

2017(Voc)

Time : 3 hours

Full Marks : 100

*Candidates are required to give their answers in
their own words as far as practicable.*

The questions are of equal value.

*Answer eight questions, selecting
at least one from each Group.*

Group – A

1. (a) State and prove Euler's theorem for homogeneous function of two variables.
(b) If $y = e^{a \sin^{-1} x}$, then prove that $(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} - (x^2 + a^2)y_n = 0$.

2. (a) If $u = \sin^{-1} \frac{x}{y} + \tan^{-1} \frac{y}{x}$ then prove
 $x \cdot \frac{\partial u}{\partial x} + y \cdot \frac{\partial u}{\partial y} = 0$.

(b) Evaluate :

$$\lim_{x \rightarrow 0} \sin x \cdot \log x$$

3. (a) Prove that $\frac{1}{p^2} = u^2 + \left(\frac{\partial u}{\partial \theta}\right)^2$.

(b) Prove that the sum of the intercepts of the tangent to $\sqrt{x} + \sqrt{y} = \sqrt{a}$ upon co-ordinate axes is constant.

Group - B

4. Integrate any two of the following :

(a) $\int \frac{dx}{\sin x(3 + 2\cos x)}$

(b) $\int \frac{(2x+1)dx}{(x+2)(x-3)^2}$

(c) $\int \frac{dx}{a + b \cos x}$

(d) $\int_0^{\pi/2} \log(\sin x)dx$

5. Find the area of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

6. Find the perimeter of the cardiode :

$$y = a(1 + \cos\theta)$$

Group - C

7. Solve any two of the following :

(a) $y dx - x dy = x \cdot y dx$

(b) $x^2 y dx - (x^3 + y^3) dy = 0$

(c) $\cos^2 x \frac{dy}{dx} + y = \tan x$

(d) $x \frac{dy}{dx} + y = y^2 \log x$

8. (a) Solve any one of the following :

(i) $p^2 + 2xp - 3x^2 = 0$

(ii) $y = 2px + p^2$

(iii) $p^2 - py + x = 0$, where $p = \frac{dy}{dx}$

(b) Find the orthogonal trajectories of the family of parabolas $y = ax^2$.

9. Solve any two of the following :

(a) $(D^2 - 4)y = x^2$

(b) $(D^2 + D + 1)y = \sin 2x$

(c) $(D^2 - 5D + 6)y = e^{4x}$

Group - D

10. (a) Define scalar triple product and give its geometrical meaning.

(b) Prove :

$$\vec{a} \times (\vec{b} \times \vec{c}) + \vec{b} \times (\vec{c} \times \vec{a}) + \vec{c} \times (\vec{a} \times \vec{b}) = 0$$

11. (a) Evaluate :

$$\frac{d}{dt}(\vec{u} \times \vec{v})$$

(b) If $\vec{r} = \vec{a} \cdot \cos wt + \vec{b} \cdot \sin wt$, then prove

$$\vec{r} \times \frac{d\vec{r}}{dt} = w \vec{a} \times \vec{b}.$$

12. (a) Prove :

$$\operatorname{Div}(\vec{a} \pm \vec{b}) = \operatorname{div.} \vec{a} \pm \operatorname{div.} \vec{b}$$

(b) Prove :

$$\operatorname{Grad}(\phi \cdot \psi) = \phi \operatorname{grad} \psi + \psi \operatorname{grad} \phi$$

Group - E

13. (a) Find the equation of the line of action of the resultant of a system of coplanar forces acting on a rigid body.

(b) Forces P, Q, R act along the lines $x = 0$, $y = 0$ and $x \cos\theta + y \sin\theta = p$, the axes being rectangular. Find the magnitude of the resultant and its line of action.

14. (a) State and prove converse of principle of virtual work.

(b) Discuss the forces which can be omitted in forming the equation of virtual work.

15. (a) Show that two S. H. M. of the same period and in the same straight line can be compounded.

(b) If a and T be respectively the amplitude and periodic time, then prove that :

$$\int_0^T v^2 \cdot dt = \frac{2\pi^2 a^2}{T}$$

16. Find the radial and transverse accelerations of a particle describing a smooth curve.



2016

Time : 3 hours

Full Marks : 100

Candidates are required to give their answers in their own words as far as practicable.

The questions are equal value.

Answer Eight questions, selecting at least one from each group.

Group - A

1. a) State and prove Leibnitz's theorem to find the n^{th} derivative of product of two functions.
b) If $y = (x^2 - 1)^n$, then prove that

$$(x^2 - 1)y_{n+2} + 2xy_{n+1} - n(n+1)y_n = 0$$
2. a) If $u = \sin^{-1} \frac{x^2 + y^2}{x+y}$, show that

$$x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \tan u$$

b) Evaluate $\lim_{x \rightarrow 0} x \log_e x$
3. a) Establish the formula
 $p = r \sin \phi$
b) Find the pedal equation of the curve
 $r^n = a^n \cos n\theta$

GD16-1517

(Turn over)

Group - B

4. Evaluate any two of the following:

a) $\int \frac{x-1}{(1+x)(1+x^2)} dx$

b) $\int \frac{\cos x dx}{(1+\sin x)(2+\sin x)}$

c) $\int \sqrt{1+\sec x} dx$

d) $\int_0^{\pi/2} \frac{\sqrt{\sin x} dx}{\sqrt{\sin x + \sqrt{\cos x}}}$

5. Find the whole area of loop of the curve

$$a^2 y^2 = x^2 (a^2 - x^2)$$

6. Find the length of the loop of the curve

$$3ay^2 = x(x-a)^2$$

Group - C

7. Solve any two of the following:

a) $\frac{dy}{dx} = \sin(x+y)$

b) $(x^2 + y^2) \frac{dy}{dx} = xy$

c) $\frac{dy}{dx} + y \cot x = 2 \cos x$

d) $\frac{dy}{dx} + 1 = e^{x+y}$

8. a) Solve any one of the following:
- $y = (1 + p)x + p^2$
 - $p^2 - p(e^x + e^{-x}) + 1 = 0$
 - $x = 4p + 4p^3$
- b) Find the orthogonal trajectories of the curve $r\theta = a$
9. Solve any two of the following:
- $(4D^2 + 4D - 3)y = e^{2x}$
 - $(D^2 + 1)y = \cos 2x$
 - $(D^2 - 2D + 1)y = x^2 e^{3x}$
 - $(D^2 + 4)y = \sin 3x + e^x + x^2$

Group - D

10. a) Prove that the necessary and sufficient condition for three vectors to be coplanar is that their scalar triple product is zero.
- b) Show that

$$[\bar{a} \times \bar{b}, \bar{b} \times \bar{c}, \bar{c} \times \bar{a}] = [\bar{a} \bar{b} \bar{c}]^2$$

11. a) Prove that the necessary and sufficient condition for the vector function $\bar{a}(t)$ to have constant magnitude is

$$\bar{a} \cdot \frac{d\bar{a}}{dt} = 0$$

- b) If $\bar{u}(t)$ and $\bar{v}(t)$ be two differentiable functions of scalar t , then prove that

$$\frac{d}{dt}(\bar{u} \cdot \bar{v}) = \bar{u} \cdot \frac{d\bar{v}}{dt} + \frac{d\bar{u}}{dt} \cdot \bar{v}$$

12. a) Show that

$$\text{grad}(\phi \pm \psi) = \text{grad}\phi \pm \text{grad}\psi$$

b) Prove that

$$\text{grad} \left(\frac{1}{r} \right) = -\frac{\vec{r}}{r^3}$$

Group - E

13. a) Obtain the general condition of equilibrium of a system of forces acting in one plane upon a rigid body.
 b) Prove that a system of coplanar forces acting on a rigid body is in equilibrium if the algebraic sum of their moments about each of the three non-collinear points is zero.
14. a) State and prove the principle of virtual work.
 b) Four weightless rods of equal lengths are joined together to form a rhombus ABCD with one diagonal BD. If a weight W is attached to C and the system be suspended from A. Show that the thrust on BD is equal to $\frac{W}{\sqrt{3}}$.
15. A particle starts moving from rest and moves along a straight line with an acceleration which is always directed towards a fixed point. Discuss the motion and find the period of one complete oscillation.
16. a) Find the tangential and normal velocities of a moving particle in a plane curve.
 b) An insect crawls at a constant rate u along the spoke of a cartwheel of radius a , the cart is moving with a constant velocity v . Find the acceleration along and perpendicular to the spoke.



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2015

Time : 3 hours

Full Marks : 100

Candidates are required to give their answers in
their own words as far as practicable.

Answer **Eight** questions, selecting at least one from
each Group.

Group - A

1. a) State and prove Euler's theorem on
homogeneous function of two variables.

b) If $y = (\sin^{-1} x)^2$, then prove that

$$(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - n^2y_n = 0$$

2. a) Expand $\log(1+\tan x)$ in ascending powers
of x using Maclaurin's series.

b) Evaluate $\lim_{x \rightarrow 0} (\sin x)^{\tan x}$

3. a) For the Pedal curve $p=f(r)$, prove that

$$p = r \frac{dr}{dp}.$$

b) Show that the curve represented by

$$\left(\frac{x}{a}\right)^n + \left(\frac{y}{b}\right)^n = 2 \text{ for different values of } n,$$

have a common tangent at the point (a, b) . Find the equation of common tangent.

Group - B

4. a) Evaluate any two of the following :

i) $\int \frac{x^2 - 1}{x^4 + 1} dx$

ii) $\int \frac{\cos x}{2 \sin x + 3 \cos x} dx$

iii) $\int_0^{\pi/4} \log(1 + \tan \theta) d\theta$

b) Find ab-initio value of $\int_a^b x^2 dx$.

(2) 8

GD-478

(2) T.D.C. Part-II (Sub.) X Voc.

5. a) If $I_n = \int_0^{\pi/4} \tan x dx$, prove that

$$I_n + I_{n-2} = \frac{1}{n-1}.$$

b) Find the perimeter of the loop of the curve

$$9y^2 = (x-2)(x-5)^2.$$

6. a) Prove that $\lceil(n)\rceil = \lfloor n-1 \rfloor$.

b) Find the area of loops of the curve

$$y^2 = x(x-1)^2.$$

Group - C

7. Solve any two of the following :

a) $\frac{dy}{dx} = \sin(x+y) + \cos(x+y)$

b) $ydx - xdy = \sqrt{x^2 - y^2} dx$

c) $(1+x^2) \frac{dy}{dx} + y = \tan^{-1} x$

d) $x \frac{dy}{dx} + y^2 = y^2 \log x$

8. a) Find the orthogonal trajectories of the family of parabola $y^2 = 4ax$ for different values of a .
- b) Solve any one of the following :
- i) $y = px + p - p^2$
- ii) $p^2 y + 2px = y$

9. Solve any two of the following :

a) $\frac{d^2y}{dx^2} - 4 \frac{dy}{dx} + 3y = 2e^{3x}$

b) $(D^2 + D + 1)y = \sin 2x$

c) $(D^2 - 2D + 1)y = xe^x$

d) $(D^3 + 1)y = x^3 + \sin x$

Group - D

10. a) Prove that $\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a} \cdot \vec{c})\vec{b} - (\vec{a} \cdot \vec{b})\vec{c}$.
- b) If $\vec{a}, \vec{b}, \vec{c}$ are three non-coplanar vectors, then prove that the vectors $\vec{a} + \vec{b}, \vec{b} + \vec{c}, \vec{c} + \vec{a}$ are also non-coplanar.

(4) T.D.C. Part-II (Sub.) X Voc.

11. a) Define curl of a vector field. Give the physical significance of curl of a vector field.

b) Evaluate $\frac{d}{dt} \left[r \cdot \frac{d\vec{r}}{dt} \times \frac{d^2 \vec{r}}{dt^2} \right]$.

12. a) Prove that $\nabla \cdot (\phi \vec{u}) = \phi \nabla \cdot \vec{u} + \vec{u} \cdot (\nabla \phi)$ where ϕ is scalar point function and \vec{u} is vector point function.

b) If $\phi = x^3 + y^3 + z^3 - 3xyz$, find $\text{curl}(\text{grad } \phi)$.

Group - E

13. a) Obtain the necessary and sufficient conditions for the equilibrium of a system of coplanar forces acting on a rigid body.

b) Forces P, Q, R act along the lines $x=0, y=0$ and $x \cos \theta + y \sin \theta = p$, the axes being rectangular. Find the magnitude of resultant and equation of its line of action.

14. a) Which forces can be omitted in forming the equation of virtual work?

(Turn over)

(5)

GD-478

- b) Six equal rods AB, BC, CD, DE, EF and FA are each of weight w and are freely jointed at their extremities so as to form a hexagon. The rod AB is fixed in a horizontal position and the middle points of AB and DE are jointed by a string. Prove that the tension in string is thrice the weight of each rod.
15. a) State and explain Hooke's law. Find the work done in extending a light elastic string to double its length.
- b) A particle rests in equilibrium under the attraction of two centres of force which attract directly as the distance, their attractions per unit mass at unit distance being μ and μ' . The particle is slightly displaced towards one of them; show that the time of small oscillation is $\frac{2\pi}{\sqrt{\mu + \mu'}}$.
16. a) Find the radial and transverse acceleration of a particle moving in a plane curve.
- b) An insect crawls at a constant rate u along the spoke of a cartwheel of radius a , the cart is moving with a constant velocity v . Find the acceleration along and perpendicular to the spoke.



Special & Regular Exam - 2014

Time : 3 hours

Full Marks : 100

Candidates are required to give their answers in their own words as far as practicable.

The questions are of equal value.

Answer eight questions, selecting at least one from each Group.

Group-A

1. a) State and prove Leibnitz's theorem to find the n^{th} derivative of product of two functions:

b) If $u = \cos^{-1} \frac{x+y}{\sqrt{x+y}}$, then show that

$$x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} + \frac{1}{2} \cot u = 0$$

2. a) If $y = (x^2 - 1)^n$, then prove that

$$(x^2 - 1)y_{n+2} + 2xy_{n+1} - n(n+1)y_n = 0$$

- b) Evaluate $\lim_{x \rightarrow 0} x \log_e x$

3. a) Establish the formula

$$P = r \sin \phi$$

b) Find the pedal equal of the curve

$$\frac{2a}{r} = 1 - \cos \theta$$

Group-B

4. Evaluate any two of the following

i) $\int \frac{\cos x \, dx}{(1 + \sin x)(2 + \sin x)}$

ii) $\int \sqrt{\frac{a-x}{x}} \, dx$

iii) $\int \sin^{-1} x \, dx$

iv) $\int_0^{\pi/2} \log \sin x \, dx$

5. Find the area of the loop of the curve

$$ay^2 = x^2(a - x)$$

6. Find the length of the loop of the curve

$$3ay^2 = x(x-a)^2$$

Group - C

7. Solve any two of the following

i) $\frac{dy}{dx} = (x+y)^2$

ii) $(x^2 + y^2) \frac{dy}{dx} = xy$

iii) $(x+2y)dx + (2x-y)dy = 0$

iv) $\frac{dy}{dx} + \frac{L}{x} = \frac{e^y}{x^2}$

8. a) Solve any one of the following.

i) $y = (1+p)x + p^2$

ii) $y^2 = 2px + p^2$

iii) $x = 4p + 4p^3$

b) Find the orthogonal trajectory of the curve $r\theta = a$

9. Solve any two of the following:

i) $(4D^2 + 4D - 3)y = e^{2x}$

ii) $(D^2 + 1)y = \cos 2x$

iii) $(D^2 + 4)y = \sin 3x + e^x + x^2$

iv) $(D^4 + 1)y = e^x \cos x$

Group - D

10. a) Define scalar product of three vectors and give its geometrical meaning.
- b) Prove that the necessary and sufficient condition for three vectors to be coplanar is that their scalar triple product is zero.

14. a)

11. a) If $\vec{u}(t)$ and $\vec{v}(t)$ be two differentiable functions

of scalar t, then prove that $\frac{d}{dt}(\vec{u} \cdot \vec{v}) = \vec{u} \cdot \frac{d\vec{v}}{dt} + \frac{d\vec{u}}{dt} \cdot \vec{v}$

- b) Prove that the necessary and sufficient condition for the vector $\vec{u}(t)$ to have constant

direction is that $\vec{u} \times \frac{d\vec{u}}{dt} = 0$

12. a) Prove that

$$\nabla \times (\vec{u} \pm \vec{v}) = \nabla \times \vec{u} \pm \nabla \times \vec{v}$$

- b) Prove that

$$\text{Grad} \left(\frac{1}{r} \right) = -\frac{\vec{r}}{r^3}$$

b)

15. a)

Group-E

13. a) State and prove the converse of the principle of virtual work.

16.

- b) Two equal uniform rods AB and AC of length $2b$ are freely joined at A, rest on a smooth vertical circle of radius r . If 2θ be the angle between them, prove that $b \sin^3\theta = a \cos\theta$

14. a) Prove that a system of coplanar forces acting on a rigid body is in equilibrium if the algebraic sum of their moments about each of the three non-collinear points is zero.

- b) Find the equation of the line of action of the resultant of a system of coplanar forces acting upon a rigid body.

15. a) Show that the simple harmonic motion is oscillatory and periodic, the period is independent of amplitude.

- b) A particle whose mass is m is acted by a force

$m\mu \left[x + \frac{a^4}{x^3} \right]$ towards the origin 0, if it starts from

rest at a distance a , show that it will arrive at

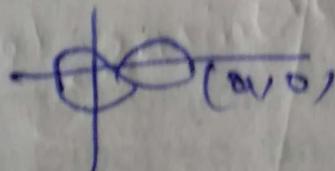
the origin in time $\frac{\pi}{4\sqrt{\mu}}$.

16. a) Find the tangential and normal acceleration of a particle moving in a curve.

- b) Prove that if the tangential and normal acceleration of a particle describing a curve be in contact throughout the motion the angle ψ through which the direction of motion turns in time t is given by

$$\psi = A \log(1 + Bt)$$

Where A and B are constants.



$$a_T^2 = n^2 (an)$$

$$y = n \frac{tan}{\sqrt{a}}$$

~~at $r = 0.05$~~ \Rightarrow ~~at $r = 0.05$~~ \Rightarrow ~~at $r = 0.05$~~

$$at = n^2 \cdot 0.05^2$$

$$a_T^2 = 2 \pi m \cdot 0.05$$

$a_T = n^2 r$
 $a_T = 0.05$
 $a_T = 2 \pi m \cdot 0.05$

car is moving with a constant velocity v . Find the acceleration along the spoke.

2013

Group-A

- 1(a) State and prove Euler's theorem on homogeneous function of two independent variables.
- b) Find the nth derivative of the function $e^{ax} \sin(bx+c)$.
- 2(a) If $u = \sin^{-1} \frac{xy}{x+y}$, show that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = 2u$
- b) Evaluate $\lim_{x \rightarrow 0} \left(\frac{1}{x} - \cot x \right)$
- 3(a) Establish the formula $\tan \phi = r \frac{dy}{dx}$
Find the pedal equation of the $r^n = a^n \cos n\theta$.

Group-B

- 4) Evaluate any two of the following:
- a) $\int \frac{(x-1)dx}{(x+1)(x+2)}$
- b) $\int \sin^{-1} x dx$

$$\int \frac{\sin x}{\sqrt{1+\sin x}} dx$$

d) $\int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x + \sqrt{\cos x}}} dx$

5) Find the area of the loop of the curve

$$x^2y^2 = 21^2(0^2 - 21^2)$$

6) Find the whole length of the curve:

$$9y^2 = (x-2)(x-5)^2$$

7) Solve any two of the following:

a) $y - x \frac{dy}{dx} = 0 \left(y^2 + \frac{dy}{dx} \right)$

b) $\frac{dy}{dx} = \sin(x+y)$.

c) $x^2y dx = (x^3 + y^3) dy = 0$

d) $\frac{dy}{dx} + 1 = e^{x+y}$.

8 a) Solve any one of the following:

i) $p^2 - p(e^x + e^{-x}) + 1 = 0$

ii) $y = 2px + p^2$

iii) $y = px + p - p^2$

b) Find the orthogonal trajectories of the curve

$$r^n = \sin n\theta$$

9) Solve any two of the following:

a) $(D^2 + 1)y = \sin 2x$

$$j) (D^2 - 4D + 4)y = x^2 + e^x + \cos 2x$$

$$c) (D^2 - 5D + 8)y = x^2$$

$$d) (D^2 - 2D + 1)y = x^2 e^{3x}$$

Group-D

10 a) Define vector product of three vectors and find the expression for the product $\vec{a} \times (\vec{b} \times \vec{c})$.

$$b) \text{ Show that } [\vec{a} \times \vec{b}, \vec{b} \times \vec{c}, \vec{c} \times \vec{a}] = [\vec{a} \vec{b} \vec{c}]^2$$

11 a) Prove that the necessary and sufficient condition for the vector function $\vec{r}(t)$ to constant magnitude is $\vec{a} \cdot \frac{d\vec{r}}{dt} = 0$.

b) Prove that

$$\vec{a} \times (\vec{b} \times \vec{c}) + \vec{b} \times (\vec{c} \times \vec{a}) + \vec{c} \times (\vec{a} \times \vec{b})$$

12 a) Show that:

$$\operatorname{div}(\vec{U} \pm \vec{V}) = \operatorname{div} \vec{U} \pm \operatorname{div} \vec{V}$$

b) If $\vec{r} = \vec{i} \cos 2\pi t + 3\vec{j} \sin 2\pi t$, find $\frac{d\vec{r}}{dt}$ when $t=0$.

Group-E

13 a) State and prove the principle of virtual work for any system of forces in one plane.

b) Four weightless rods of equal lengths are joined together to form a rhombus ABCD with one diagonal BD. If a weight w is attached to C and the system be suspended

from A. Show that the thrust in BD is equal to $\frac{w}{\sqrt{3}}$.

- 14) Prove that any system of coplanar forces acting on a rigid body is equivalent to a single force acting at an arbitrary point in the plane of forces together with a couple.
- b) Obtain the general condition of equilibrium of a system of forces acting in one plane upon a rigid body.
- 5 a) Show that two SHMs of the same period and in the same straight line may be compounded.
- b) A particle starts with a velocity V and moves under a retardation equal to K times the space described. Prove that the space described before it comes to rest is in equal to $\frac{V}{\sqrt{K}}$.
- 6 a) Find the tangential and normal acceleration of a particle moving in a plane curve.
- b) The co-ordinates of a moving point at time t . are given by $x = a(\theta + \sin \theta)$, $y = a(\theta - \cos \theta)$. Prove that its acceleration is constant and find the direction of motion at time t .

2012(Voc)

Time : 3 hours

Full Marks : 100

*Candidates are required to give their answers in
their own words as far as practicable.*

The questions are of equal value.

*Answer eight questions, selecting at least
one from each Group.*

Group - A

1. (a) If $y = \sin mx + \cos mx$; then prove that

$$y_n = m^n [1 + (-1)^n \sin 2mx]^{\frac{1}{2}}$$

- (b) If $y = (x^2 - 1)^n$, then prove that $(x^2 - 1)y_{n+2} + 2xy_{n+1} - n(n + 1)y_n = 0$.

2. (a) State and prove Maclaurin's series.

- (b) Apply Maclaurin's series to prove that

$$\log_e(1 + x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots \text{to } \infty$$

3. (a) Evaluate $\lim_{x \rightarrow 0} (\cos x)^{\cot^2 x}$.

(b) Prove that the length of the tangent =

$$\frac{y}{y_1} \sqrt{1+y_1^2} \text{ and the length of the normal =}$$

$$y \sqrt{1+y_1^2} \text{ where } y_1 = \frac{dy}{dx}.$$

Group - B

4. Evaluate any two of the following :

(a) $\int \frac{dx}{(2+x)\sqrt{1+x}}$

(b) $\int \frac{dx}{\sqrt{2ax-x^2}}$

(c) $\int \sqrt{\frac{x}{a-x}} dx$

(d) $\int \tan^{-1} x \cdot dx$

5. Find the area of the loop of the curve $x^3 + y^3 = 3axy$.

6. Prove that the total length of the cardioid $r = a(1 - \cos\theta)$ is $8a$.

Group - C

7. Solve any two of the following :

$$(a) (12x + 5y - 9)dx + (5x + 2y - 4)dy = 0$$

$$(b) (2x - y)dy = (2y - x)dx$$

$$(c) \frac{dy}{dx} = x^2 - \frac{1}{x} \cdot y$$

8. Solve any two of the following :

$$(a) 2 \frac{d^2y}{dx^2} + 5 \frac{dy}{dx} + 2y = 5 + 2x$$

$$(b) \frac{d^2y}{dx^2} + y = 12 \sin 2x$$

$$(c) \frac{d^2y}{dx^2} - 8 \frac{dy}{dx} + 9y = 40 \sin 5x$$

9. Solve any one of the following :

$$(a) x = 4p + 4p^3$$

$$(b) y = p^2x + p$$

Group - D

10. (a) Prove that the necessary and sufficient condition for three vectors to be coplanar is that their scalar triple product is zero.

- (b) Define scalar triple product and give its geometrical interpretation.
11. (a) If $\vec{u}(t)$ and $\vec{v}(t)$ be two differentiable functions of the scalar t , then prove that:

$$\frac{d}{dt} \left(\vec{u} \cdot \vec{v} \right) = \vec{u} \cdot \frac{d\vec{v}}{dt} + \frac{d\vec{u}}{dt} \cdot \vec{v}$$

- (b) Prove that the necessary and sufficient condition for the vector $\vec{u}(t)$ to have constant direction is that $\vec{u} \times \frac{d\vec{u}}{dt} = 0$.
12. (a) Prove that $\nabla \times (\vec{u} \pm \vec{v}) = \nabla \times \vec{u} \pm \nabla \times \vec{v}$.
- (b) $\text{grad}(\phi\Psi) = \phi \text{grad} \Psi + \Psi \text{grad} \phi$.

Group - E

13. (a) State and prove the converse of the principle of Virtual Work.
- (b) A light wire in the shape of a quadrant of an ellipse cut off by the principal axes has two weights fixed at its ends and rests on a smooth peg ; show that the eccentric angle

of the point of contact with the peg lies

between $\frac{\pi}{4}$ and $\frac{\pi}{3}$.

14. (a) Prove that a system of coplanar forces acting on a rigid body is in equilibrium if the algebraic sum of their moments about each of the three non-collinear points is zero.
- (b) Find the equation of the line of action of the resultant of a system of coplanar forces acting upon a rigid body.
15. A particle starts from rest and moves along a straight line with an acceleration which is always directed towards a fixed point and varies as the distance from the fixed point. Discuss the motion and find the period of one complete oscillation.
16. (a) Find expressions for radial and transverse velocities of a particle moving in a plane curve.
- (b) A particle moves in a plane under a constant acceleration μa parallel to OX and an acce-

2010

Group-A

- 1 a) State and prove Euler's theorem on homogeneous function of two independent variables.
 b) Show that $x \cdot \frac{\partial u}{\partial x} + y \cdot \frac{\partial u}{\partial y} = n$, where

$$U = \sin^{-1}(x) + \tan^{-1}(y)$$

- 2) If $y = e^{nx^2}$, then prove the following
 $y_{n+1} - 2ny_n - 2ny_{n-1} = 0$

- 3) Evaluate $\lim_{x \rightarrow 0} \left(\frac{1}{x} - \cot x \right)$

- 3 a) Establish the formula $\tan \phi = \pm \frac{dy}{dx}$

Find the

$\frac{dx}{dy} = 1$ that cut orthogonally

Group-B

- 4) Evaluate any two of the following

a) $\int 1 + \sec x \, dx$

b) $\int \frac{\sin x}{1 + \sin x} \, dx$

c) $\int \sin^n x \, dx$

d) $\int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sin x + \cos x} \, dx$

- 5) find the area of the loop of the curve

$$a^2 y^2 = x^2(a^2 - x^2)$$

2010 - ②

6) Find the whole length of the loop of the curve
 $y^2 = (x-2)(x-5)^2$

JDC4b-(

7) Solve any two following of the differential equations:

$$(i) y - x \frac{dy}{dx} = a \left(y^2 + \frac{dy}{dx} \right)$$

$$b) \frac{dy}{dx} = \tan x$$

$$c) \frac{dy}{dx} + 2y \tan x = \sin x$$

$$d) \frac{dy}{dx} + x \sin y = x^3 \cos y$$

8. a) Solve any one of the following:

$$(i) P^2 - (a+b)P + ab = 0, \text{ where } P = \frac{dy}{dx}$$

$$(ii) y = 2Px + P^2$$

b) Find the orthogonal trajectories of
 $y^n = a^n \cos nx$.

g) Solve any two of the following:

$$a) (D^2 - 4D + 3)y = 2e^{3x}$$

$$b) (D^2 - 5D + 6)y = \sin x + \cos x$$

$$c) (D^2 + 9)y = x^2 + 2x$$

~~a) Resultant of two vectors. Interpret the result geometrically.~~

~~a) Find the constant α such that the vectors $3\vec{i} + \vec{j} - 2\vec{k}$, $-\vec{i} + 3\vec{j} + 4\vec{k}$ and $\alpha\vec{i} - 2\vec{j} - 6\vec{k}$ are coplanar.~~

~~ii) Show the following:~~

$$\overrightarrow{a} + \overrightarrow{b}, \overrightarrow{b} + \overrightarrow{c}, \overrightarrow{c} + \overrightarrow{a} = 2[\overrightarrow{a} \overrightarrow{b} \overrightarrow{c}]$$

~~iii) If $\vec{r} = 2\cos 2\pi t \hat{i} + 3\vec{j} \sin 2\pi t \hat{j}$, find the $\frac{d^2}{dt^2}$, when $t=0$.~~

~~2) Prove the following:~~

$$\text{curl}(\vec{u} \pm \vec{v}) = \vec{u} \cdot \nabla \pm \vec{v} \cdot \nabla$$

b) $\text{curl}(\vec{u} \pm \vec{v}) = \text{curl } \vec{u} \pm \text{curl } \vec{v}$

Group E - F

~~3(a) State and prove the converse of the principle of virtual work for any system of coplanar forces.~~

~~b) The middle points of opposite side of a jointed quadrilateral are connected by light rods of length l and l' . If the T and T' be the tensions in these rods, prove that~~

$$\frac{T}{l} + \frac{T'}{l'} = 0$$

285

14a) Prove that any system of coplanar forces acting on a rigid body is equivalent to a single force acting at an arbitrary point in the plane of the forces together with a couple.

b) Obtain the general conditions of equilibrium of a system of forces acting in one plane upon a rigid body.

15a) Find the tangential and normal velocities of a particle moving in a plane curve.

b) If the radial and transverse velocities of a particle are always proportional to each other

~~then find the equation of the path.~~

Equiangular spiral.

16a) Prove that the period of S.H.M is independent of amplitude.

b) If a particle is making simple harmonic oscillation, the period being 2 seconds and amplitude being 3 feet, find the maximum velocity and the maximum acceleration.

Just

2008

Full Marks : 100

Time : 3 hours

The questions are of equal value

Answer eight questions, selecting at least one from each Group

GROUP—A

1. (a) State and prove Maclaurin's series. (6)

(b) If $f = (\sin^{-1} x)^2$, prove that

$$(1-x^2)y_{n+2} - x(2n+1)y_{n+1} - n^2 y_n = 0$$

2. (a) State and prove Euler's theorem for homogeneous function of three variables. (6)

(b) Evaluate :

$$\lim_{x \rightarrow 0} (\cos x)^{\cot^2 x} \quad (6) \text{ Cf Sec 1}$$

3. (a) Find the length of the polar subtangent and subnormal of a curve.

(b) Find the pedal equation of the curve $r^2 = a^2 \cos 2\theta$.

Group-B

(2)

evaluate any two of following:-

$$\int \frac{dx}{\sin x (3+2\cos x)}$$

$$\int \log x \cdot dx$$

$$(i) \int \sqrt{\frac{a+x}{a-x}} dx$$

$$(ii) \int \sqrt{(x-a)(b-x)} dx$$

stain the area included b/w the curve

$$x^2(a-x) = x^3 \& \text{ its asymptotes.}$$

find the length of arc of parabola

2 year cut off by its latus rectum.

Group-C

Solve any two differential eq^n:-

$$\frac{dy}{dx} = e^{x+y} + x^2 e^y$$

$$\frac{dy}{dx} + 1 = e^{x+y}$$

$$\frac{dy}{dx} + 1 = e^{x+y}$$

$$x^2 y dx - (x^3 + y^3) dy = 0$$

$$\int \frac{dy}{dx} + y \sec x = \tan x$$

+ T S) solve any one of the following

(i) solve any one where $P = \frac{dy}{dx}$

$$P^2 + 2Px - 3x^2 = 0$$

$$(ii) P^2 + 2Px - 3x^2 = 0$$

$$(iii) y = Px + \sin^{-1} P$$

Find orthogonal trajectory of parabola

$$y^2 = 4a(x+a)$$

- (3)
- g(a) Solve any two of following
 $D = \frac{d}{dx}$
- * (i) $(D^2 + 3D + 2)y = e^{2x}$
 (ii) $(D^2 + 1)y = 2 \cos 2x$
 (iii) $(D^2 - 4D + 4)y = x^2 + e^x + \cos 2x$
 (iv) $(D^2 + 1)y = xe^{2x}$
- Group-I.

- 10(a) Prove that the necessary & sufficient condition that three non parallel & non-zero vectors are coplanar is $[\vec{a}, \vec{b}, \vec{c}] = 0$
- * (b) Find the expression of $(\vec{a} \times \vec{b}) \cdot (\vec{c} \times \vec{d})$.

11(a) Prove that

$$(\vec{b} + \vec{c}) \cdot (\vec{a} \times \vec{d}) + (\vec{c} \times \vec{a}) \cdot (\vec{b} \times \vec{d}) + (\vec{a} \times \vec{b}) \cdot (\vec{c} \times \vec{d}) = 0$$

- (b) If $\vec{u}(t)$ and $\vec{v}(t)$ be two differentiable functions of the scalar t , then show that

$$\frac{d}{dt}(\vec{u} \times \vec{v}) = \vec{u} \times \frac{d\vec{v}}{dt} + \frac{d\vec{u}}{dt} \times \vec{v} \quad (6)$$

12. (a) If \vec{a} and \vec{b} are two vector point functions, then prove that

$$\nabla \times (\vec{a} \pm \vec{b}) = \nabla \times \vec{a} \pm \nabla \times \vec{b} \quad (6)$$

- (b) If $\vec{r} = (x, y, z)$, then show that $\text{div } \vec{r} = 3$.

GROUP—E

13. Prove that the necessary and sufficient condition that a system of coplanar forces acting on a rigid body be in equilibrium are that the sums of the resolved parts of the system of forces along any two perpendicular lines must separately vanish and the sum of the forces about any point in the plane must be zero.

14. (a) Enumerate the forces which may be omitted in forming the equation of virtual work.

- (b) The middle points of the opposite sides of a jointed quadrilateral are connected by light rods of lengths l and l' . If T and T' be the tensions in these rods, prove that

$$\frac{T}{l} + \frac{T'}{l'} = 0$$

(15)

evaluate any

 dx $\sin x (3+2x)$ $\int \log x \cdot dx$

ans the qu

15. (a) A particle moves in a straight line, its acceleration is proportional to the distance from a fixed point in the line and is always directed towards the point. Show that the periodic time is independent of amplitude.

- (b) A particle starts with a velocity v and under a retardation equal to $k t$ find the space described.

Prove that the space traversed before it comes to rest is equal to $\frac{v^2}{2k}$.

16. (a) Find the radial and transverse accelerations of a particle moving in a plane curve.
- (b) If the radial and transverse velocities of the particle are always proportional to each other, show that the equation to the path is an equiangular spiral.

$$\nabla \cdot (\vec{u} \pm \vec{v}) = \nabla \cdot \vec{u} \mp \nabla \cdot \vec{v}$$

$$\nabla \cdot (u \pm v) = \left(\frac{i \sqrt{4}}{5x} + T \right)$$

$$= \left(\frac{i \cdot 54}{5x} + T \right) r^2$$

 dy/dx

1522A

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X-VUL (Math) (IIS)

2007

Full Marks : 100

Time : 3 hours

The questions are of equal value.

Answer eight questions, selecting at least one from each Group

GROUP-A

1. (a) If u and v are two functions possessing derivatives of the n th order, then show that

$$(uv)_n = u_n v + {}^n C_1 u_{n-1} v_1 + {}^n C_2 u_{n-2} v_2 + \dots + {}^n C_r u_{n-r} v_r + \dots + {}^n C_n u v_n$$

where suffix denotes differentiation with respect to x .

- (b) If $u = \cos^{-1} \frac{x+y}{\sqrt{x+y}}$, show that

$$x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} + \frac{1}{2} \cot u = 0$$

2. (a) State and prove Euler's theorem on homogeneous function of two variables.

- (b) Evaluate :

$$\lim_{x \rightarrow 0} \log_{10} x \tan 2x$$

Q. (a) Prove that $p = r \sin \theta$ of perpendicular from r be the radius vector to the tangent which intersects or.

(b) Find the pedal equ

2

GROUP

4. Evaluate any two of the

$$(i) \int \frac{\cos x dx}{(1 + \sin x)(2 + \sin x)}$$

$$(ii) \int \sqrt{\frac{a-x}{x}} dx$$

$$(iii) \int_a^b \frac{dx}{\sqrt{(x-a)(b-x)}}$$

5. Find the area included curve

$$y^2 = x^2 \frac{a+x}{a-x} \text{ and its asymptotes}$$

6. Find the length of the curve

$$9ay^2 = (x - 2a)(x - 5a)^2$$

Continued)

8/Z-250/51

7. Solve any two of the following differential equations :

$$(i) \frac{dy}{dx} = (x+y)^2$$

$$(ii) (x^2 + y^2) \frac{dy}{dx} = xy$$

$$(iii) \frac{dy}{dx} + y \tan x = \sec x$$

$$(iv) (x+2y)dx + (2x-y)dy = 0$$

8. (a) Solve any one of the following

$$(i) y + px = x^4 p^2$$

$$(ii) y = 2px + y^2 p^3$$

where $p = \frac{dy}{dx}$

(b) Find the orthogonal trajectories of the family of parabolas $y = ax^2$.

9. Solve any two of the following :

$$(i) (D^2 - 4D + 3)y = 2e^{3x}$$

$$(ii) (D^2 + 1)y = \sin 2x$$

$$(iii) (D^2 + 4)y = \sin 3x + e^x + x^2$$

8/Z-250/51

Turn Over

GROUP—D

10. (a) Define scalar product of give geometrical interpretation of product.

(b) Prove that

$$[\vec{a} + \vec{b}, \vec{b} + \vec{c}, \vec{c} + \vec{a}]$$

11. (a) Prove that the necessary condition for the vector function to have constant magnitude is

$$\vec{a} \cdot \frac{d\vec{a}}{dt} = 0$$

(b) Prove that

$$\frac{d}{dt} [\vec{a}, \vec{b}, \vec{c}] = \left[\frac{d\vec{a}}{dt}, \vec{b}, \vec{c} \right] + \left[\vec{a}, \frac{d\vec{b}}{dt}, \vec{c} \right] + \left[\vec{a}, \vec{b}, \frac{d\vec{c}}{dt} \right]$$

12. (a) Prove that

$$\text{curl}(\text{grad}\phi) = 0$$

(b) If $\vec{r} = xi\hat{i} + yj\hat{j} + zk\hat{k}$, then prove that

$$\text{grad}\left(\frac{1}{r}\right) = -\frac{\vec{r}}{r^3}$$

8/2-250/51

(0)

GROUP—E

13. (a) Prove that any system of coplanar forces acting on a rigid body is equivalent to a single force acting at an arbitrary point in the plane of forces together with a couple.

(b) Obtain the general conditions of equilibrium of a system of forces acting in one plane upon a rigid body.

14. (a) State and prove the principle of virtual work for any system of forces in one plane.

(b) Two equal uniform rods AB and AC , each of length $2b$, are freely joined at A and rest on a smooth vertical circle of radius a . If 2θ be the angle between them, prove that

$$b \sin^3 \theta = a \cos \theta$$

15. (a) Show that two simple harmonic motions of the same period and in the same straight line may be compounded.

(b) A particle, whose mass is m , is acted upon by a force $m\mu \left[\lambda + \frac{a^4}{x^3} \right]$ towards the origin O ; if it starts from rest at a distance a , show that it will arrive at the origin in time $\frac{\pi}{4\sqrt{\mu}}$.

2006.

Group-A

Maths.

Find the nth derivative of the function

$$y = e^{ax} \sin(bx+c)$$

State and prove MacLaurin's series.

State and prove Euler's theorem on homogeneous function of 3 variables.

Evaluate $\lim_{x \rightarrow 0} \left(\frac{\tan x}{x} \right)^{1/x}$.

Prove that $\sin \Phi = r \cdot \frac{d\theta}{ds}$

Find the Pedal eqⁿ of above curve.

Group-B

Evaluate any two of the following:

$$(i) \int \frac{x^2 - 1}{x^4 + 1} dx \quad (iii) \int \frac{x^2 + 1}{\sqrt{x^2 + 4}} dx$$

$$(ii) \int_0^{\pi/2} \log \sin x dx$$

Find the area of loop of curve
 $ay^2 = x^2(a-x)$

Find the length of arc of parabola
 $y^2 = 4ax$ cut off by its latus rectum.

Group-C

Solve any two of following diffⁿ eqⁿ:

$$(i) \sin^{-1} \left(\frac{dy}{dx} \right) = x+y$$

$$(ii) \sin^{-1} \left(\frac{dy}{dx} \right) = x+y \quad (iii) \frac{dy}{dx} + y \tan x = \sec x$$

8. Solve any one of the following where

$$\rho = \frac{dy}{dx}$$

(i) $\rho^2 + 2\rho - 3x^2 = 0$ (ii) $y = (1+\rho)x + \text{approx}$
 (b) Find the orthogonal trajectory of
 $y^2 = 4a(x+a)$

q. Solve any two of the following where
 $D = \frac{d}{dx}$

(i) $(4D^2 + 4D - 9) = e^{2x}$ (iii) $(D^2 + 4)y = e^x$
 (ii) $(D^2 + 1)y = \cos 2x$

Group - D.

10(a) Prove that the necessary & sufficient condition that the three non-parallel and non-zero vectors are coplanar

$$[\vec{a}, \vec{b}, \vec{c}] = 0$$

(b) Find the expression of $(\vec{a} \times \vec{b}) \cdot (\vec{c} \times \vec{d})$

11(a) Prove that
 $\vec{a} \times (\vec{b} \times \vec{c}) + \vec{b} \times (\vec{c} \times \vec{a}) + \vec{c} \times (\vec{a} \times \vec{b}) = 0$

(b) If $\vec{u}(t)$ & $\vec{v}(t)$ be two differentiable functions of scalar t , then show that

$$\frac{d}{dt} (\vec{u} \times \vec{v}) = \vec{u} \times \frac{d\vec{v}}{dt} + \frac{d\vec{u}}{dt} \times \vec{v}$$

12(a) If \vec{a} & \vec{b} are two vector point functions then prove that

$$\vec{v} \times (\vec{a} \pm \vec{b}) = \vec{v} \times \vec{a} \pm \vec{v} \times \vec{b}$$

(b) If $\vec{r} = (x_1, y_1, z_1)$, then show that
 $\operatorname{div} \vec{r} = 3$

Group E.

prove that the necessary & sufficient condition that a system of coplanar forces acting on a rigid body in equilibrium that the sum of the resolved parts of the system of forces along any two perpendicular lines must separately vanish and the sum of the forces about any point in the plane must be zero.

enumerate the forces which may be omitted in forming the eqn of virtual work.

(1) If the middle points of opposite sides of a joined quadrilateral are connected by light rods of length l & l' . If T & T' be the tension in this rods prove that $\frac{T}{l} + \frac{T'}{l'} = 0$

(a) A particle moves in a straight line such that its acceleration is proportional to its distance from a fixed point in the straight line and is always directed towards the fixed point. Show that the periodic time is independent of amplitude.

(b) A particle starts with velocity v_0 under a retardation equal to k . The space described. Prove that space transverse before it comes to rest is equal to $\frac{v_0^2}{2k}$.

16(a) Find the radial & transverse acceleration of a particle moving on a plane curve.

(b) If the radial & transverse velocities of a particle are always proportional to each other, show that the equation to the path is: equiangular spiral.

— x —

2005

Math

State and prove Leibnitz's theorem on successive differentiation.

If $u = \cos^{-1} \frac{x+y}{\sqrt{x+y}}$, then show that

$$x \cdot \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} + \frac{1}{2} \cancel{\cot u} \cancel{\cos^2 x} = 0$$

(i) If $y = e^{a \sin^{-1} x}$, then prove that
 $(1-x^2)y_2 + xy_1 - a^2y = 0$

(ii) Evaluate

$$\lim_{x \rightarrow 0} \frac{e^x + e^{-x} - 2}{e^{x^2}}$$

(iii) Prove that $\tan \theta = r \frac{d\theta}{dr}$, where

symbol have their usual meaning.

Group-B

A) Integrate any one of the following:-

(i) $\int_0^2 x^2 dx$ (ii) $\int_a^b e^x dx$

B) Obtain the reduction formula for -

$$\int e^{ax} \cos^n x \cdot dx$$

C) Determine the length of an arc of the cycloid $x = a(\theta + \sin \theta)$,
 $y = a(1 - \cos \theta)$ measured from the vertex.

Find the volume of solid generated by revolving the cardioid $r = a(1 + \cos\theta)$ about the initial line.

Group - C.

7. Solve any ~~two~~ ^{three} of the following differential equation:

$$(i) \frac{dy}{dx} + 1 = e^{x-y}$$

$$(ii) (x-y)^2 dy = dx$$

$$(iii) \frac{dy}{dx} = \frac{x+y}{x-y}$$

8. (a) Solve any one of the following where $P = \frac{dy}{dx}$

$$(i) y = x(P + P^3)$$

$$(ii) y = 2P + P^2$$

(b) Find the orthogonal trajectory of the curve $y^2 = 4ax$

9. Solve any one of the following where $D = \frac{d}{dx}$

$$(i) (D^2 - 3D + 2)y = e^x$$

$$(ii) (D^2 - 1)y = \cos 2x$$

$$(iii) (D^3 + 2D^2 + D)y = e^{2x} + x - 7x^2$$

$$(iii) (D^3 + 2D^2 + D)y = e^{2x} + x - 7x^2$$

Group - I

Q(a) Prove that $\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a} \cdot \vec{c})\vec{b} - (\vec{a} \cdot \vec{b})\vec{c}$

b) Show that $[\vec{a} \times \vec{b}] \cdot [\vec{b} \times \vec{c}] = [\vec{a} \cdot \vec{b} \cdot \vec{c}]^2$

1. (a) If $\vec{u}(t)$ and $\vec{v}(t)$ be two differentiable functions of the scalar t , then

Show that $\frac{d}{dt}(\vec{u} \cdot \vec{v}) = \vec{u} \cdot \frac{d\vec{v}}{dt} + \vec{v} \cdot \frac{d\vec{u}}{dt}$

(b) Prove that

$$\frac{d}{dt}(\vec{r} \times \frac{d\vec{r}}{dt}) = \vec{r} \times \frac{d^2\vec{r}}{dt^2}$$

12. (a) Prove that $d\vec{w}(\vec{u} \pm \vec{v}) = d\vec{w}\vec{u} \pm d\vec{w}\vec{v}$

(b) Prove that $\text{curl } \vec{v} = \text{grad } d\vec{w} \cdot \vec{v} - \vec{v}^2 \vec{v}$

Group - E

13 (a) Find the necessary and sufficient condition for a system of coplanar forces to be in equilibrium.

(b) Three forces P, Q, R act along the sides of a triangle formed by lines $x-y=1$, $y-x=1$ & $y=2$. Find the equation of the line of action of the resultant.

4(a) State and prove that principle of virtual work for a coplanar force system.

(b) The middle points of opposite sides of a jointed quadrilateral are connected by light rods of length l and l' . If T and T' be the tension in these rods, prove that

$$\frac{T'}{l'} + \frac{T}{l} = 0$$

15(a) State & explain Hooke's law.

(b) Define simple Harmonic Motion.
Find the periodic time, amplitude & frequency.

16(a) Find radial & transverse velocity of a particle moving in a plane curve

(b) A particle starts from the origin and the components of its velocity parallel to the axes of co-ordinates at time are $2t^{-3}$ and at . Find the path.

1. 4)

ASHA SCHOOL
GANDHI CH

9. Solve for y

$$D = \frac{d}{dx}$$

(i) $(D^2 - 3D + 2)y = e^x$

(ii) $(D^2 - 1)y = \cos 2x$

(iii) $(D^3 + 2D^2 + D)y = e^{2x} + x - x^2$

GROUP-D

10. (a) Prove that

$$\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a} \cdot \vec{c})\vec{b} - (\vec{a} \cdot \vec{b})\vec{c}$$

(b) Show that

$$|\vec{a} \times \vec{b} \cdot \vec{b} \times \vec{c} \cdot \vec{c} \times \vec{a}| = |\vec{a} \cdot \vec{b} \cdot \vec{c}|^2$$

11. (a) If $\vec{u}(t)$ and $\vec{v}(t)$ be two differentiable
the scalars t, then show that

$$\frac{d}{dt}(\vec{u} \cdot \vec{v}) = \vec{u} \cdot \frac{d\vec{v}}{dt} + \vec{v} \cdot \frac{d\vec{u}}{dt}$$

(b) Prove that

$$\frac{d}{dt}\left(\vec{r} \times \frac{d\vec{r}}{dt}\right) = \vec{r} \times \frac{d^2\vec{r}}{dt^2}$$