

$$AIC = \log(S^2) + \frac{2k}{n}$$

(a). The restricted model is preferred when

$$AIC_0 < AIC_1, \text{ which denotes } \log(S_0^2) + \frac{2P_0}{n} < \log(S_1^2) + \frac{2P_1}{n}$$

$$\text{then } \log(S_0^2) - \log(S_1^2) < \frac{2(P_1 - P_0)}{n},$$

$$\log\left(\frac{S_0^2}{S_1^2}\right) < \frac{2(P_1 - P_0)}{n}, \text{ exponent both sides (monotonic transformation)}$$

$$\Leftrightarrow \frac{S_0^2}{S_1^2} < e^{\frac{2(P_1 - P_0)}{n}} \quad ①$$

$$(b). \quad n \rightarrow \infty, \quad \frac{2}{n}(P_1 - P_0) \rightarrow 0, \text{ then } e^{\frac{2}{n}(P_1 - P_0)} \approx \frac{2}{n}(P_1 - P_0) + 1$$

$$① \text{ becomes } \frac{S_0^2}{S_1^2} < \frac{2}{n}(P_1 - P_0) + 1$$

$$\text{then } \frac{S_0^2 - S_1^2}{S_1^2} < \frac{2}{n}(P_1 - P_0) \quad ②$$

$$(c). \quad S_0^2 = \frac{e_R' e_R}{n - P_0}, \quad S_1^2 = \frac{e_U' e_U}{n - P_1}$$

$$\text{then } ② \text{ becomes } \frac{e_R' e_R / (n - P_0) - e_U' e_U / (n - P_1)}{e_U' e_U / (n - P_1)} < \frac{2}{n}(P_1 - P_0)$$

$$\text{as } n \rightarrow \infty, \text{ then } (n - P_0), (n - P_1) \rightarrow n$$

$$\text{then } ② \text{ becomes } \frac{e_R' e_R - e_U' e_U}{e_U' e_U} < \frac{2}{n}(P_1 - P_0) \quad ③$$

$$(d). \quad F = \frac{(e_R' e_R - e_U' e_U) / (P_1 - P_0)}{e_U' e_U / (n - P_1)}$$

$$③ \text{ becomes } \frac{(e_R' e_R - e_U' e_U) / (P_1 - P_0)}{e_U' e_U / (n - P_1)} < \frac{2(P_1 - P_0)}{n} \frac{n - P_1}{P_1 - P_0}$$

$$\text{then } F < \frac{2(n - P_1)}{n}, \text{ as } n \rightarrow \infty, (n - P_1) \rightarrow n$$

$$\text{then } F < 2$$