T_1 - T_2}$

NOTE: In case of heat engine, the efficiency cannot be more than 100% but in case of a refrigerator, the coefficient of performance can be higher than 100% .

SECOND LAW OF THERMODYNAMICS

Lord Kelvin's Statement: "It is impossible to get a continuous supply of work from a body by cooling it to a temperature lower than that of its surrounding."

Planck's Statement: "It is impossible to construct an engine which, working in a complete cycle, will produce no effect other than the absorption of heat from the reservoir."

Kelvin-Planck Statement: "It is impossible to construct an engine which, operating in a cycle, has the sole effect of extracting heat from a reservoir and performing an equivalent amount of work."

Clausis's Statement: "Heat cannot flow itself from a colder body to a hotter body."

CARNOT'S THEOREM

Statement: All reversible engines working between the same two temperatures have the same efficiency whatever be the nature of the working substance.

PROOF:

$$\frac{Q_1'-Q_2'}{Q_1'} > \frac{Q_1-Q_2}{Q_1}$$

or $$\frac{w}{Q_1'} > \frac{w}{Q_1}$$

or $$Q_1 > Q_1'$$

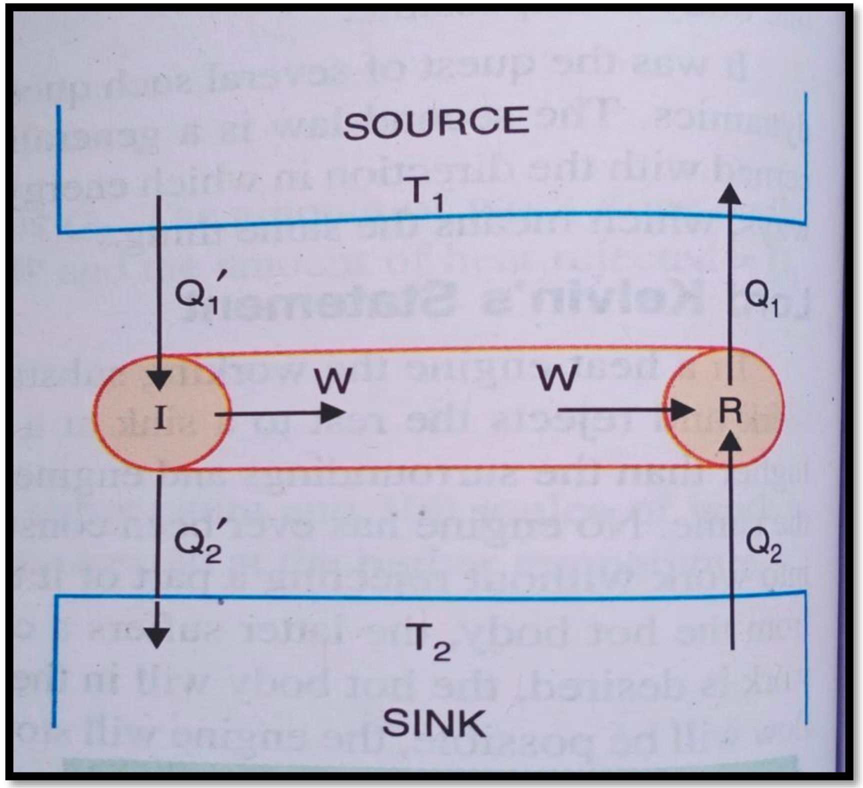
and $$Q_1' - Q_2' = Q_1 - Q_2$$

$$\Rightarrow Q_2 - Q_2' = Q_1 - Q_1'$$

therefore, $Q_2 > Q_2'$

Now if I and R are coupled together and R works in a reverse direction

Then Gain of heat by the source at T_1 = $Q_1 - Q_1'$



Loss of heat by the sink at T_2 = $\boldsymbol{Q_2 - Q_2'}$

Which is contradicting to the second law of thermodynamics.

Hence no engine can be more efficient than a perfectly reversible engine working between the same temperature.