Week 04

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Problem 1

```
The Expected Value of Classical Brownian Motion is 5.00
The Standard Deviation of Classical Brownian Motion is 1.00
The Expected Value of Arithmetic Return is 5.01
The Standard Deviation of Arithmetic Return is 5.04
The Expected Value of Geometric Brownian Motion is 8.24
The Standard Deviation of Geometric Brownian Motion is 11.02
```

1. Classical Brownian motion

$$P_t = P_{t-1} + r_t,$$
$$r_t \sim N(0, \sigma^2)$$

So the expected value of P_t isP_{t-1},the standard deviation of P_t is σ.

simulate: t = 1, P_0 = 5, nsim = 12000, σ=1.

Expectation: Expected value: 5, standard deviation: 1. Match.

2. Arithmetic Return System

$$P_t = P_{t-1}(1 + r_t),$$
$$r_t \sim N(0, \sigma^2)$$

So the expected value of P_t isP_{t-1},the standard deviation of P_t is $P_{t-1} * \sigma$.

simulate: t = 1, P_0 = 5, nsim = 12000, σ=1.

Expectation: Expected value: 5, standard deviation: 5. Match.

3. Geometric Brownian Motion

$$P_t = P_{t-1} * \mathrm{e}^{r_t},$$
$$r_t \sim N(0, \sigma^2)$$

So the expected value of P_t isP_{t-1},the standard deviation of P_t is $P_{t-1} * \mathrm{e}^{\sigma}$.

simulate: t = 1, P_0 = 5, nsim = 12000, σ=1.

Expectation: Expected value: 5, standard deviation: 13.5. Nearly match.

Problem 2

Calculate the arithmetic returns for INTC

```
In [72]: INTC_return
Out[72]:
array([[ 0.01137799],
       [-0.11678574],
       [-0.00101087],
       [-0.02286987],
       [-0.00807787],
       [ 0.00396747],
       [ 0.0191347 ],
       [ 0.01122445],
       [ 0.00625635],
       [ 0.01062978],
       [ 0.00535553],
       [ 0.01212476],
       [ 0.01237237],
       [-0.0067895 ],
       [-0.0085938 ],
       [-0.00453112],
       [-0.0043538 ],
       [ 0.00019873],
       [ 0.00576313],
       [-0.00750842],
       [-0.01094962],
       [-0.00322061],
       [ 0.00626012],
       [-0.01464988],
       [ 0.01344195],
       [-0.01969453],
       [ 0.02501028],
       [-0.01599998],
       [-0.01219517],
       [ 0.01851855],
       [-0.00505051],
       [ 0.03532998],
       [ 0.03098643],
       [-0.01559824],
       [-0.02454107],
       [ 0.00217909],
       [-0.01166239],
       [-0.00599998],
       [ 0.01951705],
       [ 0.00315776],
       [-0.00413148],
       [-0.02015015],
       [ 0.02358875],
       [ 0.00393935],
       [ 0.00667059],
       [ 0.01227825],
       [-0.00346554],
       [ 0.00135246],
       [-0.00173645],
       [-0.00463861],
       [ 0.03320387],
       [-0.00131554],
       [ 0.01373729],
       [ 0.00259884],
       [-0.0105536 ],
       [ 0.03312127],
       [ 0.01267888],
       [-0.00304057],
       [-0.0143524 ],
       [ 0.01383331]])
```

Calculate VaR

1. Using a normal distribution.

```
VaR is -1.88 using normal distribution
```

2. Using a normal distribution with an Exponentially Weighted variance ($\lambda = 0.94$)

```
VaR is -1.43 using normal distribution with Exponentially Weighted variance
```

3. Using a MLE fitted T distribution.

```
VaR is -1.50 using normal distribution
```

4. Using a Historic Simulation.

```
VaR is 3065.15 using historic simulation
```

Download new data

```
Date
2022-01-19   -0.020818
2022-01-20   -0.029467
2022-01-21    0.000000
2022-01-24   -0.001922
2022-01-25   -0.018098
2022-01-26    0.013529
2022-01-27   -0.070420
2022-01-28   -0.006660
2022-01-31    0.022837
2022-02-01    0.002663
2022-02-02    0.011440
2022-02-03   -0.024843
2022-02-04   -0.005592
2022-02-07    0.003541
2022-02-08    0.013076
2022-02-09    0.022536
2022-02-10   -0.021038
Name: Close, dtype: float64
```

```
The mean of returns is -0.01
```

Method 1,2 and 3 generate similar result, but historic simulation is different. VaR is small, the risk in this stock is low.

Problem 3

Using historic method to calculate VaR of 3 portfolios. Take the actual profit or loss of the portfolio and do not assume any distribution. The estimation of the actual distribution of changes in the risk factors is not required.

In the absence of structural breaks, historic method tends to function better than alternative methods. It is less sensitive to the odd outlier and does not absorb estimation error in the same way as parametric methods.

Portfolio A

```
0     2.226864e+07
1     2.226861e+07
2     2.226859e+07
3     2.226848e+07
4     2.226848e+07
          ...
56    2.226844e+07
57    2.226844e+07
58    2.226840e+07
59    2.226847e+07
60    2.226845e+07
Name: TSLA, Length: 61, dtype: float64
```

Portfolio B

```
0     2.007963e+07
1     2.007962e+07
2     2.007971e+07
3     2.007971e+07
4     2.007967e+07
          ...
56    2.007969e+07
57    2.007967e+07
58    2.007963e+07
59    2.007969e+07
60    2.007967e+07
Name: GOOGL, Length: 61, dtype: float64
```

Portfolio C

```
0     1.927917e+07
1     1.927917e+07
2     1.927925e+07
3     1.927925e+07
4     1.927923e+07
          ...
56    1.927925e+07
57    1.927922e+07
58    1.927919e+07
59    1.927924e+07
60    1.927922e+07
Name: GOOG, Length: 61, dtype: float64
```

total VaR:

```
0     6.163394e+07
1     6.163394e+07
2     6.163394e+07
3     6.163394e+07
4     6.163394e+07
          ...
56    6.163394e+07
57    6.163394e+07
58    6.163394e+07
59    6.163394e+07
60    6.163394e+07
Name: BAC, Length: 61, dtype: float64
```